TECHNICAL ELECTRICITY

KUKDE

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PREFACE

This Text is prepared carefully to cover the syllabus prescribed for the students of all Polytechnic Institutes. The standard maintained is fairly high. The language of the Text is simple and the emphasis is on the fundamental principles of Electrical Engineering.

This book contains 400 diagrams and illustrations and about 70 worked examples which help to simplify the study of the subject. At the end of the book there is a collection of useful Numerical Problems, many of which are borrowed from Examination Papers. To gain confidence the students are advised to attempt these problems.

Thanks are due to messrs. Mather and Platt, W. H. Allen, Sons and Co., Evershed and Vignoles of London, Ateliers de Construction Oerlikon of Switzerland and others for the gracious permission to use their illustrations and written material. Thanks are also due to the Management of the Aryabhushan Press, Poona, for their excellent co-operation.

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CHAPTER I

THE ELECTRIC CIRCUIT

1. The Electric current: All matter is made up of atoms. They are built up of positive electricity called *protons* and negative electricity called *electrons*. In the normal state, the atom of any element consists of equal number of protons and electrons. Hence in the normal state an atom is said to possess no electrical charge.

The nucleus of most of the atoms consists of all its protons and some of its electrons. The remaining electrons of the atom revolve round the nucleus. These free electrons are held in their orbit by a certain attractive force. They move in a haphazard manner from atom to atom. But when a difference of electrical pressure is applied at the ends of a conductor the movement of electrons in the conductor from atom to atom becomes a steady flow. This flow constitutes an electric current. The flow of electrons is from negative to positive, i. e. opposite to the conventional direction of current which has always been regarded as being from positive to negative.

In the case of elements which are used as insulators (mica, rubber, glass etc.) there is a great force of attraction between the free electrons and the nucleus. Hence in these cases the potential difference has to be very great to effect a flow of electrons.

In all cases the movement of electrons is impeded or obstructed by collision of the electrons with the molecules of that element. This obstruction to the flow of electrons is called *resistance*.

The electron is a very small unit of quantity of electricity. The practical unit of quantity of electricity is the *coulomb*.

$$1 \textit{ coulomb} = 6{\cdot}29 \times 10^{18} \text{ electrons.}$$

The practical unit of current is the ***ampere*** and is the rate of flow of quantity of electricity; symbol used is I.

$$1 \textit{ ampere} = 1 \textit{ coulomb} / \textit{second}.$$

If the atoms of a body lose some of its free electrons the body is said to be positively charged. Conversely, if all the atoms of a body

absorb a few extra electrons the body is said to be negatively charged. Ionisation is the process by which an atom loses some of its free electrons.

2. Resistance: Resistance is the property of a material which opposes the passage of electric current through it. The resistance of a conductor

(a) varies directly as its length,

(b) varies inversely as its cross-sectional area,

(c) depends on the material and

(d) depends on its temperature.

According to (a) and (b)

$$\text{resistance} \propto \frac{l}{A}$$

and according to (c) we must use a factor ρ (rho) which is called the ***specific resistance*** or ***resistivity*** of the material. Therefore

$$\textit{resistance}\ (R) = \rho\,\frac{\textit{length}\ (l)}{\textit{area}\ (A)}$$

or using symbols only,

$$R = \rho\,\frac{l}{A} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

The specific resistance ρ is defined as the resistance of a cube of material, the measurement being made between the two opposite faces of the cube. The value of ρ depends upon the units of l and A. The value given is either ***per cm cube*** or ***per inch cube*** at a ***specified temperature.***

The practical unit of resistance is the ***ohm.*** A column of mercury, weighing 14·452 grams and 106·3 cm long, having a uniform cross-section and held at the temperature of melting ice, has a resistance of 1 ohm. This is the ***standard ohm.*** Abbreviation for ohm is the greek letter Ω.

It is often convenient to use the multiple or the sub-multiple of the ohm in certain special cases. When the resistance is very high use is made of the ***megohm*** and when the resistance is very low, the ***microhm.***

$$1\ \textit{megohm} = 10^6\ \text{ohms};$$

$$1\ \textit{microhm} = 10^{-6}\ \text{ohm}.$$

3. Specific Resistance: Among metals silver has the lowest value for ρ. Hence silver is the best conducting material. But being costly it is used only in certain special cases. Next comes copper, and since it is one of the cheaper metals, it is used universally as a conducting material.

The value of ρ for copper is $1{\cdot}724 \times 10^{-6}$ ohm per cm cube at 20°C. The corresponding value of ρ per inch cube is

$$0{\cdot}67879 \times 10^{-6} \text{ ohm.}$$

4. Effect of Temperature on Resistance: All pure metals increase in resistance with the rise in temperature. But carbon, insulating materials and electrolytes decrease in resistance with the rise in temperature. But alloys of most metals increase very slightly in resistance with rise in temperature. Those alloys which are used for electrical work have practically a constant resistance at all temperatures.

If a piece of wire has a resistance of R_0 at 0°C, its resistance at 1°C will be $(R_0 + x)$, at 2°C it will be $(R_0 + 2x)$ ohms and so on, where x is the increase in resistance per degree rise in temperature. The ratio $\frac{x}{R_0}$ is called the *temperature coefficient* of the metal. The symbol used is the greek letter $\propto$. For a rise of t°C the resistance will be

$$R_t = (R_0 + xt)$$

Since $\propto = \frac{x}{R_0}$ we substitute the value of x in the above expression

$$R_t = (R_0 + R_0 \propto t)$$

$$R_t = R_0 (1 + \propto t) \qquad \ldots \qquad \ldots \qquad \ldots \qquad (2)$$

Fig. 1 shows the graph of resistance and Fig. 2 shows the same graph over a wide limit.

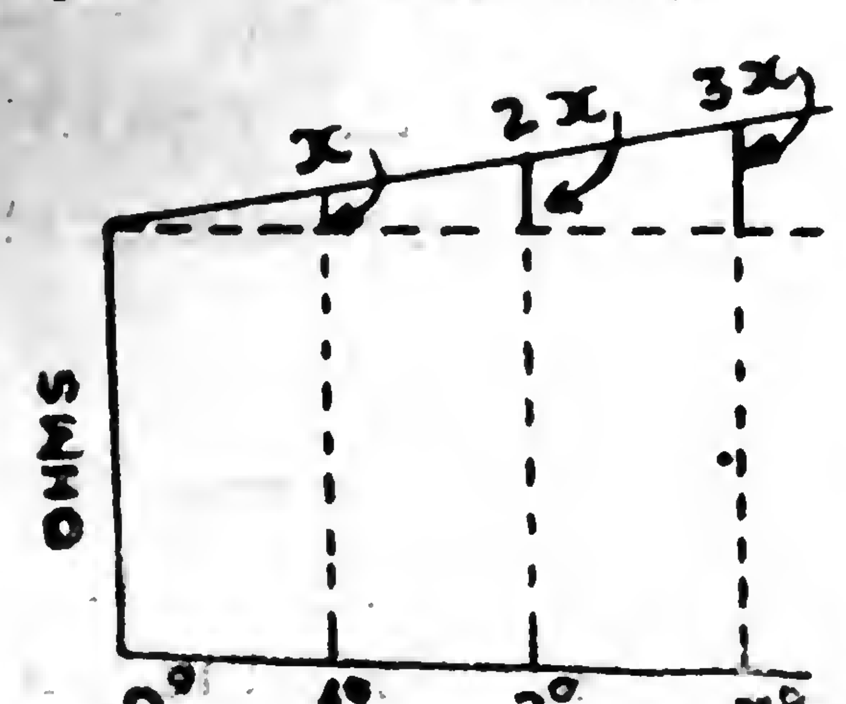


Fig. 1

Fig. 2

If the slope of the graph of Fig. 2 is continued accurately to the left it will cut the x-axis at a point corresponding to $-234{\cdot}5^\circ$ C or -390° F. This gives a very convenient method of calculating the resistance at any temperature t_2 if the temperature at t_1 is known. Thus

$$\frac{R_2}{R_1} = \frac{234{\cdot}5 + t_2}{234{\cdot}5 + t_1} \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

where R_1 is the resistance at t_1 temperature and R_2 is the resistance at t_2 temperature.

This can be proved easily by taking similar triangles of Fig. 2.

For copper the value of α at 0° C is 0·00427 and at 20° C the value is 0·00393. Thus the temperature must be stated when giving the value of α.

Example: A copper wire has a resistance of 10 ohms at 0° C. Find the resistance at 60° C. α at 0° C is 0·00427.

Solution: By using Eq. (2),

$$R_{60} = R_0 (1 + \alpha\, 60) = 10 (1 + 0{\cdot}00427 \times 60)$$
$$= 12{\cdot}562 \text{ ohms.}$$

OR, by using Eq. (3)

$$\frac{R_{60}}{R_0} = \frac{234{\cdot}5 + 60}{234{\cdot}5 + 0}$$

$$R_{60} = 10 \times \frac{234{\cdot}5 + 60}{234{\cdot}5} = 12{\cdot}562 \text{ ohms.}$$

Often the increase per degree rise in temperature is referred to the resistance at a temperature other than 0° C, say at $t_1{}^\circ$ C, then the resistance at $t_2{}^\circ$ C becomes

$$R_2 = R_1 [1 + \alpha (t_2 - t_1)] \quad \ldots \quad \ldots \quad (4)$$

where α is the temperature coefficient at $t_1{}^\circ$ C, R_1 is the resistance at $t_1{}^\circ$ C and R_2 is the resistance at $t_2{}^\circ$ C.

5. Potential Difference: The potential difference between two points of an electric circuit is the electrical pressure or voltage available to send a current between the two points. The practical unit of

potential difference (p. d.) is the *volt*. The electrical pressure generated in a cell or a dynamo is called its *electromotive force* (e. m. f.) and its unit is also the volt.

6. **Ohm's Law:** If the temperature remains unchanged, Ohm's law may be stated as "the ratio of p. d. between any two points of a circuit to the current flowing in it is constant".

Thus, stated in the form of an equation, it becomes

$$\frac{V}{I} = R \text{ (a constant)} \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

where V is the p. d. in volts, I is the current in amperes and R is the resistance in ohms.

By transposing, Eq. (5) becomes

$$I = \frac{V}{R} \quad \text{or} \quad V = IR$$

Thus from the last three equations we get the following definitions:—

(a) The *ampere* is that current which flows in a circuit of 1 ohm resistance when a p. d. of 1 volt is maintained across the circuit.

(b) The *volt* is the electrical pressure required to maintain a current of 1 ampere in a circuit of 1 ohm resistance.

(c) The *ohm* is the resistance of a circuit in which a current of 1 ampere flows when a p. d. of 1 volt is maintained across the circuit.

A better definition of ampere depends upon the wellknown electrolytic effect of current on metals in chemical solution. The fact is that 96500 coulombs will liberate 1 gram equivalent of any metal in chemical solution. For instance, silver which has an atomic weight of 108 and valency of 1, if used as the substance, then 1 ampere will liberate in one second a given amount of silver i. e.

$$\text{weight of silver} = \frac{\text{atomic weight}}{96500 \times \text{valency}}$$

$$= \frac{108}{96500 \times 1} = 0{\cdot}001118 \text{ g}$$

The number 0·001118 is the *electro-chemical equivalent* of silver.

7. Series and Parallel Connections: An electrical circuit usually has more than one resistance. These resistances are connected either in series or in parallel. Fig. 3 shows the series connection where there is only one path for the current to pass from P_1 to P_2. The resistance values of the units are r_1, r_2 and r_3. The total resistance of the whole circuit is given by

$$R = r_1 + r_2 + r_3 \quad \dots \quad \dots \quad \dots \quad \dots \quad (6)$$

In Fig. 4 the three units are connected in parallel. The total current passing from P_1 to P_2 has three paths and the equivalent resistance of the whole circuit is given by

$$R = \frac{1}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}} \quad \text{or} \quad \frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} \quad \dots \quad (7)$$

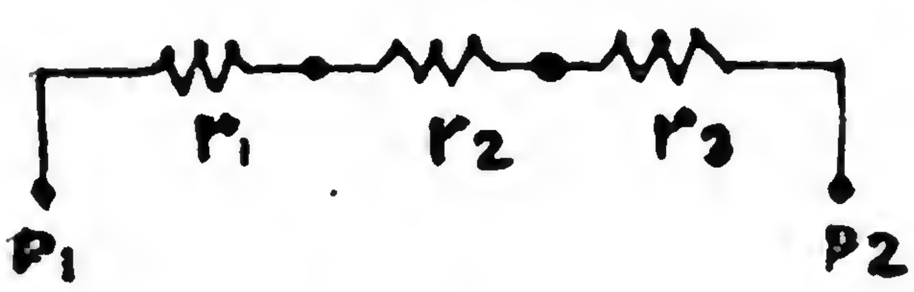


Fig. 3

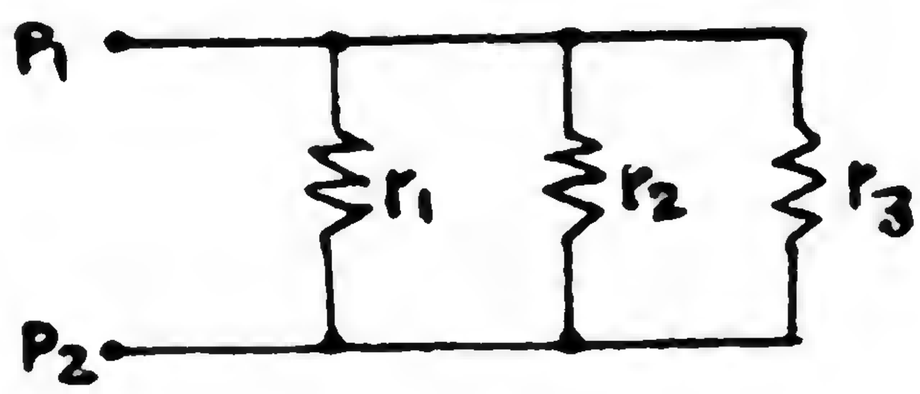


Fig. 4

In Fig. 5 the three resistances are in a sort of mixed connection. Here r_2 and r_3 are in parallel and the combination is in series with r_1. The equivalent resistance of the whole circuit, i. e. between the points P_1 and P_2 is given by

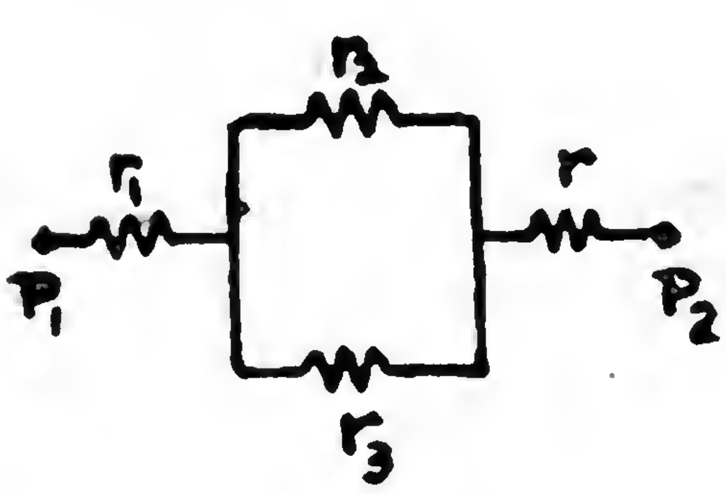


Fig. 5

$$R = r_1 + \frac{1}{\frac{1}{r_2} + \frac{1}{r_3}} + r$$

In a series circuit the current in each unit is the same, but the p. d. across the units will have different values if the unit resistances are not of the same value. In parallel circuit the p. d. across each unit is the same, but the current through each unit will have different values if the unit resistances have different values.

A *shunt* is a thin but a stiff metal strip made of an alloy called manganin. It is used in parallel with galvanometers or other measuring instruments to measure larger currents by the instruments. See Fig. 6.

Suppose I is the current to be measured by the instrument. Let the resistance of the instrument be G and that of the shunt S ohms. From the figure,

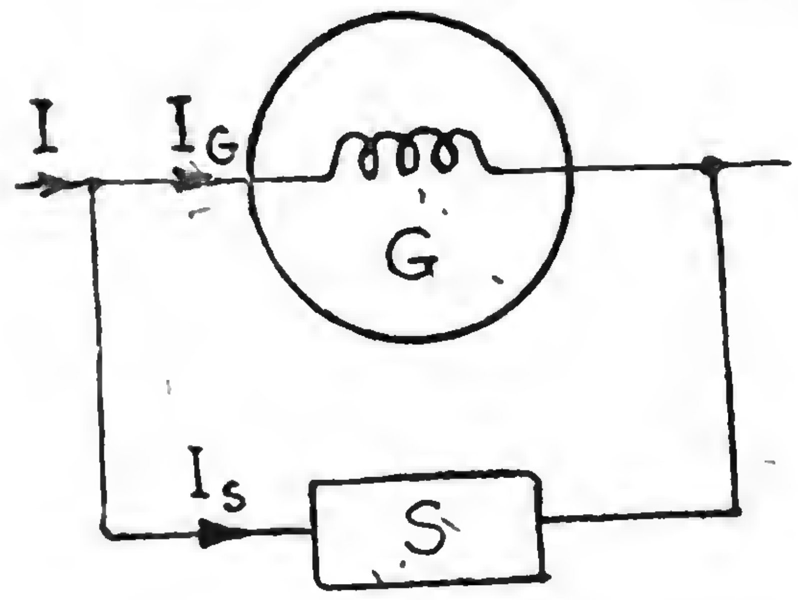


Fig. 6

$$I = I_G + I_S$$

and $$S \times I_S = G \times I_G$$

$$\therefore \quad I_S = I_G \times \frac{G}{S}$$

$$I = I_G + I_G \frac{G}{S}$$

$$I = I_G \frac{(S+G)}{S}.$$

The factor $\left(\frac{S+G}{S}\right)$ is called the *multiplying power* of the shunt and is usually made equal to 10, 100 or 1000.

Example : An ammeter has a resistance of 3 ohms and reads upto 100 milli-amperes. Calculate the shunt resistance for a full scale deflection corresponding to 10 A.

Solution : Multiplying factor of shunt $= \frac{10}{0{\cdot}1} = 100$

$$\therefore \quad 100 = \left(\frac{S+G}{S}\right); \quad G = 3 \text{ ohms.}$$

$$\therefore \quad 99S = 3 \qquad \therefore \quad S = 0{\cdot}0303 \textit{ ohm.}$$

8. Kirchhoff's Laws : There are certain networks the solutions of which are not possible by the use of Ohm's Law. One such circuit is that of a Post-office Box shown in Fig. 7.

First Law of Kirchhoff says that the algebraic sum of all the currents meeting at a point is zero.

The second Law says that the algebraic sum of all the p. ds. round a closed circuit is zero.

An example will now be worked out to illustrate the application of these laws.

Example : Find the current in each branch of the circuit shown in Fig. 7. The resistances are in ohms.

Solution : By applying the first law, the directions of currents in the circuit are marked in Fig. 8. The direction of current in the

branch BD is assumed to be from B to D.

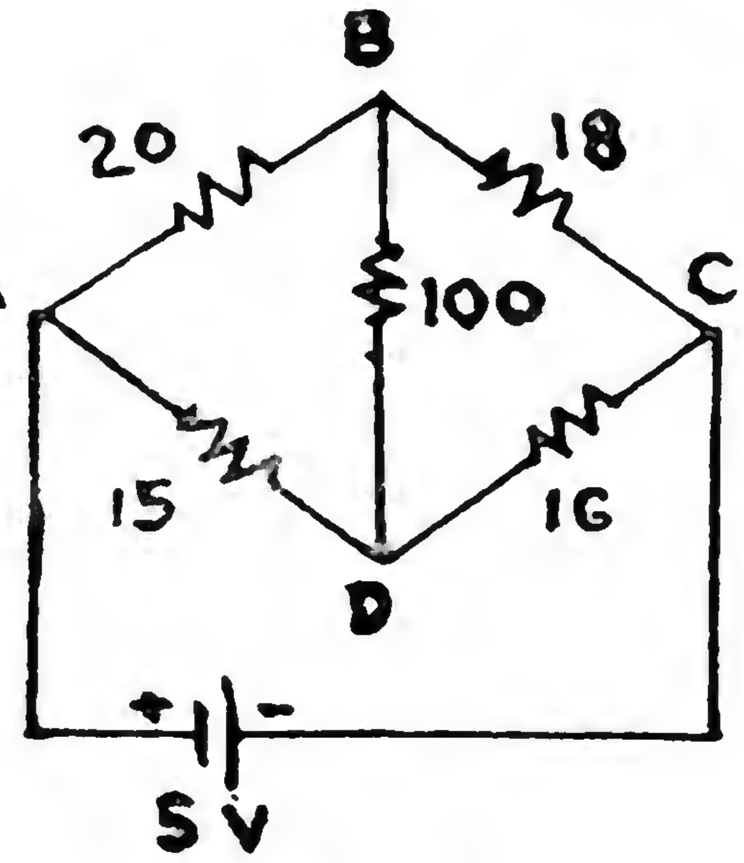


Fig. 7

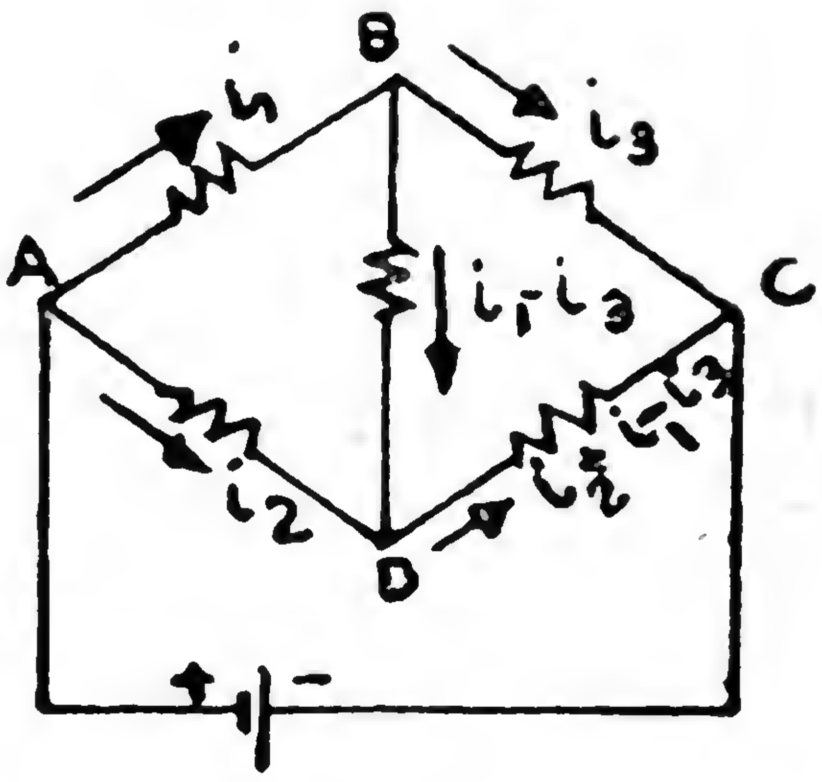


Fig. 8

Since there are three unknown currents, three equations must be written according to the second law, in such a manner that all the branch p. d.s and the e. m. f.s appear at least once in the three equations. Thus

$$20i_1 + 18i_3 - 5 = 0 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \text{(i)}$$

$$15i_2 - 16(i_1 + i_2 - i_3) - 5 = 0 \quad \ldots \quad \ldots \quad \text{(ii}$$

$$20i_1 - 100(i_1 - i_3) - 15i_2 = 0 \quad \ldots \quad \ldots \quad \text{(iii)}$$

The + sign is given to a p. d. or an e. m. f. if the progression in any closed circuit happens to be from a higher to a lower potential point, and the − sign if it is from a lower to a higher potential point.

From (ii) $i_2 = \dfrac{16i_3 - 16i_1 + 5}{31}$

Substituting the value of i_2 in (iii) and solving (i) and (iii) simultaneously, we get

$$i_1 = 0{\cdot}1307 \text{ amp. and } i_3 = 0{\cdot}1325 \text{ amp.}$$

The current in BD $= 0{\cdot}1307 - 0{\cdot}1325 = -0{\cdot}0018$ amp.

In other words the current in BD flows from D to B.

9. Conductivity: The reciprocal of resistivity is called *conductivity*. The symbol used is the greek letter γ. It is defined as the specific conducting power of a material. The reciprocal of resistance is called *conductance*. Its unit is the *mho* and the symbol used is G. So that

$$\gamma = \frac{1}{\rho}$$

$$G = \frac{1}{R} = \frac{\gamma A}{l}$$

since $$R = \rho \frac{l}{A}$$

The rate of change of potential with respect to distance is called *potential gradient*, symbol g. Thus by definition

$$g = \frac{V}{l}$$

Since $$I = \frac{V}{R}$$

$$I = \frac{\gamma VA}{l} = \gamma gA \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

Also $$I = \frac{gA}{\rho}$$

$$\therefore \quad \rho = \frac{g}{I/A} = \frac{\textit{potential gradient}}{\textit{current density}}$$

G x S
m − f

Hence resistivity is a ratio of potential gradient in a conductor to the current density in it.

10. Energy, Work and Power: Energy is the capacity for doing work, and work is said to be done when a force, acting on a body, causes motion of the body. Both energy and work are measured in terms of the same unit. Energy appears in several forms such as (a) mechanical, (b) electrical, (c) heat, (d) chemical etc.

The foot-pound (ft-lb) is the unit of work or energy and it is equal to the work done in lifting a mass of one pound one foot vertically against gravity.

The erg is the c. g. s. unit of work or energy. It is the work done when a force of one dyne acts for a distance of one centimetre.

Energy can be transformed from one form into another, and in any such transformation the total amount of energy remains unchanged. The word WORK should only be used where mechanical movement takes place.

The practical unit of energy used in engineering is the *joule*. One joule is the energy transformed between two points in an electrical circuit when the potential difference between the points is one volt and

one coulomb is transported from one point to the other.

$$1 \; joule = 10^7 \text{ ergs} = 0{\cdot}7375 \text{ ft. lb.}$$

$$energy = V \times Q \text{ joules} \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

But $Q = It$, where I is the current in amperes and t the time in seconds, therefore

$$energy = VIt \text{ joules.}$$

Also, since $V = IR$ and $I = \frac{V}{R}$

$$energy = \frac{V^2}{R} t \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

Power is the rate of doing work. The most commonly used mechanical unit is the horse-power (h. p.). The unit of power in electrical engineering is the *watt*

$$power = V \times I \text{ watts} \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

$$= I^2 R \quad \text{"} \quad \ldots \quad \ldots \quad \ldots \quad (12)$$

$$= \frac{V^2}{R} \quad \text{"} \quad \ldots \quad \ldots \quad \ldots \quad (13)$$

Note that *one joule = one watt/second.*

The two expressions, (i) $power = I^2 R$ and (ii) $energy = (I^2 Rt)$ are the mathematical statements of Joule's Law, which states that the heat developed in an electric circuit is given by the expression

$$heat \propto I^2 Rt$$

$$= 0{\cdot}24 \, (I^2 Rt) \text{ calories}$$

Where *calory* is the heat unit of energy.

$$1 \; calory = 4{\cdot}19 \text{ joules}$$

$$1 \; joule = 0{\cdot}24 \text{ calory.}$$

One calory is the heat required to raise the temperature of one gram of water by 1° C.

The *British Thermal Unit* is also a unit of heat energy and is equal to the amount of heat required to raise the temperature of one pound of water by 1° F. It is equal to 778 ft-lb. and is most commonly known as the *mechanical equivalent of heat.*

1 B. T. U. = 778 ft-lb. = 252 calories.

Larger units of power and energy are often used for convenience, such as

1 *kilowatt* = 1000 watts

1 *megawatt* = 10^6 watts

1 *watt-hour* = 3600 joules

1 *kilowatt-hour* = 1000 watt-hours
= 1 Board of Trade Unit.

Example: Find the hot resistance of a 150 watt 250 volt filament lamp.

Solution: Current taken $= \left(\dfrac{\text{watts}}{\text{volts}}\right) = \dfrac{150}{250} = 0{\cdot}6$ amp.

$$\text{resistance of filament} = \frac{250}{0{\cdot}6} = 416{\cdot}67 \textit{ ohms}.$$

Example: If electrical energy is sold at 5 naiye paise per B. O. T. (Board of Trade) unit, calculate the cost of electric energy to light for 6 hours 50 lamps of the last problem.

Solution: Total power = 150 × 50 = 7500 watts

total energy = 7500 × 6 = 45000 watt-hours
= 45 kilowatt-hours (kWH)

cost of energy = 45 × 5 = 225 nP
= Rs. 2–25 nP.

In order to measure power a *standard* is necessary. But

$$power = V{\cdot}I = I^2 R = \frac{V^2}{R};$$

i. e. power involves two quantities out of 3 of Ohm's Law. This shows that at least two of these quantities must be chosen to have their respective *standard units* for accurate measurements in laboratories.

To this end a column of mercury weighing 14·452 g and 106·3 cm long is chosen as a *Standard Ohm*. This column must have a uniform cross-section and be held at the temperature of melting ice.

Similarly, the *Weston Cadimum Cell*, whose e. m. f. is 1·0183 volt, has been chosen as a *Standard Cell* for comparing or measuring

the p. d.s of other sources. This cell is used in precision measurements in such a manner that during the test it gives out practically no current. Thus the e. m. f. of this cell remains constant for a very long period.

11. Conducting Materials—Alloys—Insulating Materials: Two materials which are chiefly used for conduction of electric current are *copper* and *aluminium.* Copper is the second best conductor and aluminium is fourth in the list, silver and gold being first and third respectively.

Carbon is used in incandescent and arc lamps, for brushes in electrical machines and for variable rheostats such as carbon piles.

Alloys of certain metals are extensively used for (a) Standard Resistances, (b) Regulating Resistances and (c) over-head line conductors, where very long spans on extra-high voltage transmission are encountered. In alloys it is found that their resistivities are greater than the average of their constituents, while their temperature coefficients are much lower. The names of some of the common alloys are :—German silver, manganin, platenoid, phosphur - bronz etc., and some are known by their "trade" names, such as "EUREKA", "NICHROME" etc.

Insulators are substances whose specific resistances are much greater than that of copper. For instance rubber has a sp. resistance which is 10^{22} times that of copper. Hence it may be stated that insulating materials are poor conductors of electricity. These are divided into groups, viz. (a) hygroscopic (those that absorb moisture) and (b) non-hygroscopic.

Hygroscopic materials are (i) *paper,* used in cable manufacture; (ii) *silk,* (iii) *cotton,* used for machine windings and certain wire coverings; (iv) *asbestos,* used where fire-proofing is required, and (v) *fibre,* used for insulating coils from machine and apparatus frames. Non-hygroscopic ones are (i) *India rubber,* used ordinarily for wires and cables; (ii) *mica* used for insulating commutator segments; (iii) *glass,* used as containers for primary and secondary cells; (iv) *bitumen,* used in cables; (v) *ebonite,* used in instruments, keys and switches; (vi) *porcelain* used for line insulators and bushings. Only a few of both types of insulating materials have been mentioned here.

Unfortunately, the insulating properties of most of these materials deteriorate with a rise of temperature. Hence it is the standard practice to determine the output capacity of a machine or an apparatus by the type of insulation used, and consequently it also determines the safe limit of temperature rise. For an insulating material the relation between resistance at a certain fixed temperature and at a certain higher temperature is given by

$$\text{Log } R_{t_2} = \log R_{t_1} - \frac{\log 2}{t}(t_2 - t_1) \quad \dots \quad \dots \quad (14)$$

where t_1 is the initial, t_2 is the higher temperature, and t is the rise of temperature necessary to halve the insulation resistance, or

$$R_2 = R_1 / K^{(t_2 - t_1)} \quad \dots \quad \dots \quad \dots \quad (15)$$

[where K is a constant depending upon the insulating material used.]

Example: The insulation resistance of a cable is 1·5 megohms at 25°C. What will be its insulation resistance at 60°C, if the insulation resistance is halved by a rise of 30°C?

Solution:

$$\text{Log } R_{t_2} = \log(1{\cdot}5 \times 10^6) - \log 2\left(\frac{60-25}{30}\right)$$

$$= 6{\cdot}17609 - 0{\cdot}3010 \times \frac{35}{30}$$

$$= 5{\cdot}8249$$

$\therefore \quad R_{t_2} =$ *668100 ohms*, or *0·6681 megohm.*

The primary function of insulators is to restrict the current to its legitimate path, i. e. along the conducting wires, and not to allow even a thousandth part of it (current) to find a path where it will be useless or harmful.

The current carrying capacity of conductors is the maximum current which can be allowed to flow without causing any harmful heating, or which is liable to cause damage to the insulating material, A thumb rule of "1000 amperes per square inch of conductor cross-sectional area" is sometimes used for large wires. For small ones an empirical formula is recommended by the Institute of Electrical Engineers, viz.,

$$I = 2{\cdot}6\, A^{0{\cdot}82}$$

where I is the current in amperes and A is the cross-sectional area of a conductor in thousandth of a square inch. But it is always advisable to use Wire Tables given in Hand-books. For the current density varies from 4000 amperes per sq. inch in small wires to about 900 for large ones.

12. Fuses and the Law of Fusing Current: A fuse consists of a wire or a thin strip of metal and is used for safety purposes in direct current and low voltage alternating current circuits. The fuse material may be (a) lead, (b) tin or (c) copper. (a) and (b) are used for low currents only. The function of a fuse is to melt and cause a break in the circuit when the current exceeds in that circuit above a predetermined value, thus saving a machine or apparatus from being damaged by excessive amount of current. Ordinarily, the fuse should melt when 125 % of full load current has been passing for about 1 minute. It is an inherent quality of a fuse that as the overload increases, the time interval of fusing shortens.

Fuse wires are fixed on porcelain bases, which are either detachable or fixed to the fuse holder. Fuse wires are (i) completely enclosed, (ii) semi-enclosed or (iii) open. The "cartridge" type of fuse, which is completely enclosed, is very reliable. Care should be taken that the molten metal of the fuse wire does not cause fire or damage to anything in its vicinity. The value of the current at which the wire melts is called its "*fusing current*". This depends upon (a) the melting point of the material, (b) the cross-sectional area and (c) the surface area of the wire. The length of the wire and the size of the terminals to which the fuse wire is fixed also have some influence on the value of the fusing current.

A current passing in a fuse wire produces heat, and if the temperature remains constant

heat produced per second = *heat dissipated per second* by radiation. i. e.

$$I^2R = \text{surface area of the fuse wire} \times \text{a constant},$$
$$= d \times l \times \text{a constant},$$

where d = diameter of the wire and l = length of the wire.

$$\text{But} \quad R = \rho \frac{l}{A} = \frac{l}{d^2} \times \text{another constant},$$

$$\therefore \quad I^2 \frac{l}{d^2} \times k_2 = d \times l \times k_1$$

or $\quad I^2 = d^3 \times k_3$ (k_1, k_2 and k_3 being constants,)

i. e. $I = \sqrt{d^3} \times$ a constant. ... ... (16)

The above relation is known as Preece's Law of fusing currents but the law is not very reliable. For, there are many factors which influence I, the fusing current, such as the surface condition of the wire, the size of the terminals used, the length of the wire, and the type of the fuse holder.

CHAPTER II

ELECTRO-MAGNETISM

Introductory Note: In the study of magnetic and the dielectric circuits the beginner should study the units of quantities in the two c. g. s. systems. These two systems are (i) the electromagnetic (e. m. u.) and (ii) the electrostatic (e. s. u.). The Table below gives unit names for the most common quantities in the two systems as well as in the practical (m. k. s.) system.

Quantity	System of Units		
	Practical	E. M. U.	E. S. U.
Potential (V)	Volt	Abvolt	Statvolt
Current (I)	Ampere	Abampere	Statampere
Resistance (R)	Ohm	Abohm	Statohm
Inductance (L)	Henry	Abhenry	Stathenry
Charge (Q)	Coulomb	Abcoulomb	Statcoulomb
Capacitance (C)	Farad	Abfarad	Statfarad

$$1 \text{ volt} = 10^8 \text{ abvolts} = \frac{1}{300} \text{ statvolt}$$

$$1 \text{ ampere} = \frac{1}{10} \text{ abampere} = 3 \times 10^9 \text{ statamperes}$$

$$1 \text{ ohm} = 10^9 \text{ abohm} = \frac{1}{9 \times 10^{11}} \text{ statohm}$$

$$1 \text{ henry} = 10^9 \text{ abhenrys} = \frac{1}{9 \times 10^{11}} \text{ stathenry}$$

$$1 \text{ coulomb} = \frac{1}{10} \text{ abcoulonb} = 3 \times 10^9 \text{ statcoulombs}$$

$$1 \text{ farad} = 10^{-9} \text{ abfarads} = 9 \times 10^{11} \text{ statfarads}$$

1. Magnetism : Assuming that a student has prior knowledge of magnetism, we give below a summary of it to refresh his memory :-

(a) Like poles repel and unlike poles attract.

(b) A unit magnet pole is one which when placed 1 cm from an equal and similar pole repels it with a force of 1 dyne.

(c) Coulomb's Law states that if two like poles of m_1 and m_2 units are placed r cm apart *in air* then the force of repulsion between them is given by

$$force = \frac{m_1 \times m_2}{r^2} \text{ dynes} \quad \dots \quad \dots \quad \dots \quad (1)$$

(d) The region surrounding a magnetised body in which a magnet pole experiences a mechanical force, is called *magnetic field.* The symbol used is the greek letter Φ.

(e) If at a point in a magnetic field a unit pole experiences a force of 1 dyne, the magnetic field at the point is said to be of unit strength or of *unit field intensity.* The unit of field intensity is the *oersted* and the symbol used is H.

If a magnet pole of strength m units is placed in a magnetic field having an intensity of H oersteds the force acting on the pole is given by

$$force = m \times H \; dynes \quad \dots \quad \dots \quad \dots \quad (2)$$

$$H = \frac{force}{m} \; oersteds$$

Hence *oersteds = dynes per unit pole.*

(f) Since the magnetic field is invisible, it is customary to represent it with lines called *lines of force.* These lines always tend to shorten their length. Each line is called a *maxwell.* The *field density* or the *flux density* is the measure of the number of lines per sq. cm. The area considered must be at right angles to the direction of lines of force.

Hence flux is a vector quantity having magnitude and direction. The unit of flux density is the *gauss* and the symbol used is B. 1 gauss = one line of force per sq. cm.

2. Flux Produced by a Current : When a current flows in a conductor a magnetic field or flux is created surrounding its entire length. Fig. 1 shows only a few groups of concentric lines of force. The arrows on the lines indicate the direction of force on a small compass needle placed at the point. Fig. 2 shows the field as seen

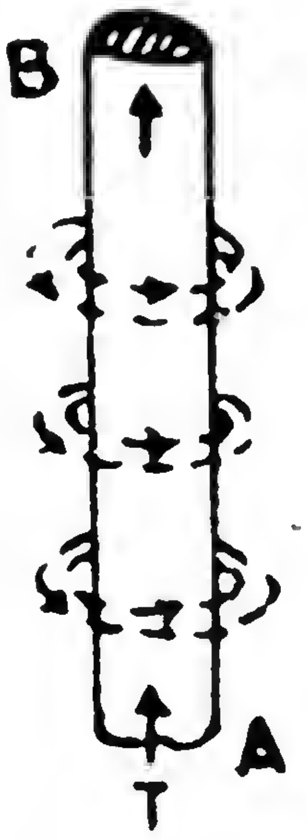
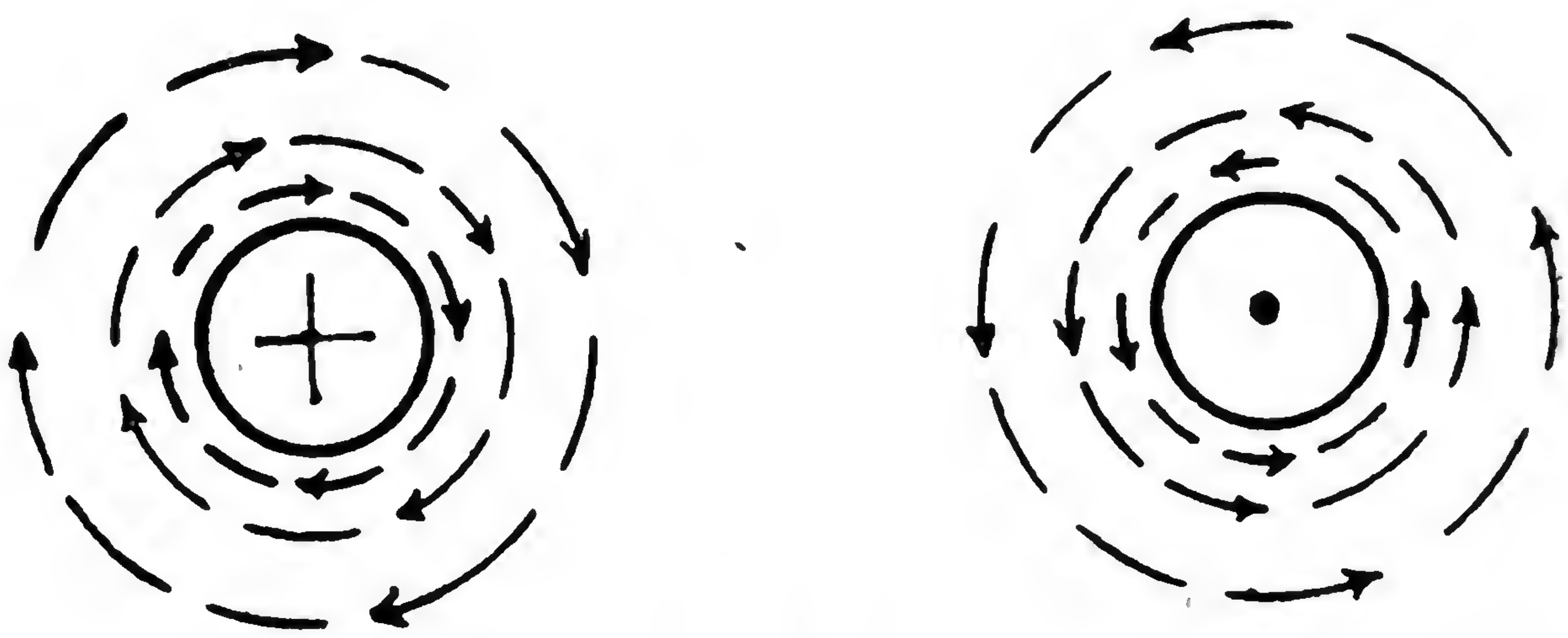


Fig. 1



End A Fig. 2 End B

from ends A and B respectively. If the current-carrying conductor is bent into a loop as shown in Fig. 3 the direction of flux at a point is at right angles to the axis at that point, and the flux passes through the loop. If the conductor is wound spirally in the form of a helix

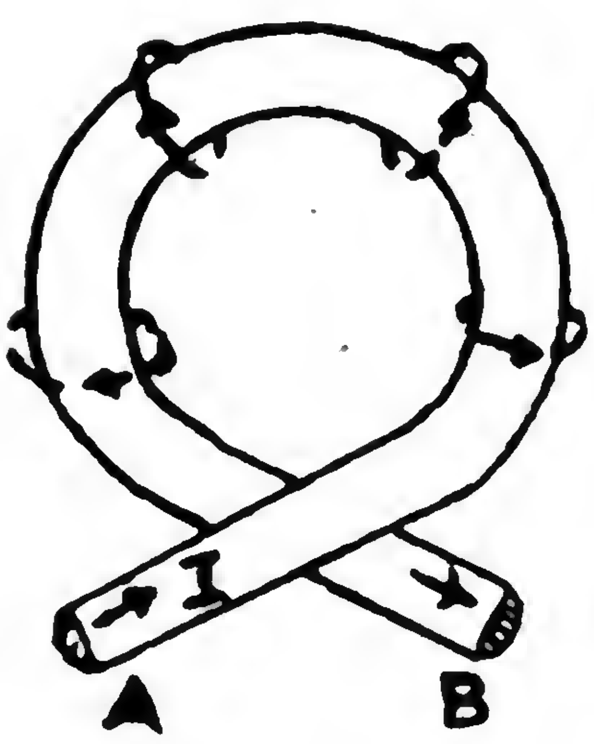


Fig. 3

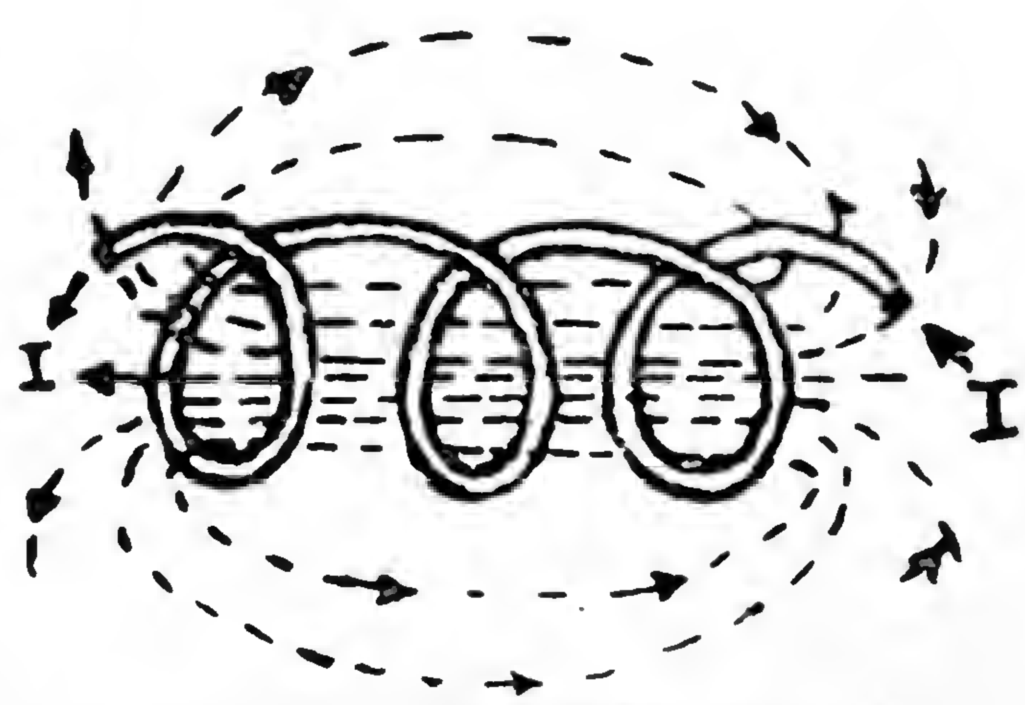


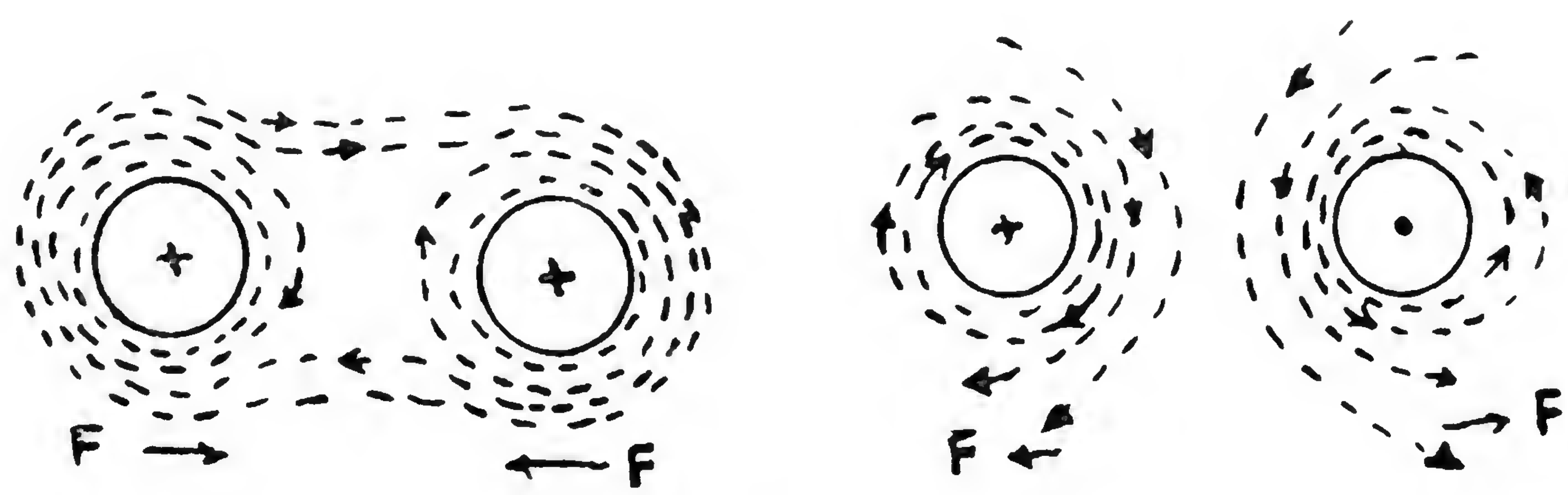
Fig. 4

the magnetic flux is as shown in Fig. 4. It is very similar to the flux of a bar magnet.

A simple rule to determine the direction of flux round the conductor is to place the palm of the *right hand* over (or under) the conductor in such a manner that the outstretched thumb points in the direction of current. If now the four fingers attempt to grip the conductor, the direction of the movement of the fingers indicates the direction of flux.

The direction of flux inside a coil is determined by placing the fingers of the *right hand* over the coil in such a manner that if in gripping the coil the fingers move in the direction of current flowing in the turns, the outstretched thumb points in the direction of flux, or it indicates the end of the coil where the N-pole is situated.

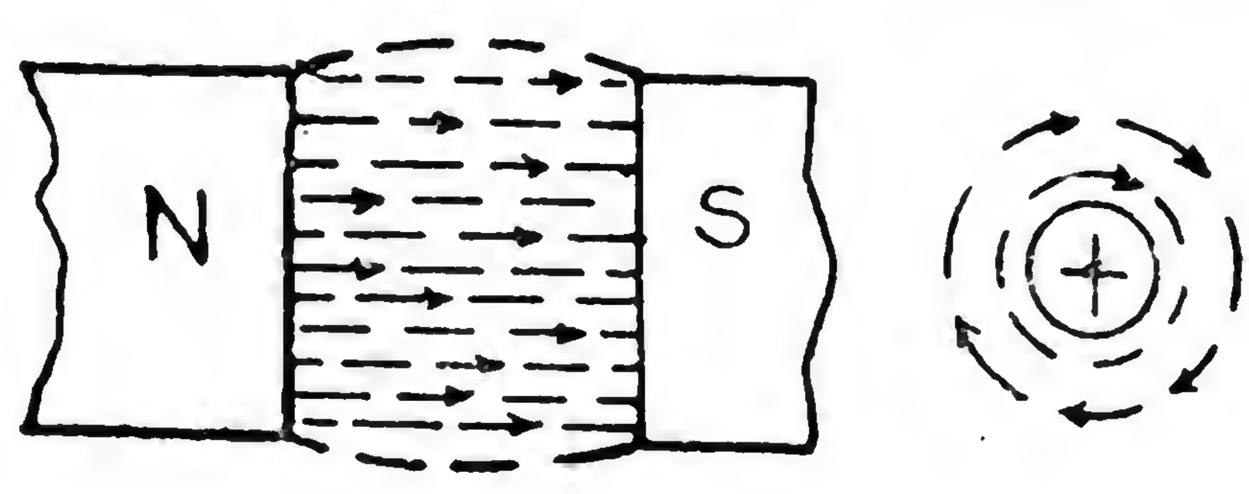
When two parallel conductors, separated by a short distance, carry a current the pattern of magnetic flux is as shown in figs. 5 (a) and (b). In (a) the current in the conductors flows in the same direction, and in (b) the direction of current in one is opposite to that of the other. In (a) the force between the two conductors is attraction



(a) Fig. 5 (b)

and in (b) it is repulsion. This is due to the fact that (i) magnetic lines repel one another and (ii) that they tend to become as short as possible. In Fig. 5 lines of force are crowded on one side of conductors, so that a conductor if free to move will be pushed in the direction of arrow *F* shown near that conductor.

3. Force Acting on a Current-Carrying Conductor in a Magnetic Field: Fig. 6 (a) shows a magnetic field of uniform



(a) Fig. 6 (b)

density and Fig. 6 (b) the field surrounding a current-carrying conductor. When this conductor is in the field of Fig. 6 (a) with its length at right angles to the direction of the main field the resultant field is as shown in Fig. 7. It is seen that the direction of force on the conductor is downwards.

This direction is determined by Fleming's Left Hand Rule which says

"Hold the thumb, the fore-finger and the middle finger of the *left hand* mutually at right angles to each other. Point the fore-finger in the direction of flux and the middle finger in the direction of current. The thumb then points in the direction of force or motion."

See Fig. 8. This is the principle on which electric motors work.

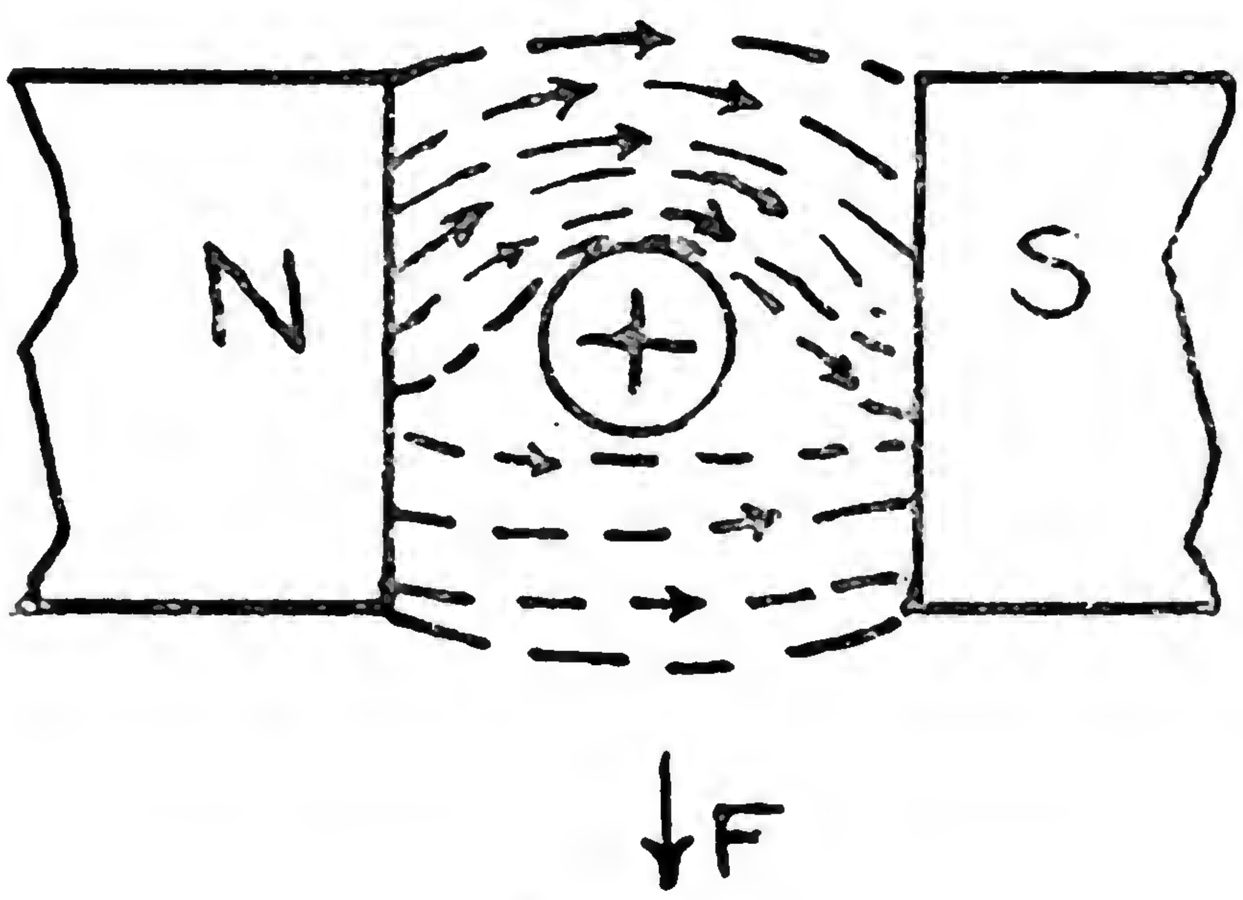


Fig. 7

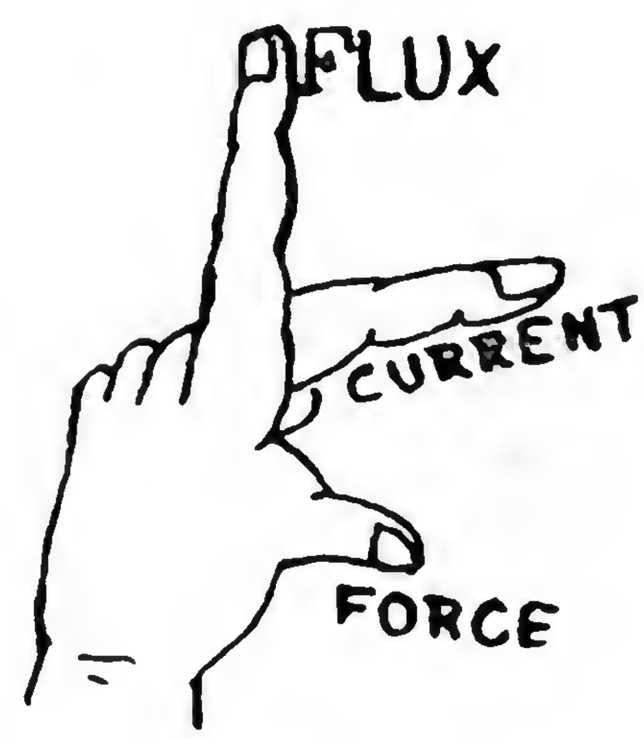


Fig. 8

If B is the flux density, l the active length of the conductor and I the current in the conductor, ***all the quantities being in c. g. s. units***, the force acting on the conductor is given by

$$force = B \cdot l \cdot I \text{ dynes} \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

This expression is fundamental and most important.

Fig. 9 shows the face of a S-pole, having length l and width d. If a conductor, carrying current I in the direction shown, moves a distance d, the work given out by the arrangement (i. e. by a motor) is

$$work = force \times distance$$
$$= B{\cdot}l{\cdot}I \times d \text{ ergs}$$

But ld is the area of the pole face and B is the flux density, therefore the total flux of the pole $\Phi = Bld$

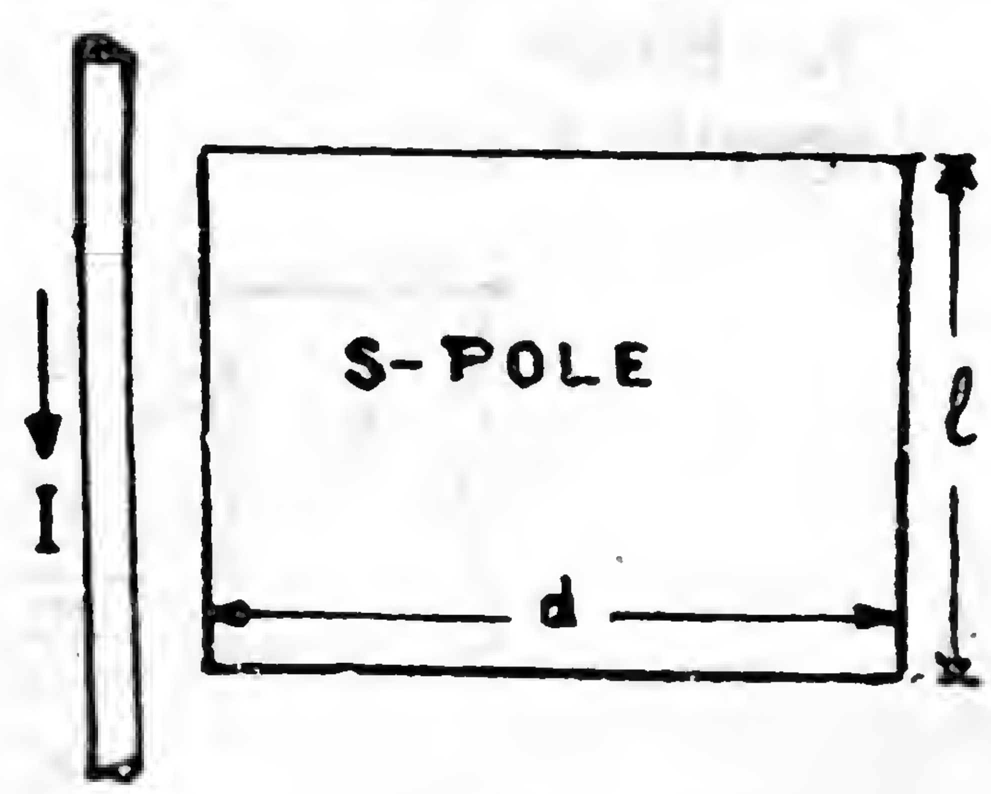


Fig. 9

Hence

$$work = \Phi \times I \text{ ergs} \quad \ldots \quad \ldots \quad (4)$$

If the conductor is moved a distance d against the force by an outside agency the work required is also ΦI. The machine is now a *generator*, the mechanical work being supplied by its prime mover.

4. Field Intensity (H): Ampere's Rule states that the field intensity at a point P due to an element dx of a conductor carrying current I is

$$dH = \frac{I\,dx\cos\theta}{s^2} \quad \ldots \quad \ldots \quad \ldots \quad (5a)$$

where s is the distance of P from dx

dx is the length of the elemental conductor,

θ is the angle through which dx must be turned to make it normal to the line from P. Using Ampere's Rule following four cases are treated.

Case 1. Field intensity near a long straight conductor

Fig. 10 shows an element dx of a long straight conductor carrying current I and P is any point distant s cm from dx. r is a perpendicular from P and ab is a portion of the conductor.

The total field intensity at P is obtained by integrating the effect of every element of the conductor. The contribution of dx is

$$dH = \frac{I\,dx\cos\theta}{s^2}$$

From the geometry of the figure

$$s = r\cos\theta;\quad x = r\tan\theta;\quad \text{and}\quad dx = \sec^2\theta\,d\theta$$

Considering the length ab only, the intensity at P is

$$H = \int_{-\beta}^{\alpha} I \times \frac{r\sec^2\theta\cdot d\theta}{r^2\sec^2\theta}\cos\theta = \frac{I}{r}\int_{-\beta}^{\alpha}\cos\theta\,d\theta$$

$$\therefore\quad H = \frac{I}{r}(\sin\alpha + \sin\beta) \quad \ldots \quad \ldots \quad (5b)$$

For the whole length of the conductor angles α and β are nearly 90°, hence the total field intensity at P becomes

$$H = \frac{2I}{r} \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

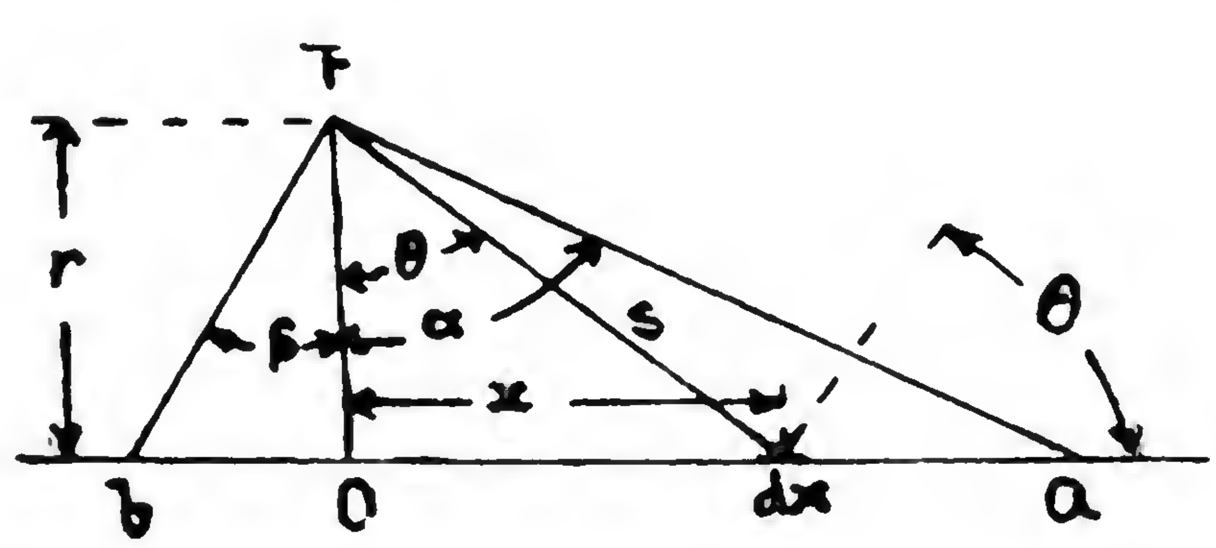


Fig. 10

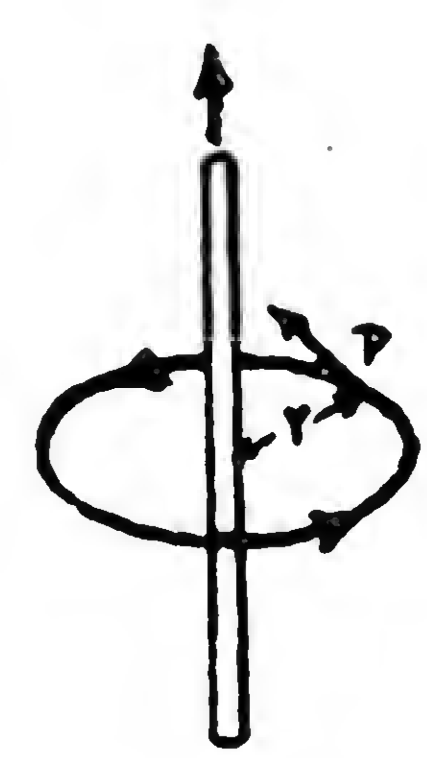


Fig. 11

The same result is obtained by considering Fig. 11, which shows an isolated conductor carrying current I. P is a point on one of the loops of r cm. radius.

Let H be the field intensity at P. If a magnet pole of m units is placed at P it will be acted upon by a force of mH dynes and when the pole is moved once round the loop the work done is

$$\begin{aligned} work &= force \times distance \\ &= mH \times 2\pi r \text{ ergs} \qquad \ldots \qquad \ldots \qquad (i) \end{aligned}$$

But the flux of the magnet pole is $4\pi m$ lines, and in going round the conductor once each line cuts the conductor once. According to Eq. (4) therefore

$$\begin{aligned} work\ done &= \Phi I \text{ ergs} \\ &= 4\pi m \times I \text{ ergs} \qquad \ldots \qquad \ldots \qquad (ii) \end{aligned}$$

Equating (i) with (ii),

$$mH \times 2\pi r = 4\pi m \times I$$

$$H = \frac{2I}{r} \quad \text{as before.}$$

Case 2. Field intensity on the axis of a round thin coil carrying current I.

Fig. 12 shows a very thin coil of T turns and P is any point on its axis a cm. from the centre of the coil. The radius of the coil is r cm. and the distance of P from any point on the circumference of the coil is s cm.

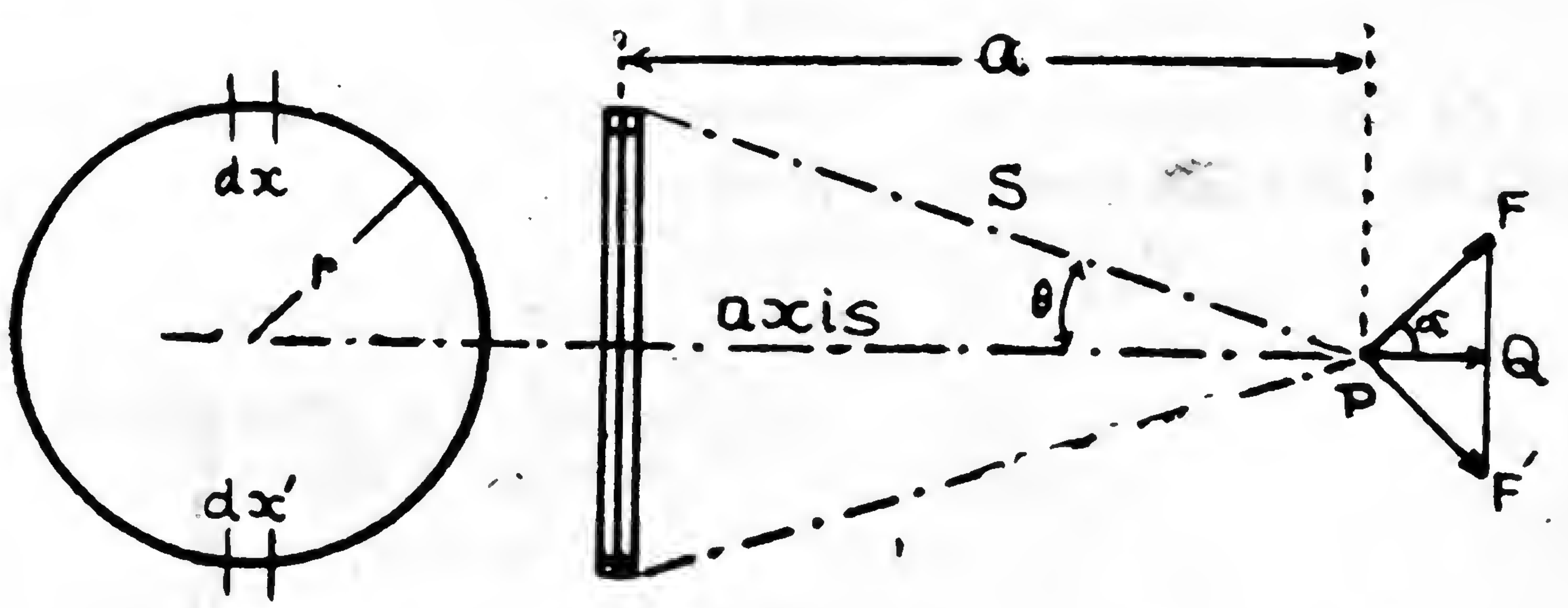


Fig. 12

Consider two diametrically opposite elements of the coil dx and dx'. By Ampere's Rule the contribution of dx at P is

$$dH = \frac{IT}{s^2}\,dx.$$

This is shown as PF in the figure and that due to dx' is shown by PF'. The components of these two vector quantities at right angles to the axis cancel each other, since they are equal and opposite in direction. But the components of these along the axis are in the same direction and therefore additive. Each component is equal to PQ. From the figure

$$PQ = PF \cos \alpha = PF \sin \theta$$

$$\Sigma dH = \frac{IT}{s^2} (\Sigma ds) \sin \theta$$

But $s = \sqrt{(r^2 + a^2)}$ and $\sin \theta = \frac{r}{s} = \frac{r}{\sqrt{(r^2 + a^2)}}$

$$\therefore \quad H = \frac{IT}{s^2} (2\pi r) \sin \theta$$

$$H = \frac{2\pi r^2 IT}{(r^2 + a^2)^{3/2}} \quad \ldots \quad \ldots \quad (7)$$

H is in oersteds if I, r and a are in c. g. s. units.

Case 3. Field intensity at the centre of a thin coil carrying a current I.

If the point P of the last case is brought to the centre of the coil, a is equal to zero. Hence the field intensity at the centre of the coil becomes

$$H = \frac{2\pi IT}{r} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

If all the quantities are in c. g. s. units H is in oersteds.

Case 4. Field intensity at any point on the axis of a current carrying coil.

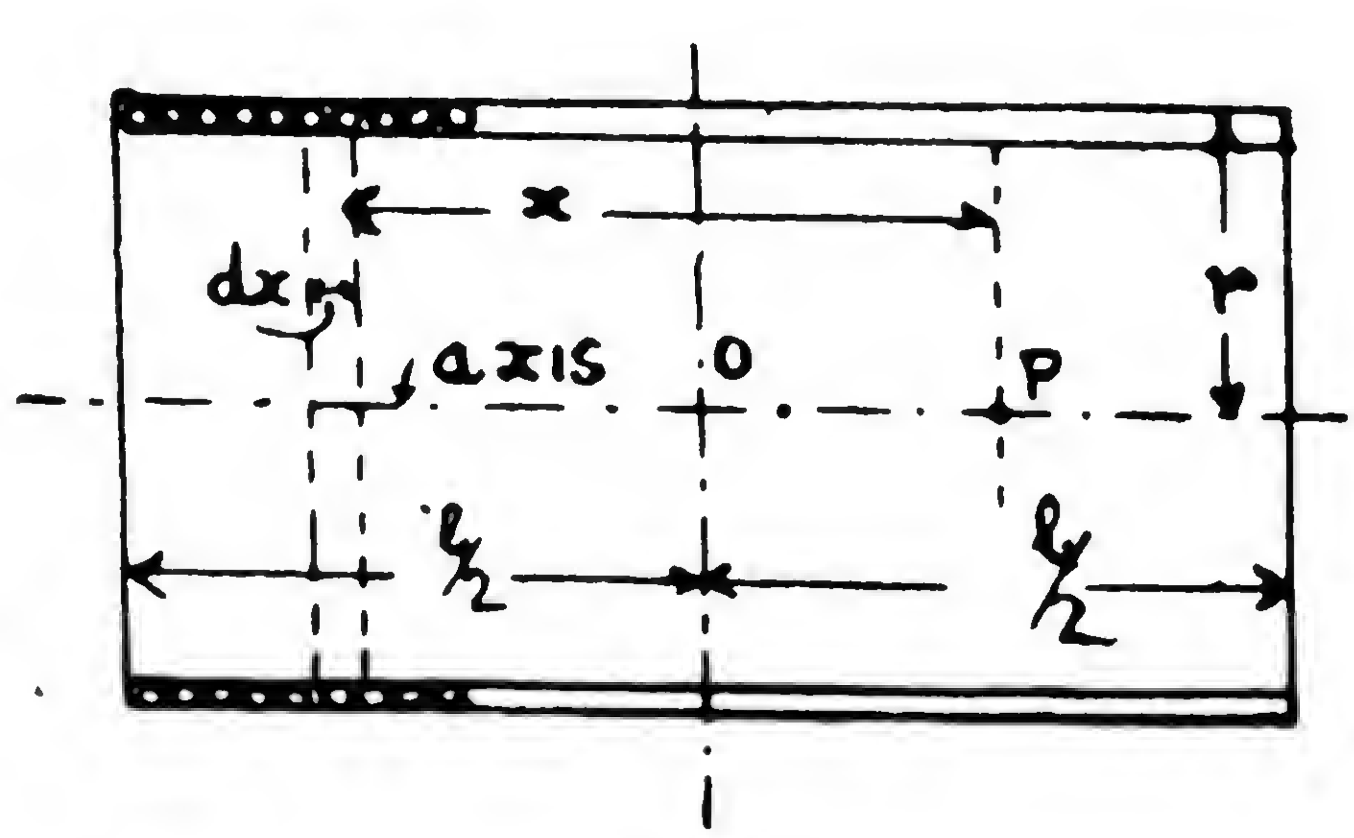


Fig. 13

The coil of Fig. 13 is l cm long and its radius is r cm. It is uniformly wound with T turns and the current is I c. g. s. units (i. e. I abamperes). Its centre point O on the axis is considered as the origin and P is any point on the axis m cm from O.

Consider an element of the coil of length dx and at a distance x from P. The field intensity at P due to dx is

$$dH = 2\pi r^2 I \, (T/l) \, dx$$

where $(T/l)\,dx$ is the number of turns in the element dx. The total field intensity at P due to all the elements of the coil is obtained by integrating the above expression when x varies between the limits $(l/2 + m)$ and $-(l/2 - m)$

$$H = \frac{2\pi r^2 I T}{l} \int_{-(l/2-m)}^{(l/2+m)} \frac{dx}{(r^2+x^2)^{3/2}}$$

$$H = \frac{2\pi IT}{l}\left(\frac{l/2+m}{\sqrt{[r^2+(l/2+m)^2]}} + \frac{l/2-m}{\sqrt{[r^2+(l/2-m)^2]}}\right) \quad \ldots \quad (9)$$

In this connection two points are of special interest one is the central point O, where $m = 0$ and the other is the end of the coil when $m = l/2$. Substituting these values, the field intensity at the centre of the coil is

$$H_0 = \frac{4\pi IT}{\sqrt{(d^2+l^2)}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

and at the ends

$$H_e = \frac{2\pi IT}{\sqrt{(d^2+l^2)}} = \frac{1}{2} H_0 \quad \ldots \quad \ldots \quad (11)$$

where $d = 2r$.

The field intensity is fairly uniform over the major length of the coil, but it rapidly falls in value towards the end as shown in Fig. 14.

When the coil is long compared with its diameter, the field intensity inside the coil is

$$H = \frac{4\pi IT}{l} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (12)$$

H is in oersted if I and l are in c. g. s. units.

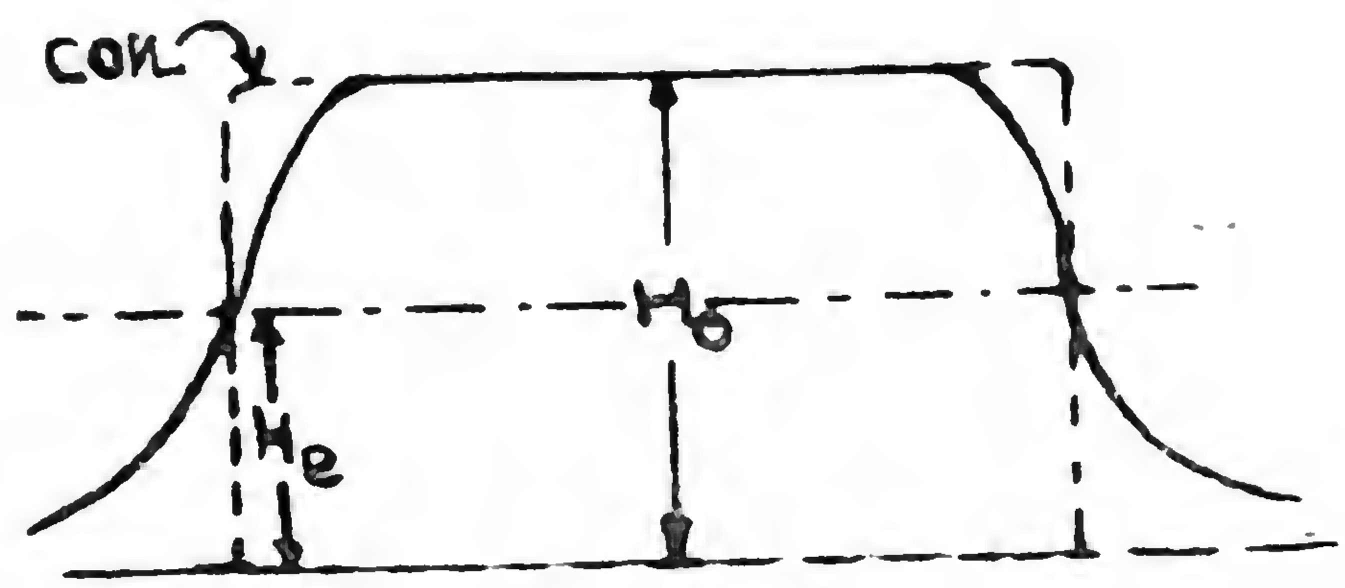


Fig. 14

Definition of H: Unit field intensity is said to exist at a point in a magnetic field if it exerts a force of 1 dyne on a unit pole placed at the point. See Eq. (2).

5. Force Between Two Parallel Current-Carrying Conductors: Fig. 15 shows two conductors spaced r cm carrying current I (c. g. s. units) in the same direction. The field intensity at any point on conductor A due to the current in conductor B is given by Eq. (6), namely

$$H = \frac{2I}{r} \text{ oersteds}$$

and the force acting on A as given by Eq. (3) is

$$force = B \cdot l \cdot I \text{ dynes}$$

But in air $B = H$, therefore

$$force = H \cdot l \cdot I \text{ dynes}$$

substituting the value of H

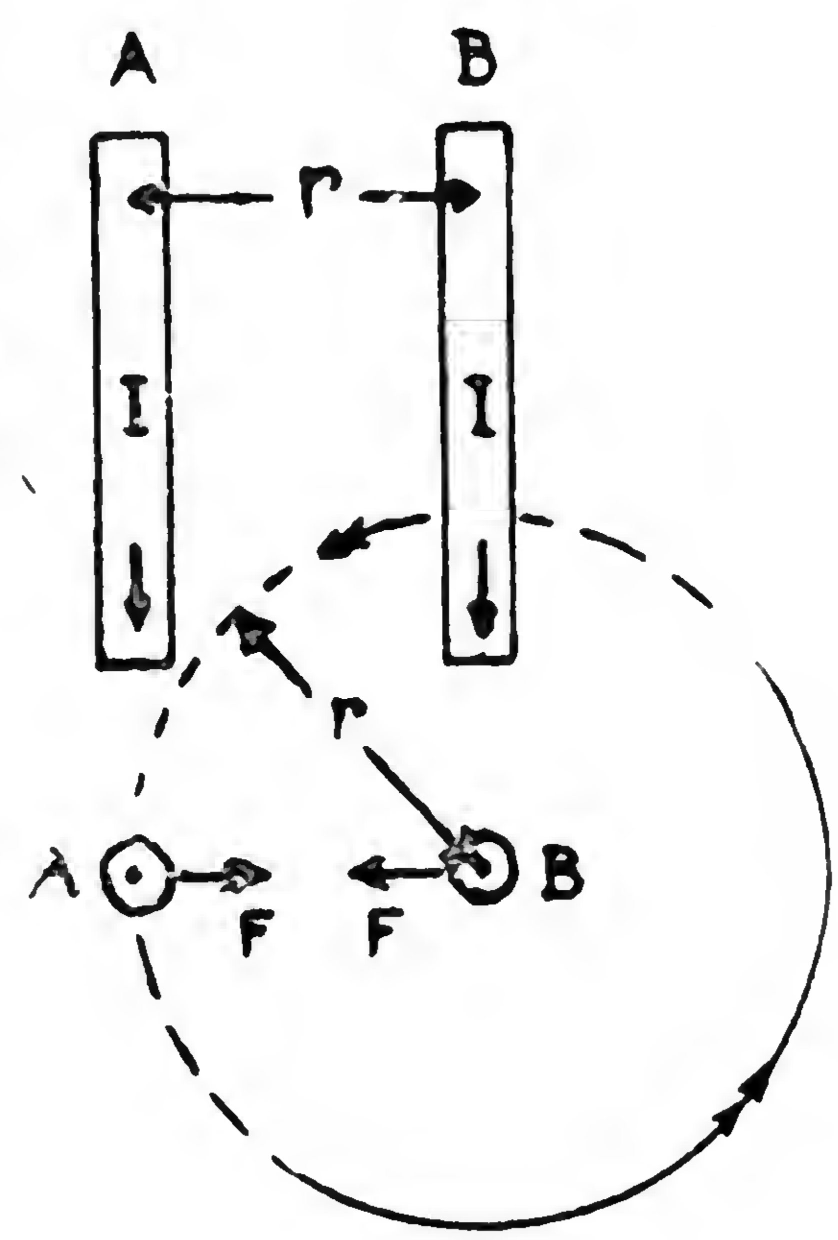


Fig. 15

$$force = \frac{2I}{r} l \cdot I$$

$$force = \frac{2I^2 l}{r} \text{ dynes} \qquad \dots \quad \dots \quad \dots \quad (13)$$

I, l and r being in c. g. s. units.

Example: Let A and B be two bus-bars carrying 500 amperes and separated by a distance of four inches. These are clamped at every yard. The force on any one is as given by Eq. (13).

In c. g. s. units,

$$500 \text{ amps} = 500/10 = 50 \text{ abamperes}$$

$$4 \text{ inches} = 4 \times 2{\cdot}54 \text{ cm}$$
$$1 \text{ yard} = 36 \times 2{\cdot}54 \text{ cm}$$
$$\therefore \quad force = \frac{2 \times (50)^2 \times 36 \times 2{\cdot}54}{4 \times 2{\cdot}54} \text{ dynes}$$
$$= 45000 \text{ dynes per yard, or}$$
$$= 45000/981 = 45{\cdot}9 \text{ g weight, or}$$
$$= 45{\cdot}9/453{\cdot}6 = 0{\cdot}101 \text{ lb. weight.}$$

When currents are in the same direction the force between the two is attraction by the Left-hand Rule and repulsion when the direction of current in one is opposite to that in the other. See Fig. 5.

6. The Law of Magnetic Circuit : Magnetic circuit is the path in which a number of magnetic lines of force are created and ***magneto-motive force*** is the cause that creates this magnetic flux Φ.

Reluctance is the property of the magnetic path that opposes the creation of magnetic flux. Reluctance is analogous to resistance of the electric circuit, namely

$$reluctance = \frac{l}{\mu A}$$

where l is the length of magnetic path,

A is the cross-sectional area of path and

μ is the permeability of the medium through which the magnetic lines pass.

Since $B = \mu H$ and $\Phi = B \cdot A$, we have $H = \dfrac{\Phi}{\mu A}$. Substituting the value of H in Eq. (12)

$$\frac{\Phi}{\mu A} = \frac{4\pi I T}{l}$$

$$\therefore \quad \Phi = \frac{4\pi I T}{\dfrac{l}{\mu A}} \qquad \dots \quad \dots \quad \dots \quad \dots \qquad (14)$$

The numerator of Eq. (14) is the ***magnetomotive force*** (*m. m. f.*) and the denominator is the reluctance. Eq. (14) is the law of magnetic circuit and resembles Ohm's Law, namely

$$flux = \frac{m.m.f.}{reluctance}; \quad current = \frac{e.m.f.}{resistance}.$$

Magnetomotive force ($m.m.f.$) is the work done in ergs in carrying a unit pole once around the magnetic circuit. From Eq. (4)

$$work\ done = \Phi I \text{ ergs}$$

In the case of a solenoid of T turns and carrying current I (c. g. s.) if a unit a unit pole is taken round once, the number of lines cut in the process is

$$4\pi T = \Phi \text{ and } \Phi I = work\ done = m.\,m.\,f.$$

$$\therefore \quad m.\,m.\,f. = 4\pi IT \qquad \ldots \qquad \ldots \qquad (15)$$

The unit of m. m. f. is the *gilbert* and is proportional to the product of current and the number of turns.

In practice the current is always stated in amperes so that when I is in amperes

$$m.\,m.\,f. = \frac{4\pi IT}{10}$$

The quantity IT is called "ampere-turns", hence

$$m.\,m.\,f. = \frac{4\pi}{10} \times ampere\text{-}turns$$

$$= 1{\cdot}257 \text{ amp-turns.} \qquad \ldots \qquad \ldots \qquad (16)$$

Since $H = \dfrac{4\pi IT}{l}$, and $m.\,m.\,f. = 4\pi IT$

$$\therefore \quad H = \frac{m.\,m.\,f.}{l} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (17)$$

$H =$ gilberts per centimetre.

$H = 1{\cdot}257$ amp-turns per centimetre.

Permeability (μ) expresses the relative magnetic conductance of materials. It is a ratio

$$\mu = \frac{B}{H} \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad (18)$$

μ for air is taken as 1 and for iron or steel its value rises to 2000 and even higher in certain special brands of steel. μ is not constant for a sample of iron or steel but varies with flux density and temperature. Therefore in air $B = H$.

If in an air-cored solenoid an iron core is introduced the number of lines of force are increased enormously, other things remaining the same. See Eq. (14).

Further, since $B = \Phi/A$, Eq. (18) reduces to

$$\Phi = \mu HA \quad \ldots \quad \ldots \quad \ldots \quad (19)$$

Thus H is also *magnetic gradient* as Eqs. (17) and (19) suggest. See Eq. (8) Chapter I.

7. The Magnetisation Curve: Except at low values of induction the flux in iron or steel is not proportional to m. m. f. or H, because μ varies as flux density. Therefore to know how Φ varies with m. m. f. or B varies with H, a sample piece is tested and the results give the B–H curve i. e. the magnetisation curve. Fig. 16 shows B–H curves for four different materials.

The ordinates are marked in kilogauss (i. e. 1000 lines per cm^2) and the abscissa in amp–turns per cm length of magnetic path. If the values of B in gauss are divided by corresponding values of H in oersteds, the results are the values of permeability (μ) at the corresponding values of B for the specimen.

In most magnetic circuits there is always an air–gap or two. The flux tends to occupy a larger area in air than in iron. This is called *fringing*. Further, all magnetic lines do not pass through the specified region of the air-gap. So the useful flux in air–gap is usually less than the flux created in iron. This is known as leakage. The ratio *total flux / useful flux* is called *leakage coefficient*. Its value varies between 1·5 to 1·1 and depends upon the permeability of the iron path. In many instances the flux in the air–gap is the useful flux.

Example: A cast-steel ring, having a cross-sectional area of 25 sq. cm and mean length of 180 cm, has a coil of 400 turns wound on it. Calculate (i) the current required in the coil to produce a flux of 0·3 megalines in the ring, and (ii) the permeability of cast-steel at that density.

Solution: (i) Flux density $B = \dfrac{\Phi}{\text{area}} = \dfrac{300000}{25} = 12000$ gauss.

Referring to Fig. 16 the ampere-turn per cm required for cast-steel to

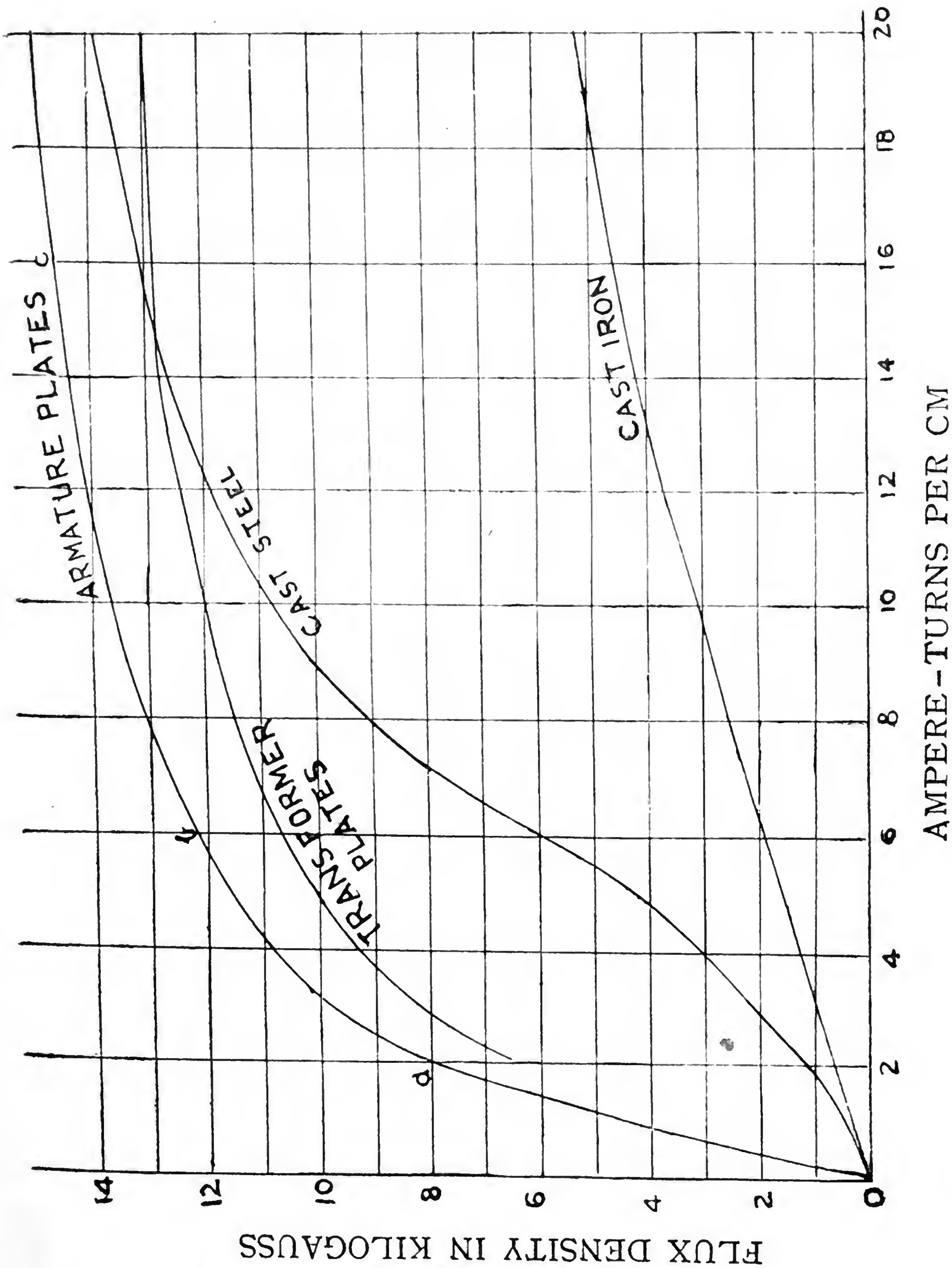

ARMATURE PLATES
CAST STEEL
TRANSFORMER PLATES
CAST IRON
FLUX DENSITY IN KILOGAUSS
AMPERE-TURNS PER CM
0
2
4
6
8
10
12
14
16
18
20

Fig. 16

give a flux density of 12000 gauss is from graph of cast-steel is 12·3 amp-turns per cm length of iron path. Therefore

$$\text{total amp-turns} = 12{\cdot}3 \times 180 = 2214$$

$$\therefore \quad current = \frac{amp\text{-}turns}{turns} = \frac{2214}{400} = 5{\cdot}535\ A.$$

$$(\text{ii}) \quad \mu = \frac{B}{H}\,;\ H = \frac{4\pi IT}{10l} \text{ and } \frac{IT}{l} = \text{amp-turns/cm}$$

Substituting the value of $\left(\frac{IT}{l}\right) = 12{\cdot}3$

$$H = \frac{4\pi}{10} \times 12{\cdot}3 = 15{\cdot}46 \text{ oersteds}$$

$$\therefore \quad \mu = \frac{12000}{15{\cdot}46} = 776.$$

Example: A saw-cut of 3 mm width is made in the ring of the last example. Calculate the current the coil must take to produce the same flux density in the iron path. Take leakage coefficient as 1·3.

Solution: The amp-turns required for the iron path is the same namely 2214.

$$\text{Leakage coefficient} = 1{\cdot}3 = \frac{\text{flux in iron}}{\text{flux in air-gap}}$$

$$\therefore \quad \text{flux in air-gap} = \frac{300000}{1{\cdot}3} = 230770 \text{ lines.}$$

$$\text{Using} \quad \Phi = \frac{4\pi IT}{10l} \times \mu A\,;\ l = 0{\cdot}3\,;\ A = 25 \text{ and } \mu = 1.$$

$$IT = \frac{230770 \times 0{\cdot}3 \times 10}{25 \times 4\pi} = 2200 \text{ for the air-gap}$$

$$\therefore \quad \text{total ampere-turns} = 2214 + 2200 = 4414$$

$$\therefore \quad \text{current in coil} = \frac{4414}{400} = 11{\cdot}035\ A.$$

But now suppose that the current in the coil of the ring remained at 5·535 A, i. e. the total amp-turns provided = 2214, what would be the flux in the iron and in the air-gap, the leakage coefficient being equal to 1·3 ?

Such a problem has to be attempted by assuming a flux density or flux in the air-gap. And by "trial are error" method one has to get as close to correct answer as is possible with about 3 attempts. The reason is that the permeability of iron path varies as flux density.

Solution : With the addition of an air-gap the flux will be considerably reduced, because the air-gap takes nearly half the m. m. f. provided (i. e. the ampere-turns). Hence as first approximation we assume the flux in the air-gap to be 150000 lines. Hence

$$\text{air-gap amp-turns} = \frac{150000}{25} \times \frac{0 \cdot 3}{4\pi} \times 10 = 1432$$

The flux in iron $= 1 \cdot 3 \times 150000 = 195000$ lines

$$\therefore \quad \text{density } B = \frac{195000}{25} = 7800 \text{ gauss.}$$

From graph for cast-steel ampere-turns per cm $= 7$.

For iron path amp-turns $= 7 \times 180 = 1260$.

$\therefore$ Total ampere-turns $= 1432 + 1260 = $ *2692*

which is more than what is provided. Hence the flux must be less than 150000 in the air-gap.

Let us now assume the air gap flux $= 120000$ lines.

$$\therefore \quad \text{amp-turns for gap} = \frac{120000}{25} \times \frac{0 \cdot 3}{4\pi} \times 10 = 1145 \text{ amp-turns.}$$

$$\text{Flux density in iron} = \frac{120000 \times 1 \cdot 3}{25} = 6240 \text{ gauss.}$$

From graph for cast steel ampere-turns per cm $= 6$.

For iron path amp-turns $= 6 \times 180 = 1080$ amp-turns.

Total ampere-turns $= 1145 + 1080 = $ *2225 amp-turns.*

This is very nearly equal to the amp-turns provided on the coil. The answer therefore is

flux in iron $= 120000 \times 1 \cdot 3 = $ *156000 lines*

flux in gap $= $ *120000 lines*

8. Hysteresis: Let the magnetising force H on an unmagnetised specimen of iron be increased from zero to a certain maximum in a series of equal steps, the corresponding values of flux density B is measured. On plotting these points, see Fig. 17, we get the curve Oab. This is the normal magnetisation curve.

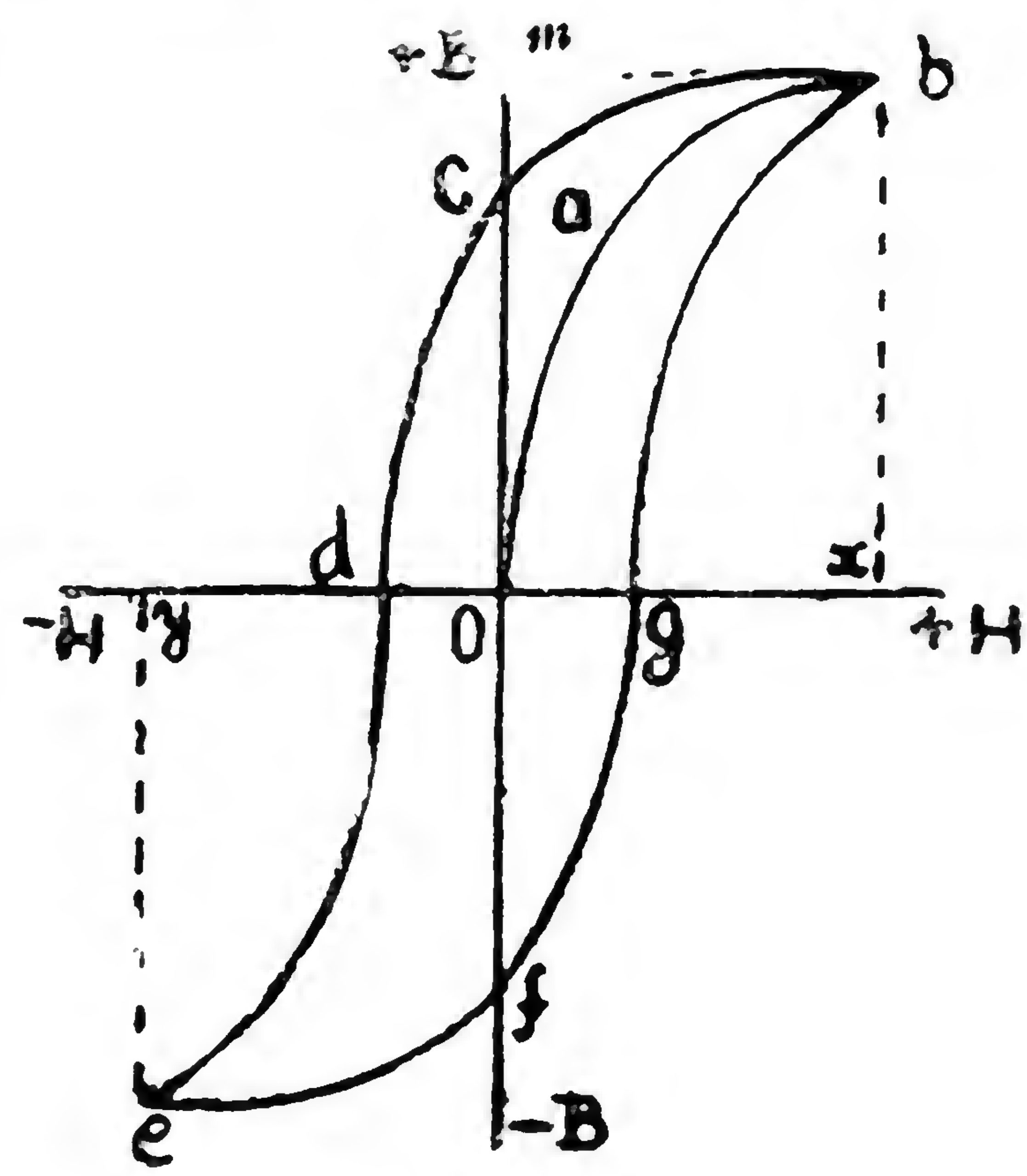


Fig. 17

If the magnetising force is now decreased the induction density B will not fall back along the line baO, but will decrease less rapidly along the line bc. In other words, the induction density lags behind the magnetising force. This "lagging" has been given the name *hysteresis.*

Oc is the measure of ***residual magnetism.*** To destroy this residual magnetism H must be reversed and increased to an amount Od, called the ***coercive force.*** If H is further increased until it reaches the same maximum, the induction density also reaches an amount $ye = bx$, i. e. the iron is now magnetised in the opposite direction. The trace of the curve is de.

If now H is reduced to zero in equal steps, B reduces to Of, the residual magnetism. Next reversing H and increasing it to the same maximum value we obtain curve fgb. The loop $bcdefgb$ is called the ***hysteresis loop,*** and the iron is said to have gone through one cycle of magnetisation.

Initially when H is increased from zero to Ox the energy stored in the magnetic circuit is proportional to the area $OabmcO$. This energy comes from the electric circuit. When H is reduced to zero, the energy returned to the electric circuit is proportional to the area $bmcb$. The remaining portion of the energy is $OabcO$ and is lost in the iron.

Therefore the net energy expenditure for one cycle of magnetisation is proportional to the area of the loop. Hence hysteresis loss $\propto$

area of the loop *per cycle.* If f is the frequency or the number of cycles of magnetisation per second

$$hys.\ loss \propto [(\text{area of loop}) \times f \times \text{volume of iron}]\ \text{ergs} \quad \dots \quad (20a)$$

If A is the number of sq. centimetres in the loop, H' is the oersteds per cm. length and B' is the gauss per cm. length of ordinate,

$$hys.\ loss = \frac{1}{4\pi}(A \times B' \times H')f \times volume \times 10^{-7}\ \text{watts} \quad \dots \quad (20b)$$

Example: The area of hysteresis loop for a sample of sheet steel is 6·5 cm.^{2} The scales used are

1 cm. = 5 amp-turns per cm.

1 cm. = 2000 gauss

This material is used for an armature core which weighs 11 kg.

Find the hysteresis loss (per second). The frequency of magnetisation is 50 cycles per second and the specific gravity of the material is 7·7.

Solution: Volume of core in cc. $= \frac{11 \times 1000}{7{\cdot}7} \left(\frac{\text{weight}}{\text{sp. gr.}}\right)$

$H' = 5$ amp-turns per cm. $= \frac{4\pi}{10} \times 5 = 2\pi$ oersteds

$B' = 2000$ gauss and $A = 6{\cdot}5$ sq. cm. Hence

$$hys\ loss = \frac{1}{4\pi}(6{\cdot}5 \times 2\pi \times 2000) \times 50 \times \frac{11 \times 1000}{7{\cdot}7} \times 10^{-7}\ \text{watts}$$

$$= 46{\cdot}43\ \text{watts.}$$

Another method of determining the hysteresis loss is due to Dr. Charles Steinmetz. He found that

$$hys.\ loss = \eta B^{1{\cdot}6}\ \text{ergs per cycle per cc.} \quad \dots \quad \dots \quad (21)$$

where η is the hysteresis constant for the material. Its value varies between 0·001 and 0·003 for steel stampings. For best grade silicon sheets it is about 0·00045 and for parmalloy it is 0·0001. B is the maximum flux density in gauss.

Hence, if f is the number of cycles of magnetisation per second

$$hys.\ loss = \eta B^{1{\cdot}6} f \times 10^{-7}\ \text{watts per cc.} \quad \dots \quad \dots \quad (22)$$

Example: An armature of a 2-pole d. c. machine rotates at a speed of 1500 r. p. m. in a magnetic field density of 7500 gauss.

Calculate the hysteresis loss in watts. Hysteresis constant $\eta=0{\cdot}0025$, weight of armature stampings is 250 lb. and the specific gravity of the material is 7·5.

Solution : Total volume of stampings $= 250 \times \dfrac{453{\cdot}6}{7{\cdot}5}$ cc.

$$f = \frac{1500}{60} = 25 \text{ cycles per second. Hence}$$

$$hys.\ loss = 0{\cdot}0025 \times (7500)^{1{\cdot}6} \times 25 \times \left(250 \times \frac{453{\cdot}6}{7{\cdot}5}\right) \times 10^{-7}$$

$$= 150 \text{ joules per second or watts.}$$

9. **Electro-Magnetic Induction:** Faraday's Law states that "the induced e. m. f. in an electric circuit is proportional to the *rate of change* in the number of lines of force linked with the circuit."

The change of flux takes place either

(i) By the movement of conductor in a magnetic field. This produces *dynamically* induced e. m. f., or

(ii) By the variation of current in the circuit. This produces *statically* induced e. m. f.

Thus if a conductor is moved in a magnetic field an e. m. f. is induced in the conductor. If the conductor forms a part of a closed circuit a current will flow. But when the current flows in the conductor it experiences a force, according to Eq. (3). The direction of this force is such as to oppose the motion of the conductor. Work therefore has to be done to produce electrical energy. The above statement is known as *Lenz's Law*.

There is a definite relationship between

(i) the direction of induced e. m. f.,

(ii) the direction of motion of conductor, and

(iii) the direction of magnetic flux.

This is well remembered by Fleming's Right-hand Rule by holding the three fingers of the right hand mutually at right angles to each other as shown in Fig. 18.

The thumb in the direction of motion and the forefinger in the direction of flux, then the middle finger points the direction of e. m. f. This is the principle on which electric generators work.

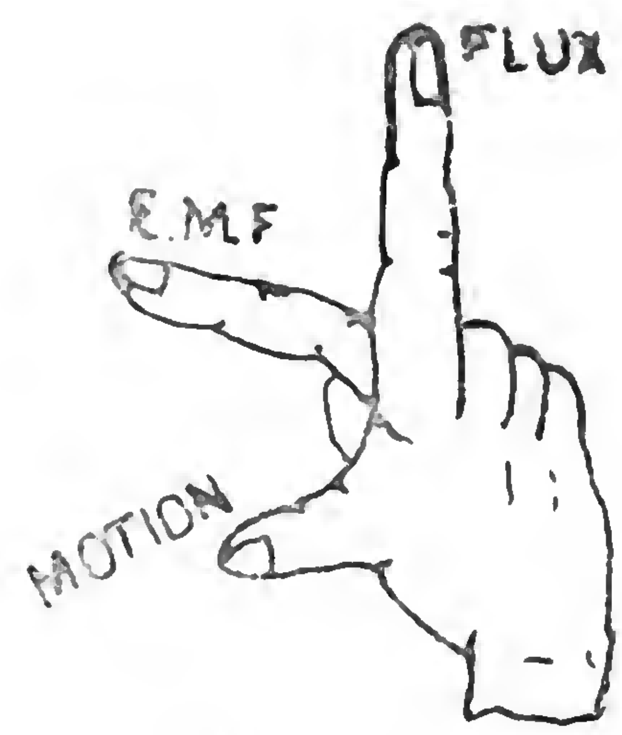


Fig. 18

If B is the flux density in which a conductor of *active length* of l cm. moves with a velocity of v cm/second, the induced voltage generated in the conductor is

$$(\textit{induced e. m. f.})\ e = B.\ l.\ v \text{ abvolts} \quad \dots \quad \dots \quad (23a)$$

B being in lines per sq. cm. (i. e. gauss)

since $l \times v = area$, and $B \times l \times v =$ total *number of lines*, (Φ in maxwells),

$$(\textit{induced e. m. f.})\ e = \textit{lines cut per second.}$$

In practical units

$$e = B.\ l.\ v \times 10^{-8} \text{ volts} \quad \dots \quad \dots \quad \dots \quad (23b)$$

[1 volt = 10^8 abvolts]

Also

$$(\textit{induced e. m. f.})\ e = \frac{d\Phi}{dt} \times 10^{-8} \text{ volts} \quad \dots \quad (24)$$

where $\dfrac{d\Phi}{dt}$ is the rate of change of flux.

Hence "if a conductor cuts one line of force in one second the induced voltage generated in it is one abvolt".

10. Self-Inductance: This is the property of a circuit by virtue of which an e. m. f. is induced in the circuit whenever its current changes.

An air-cored coil is shown in Fig. 19 carrying a current which produces lines of force. These lines form complete loops with the coil. In other words these lines are *linked* with the coil. The product of the number of turns of the coil and the number of lines of force is called *flux linkages*. Hence if the current in the coil changes, the number of lines of force linking the coil will also change. Therefore according to Faraday's Law an

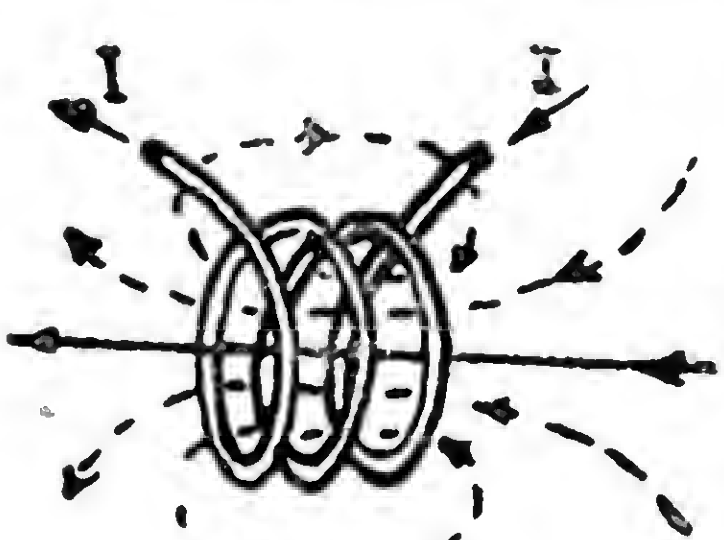


Fig. 19

e. m. f. will be induced in the coil. This is called *self-induced e. m. f.* or better the *e. m. f. of self-induction.*

Further, if the current increases, this e. m. f. will oppose the increase in current, i. e. it will oppose the applied voltage across the coil. And if the current decreases, the induced e. m. f. will tend to maintain the original value of current, i. e. it will act in the same direction as that of the applied voltage according to Lenz's Law. The e. m. f. of self-induction takes place only at instants when there is rate of change of current. Once the current is steady there is no rate of change of flux $\left(\dfrac{d\Phi}{dt}\right)$ and therefore no e. m. f. of self-induction.

$$\therefore \quad e.\,m.\,f.\text{ of self-induction}, \quad e = T\frac{d\Phi}{dt} \text{ abvolts} \quad \ldots \quad (25)$$

where T = number of turns of coil and Φ = lines in maxwells. See Eq. (24)

multiplying and dividing by I (c. g. s. unit of current)

$$e = \left(\frac{T\Phi}{I}\right)\frac{di}{dt} \text{ abvolts} \quad \ldots \quad \ldots \quad \ldots \quad (26)$$

Now $T\Phi$ = number of flux linkages

$\dfrac{di}{dt}$ = rate of change of current (abamperes per second)

$\dfrac{T\Phi}{I}$ = *flux linkages per unit current* which is constant for a particular coil, calling it L,

$$L = \frac{T\Phi}{I} = \text{coefficient of self-induction.}$$

$$\therefore \quad e = L\frac{di}{dt} \text{ abvolts}$$

$$\text{or} \quad e = L\frac{di}{dt}\, 10^{-8} \text{ volts} \ldots \quad \ldots \quad \ldots \quad \ldots \quad (27)$$

The unit of L in e. m. c. g. s. system is the *abhenry,* and $\left(\dfrac{di}{dt}\right)$ is the rate of change of current in *abamperes per second.*

In practical units

$$e = L\frac{di}{dt} \textit{ volts} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (28)$$

where L is in henrys, $\left(\dfrac{di}{dt}\right)$ = *amperes per second.*

Hence, from Eqs. (27) and (28), 1 henry = 10^9 abhenrys, and

$$L = \frac{T\Phi}{I} \times 10^{-8} \text{ henry} \quad \ldots \quad \ldots \qquad (29\,a)$$

where I is in amperes and Φ in maxwells.

By Eq. (14) the flux in a solenoid is $\Phi = \dfrac{4\pi IT}{10l} \times \mu A$, I being in amperes. Substituting the value of Φ in Eq. (29 *a*)

$$L = \frac{0{\cdot}4\pi T^2}{l} \times \mu A \times 10^{-8} \text{ henrys} \quad \ldots \qquad (29b)$$

l and A being in cm. units.

(i) In the case of air-cored solenoids $\mu = 1$, so that L is constant for a particular solenoid, i. e. inductance is constant though current in the solenoid changes.

(ii) In iron-cored solenoids L changes since μ is changing with the flux density B. If the air-gap reluctance is very much greater than that of the iron path, L is fairly constant for low values of B. The value of L varies for values of B which are beyond the saturation point, i. e. above the "knee" of the magnetisation curve.

Example : A steel ring has a mean diameter of 20 cm. a cross-sectional area of 6 cm^2 and has a 500 turn coil wound on it. The ring requires 8 amp-turns per cm. to produce a flux density of 9 kilo-gauss in the ring. Find (a) the exciting current, (b) the inductance and (c) the permeability.

Solution : Total amp-turns = 8 × mean length in cm.

$$= 8 \times 20\,\pi = 502{\cdot}4.$$

$$\therefore \quad \textit{current} = \frac{\text{amp-turns}}{\text{turns}} = \frac{502{\cdot}4}{500} = 1 \text{ amp (appr.)}$$

Total flux = B × *area* = 9000 × 6 = 54000 maxwells

$$\therefore \quad L = \frac{\Phi\,T}{I} \times 10^{-8} = \frac{54000 \times 500}{1} \times 10^{-8}$$

$$= 0{\cdot}27 \text{ henry}$$

magnetising force $H = 1{\cdot}25 \times$ amp-turns per cm.

$= 1{\cdot}25 \times 8 = 10$ oersteds

$$\therefore \quad \mu = \frac{B}{H} = \frac{9000}{10} = 900.$$

11. Mutual Inductance: Fig. 20 shows two coils A and B wound on one magnetic circuit having no air-gap. Coil A has T_1 and B T_2 turns. When a current I_1 amperes flows in A it produces a flux Φ_1 in coil A. Due to magnetic leakage the flux in B is $K\Phi_1$ where $K < 1$.

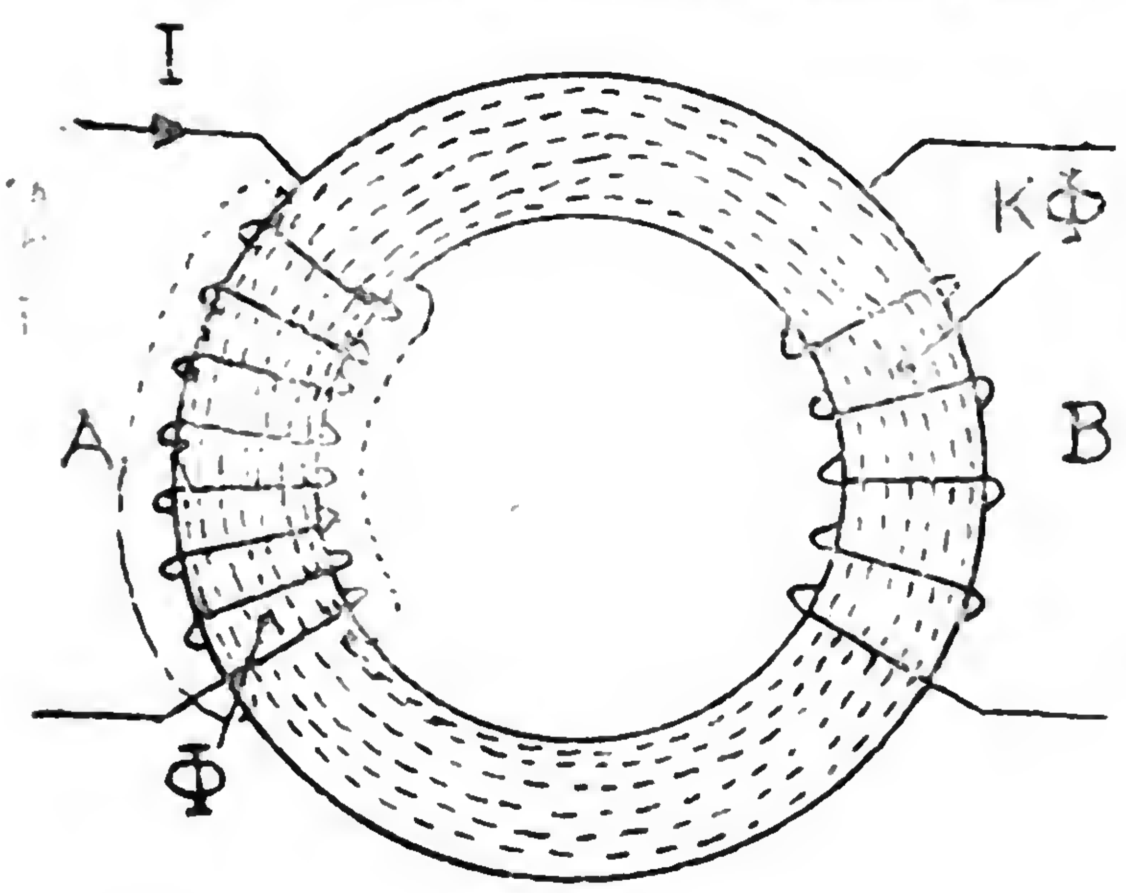


Fig. 20

If there is a rate of change of current in A an e. m. f. is induced not only in A but also in B. Calling the induced e. m. f. in B as e_2

$$e_2 = T_2 \times \left(\begin{array}{c}\textit{flux in B due to}\\ \textit{unit current in A}\end{array}\right) \times \left(\begin{array}{c}\textit{rate of change of}\\ \textit{current in A}\end{array}\right)$$

$$= T_2 \times \frac{K\Phi_1}{I_1} \times \frac{di}{dt}$$

$$\therefore \quad e_2 = M\frac{di}{dt} \quad \cdots \quad \cdots \quad \cdots \quad (30)$$

where $\quad M = K\dfrac{\Phi_1 T_2}{I_1}$

and is called *mutual inductance* and K is the *coefficient of coupling.*

Similarly, if coil B carries a current I_2 when placed across the supply, this current will produce in coil B a flux Φ_2 and the flux that will link with A is $K\Phi_2$. If there is a rate of change of current in B an e. m. f. e_1 will be induced in A such that

$$e_1 = T_1 \times \left(\begin{array}{c}\textit{flux in A due to}\\ \textit{unit current in B}\end{array}\right) \times \left(\begin{array}{c}\textit{rate of change of}\\ \textit{current in B}\end{array}\right)$$

$$= T_1 \times \frac{K\Phi_2}{I_2} \times \frac{di}{dt}$$

$$= M\frac{di}{dt} \quad \cdots \quad \cdots \quad \cdots \quad (31)$$

where $$M = \frac{K\Phi_2 T_1}{I_2}.$$

$$\therefore \quad M^2 = \left(\frac{K\Phi_1 T_2}{I_1}\right)\left(\frac{K\Phi_2 T_1}{I_2}\right)$$

Rearranging

$$M^2 = K^2\left(\frac{\Phi_1 T_1}{I_1}\right)\left(\frac{\Phi_2 T_2}{I_2}\right)$$

The term in the first bracket is L_1 and in the second L_2. Hence

$$M = K\sqrt{(L_1 L_2)} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (32)$$

When K is very near to unity the coupling is said to be *tight* or the circuit is said to be perfectly coupled. When $K = 1$

$$M = \sqrt{(L_1 L_2)} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (33)$$

The unit of mutual inductance is the henry in practical units. In c. g. s. system of units it is abhenry. 1 henry $= 10^9$ abhenrys.

Thus *two circuits are said to possess mutual inductance of one henry, when current changing at the rate of one ampere per second in one circuit induces an e. m. f. of one volt in the other circuit.*

If two coils (having a common magnetic path) are connected in "series aiding" the total inductance is

$$L = L_1 + L_2 + 2M \qquad \ldots \qquad \ldots \qquad \ldots \qquad (34)$$

And when they are connected in "series opposing" the total inductance is

$$L = L_1 + L_2 - 2M \qquad \ldots \qquad \ldots \qquad \ldots \qquad (35)$$

"Series aiding" means the coils are connected in series across supply mains and the flux produced by each coil has the same direction. "Series opposing" means the coils are in series as before, but the flux produced by one is in opposite direction to the flux of the other.

Example: A closed iron circuit of mean length 1 metre and having a cross-sectional area of 12 sq. cm. has two coils wound on it, one of 1000 turns and the other of 100 turns. Assuming $\mu = 900$ and constant over the working range, calculate the mutual inductance between the coils. Take coefficient of coupling as 0·9.

Solution: $A = 12$; $l = 100$ cm.; $\mu = 900$; $T_1 = 1000$; $T_2 = 100$; $K = 0{\cdot}9$.

$$L_1 = \frac{0{\cdot}4\pi\, T_1^2}{l} \times \mu A \times 10^{-8} H = 1{\cdot}356\, H$$

$$L_2 = \frac{0{\cdot}4\pi\, T_2^2}{l} \times \mu A \times 10^{-8} H = \frac{1{\cdot}356}{100} H.$$

$$M = K \sqrt{(L_1 L_2)}$$

$$= 0{\cdot}9 \sqrt{\left(1{\cdot}356 \times \frac{1{\cdot}356}{100}\right)} = 0{\cdot}9 \times \frac{1{\cdot}356}{10} H$$

$$= 0{\cdot}122\ H.$$

Example: Two coils, when connected in "series aiding", have a total inductance of 860 *mH* and when connected in "series opposing" the total inductance is 140 *mH*. One coil is known to have 4 times the inductance of the other. Calculate (a) the mutual inductance, (b) inductance of each coil and (c) the coefficient of coupling.

Solution; In series aiding

$$L = L_1 + L_2 + 2M = 860 \quad \ldots \quad \text{(i)}$$

In series opposing

$$L = L_1 + L_2 - 2M = 140 \quad \ldots \quad \text{(ii)}$$

Let $L_2 = 4L_1$

$\therefore \quad 5L_1 + 2M = 860$

$\quad\quad 5L_1 - 2M = 140$

$\therefore \quad M = 180\ mH$ and $L_1 = 100\ mH \quad \therefore \quad L_2 = 400\ mH.$

Since $M = K\sqrt{(L_1 L_2)}$

$$\therefore \quad K = \frac{M}{\sqrt{(400 \times 100)}} = \frac{180}{200} = 0{\cdot}9.$$

12. Rise end Decay of Current in Inductive Circuits:

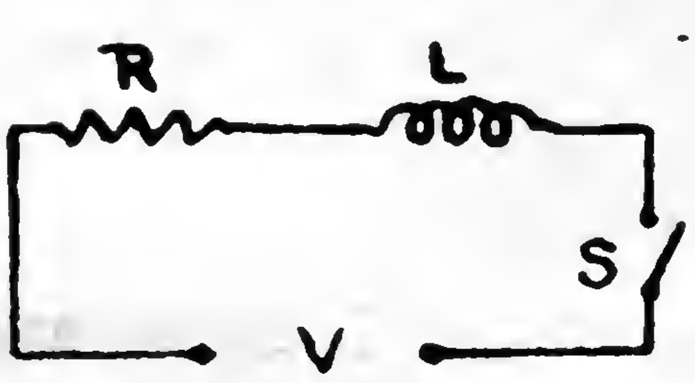


Fig. 21

When inductance is present in circuits the current in the circuit does not attain its maximum final value immediately. This is due to induced e. m. f. in the inductive part of the circuit. So that the applied voltage has to evercome the iR drop and the induced e. m. f. e which opposes the applied voltage, i. e.

$$V = iR + e$$

$$V = iR + L\frac{di}{dt} \qquad \cdots \quad \cdots \quad \cdots \qquad (36)$$

On closing the switch S of Fig. 21, $i = 0$, therefore $iR = 0$ hence

$$V = L\frac{di}{dt}$$

i. e. $$\frac{di}{dt} = \frac{V}{L} \qquad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \qquad (37)$$

This is the initial rate of change of current.

Solving the differential equation (36) we get

$$i = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$i = I\left(1 - e^{-\frac{R}{L}t}\right) \qquad \cdots \quad \cdots \quad \cdots \qquad (38)$$

where i is the value of current at any time t, I is the final steady current, R is the resistance of the circuit, L the inductance and e is the base of Napierian logarithms.

The graph of rise of current as given by Eq. (38) is shown in

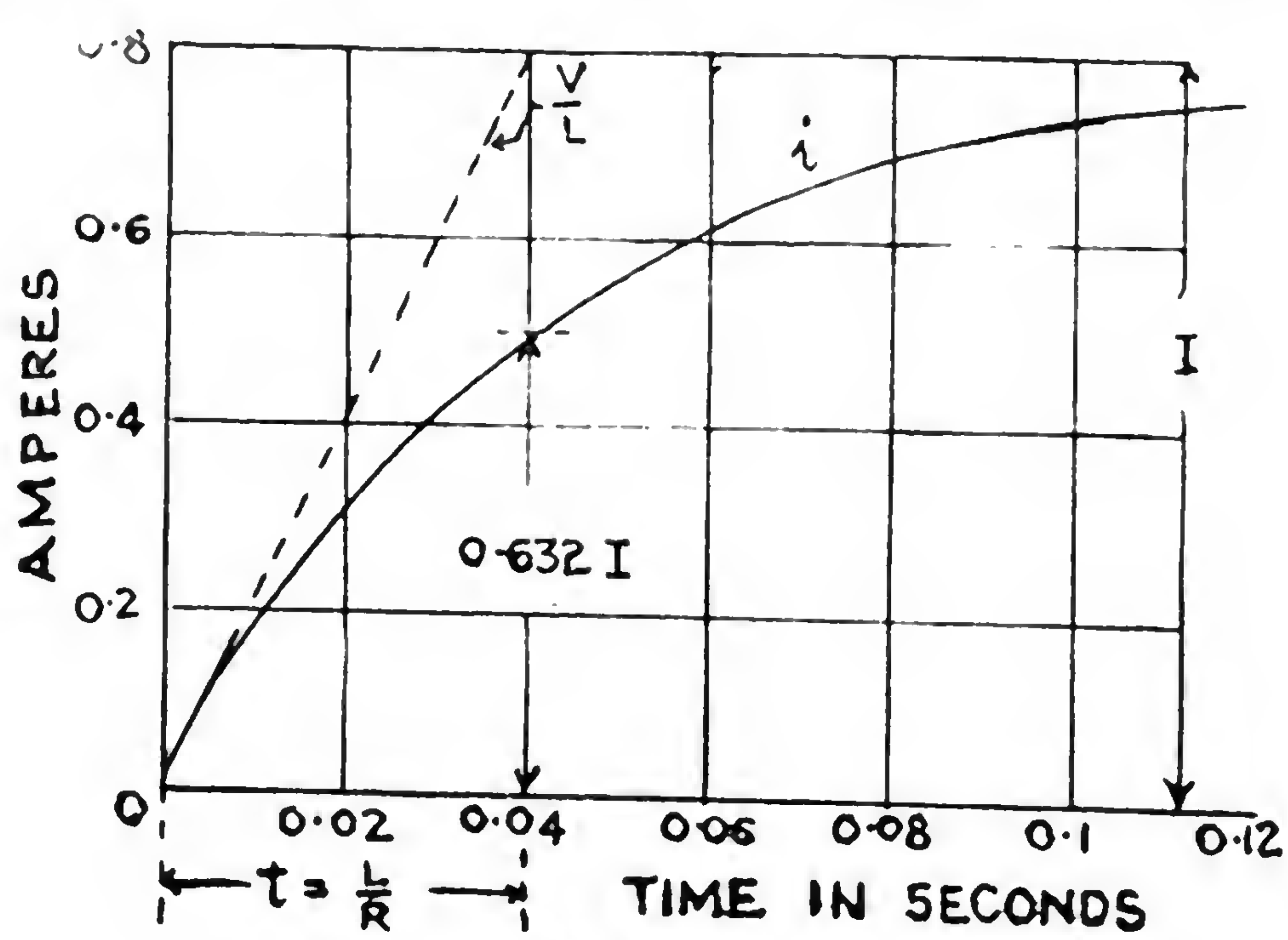


Fig. 22

Fig. 22. The fraction $\frac{L}{R}$ is called the *time constant* of the circuit and its unit is the second. For if the current continued to rise at its initial rate $\frac{V}{L}$, the final value of I is reached in $I\left(\frac{L}{V}\right)$ seconds

$$I\left(\frac{L}{V}\right) = I \times \frac{L}{IR} = \frac{L}{R} \text{ seconds.}$$

But actually in $\frac{L}{R}$ seconds i. e. when $t = \frac{L}{R}$

$$i = I\left(1 - e^{-\frac{R}{L}\times\frac{L}{R}}\right) = I\,(1 - e^{-1}) = I\left(1 - \frac{1}{2{\cdot}718}\right)$$

$$= 0{\cdot}632\ I.$$

Therefore time constant is the time taken by the current to rise to 63·2 % of its final value.

Similarly, a steady current I, when the circuit is opened, does not die immediately but takes some time to reach zero value. Since the applied voltage is zero, Eq. (36) becomes

$$0 = i\,R + L\,\frac{di}{dt} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (39)$$

$$\therefore \quad \frac{di}{dt} = -\,\frac{IR}{L} = -\,\frac{V}{L}$$

$-\,\frac{V}{L}$ is the initial rate of decrease of current.

Solving the differential equation (39)

$$i = I\,e^{-\frac{R}{L}t} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (40)$$

Fig. 23 shows the decay of current, curve II, along with the graph for the rise of current, curve I.

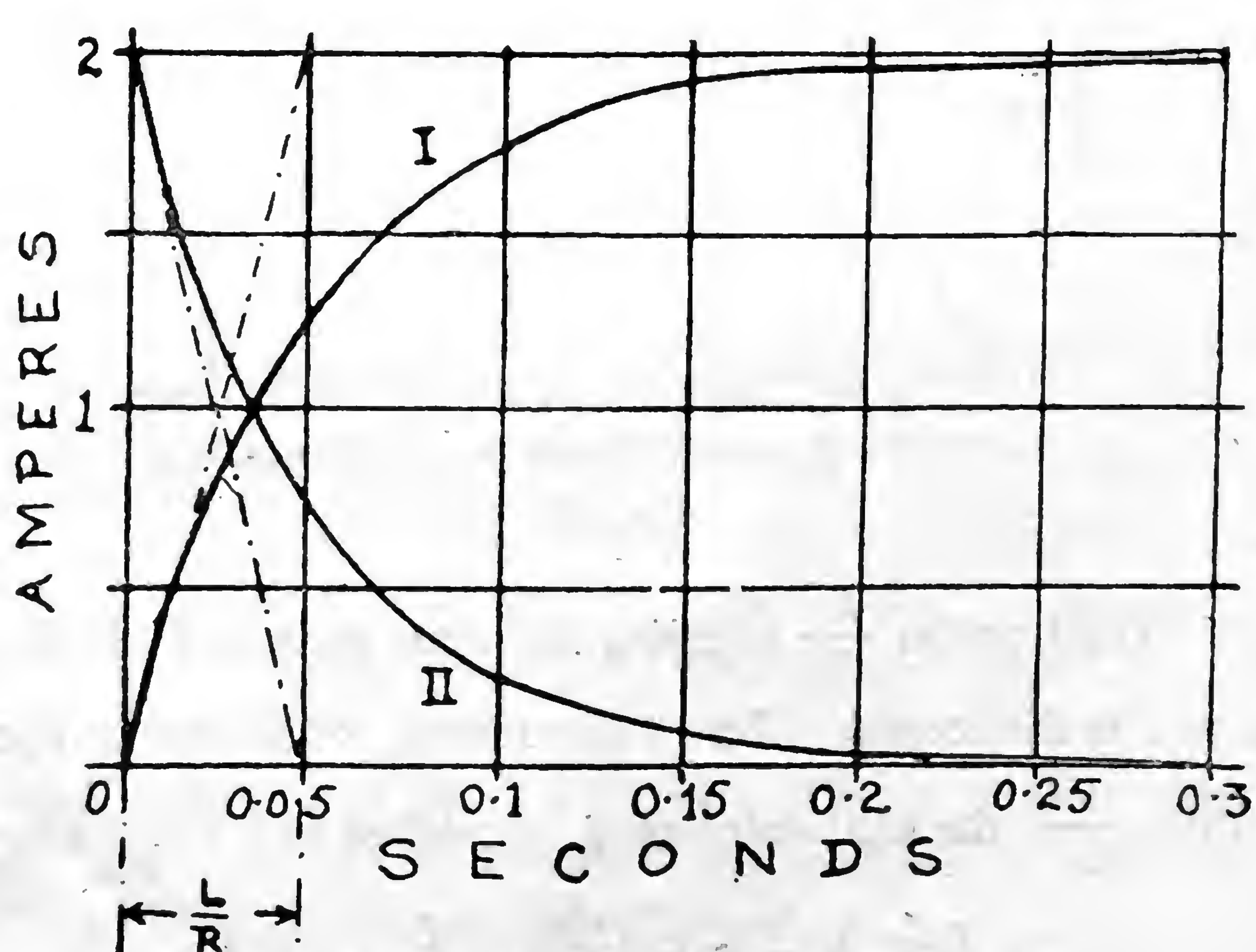


Fig. 23

When $t = \dfrac{L}{R}$, Eq. (40) becomes

$$i = I e^{-\frac{R}{L} \times \frac{L}{R}} = I e^{-1} = \frac{I}{e} = \frac{I}{2{\cdot}718} = 0{\cdot}368\ I.$$

Thus in this case in time $\dfrac{L}{R}$ the current value is 36·8 % of its initial value. The graphs of figs. 22 and 23 are for a circuit where the applied voltage $V = 100$ volts, $R = 50$ ohms and $L = 2{\cdot}5$ henrys.

Example: A shunt field winding has a resistance of 100 ohms and an inductance of 4 henrys. It is connected suddenly across 200-volt supply. Calculate

(a) the final value of current,

(b) the initial rate of change of current,

(c) the time constant of the circuit,

(d) time taken for the current to reach 80 % of its final value

and (e) the energy stored in the magnetic field.

Solution: (a) $I = \dfrac{V}{R} = \dfrac{200}{100} = 2A.$

(b) $\dfrac{V}{L} = \dfrac{200}{4} = 50$ *Amperes per second.*

(c) Time constant $= \dfrac{L}{R} = \dfrac{4}{100} = 0{\cdot}04$ second.

(d) Using $i = I\,(1 - e^{(-R/L)\,t})$

$$0{\cdot}8 I = I\,(1 - e^{-\frac{100}{4}t})$$

$$e^{-25t} = 0{\cdot}2.$$

Taking logs of both sides of equation

$$-\ 25t \log 2{\cdot}718 = \log 0{\cdot}2$$

$$-\ 25\,t \times 0{\cdot}4343 = \bar{1}{\cdot}301$$

$$25 \times 0{\cdot}4343\,t = 0{\cdot}699$$

$$\therefore\quad t = 0{\cdot}064 \text{ second.}$$

(e) Energy stored $= \frac{1}{2} L I^2$ joules

$$= \tfrac{1}{2} \times 4 \times 2^2 = 8 \text{ joules.}$$

13. Energy Stored in Magnetic Field: Writing Eq. (36) and multiplying both side by idt we get

$$v\,idt = i^2 R\,dt + Li\frac{di}{dt}\,dt$$

This is an energy equation where

$vi\,dt$ = energy supplied by the source in time dt

$i^2 R\,dt$ = energy lost in resistance R.

Hence $Li\,di$ must be the energy expended in establishing the magnetic field. Hence the total energy is

$$W = \int_0^I Li \cdot di$$

energy stored, $W = \frac{1}{2} L I^2$ (joules or ergs) ... ... (41)

when I is in amperes, L is in henrys W is in joules, and when I is in abamperes and L is in abhenrys (i. e. in c. g. s. units) W is in ergs, which is the unit of work in the c. g. s. system.

From Eq. (41) the energy stored in magnetic field can be expressed in terms of volume of field and the flux density as shown below.

By definition, $L = \frac{T\Phi}{I}$ (see Section 10), hence substituting the value of L in Eq. (41)

$$W = \frac{1}{2}\left(\frac{T\Phi}{I}\right) I^2 \text{ ergs} = \frac{1}{2}\,\Phi\, IT \text{ ergs.}$$

and substituting the value of IT from Eq. (14), $\left[IT = \frac{\Phi \cdot l}{4\pi\mu A}\right]$

$$W = \frac{\Phi^2}{8\pi} \times \frac{l}{\mu A} \text{ ergs.}$$

and since $\Phi = B \times A$, where A is the area of each pole face

$$W = \frac{B^2}{8\,\pi\,\mu} \times Al \text{ ergs} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (42)$$

Hence energy stored per unit volume of magnetic field,

$$W = \frac{B^2}{8\,\pi\,\mu} \text{ ergs per cc.} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (43)$$

since $A \times l$=volume of magnetic field, l being the distance between the magnetised surfaces. All quantities are in c. g. s. units. For air $\mu = 1$.

14. Tractive Force between Magnetised Surfaces: Fig. 24 shows an electro-magnet and its armature. Let

A = area in sq. cm. of each pole-face

B = flux density in gauss in the space between the pole-face and armature.

l = distance in cm. between a pole-face and armature.

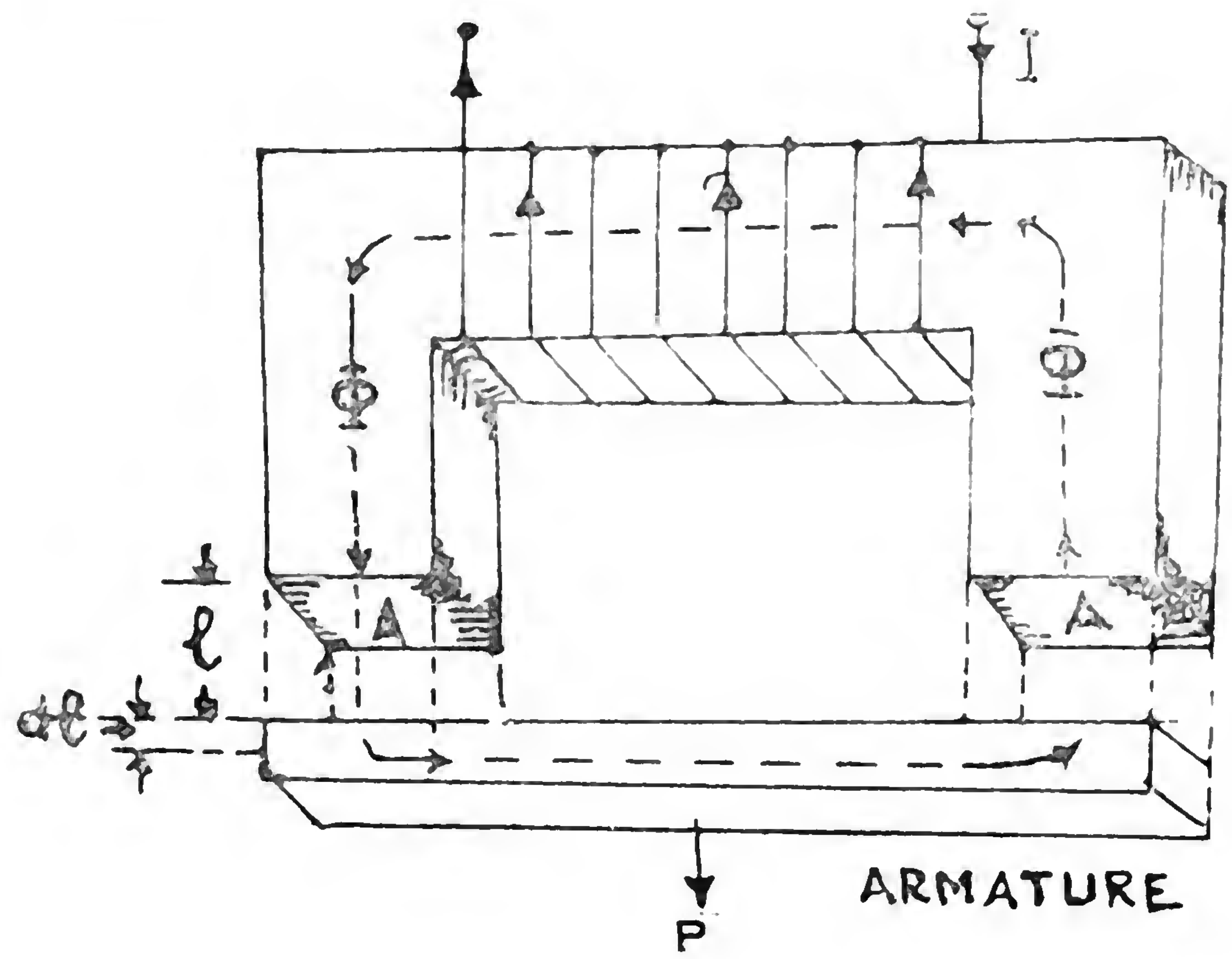


Fig. 24

The armature experiences a force of attraction at *two places*, i. e. at each pole-face. If a pull of P dynes moves the armature a small distance dl cm. away from the magnet and assuming there is no increase in current,

$$Work\ done = P \times dl \text{ ergs.}$$

This is equivalent to the extra energy stored in the magnetic field.

$$P \times dl = 2 \times \frac{B^2 A}{8\,\pi\,\mu} \times dl$$

Hence pull required per pole

$$P = \frac{B^2 A}{8\,\pi\,\mu} \text{ dynes per pole} \qquad \dots \quad \dots \quad \dots \quad (44)$$

For air $\mu = 1$.

If the pull P is expressed in lb.

$$P = \frac{B^2 A}{1118 \times 10^4 \mu} \text{lb.}$$

By transposing the terms of Eq. (44) and solving for B and taking $\mu = 1$ for air

$$B = 5\sqrt{\frac{P}{A}} \text{ gauss} \quad \dots \quad \dots \quad (45)$$

where P is in dynes and A is the area in sq. cm. of one pole-face.

Example: Estimate (a) the braking force in tons and (b) the rate of retardation in miles per hour per second in the case of a 10-ton tram-car having two magnetic track brakes with a contact area of 3·2 sq. inches per pole-face. The magnetic flux per track brake is 0·3 megalines and the coefficient of friction between the brake pole-face and the track is 0·18. Take $g = 32{\cdot}2$.

Solution: (a) Force of brakes acts vertically downwards.

$$\textit{force} = \frac{B^2 A}{8\pi\mu} \text{ dynes per brake.} \quad \left[B^2 = \frac{\Phi^2}{A^2}\right.$$

$$B = \frac{300000}{3{\cdot}2\,(2{\cdot}54)^2} \text{ gauss and } \mu = 1 \text{ for air.}$$

$$\textit{force} = \frac{(300000)^2}{(3{\cdot}2)^2 \times (2{\cdot}54)^4} \times \frac{3{\cdot}2 \times (2{\cdot}54)^2}{8\pi}$$

$$= 1{\cdot}735 \times 10^8 \text{ dynes}$$

$$\textit{force} = \frac{1{\cdot}735 \times 10^8}{981 \times 453{\cdot}6 \times 2240} = 0{\cdot}174 \text{ ton.}$$

Force due to both the brakes and taking coeff. of friction into account *braking force* $= 2 \times 0{\cdot}174 \times 0{\cdot}18 =$ *0·0626 ton.*

(b) Now braking force $= \dfrac{\text{mass} \times \text{retardation}}{g}$

mass = 10 tons and g is 32·2 ft./sec.^{2}

$$\therefore \quad \text{retardation} = \frac{0{\cdot}0626 \times 32{\cdot}2}{10} \text{ ft./sec}^2 = 0{\cdot}2016 \text{ ft./sec.}^2$$

$$= \frac{0{\cdot}2016 \times 3600}{5280} = \textit{0·137 m. p. h./sec.}$$

15. Permanent Magnet Materials: (1) Carbon tool-steel was the only material used for a number of years as a permanent magnet material. Then came (2) 5% to 6% tungsten steel, containing also very small amounts of carbon (0·8%) and manganese (0·5%).

Next was (3) cobalt-cromium steel (6%) followed by (4) 36% cobalt steel and (5) 42% cobalt steel. In recent years (since 1931) the development of Alnico alloys marked a radical improvement in permanent magnet materials.

Magnetic materials should possess *high retentivity*, *high stored energy* and *should require great coercive force.* Fig. 25 shows the demagnetisation curves of some of these materials. They lie in the second quadrant and are portions of hysteresis loops. This is the operating region of permanent magnets. The graphs clearly show the progress made by science in developing high quality materials for permanent magnets.

In Fig. 25 the materials represented are

curve I—cabon tool steel

II—5% tungsten steel

III—42% cobalt tool steel

VI—"Alnico" 6.

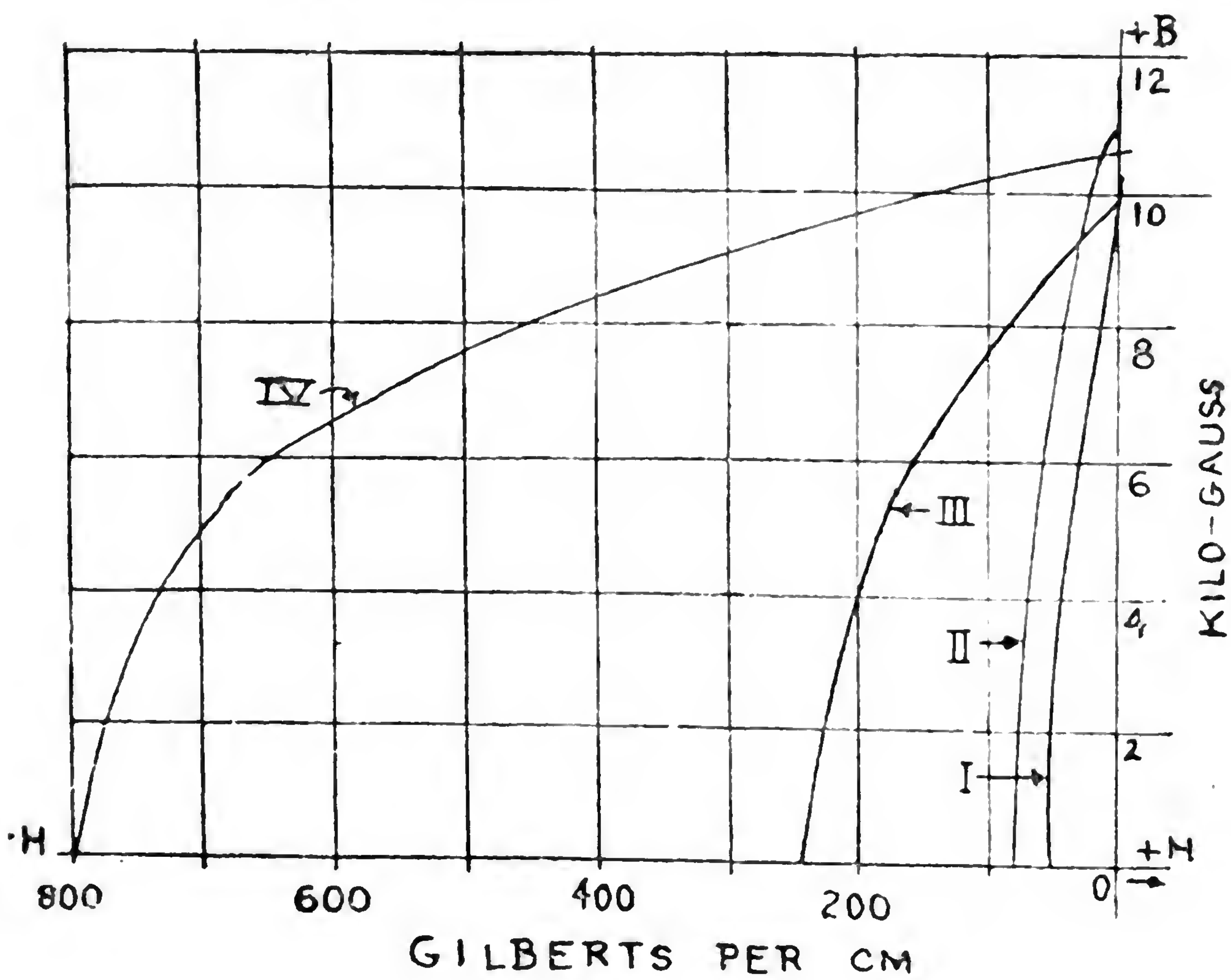


Fig. 25

CHAPTER III

THE DIELECTRIC CIRCUIT

1. The Electric Charge: Electric charge on a body is defined as either a surplus or a deficit of free electrons. When a surface of a conducting material is "charged", the charge (i. e. electrons) spreads all over the surface, but not uniformly except when the surface is a sphere. When the surface is not a sphere the density of charge (i. e. the surface density) is greater where the radius of curvature is smaller. Hence in practice tapering shapes should be avoided. In the case of insulating materials the charge does not spread at all but remains near the spot where it is produced.

The charge on an electron or proton is the ***natural unit*** and is equivalent to $1{\cdot}603 \times 10^{-19}$ coulomb. Since the ***coulomb*** is the unit of electric charge in the practical (m. k. s.) system of units and since we use the c. g. s. electrostatic (e. s. u.) system of units also, it is better to know that the unit of charge in the e. s. u. is the *statcoulomb*.

$$1 \; coulomb = 3 \times 10^{9} \text{ statcoulombs.}$$

2. Unit Charge: Coulomb's Law states that the force between two point charges, Q, and q, separated by a distance r in vacuum (or air) is given by

$$force = \frac{Q \times q}{\epsilon \, r^2} \text{ dynes} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (1)$$

where r is the distance in cm. and the greek letter ϵ is the proportionality factor and designates the ***permittivity*** of the medium. In the e. s. system of units the permittivity of vacuum or air is 1. If unit values are assigned to Q, q and r, a statcoulomb is defined as "a unit charge of electricity which will experience a force of one dyne when placed one centimetre away from another unit charge in vacuum or air".

The ratio $\dfrac{\textit{capacitance with substance as dielectric}}{\textit{capacitance with air as dielectric}}$ is called ***permittivity*** or ***dielectric constant*** (ϵ).

3. The Dielectric Field : Since one charged body exerts a force on another charged body, the intervening medium is presumed to be under stress. This is due to the presence of dielectric field in the medium. Further, it indicates storage of energy in the medium. The nature of this stored energy is static.

Like the magnetic field, the dielectric field is represented by lines of force. Their sence of direction is from a positively to a negatively charged surface. These lines do not touch or cross each other. The dielectric flux from a charge of Q units is equal to $4\pi Q$ lines and the symbol used is the greek letter ψ.

4. Field Intensity : Unit field intensity at a point in the field is said to exist when a unit charge (i. e. a statcoulomb) placed at the point experiences a force of one dyne. The symbol used for field intensity is $\mathcal{E}$.

$$\mathcal{E} = \frac{f}{Q} \text{ dynes per statcoulomb}$$

$\mathcal{E}$ is a vector quantity having magnitude and direction and so $\mathcal{E}$ is also called *electric force*. It is analogous to the quantity H of the magnetic circuit.

If a unit charge q is moved from a lower potential point (V_2) to a higher potential point (V_1) the work done is

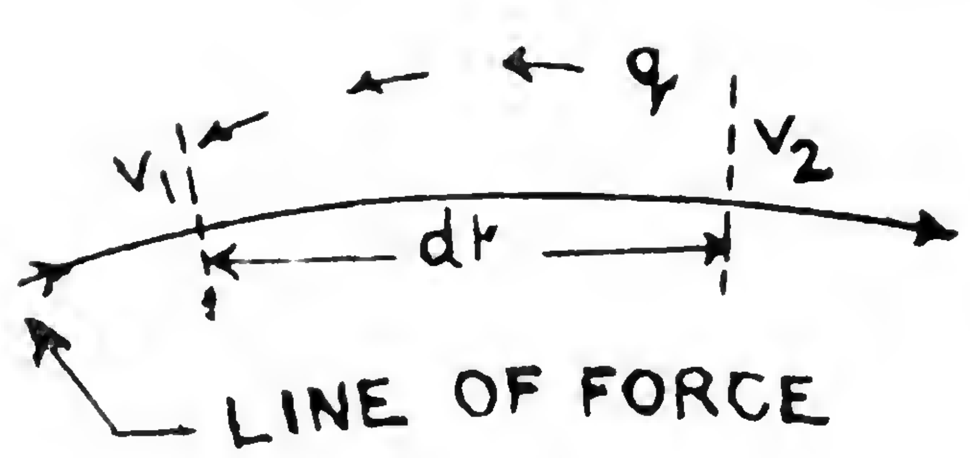


Fig. 1

$$dW = q\,\mathcal{E}\,dr$$

$$W = q\int \mathcal{E}\,dr \text{ ergs} \quad \ldots \quad \text{(i)}$$

If the potential difference between the two points is $-V$,

i. e. $(V_2 - V_1) = - V$, the work done in moving q is

$$W = - qV \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \text{(ii)}$$

Equating (i) with (ii),

$$q\int \mathcal{E}\,dr = - qV$$

$$\therefore \quad \mathcal{E} = - \frac{dV}{dr} \text{ statvolts per cm.} \quad \ldots \quad \ldots \quad \ldots (2)$$

Thus Eq. (2) shows that $\mathcal{E}$ is also potential gradient. Hence "electric intensity at a point in dielectric field is equal to the potential gradient at that point."

The value of potential gradient at which a dielectric "breaks down" is called the *dielectric strength* of the substance.

5. **The Dielectric Flux Density:** Symbol used is D and is defined as the number of dielectric lines per sq. cm. the area being at right angles to the lines

$$D = \frac{\psi}{A}$$

where ψ is the total dielectric flux in area of A sq. cm.

Analogous to Eq. (8) Chapter I, we write

$$\psi = \epsilon\, gA = \epsilon\, \mathcal{E}\, A \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

since $\mathcal{E} = g$. Therefore

$$\frac{\psi}{A} = \epsilon\, \mathcal{E}$$

$$D = \epsilon\, \mathcal{E} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

Eq. (4) is very similar to $B = \mu H$ of the magnetic circuit.

Further, when a total charge of Q units resides on an area of A sq. cm, the density of charge (σ) on the surface is

$$\sigma = \frac{Q}{A}.$$

But since $4\pi Q$ lines are given out by a charge Q

$$D = \frac{4\pi Q}{A} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

Example: A metal sphere of 4 inch radius has a surface density of 7 electrostatic units per sq. cm. Calculate the potential gradient at the surface of the sphere in volts per cm.

Solution: The area of the surface of the sphere

$$A = 4\pi(4 \times 2{\cdot}54)^2 \text{ sq. cm.}$$

The total charge on the surface

$$Q = 7 \times A \text{ statcoulombs, and}$$

$$\psi = 4\pi Q = 4\pi \times 7A$$

$$D = \psi/A = 4\pi \times 7 = 88 \text{ lines/cm}^2$$

The potential gradient at the surface is $g = \mathcal{E}$, and

$$\mathcal{E} = D/\epsilon$$

For air $\epsilon = 1$, therefore

$$g = D = 88 \text{ statvolts per cm.}$$

$$1 \text{ statvolt} = 300 \text{ volts, hence } g = 88 \times 300 = 26400 \ V/cm.$$

6. **Capacitance**: Where a large quantity of electric charge can be stored in a small volume of space between two conducting surfaces, separated by a dielectric, the arrangement is a *capacitor* or *condenser*. And the ability of a capacitor to store a large charge is called its capacitance.

Fig. 2 shows two parallel metal plates A and B and separated by a very small distance. Before the switch SW is closed no charge exists on either plate and no potential difference between the plates.

On closing the switch there is a momentory flow of electrons from plate A to plate B in the direction shown by the arrows. This direction of flow of electrons is opposite to the conventional direction of current. This transient flow or electrons is the charging current causing plate A charged positively and B negatively. As the charges on the plates increase, potential difference between the two plates rises from zero while the charging current goes on reducing and eventually becomes zero when the p. d. between the plates becomes equal and opposing the applied voltage. See Fig. 3. When the applied voltage is removed the plates stay charged and the voltage or p. d. across the plates is V volts.

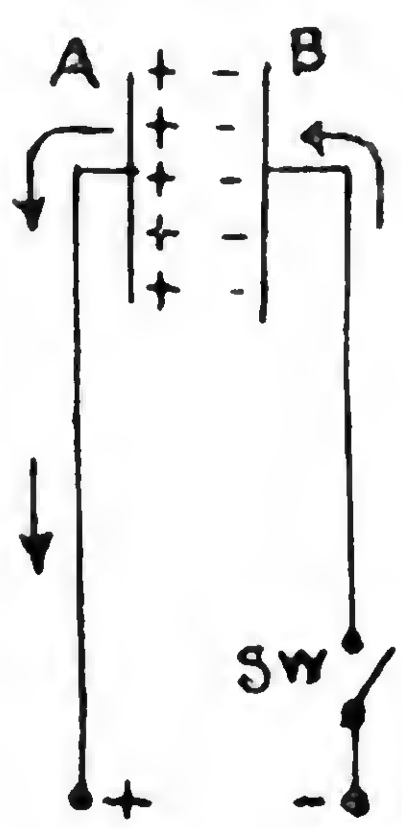


Fig. 2

If the charge on each plate is Q units then

$$Q = CV \quad \ldots \quad \ldots \quad (6)$$

$$\text{or} \quad C = \frac{Q}{V} \quad \ldots \quad \ldots \quad (7)$$

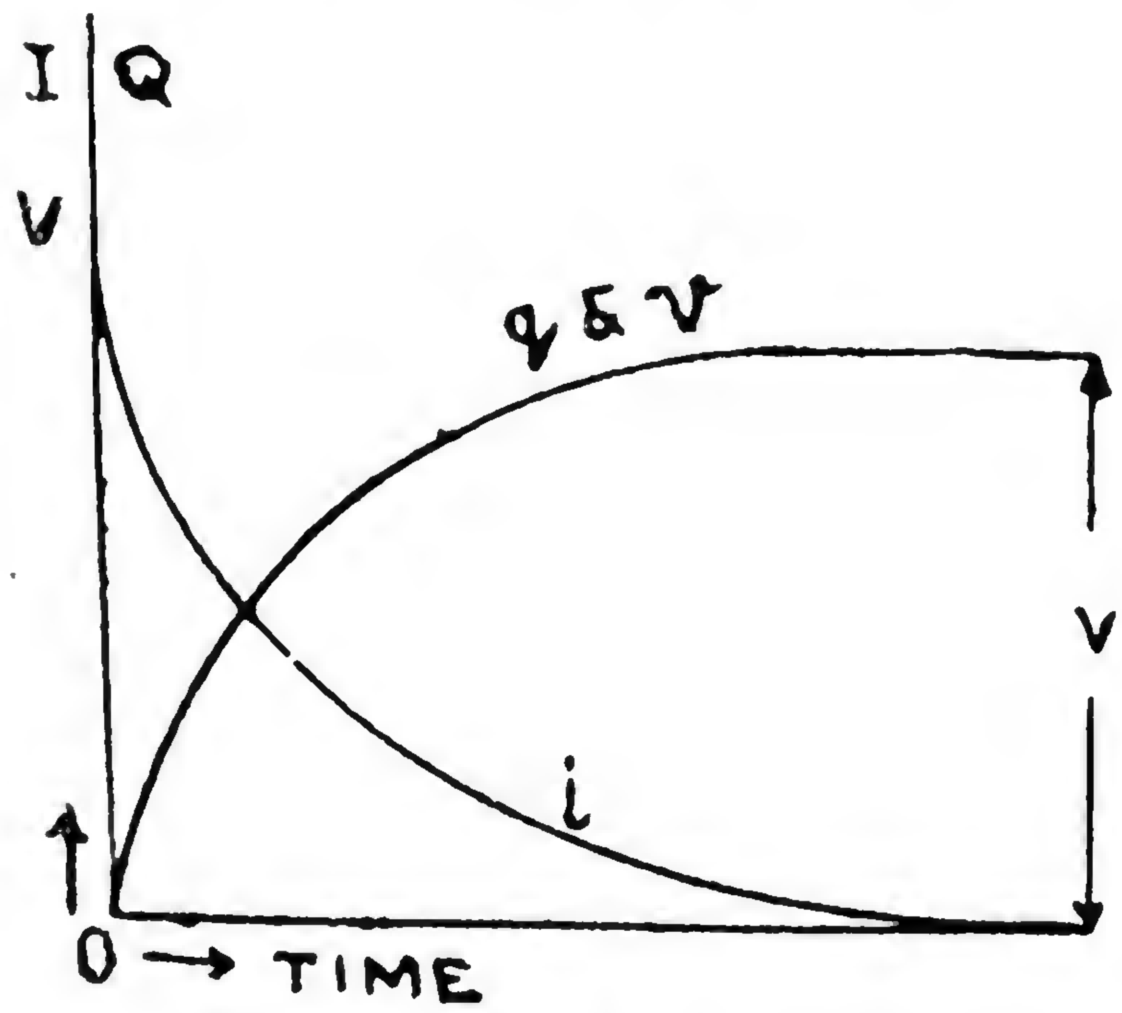


Fig. 3

where C is capacitance, Q charge on each plate and V the p. d. between the plates. The units of these quantities may be either in practical (m. k. s.) or e. s. units, i. e.

(a) C in farads, Q in coulombs and V in volts;

(b) C is statfarads, Q in statcoulombs and V in statvolts.

Now 1 coulomb $= 3 \times 10^9$ statcoulombs and 1 volt $= 1/300$ statvolt hence

$$1\ farad = \frac{3 \times 10^9}{1/300} = 9 \times 10^{11} \text{ statfarads.}$$

From Eq. (7) capacitance is a ratio charge to p. d. Hence if a capacitor is given a charge of 1 coulomb and its p. d. rises to 1 volt its capacitance is said to be 1 farad. The farad is too large a unit for practical use, therefore its submultiple the microfarad (μF) is often used

$$1\ microfarad = 10^{-6} \text{ farad.}$$

The reciprocal of capacitance is called *elastance*, symbol used is S and its unit is the *daraf*. Therefore

$$Q = V/S$$

Elastance is proportional to length of path of dielectric circuit and inversely proportional to its cross-sectional area, i. e.

$$S \propto \frac{1}{A}$$

Elastance may be defined as the property of dielectric circuit which opposes a circuit being charged. It is analogous to resistance of the electric circuit and reluctance of magnetic circuit.

7. Energy Stored in Dielectric Field: Let a capacitor of C farads be connected across a voltage of V volts. After a time t the charging current is

$$i = C\frac{dv}{dt} \qquad \dots \qquad \dots \qquad \dots \qquad \dots \qquad (8a)$$

multiplying both sides by $(v \cdot dt)$

$$vi \cdot dt = Cv \cdot dv$$

where (vi) is power and $(vi \cdot dt)$ is energy. Hence

$$total\ energy = C\int_0^V v \cdot dv$$

$$= \tfrac{1}{2} CV^2 \text{ joules} \qquad \dots \qquad \dots \qquad \dots \qquad (8b)$$

8. Calculation of Capacitance: Only in few cases it is possible to calculate accurately capacitance of any arrangement from its dimensions. The reason is that in majority cases

(a) the shape of dielectrics is irregular;

(b) the dimensions are rarely simple geometrical forms and

(c) ψ cannot be confined in a definite place since the range of values of ϵ varies between 1 and 10 only. This is in contrast with the values of μ and ρ.

A. *Capacitance of Two Parallel Plates*: In this case the flux density is assumed to be constant since the distance between the plates is very small. Let

A = area of each plate in sq. cm;

d = distance between the plates in cm;

ϵ = permittivity of the medium;

Q = the electric charge on each plate.

If V is the p. d. across the plates, then the voltage consumed is

$$V = \mathcal{E} \times d$$

But $\mathcal{E} = \dfrac{D}{\epsilon}$; $D = \dfrac{\psi}{A}$ and $\psi = 4\pi Q$, therefore

$$V = \frac{4\pi Q}{\epsilon A} \times d$$

Transposing,

$$\frac{\epsilon A}{4\pi d} = \frac{Q}{V} = C, \text{ the capacitance}$$

$$\therefore \quad C = \frac{\epsilon A}{4\pi d} \text{ statfarads} \quad \dots \quad \dots \quad \dots \quad \dots \quad (9)$$

$$C = \frac{\epsilon A}{4\pi d} \times \frac{1}{9 \times 10^{11}} \text{ farads} \quad \dots \quad \dots \quad \dots \quad (10)$$

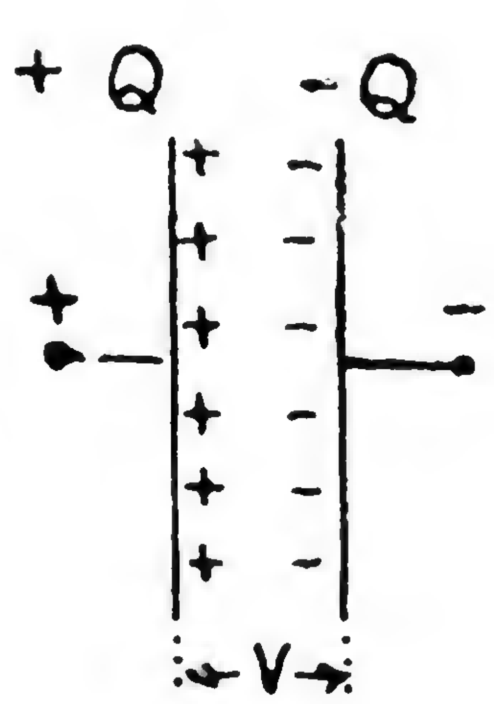


Fig. 4

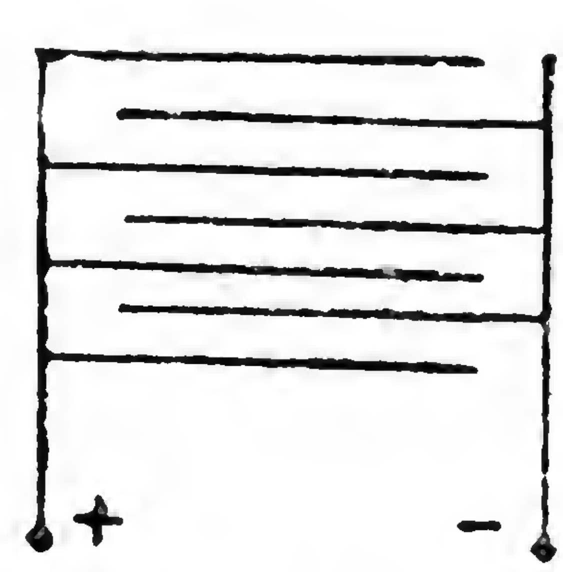


Fig. 5

It is the usual practice to use a number of plates, the total number always is odd; this increases the capacitance. If there are n plates the capacitance is

$$C = \frac{\epsilon A\,(n-1)}{4\,\pi\,d} \text{ statfarads} \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

This arrangement is equivalent to having $(n-1)$ 2-plate condensers in parallel. See Figs. 4 and 5.

B. *Capacitance of a Single-Core Cable:* In this case the flux density is not constant from core to sheath of the cable. Fig. 6 shows a single-core cable having a metal sheath. The radius of the core is r cm. and the thickness of the dielectric is R cm. The permittivity is ϵ and the length of the cable l cm.

If the charge on the core is Q units (e. s. u.), the flux density at the surface of the core is

$$D = \frac{\psi}{A} = \frac{4\,\pi\,Q}{2\,\pi\,r \cdot l}$$

$$= \frac{2Q}{r \cdot l} \text{ lines per sq. cm.}$$

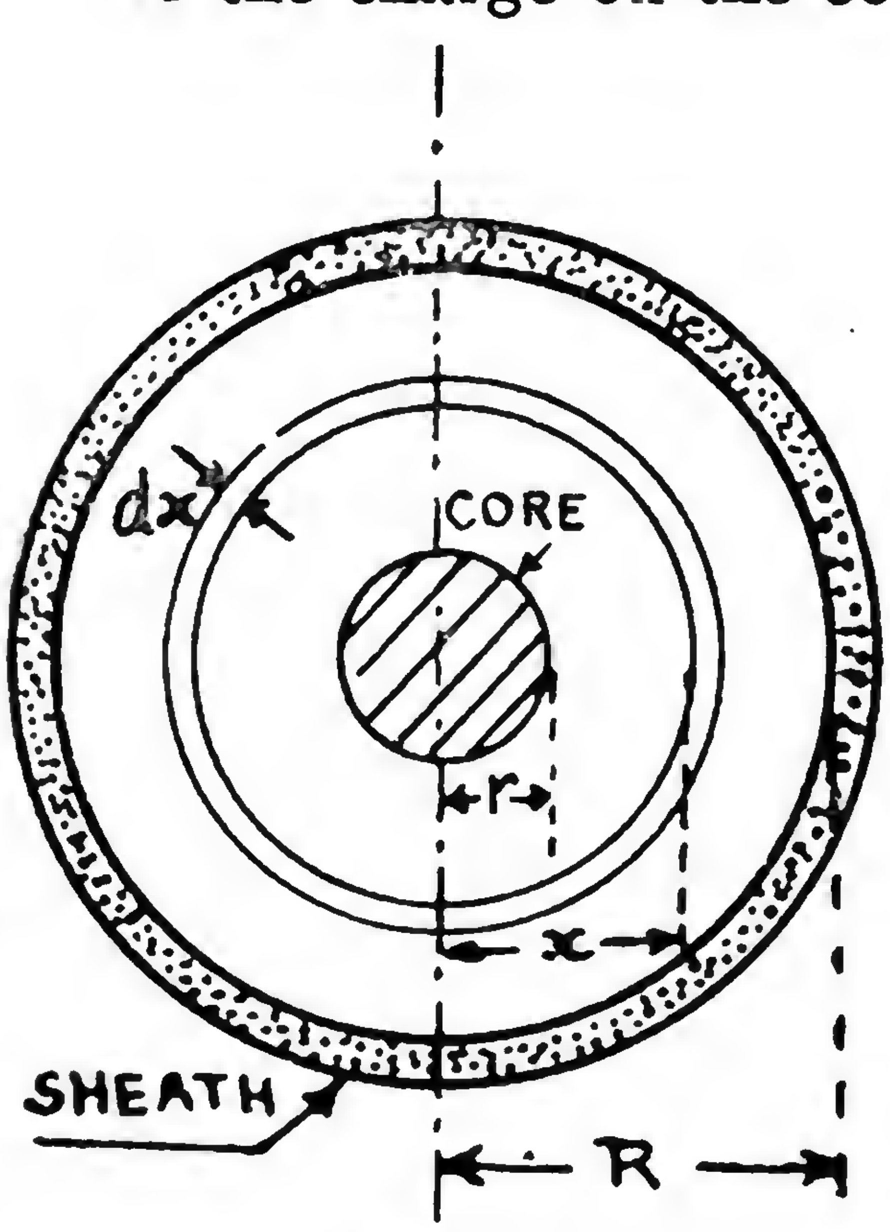


Fig. 6

and at the inner surface of the sheath, since ψ is constant

$$D = \frac{2\,Q}{R \cdot l}$$

and at any point x from the centre of the core

$$D = \frac{2\,Q}{x \cdot l}$$

In carrying a charge of Q units through a distance dx the work done is the measure of the voltage, i. e.

$$dV = \mathcal{E} \times dx$$

$$\text{But} \quad \mathcal{E} = \frac{D}{\epsilon}$$

$$\therefore \quad dV = \frac{2\,Q}{\epsilon\,xl}$$

$$V = \frac{2\,Q}{\epsilon\,l} \int_r^R \frac{dx}{x}$$

$$V = \frac{2\,Q}{\epsilon \cdot l} \log_e \frac{R}{r}$$

$$\frac{Q}{V} = C = \frac{\epsilon \cdot l}{2 \log_e \dfrac{R}{r}} \text{ statfarads} \quad \ldots \quad \ldots \quad \ldots \quad (12)$$

$$C = \frac{\epsilon \cdot l}{2 \log_e \dfrac{R}{r}} \times \frac{1}{9 \times 10^{11}} \text{ farads} \quad \ldots \quad (13)$$

For a mile length of a single-core cable the capacitance, in micro-farads, is

$$C = \frac{0{\cdot}0388\, \epsilon}{\log \dfrac{R}{r}}\ \mu F \text{ per mile} \quad \ldots \quad \ldots \quad (14)$$

C. *Capacitance of a 2-Conductor Overhead Line*: Fig. 7 shows a cross-sectional view of a line of l cm. length. The conductors are run in parallel. Ler r cm. be the radius of each conductor and the distance between their centres d cm. If the charge on A is $+Q$ units (e. s. u.) then the charge on B is $-Q$ units. The electric force $\mathcal{E}$ at any point P distant x cm from the centre of A due to the charge on A is

$$\mathcal{E}_A = \frac{2\,Q}{\epsilon\, x \cdot l}$$

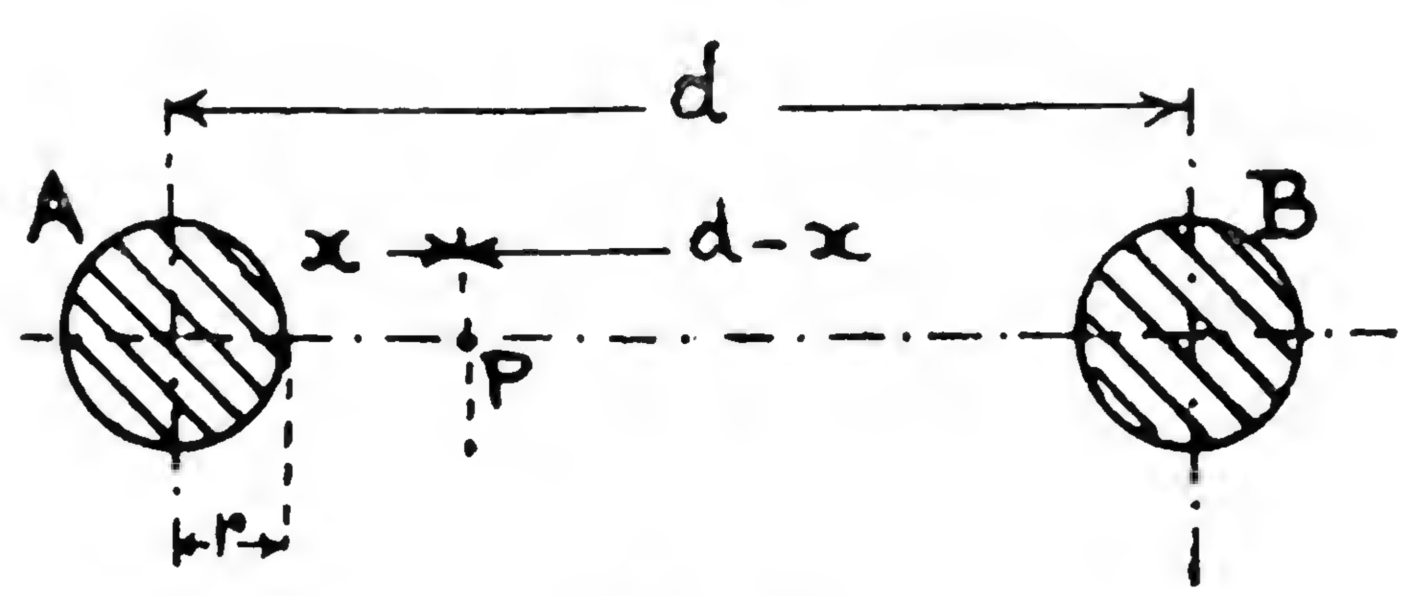


Fig. 7

Similarly, due to the charge on B, the electric force at P is

$$\mathcal{E}_B = \frac{2\,Q}{\epsilon \cdot l\,(d-x)}$$

The direction of both these forces is from A to B.

$$\therefore \quad dV = \mathcal{E} \cdot dx = \frac{2\,Q}{\epsilon \cdot l} \left(\frac{1}{x} + \frac{1}{d-x} \right) dx$$

$$\therefore \quad V = \frac{2Q}{\epsilon \cdot l}\int_{r}^{(d-r)}\left(\frac{1}{x}+\frac{1}{d-x}\right)dx$$

$$= \frac{4Q}{\epsilon \cdot l}\log_e \frac{d-r}{r}$$

But $\dfrac{Q}{V} = C$, capacitance

$$\therefore \quad C = \frac{\epsilon \cdot l}{4 \log_e \dfrac{d-r}{r}} \text{ statfarads} \quad \dots \quad \dots \quad \dots \quad (15)$$

In overhead lines d is a very large compared to r. Hence $\dfrac{d-r}{r}$ is replaced by $\dfrac{d}{r}$. If the line is one mile long

$$C = \frac{0{\cdot}0194}{\log \dfrac{d}{r}} \mu\text{F per mile} \quad \dots \quad \dots \quad \dots \quad (16)$$

9. Capacitors in Series and Parallel: In Fig. 8 there are three capacitors c_1, c_2 and c_3 connected in series. The voltage across the combination is V and across each the voltage is V_1, V_2 and V_3. Let the charge on the +ve plate of c_1 be $+Q$ and on is −ve plate $-Q$. Since the −ve plates are in metallic contact with neighbouring +ve plates the charges on the plates of c_2 and c_3 are also Q units.

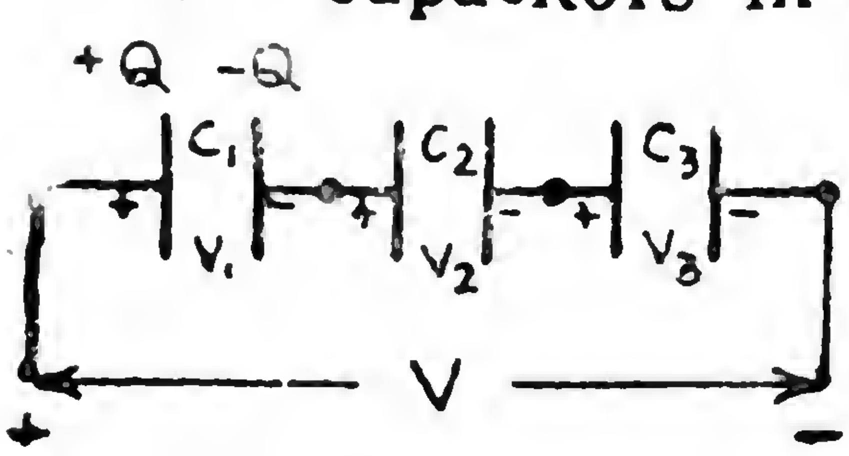


Fig. 8

$$c_1 = \frac{Q}{V_1},\ c_2 = \frac{Q}{V_2} \text{ and } c_3 = \frac{Q}{V};\ \text{also } V = V_1 + V_2 + V_3$$

The total charge is Q and the total voltage is V, therefore total capacitance is $C = \dfrac{Q}{V}$ or

$$\frac{1}{C} = \frac{V}{Q} = \frac{V_1 + V_2 + V_3}{Q} = \frac{V_1}{Q} + \frac{V_2}{Q} + \frac{V_3}{Q}$$

$$\therefore \quad \frac{1}{C} = \frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3} \quad \dots \quad \dots \quad \dots \quad \dots \quad (17)$$

Fig. 9 shows three capacitors connected in parallel and across a voltage V. The charges are different in this case namely Q', Q'' and Q''' the total charge

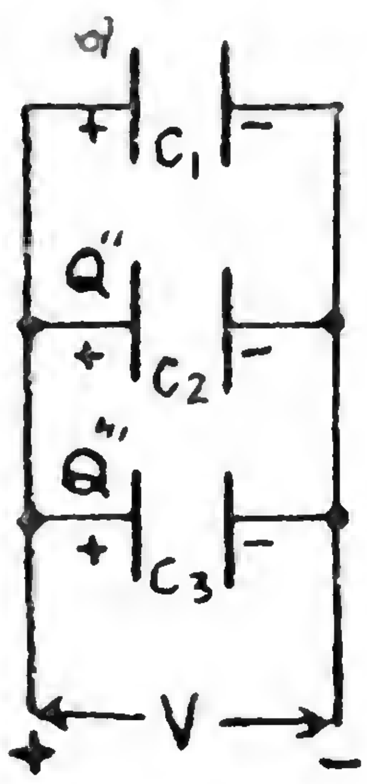


Fig. 9

$$Q = Q' + Q'' + Q'''$$

The total capacitance of the combination is

$$C = \frac{Q}{V} = \frac{Q'}{V} + \frac{Q''}{V} + \frac{Q'''}{V}$$

$$C = c_1 + c_2 + c_3 \quad \ldots \quad \ldots \quad \ldots \quad (18)$$

10. Capacitance with Two Dielectrics in Series: Fig. 10 shows a capacitor having two dielectrics of permittivities $\epsilon_1 = 5$ and $\epsilon_2 = 2$ and thicknesses of d_1 and d_2 respectively. The total capacitance may be found as follows :—

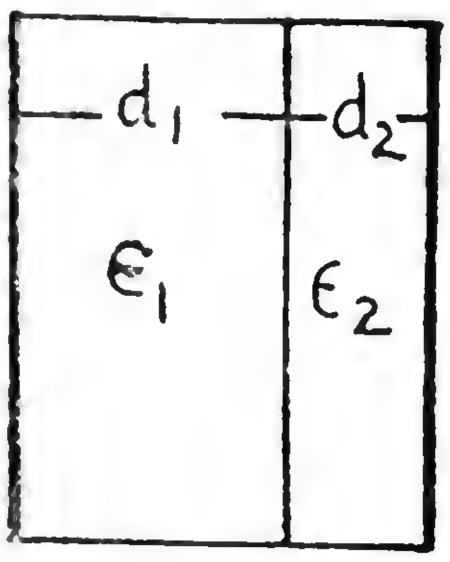


Fig. 10

Let c_1 be the capacitance of x part and c_2 of part y.

$$\therefore \quad c_1 = \frac{\epsilon_1 A}{4\pi d_1} \text{ and } c_2 = \frac{\epsilon_2 A}{4\pi d_2}$$

where A is the area of each plate. Since c_1 and c_2 are in series, the total capacitance c is given by

$$\frac{1}{C} = \frac{1}{c_1} + \frac{1}{c_2}$$

$$\frac{1}{C} = \frac{1}{\frac{\epsilon_1 A}{4\pi d_1}} = \frac{1}{\frac{\epsilon_2 A}{4\pi d_2}} = \frac{4\pi d_1}{\epsilon_1 A} + \frac{4\pi d_2}{\epsilon_2 A}$$

$$= \frac{4\pi}{A}\left(\frac{d_1}{\epsilon_1} + \frac{d_2}{\epsilon_2}\right) \quad \ldots \quad \ldots \quad \ldots \quad (19)$$

It is best to keep the reciprocal form.

Example: A 2-plate capacitor has an insulating sheet of 4 mm thickness and of permittivity 6. By increasing the distance between the sheets by 2 mm and inserting a sheet of that thickness, the capacitance is reduced to 40 % of the original value. Find the permittivity of the second sheet.

Solution: Using Eq. (9), the original capacitance C_1 is

$$C_1 = \frac{\epsilon_1 A}{4\pi d_1} = \frac{6A}{4\pi \times 4} = \frac{3A}{8\pi}$$

where A is the area of each plate. For the second case, using Eq. (19), the total capacitance C_2 is

$$\frac{1}{C_2} = \frac{4\pi}{A}\left(\frac{d_1}{\epsilon_1} + \frac{d_2}{\epsilon_2}\right) = \frac{4\pi}{A}\left(\frac{4}{6} + \frac{2}{\epsilon_2}\right)$$

$$\text{now} \quad \frac{C_1}{C_2} = \frac{100}{40} = \left(\frac{3A}{8\pi}\right) \times \left[\frac{4\pi}{A}\left(\frac{4}{6} + \frac{2}{\epsilon_2}\right)\right]$$

solving for ϵ_2, we get $\epsilon_2 = 2$.

Example: The conductors of a single phase overhead line are spaced 80 cm apart. The radius of each conductor is 5 mm. Find the capacitance per mile of the line.

Solution: Using Eq. (16),

$$C = \frac{0{\cdot}0194}{\log \frac{80}{0{\cdot}5}} = \frac{0{\cdot}0194}{2{\cdot}2041} = 0{\cdot}0088 \ \mu F/mile$$

11. Charging Current of a Capacitor: Fig. 11 shows a capacitor of C farads being charged through a resistance of R ohms in series, the supply voltage being V volts. Let

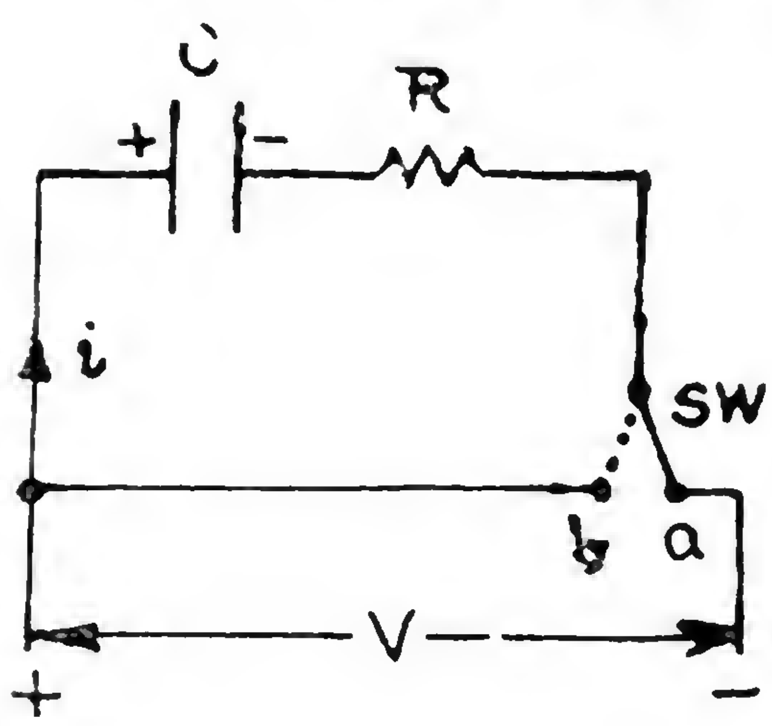


Fig. 11

i = instantaneous value of current

v = " " of p. d. across C.

The applied voltage V is consumed by iR drop and v. Hence

$$V = iR + v \qquad \ldots \quad \ldots \quad \text{(i)}$$

and by Eq. (8a) $i = C\dfrac{dv}{dt}$

$$\therefore \quad V = CR\frac{dv}{dt} + v \qquad \ldots \quad \ldots \quad \ldots \quad \text{(ii)}$$

At the instant of closing the switch as shown in the figure $v = 0$, i. e. at the instant of starting

$$V = CR\frac{dv}{dt}$$

$$\therefore \quad \frac{dv}{dt} = \frac{V}{CR}$$

$\dfrac{V}{CR}$ is the initial rate of change of voltage across the capacitor.

The solution of the differential equation (ii) is

$$v = V\left(1 - e^{-\frac{t}{CR}}\right) \quad \dots \quad \dots \quad \dots \quad (20)$$

Eq. (20) is the equation for rise of potential across the capacitor during charing period.

Rearranging (i) and solving for i,

$$i = \frac{1}{R}(V - v)$$

Substituting the value of v from Eq. (20)

$$i = \frac{V}{R} e^{-\frac{t}{CR}} \quad \dots \quad \dots \quad \dots \quad \dots \quad (21)$$

and since $\frac{V}{R} = I$, the initial current, Eq. (21) becomes

$$i = I e^{-\frac{t}{CR}} \quad \dots \quad \dots \quad \dots \quad \dots \quad (22)$$

This is the expression of the charging current. CR is the *time constant.* e is the base of Naperian logarithms.

12. Discharge Current of a Capacitor: When the switch is brought on terminal b after charging the capacitor. $V = 0$, and $v = V$

$$0 = iR + v \quad \dots \quad \dots \quad \dots \quad (\text{i})$$

$$= CR\frac{dv}{dt} + V$$

$$\frac{dv}{dt} = -\frac{V}{CR} \quad \therefore \quad v = V e^{-\frac{t}{CR}} \quad \dots \quad \dots \quad (23)$$

This is the initial rate of change of voltage across the capacitor. From (i) above

$$i = -\frac{v}{R}$$

$$i = -\frac{V}{R} e^{-\frac{t}{CR}} \quad \dots \quad \dots \quad \dots \quad (24)$$

or, $$i = -I e^{-\frac{t}{CR}} \quad \dots \quad \dots \quad \dots \quad \dots \quad (25)$$

Fig. 12 shows the two curves of Eqs. (22) and (25).

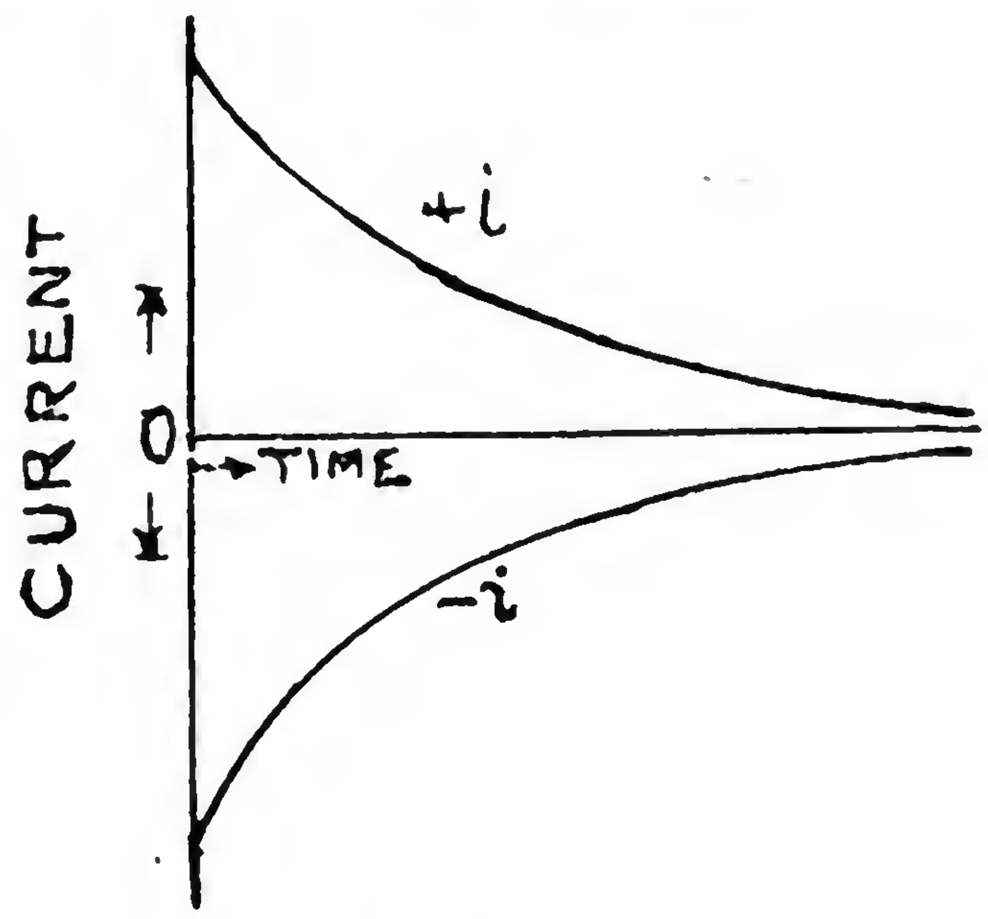


Fig. 12

Example: A 100 μF capacitor in series with 500 ohm resistor is suddenly connected to a 400 V supply mains. Calculate

(*a*) initial value of current;

(*b*) initial rate of change of voltage across the capecitor;

(*c*) time constant of the circuit;

(*d*) value of current when time is equal to time constant;

(*e*) voltage across the capacitor after 0·06 second

(*f*) charge on capacitor after 0·06 second.

Solution: (*a*) Initial current $= \dfrac{V}{R} = \dfrac{400}{500} = 0{\cdot}8\,A.$

(*b*) Initial rate of change of voltage

$$= -\frac{V}{CR} = \frac{400}{500} \times \frac{10^6}{100} = 8000 \text{ volts/second.}$$

(c) Time constant $= CR = \dfrac{100}{10^6} \times 500 = 0{\cdot}05$ sec.

(*d*) $i = Ie^{-\frac{0\cdot05}{0\cdot05}} = I\,e^{-1} = 0{\cdot}8 \times 0{\cdot}368$

$i = 0{\cdot}2944\,A.$

(*e*) $v = V(1 - e^{-\frac{t}{CR}}) = V(1 - e^{-\frac{0\cdot06}{0\cdot05}}).$

$v = 400\,(1 - e^{-1\cdot2}) = 400\,(1 - 0{\cdot}3012)$

$v = 279{\cdot}52$ volts.

(*f*) $Q = CV = \dfrac{100}{10^6} \times 400 = 0{\cdot}04$ coulomb

$\therefore \quad q = Q\,(1 - e^{-1\cdot2}) = 0{\cdot}04 \times (1 - 0{\cdot}3012)$

$q = 0{\cdot}028$ coulomb.

13. Dielectric Materials: As a dielectric, any insulating material should be considered from the point of its behaviour, e. g.

(i) its break-down voltage,
(ii) losses caused by dielectric stress,
(iii) dielectric flux density, also
(iv) its safe working temperature.

For example, thin rubber sheet withstands a dielectric stress of 175000 volts/cm. while air withstands 30000 volts/cm. Hence rubber is a better dielectric material than air. Again, the leakage current of rubber is higher than that of air. Hence air is a better insulating substance than rubber.

The following Table gives particulars of few substances. For more detailed information see any Standard Hand book for Electrical Engineers.

PROPERTIES OF DIELECTRICS

Substance	Permittivity (approximate)	Dielectric strength kV/cm	Safe temp. °C.
Mica	7	1750	500
Transformer oil	2·2	130	70
Bakelite	5	very low (5)	200
Paraffin wax	2·2	75	45
Manila paper	2	25	90
Varnished paper	2	175	90
Rubber	2·5	175	40
Glass	6	35	ambient
Air	1	30	...

CHAPTER IV

D. C. GENERATORS

1. Dynamo Parts: It is best to investigate actually a good sized d. c. machine and carefully inspect the location and purpose of each of its component parts. The following are the principal parts of a d. c. dynamo (generator or motor).

(*a*) *The armature* is cylindrical in shape and it accommodates conductors in its slots. Its shaft rests on bearings, so that it is free to rotate. The cylindrical drum is made up of thin sheet steel stampings which are rigidly clamped together and keyed to the shaft.

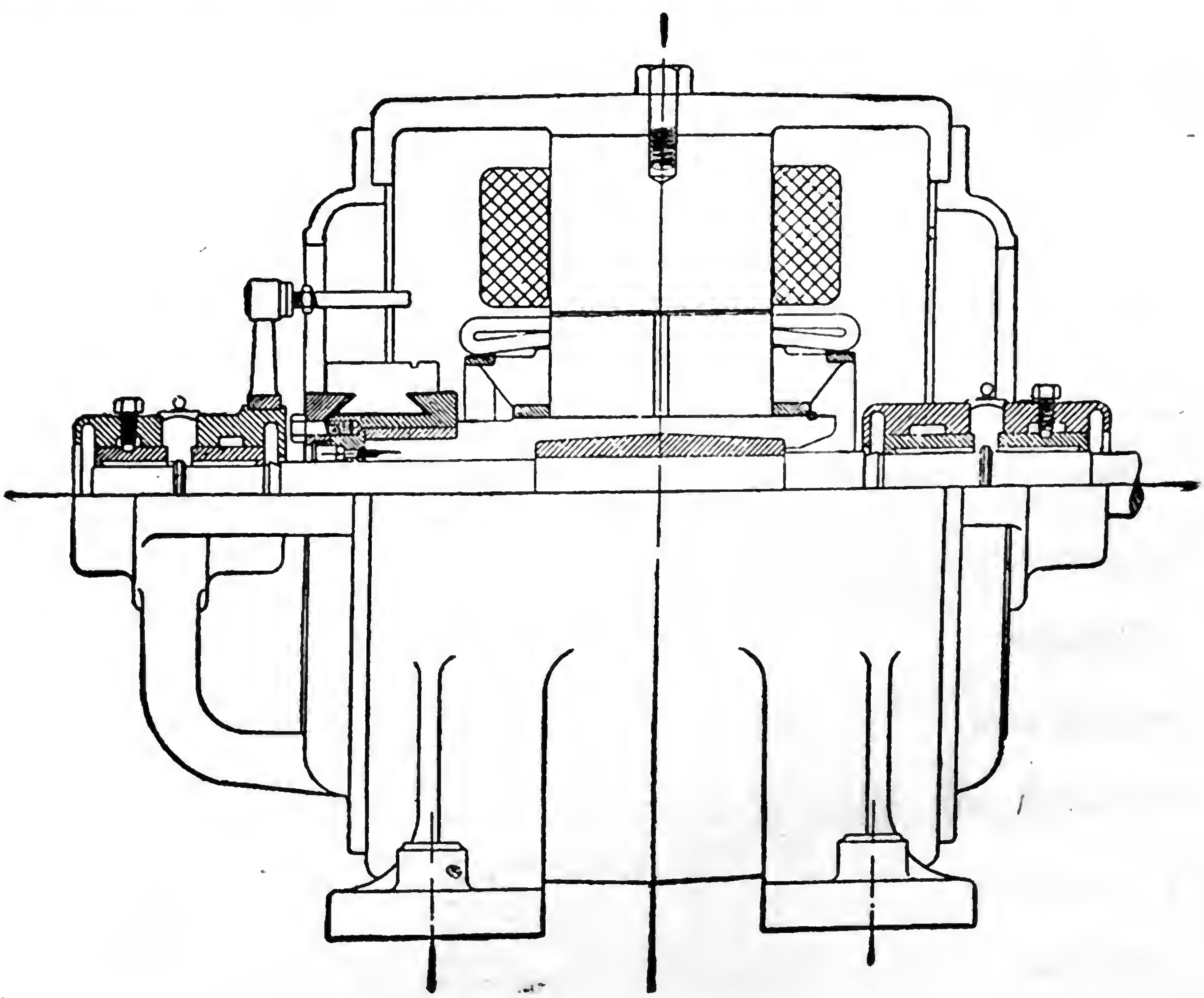


Fig. 1

(*b*) *The field magnet system* consists of field-coil windings mounted on iron cores called *field poles*. These field poles are bolted to an iron frame called the *Yoke*. There is a small *air-gap* between the poles and the armature.

When the current passes in the field coils magnetic flux is created. A cross-sectional view of a d. c. machine is shown in Figs. 1 and 2.

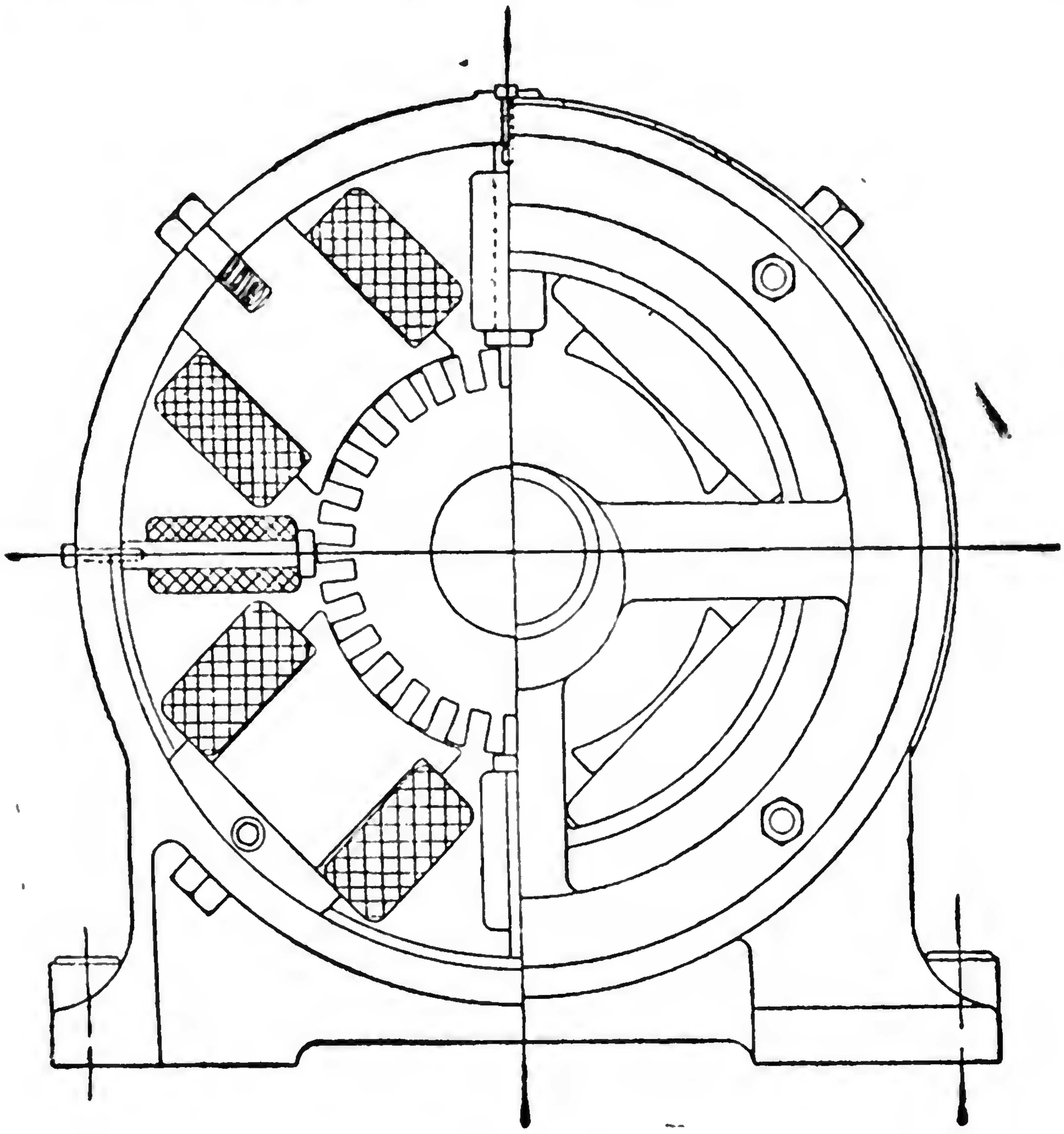


Fig. 2

(*c*) *The commutator* is situated on the armature shaft at one end. It consists of high conductivity copper segments. Each segment is insulated by mica from its neighbouring segment. Ends of

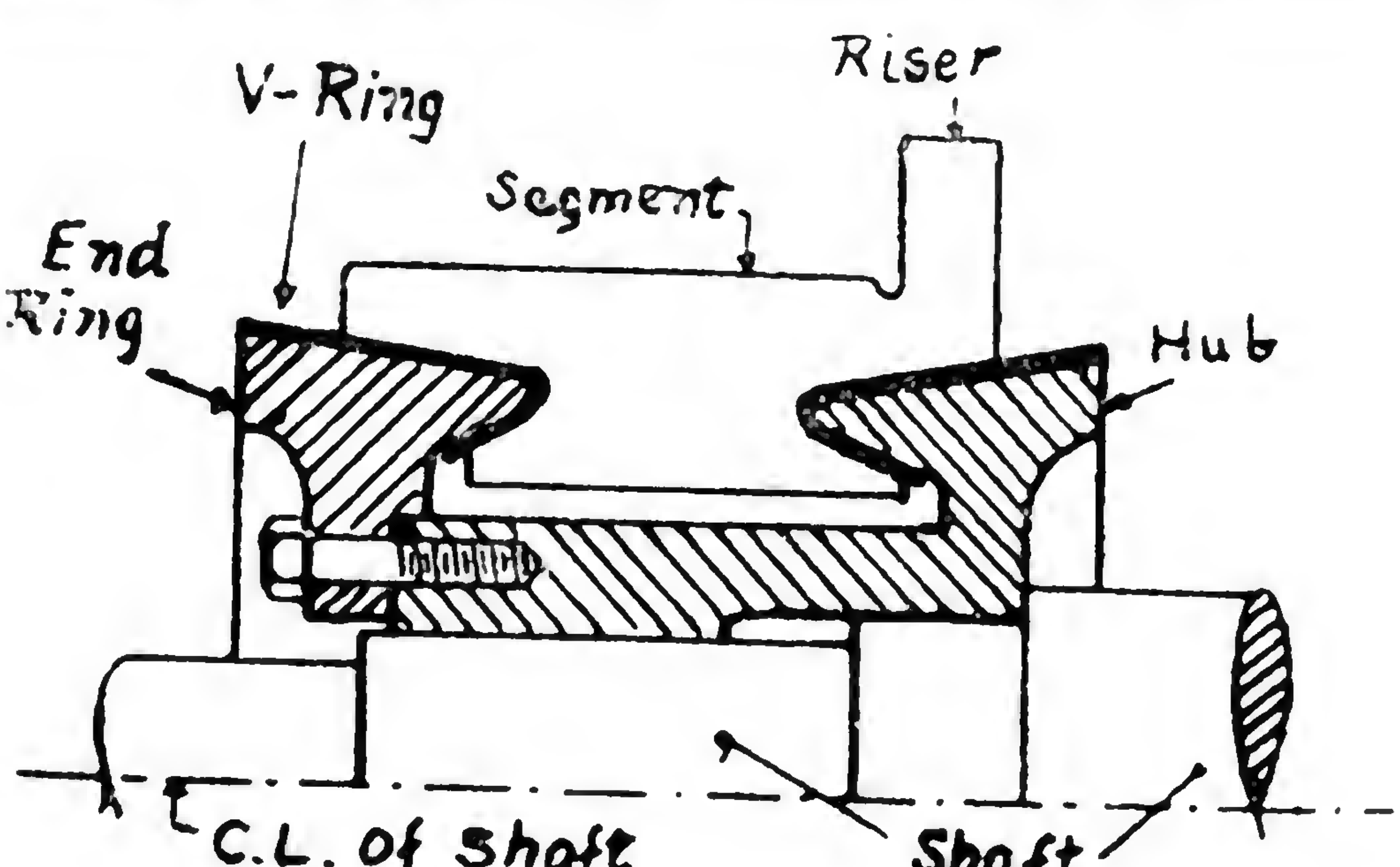


Fig. 3

armature coils are soldered to these segments. The whole commutator assembly is insulated from the armature shaft. See Fig. 3.

(*d*) *The brushes and the brush gear*: The current from the armature is collected from the commutator by means of *carbon brushes*. These brushes are housed in "boxes" which are arranged in a row and mounted on *brackets*. The brackets are fixed on a *brush-rocker* but insulated from it. The whole assembly is either mounted on a pedestal bearing or is supported from the yoke. Thus the brushes remain stationary.

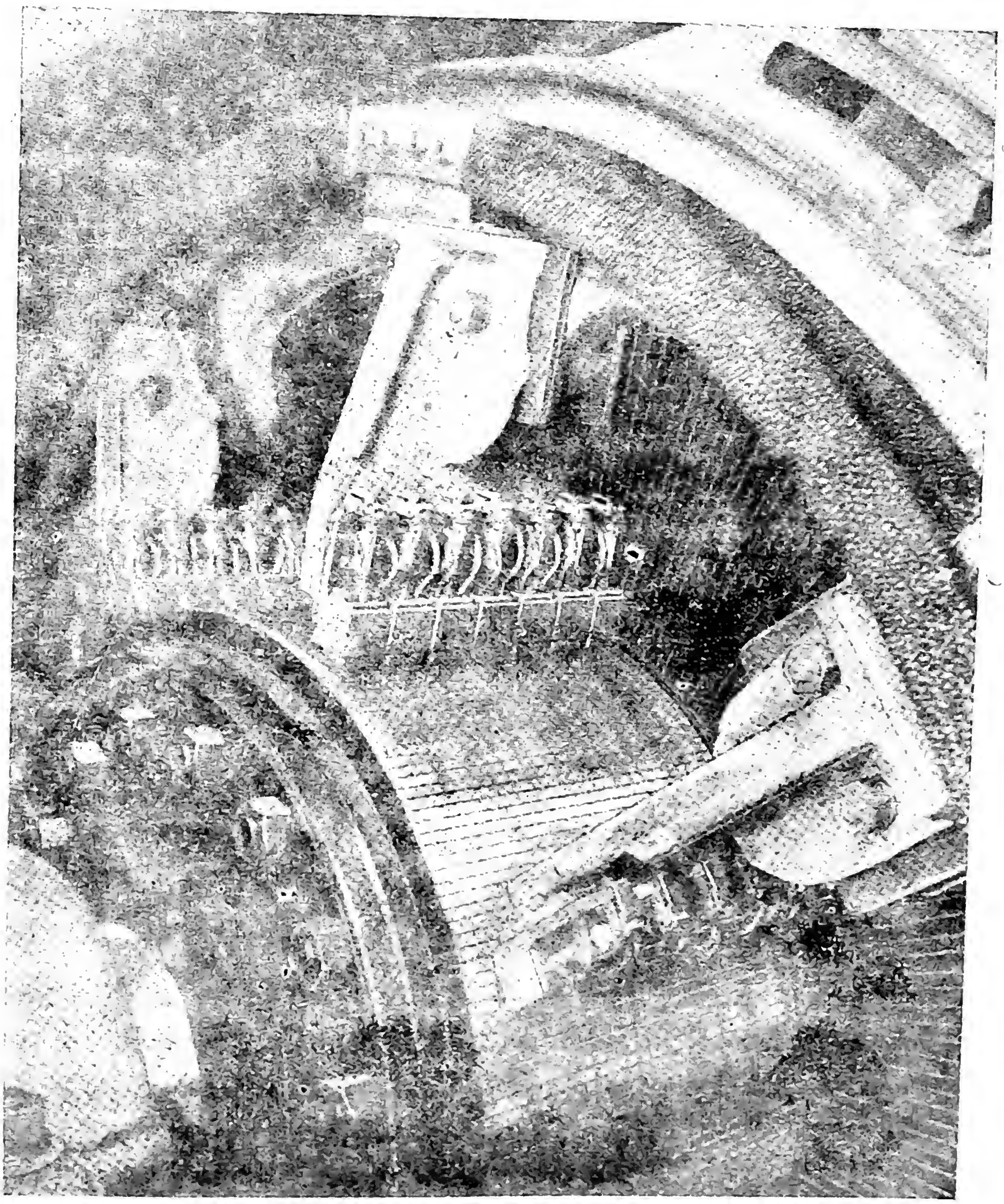


Fig. 4. Part of Brush-Gear — 400 kW 8-Pole Machine
(W. H. Allen, Sons and Co. Ltd.)

Brush sets of same polarity have a common ring of copper to which brush arms are attached. The brushes should always so rest on the commutator that they are in electrical contact with those armature conductors which lie on the neutral axis of the resultant magnetic field.

2. **Methods of Field Excitation :** D. C. machines are classified according to the method of their field excitation. Following are the methods :—

(*a*) *Separate Excitation.* This is effected by taking current from an entirely separate d. c. source of supply for the field windings.

(*b*) *Self-Excitation.* The current to the field windings is supplied by the machine itself. But there are three sub-divisions which are

(i) *Shunt Excitation.* Fig. 5 shows the arrangement. The field winding *sh* is shunted across the armature A. The field coils are of fine wire with many turns and the winding resistance is very high so that the field coils take a small current. If the terminal voltage is *V*, the current in the shunt field coils is $I_{sh} = V/R_{sh}$, where R_{sh} is the resistance of the field. This type is known as a shunt machine.

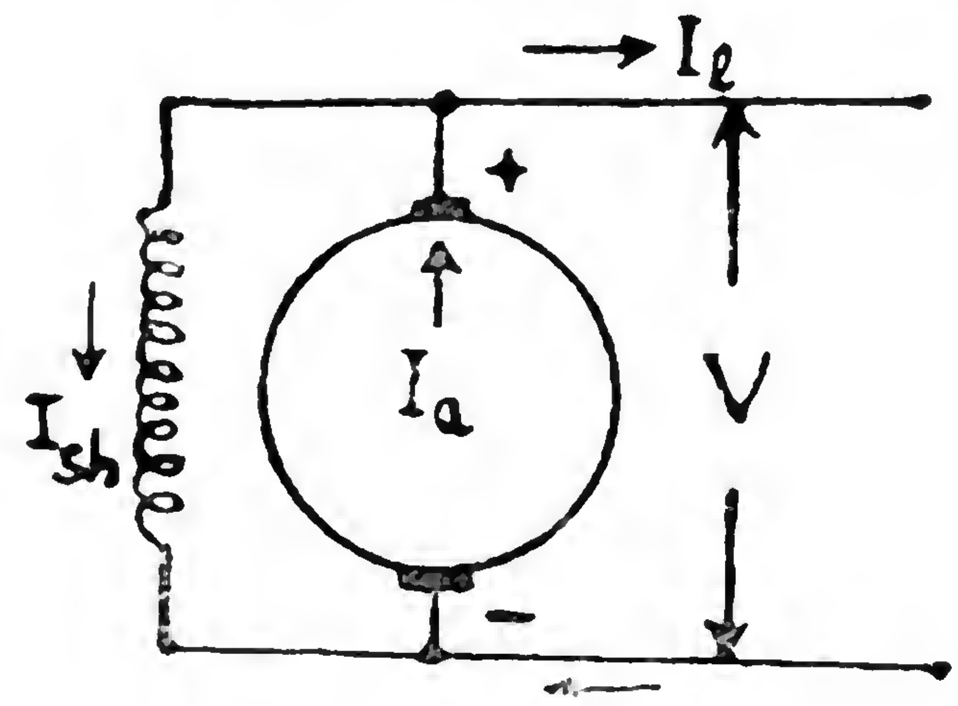


Fig. 5

(ii) *Series Excitation.* Fig. 6 shows the arrangement. The "series" name implies that the field windings are in series with the armature. So that unless there is load no current can flow through the field coils. *The armature current = field current = load current.* Since the field current is heavy, these field coils are wound with few turns of wire of large cross-sectional area. So that the resistance R_{se} of the series field coil is very low. These machines are only used for special purposes such as boosters. This type is called a series machine.

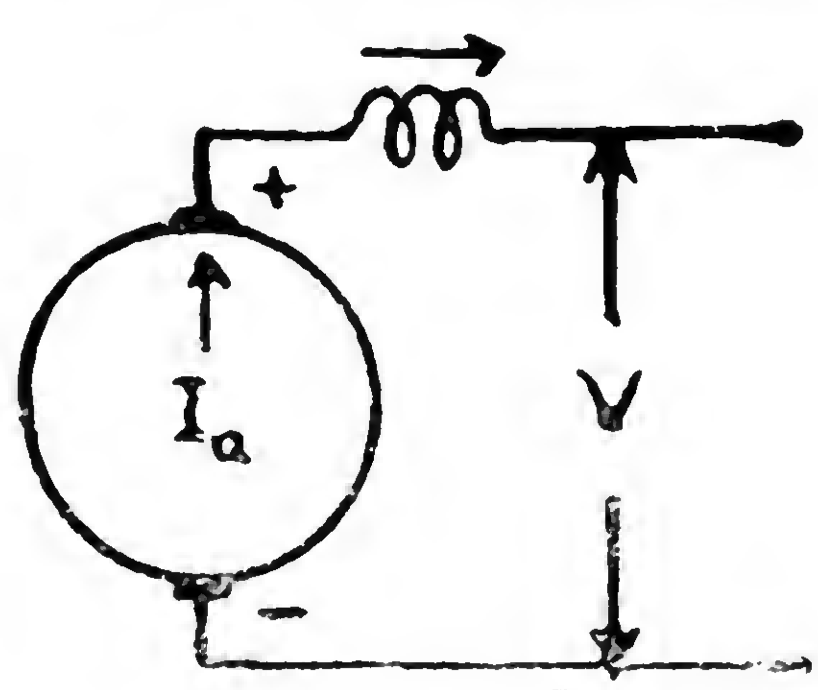


Fig. 6

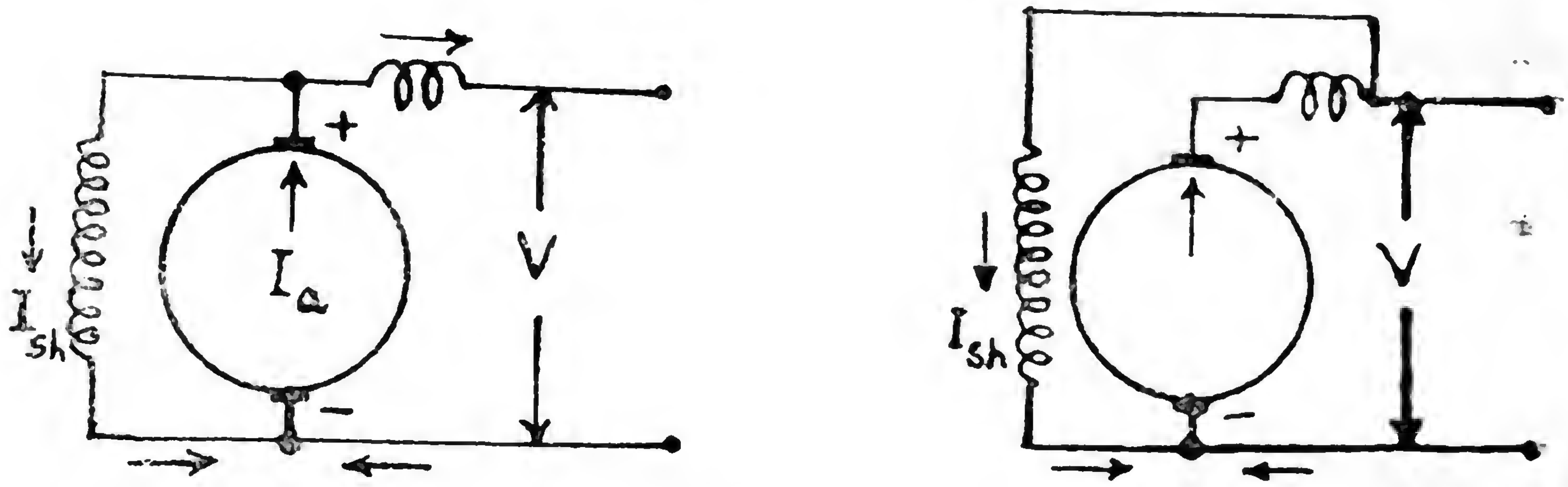
(iii) *Compound Excitation.* In compound machines the magnetic flux is produced mostly by shunt field coils and partly by series field coils. In other words this type of machine has both the types of field coils. Fig. 7 shows two methods of connecting the field coils. Connection shown in Fig. 7 (a) is called *short shunt* and Fig. 7 (b) is called *long shunt* connection.

In short shunt machines

$$I_{se} = I_l ; \; I_a = I_l + I_{sh} .$$

In long shunt machines

$$I_{se} = I_a = I_{sh} + I_l .$$



(a) Fig. 7 (b)

3. **Armature Windings:** There are two important methods of winding (or arranging) the conductors in the armature slots. These are

(a) *Wave* or *Two-Circuit Winding,* and

(b) *Lap* or *Multiple Circuit Winding.*

In simple wave winding there are two paths in parallel for the armature current *irrespective of the number of poles on the machine.* Therefore only two sets of brushes on the commutator are necessary to collect current from the armature.

In lap-wound armatures there are as many paths in parallel as the machine has poles. The total armature current divides equally in these parallel paths. Hence there must be as many brush sets as the machine has poles. Figs. 8 and 9 show the equivalent circuits of the two types of windings. Brush positions are indicated by *b*.

Following symbols stand for certain armature winding terms :—

y = theoretical pitch

y_F = front pitch

y_B = back pitch

y_R = resultant pitch

c = total number of coil-sides

Z = total number of conductors

p = total number of poles.

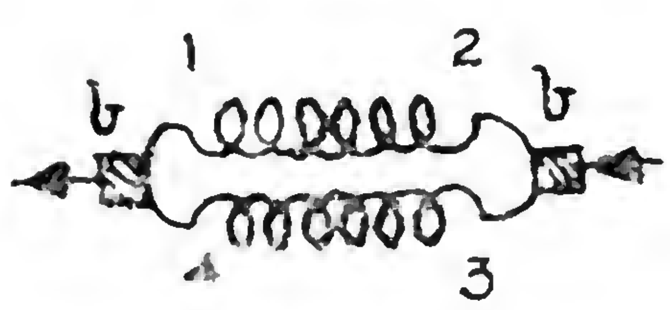


Fig. 8

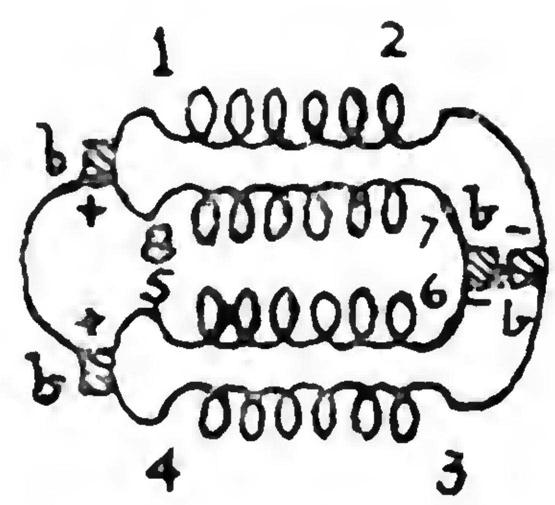


Fig. 9

4-pole LAMP winding

The theoretical pitch $y = \frac{c}{p}$, y_F and y_B are always odd numbers.

For lap winding : $y = \frac{c}{p}$ ... ... (i)

$y_F \pm y_B = \pm 2$ ... ... (ii)

c may or may not be a multiple of p.

For wave winding : $c = (y_R \times p) \pm 2$ ... (iii)

$y_F + y_B \simeq 2y$ ... ... (iv)

All armature windings must form a complete closed circuit, i. e., there must not exist an open end anywhere in the winding.

Fig. 10 shows a winding diagram for a 4-pole wave wound armature having 23 slots and 2 coil-sides per slot. It will be noticed

that for this winding $y = 11{\cdot}5;\ y_F = y_B = 11.$

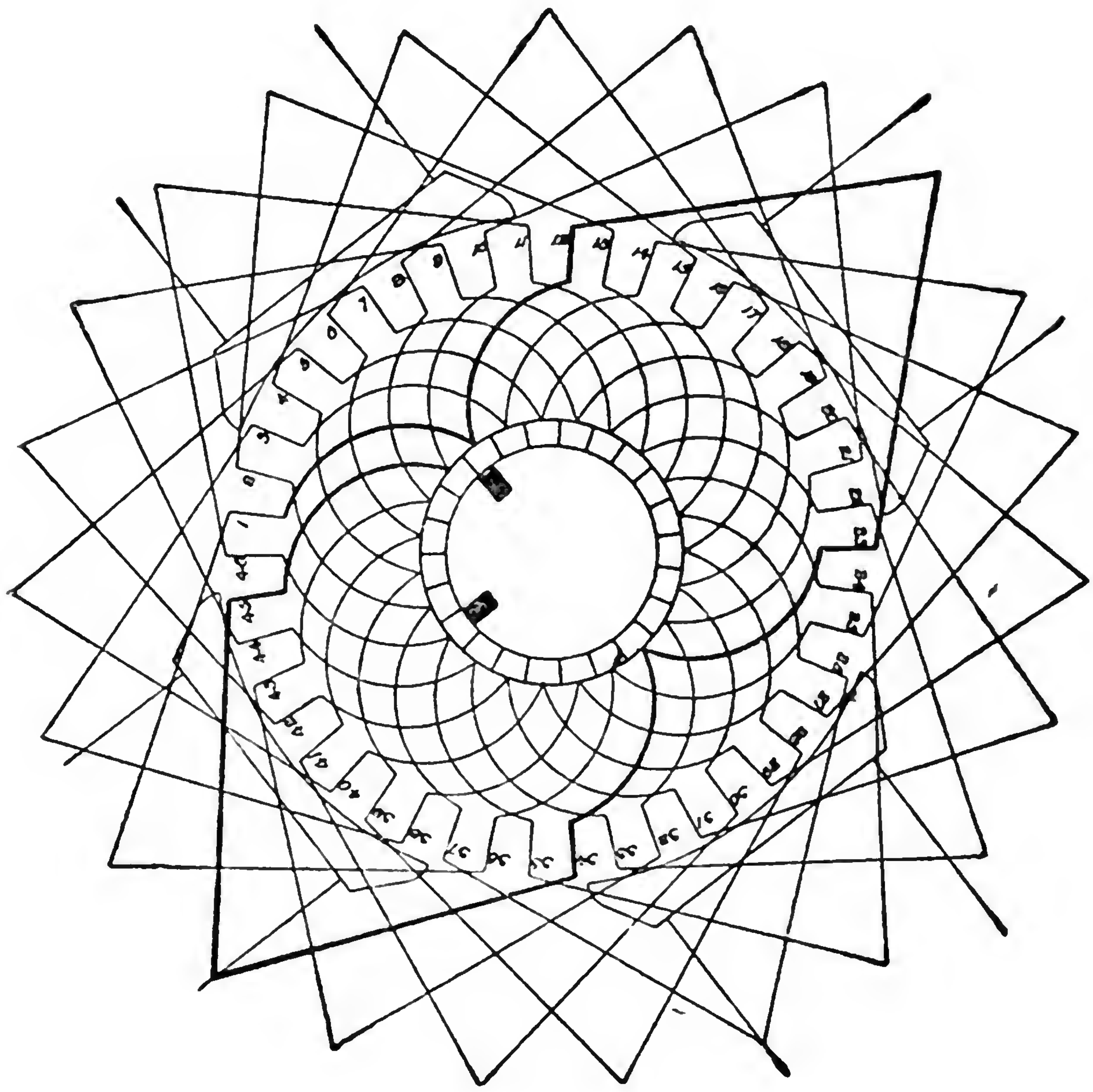


Fig. 10

Fig. 11 shows a winding diagram for a 6-pole lap wound armature having 24 slots and 2 coil-sides per slot. Here

$$y = 8;\ y_B = 7;\ y_F = -9;\ y_F - y_B = -2;\ \text{coils} = 24;$$

and commutator segments = 24. Brush sets = no. of poles.

The reason why a commutator is necessary is this :—

The voltage (and therefore the current) of a generator in the armature itself is alternating. To be able to supply unidirectional current in the external circuit, the currents from the armature coils must be collected from their respective sectors in such a manner that the brushes will always have currents flowing in them in one direction only. Thus direct current is available for the external circuit.

4. **The E. M. F. Equation:** Either Eq. (23) or Eq. (24) given in Chapter II may be used to determine the expression for the

induced e. m. f. in the armature of a d. c. machine. Let

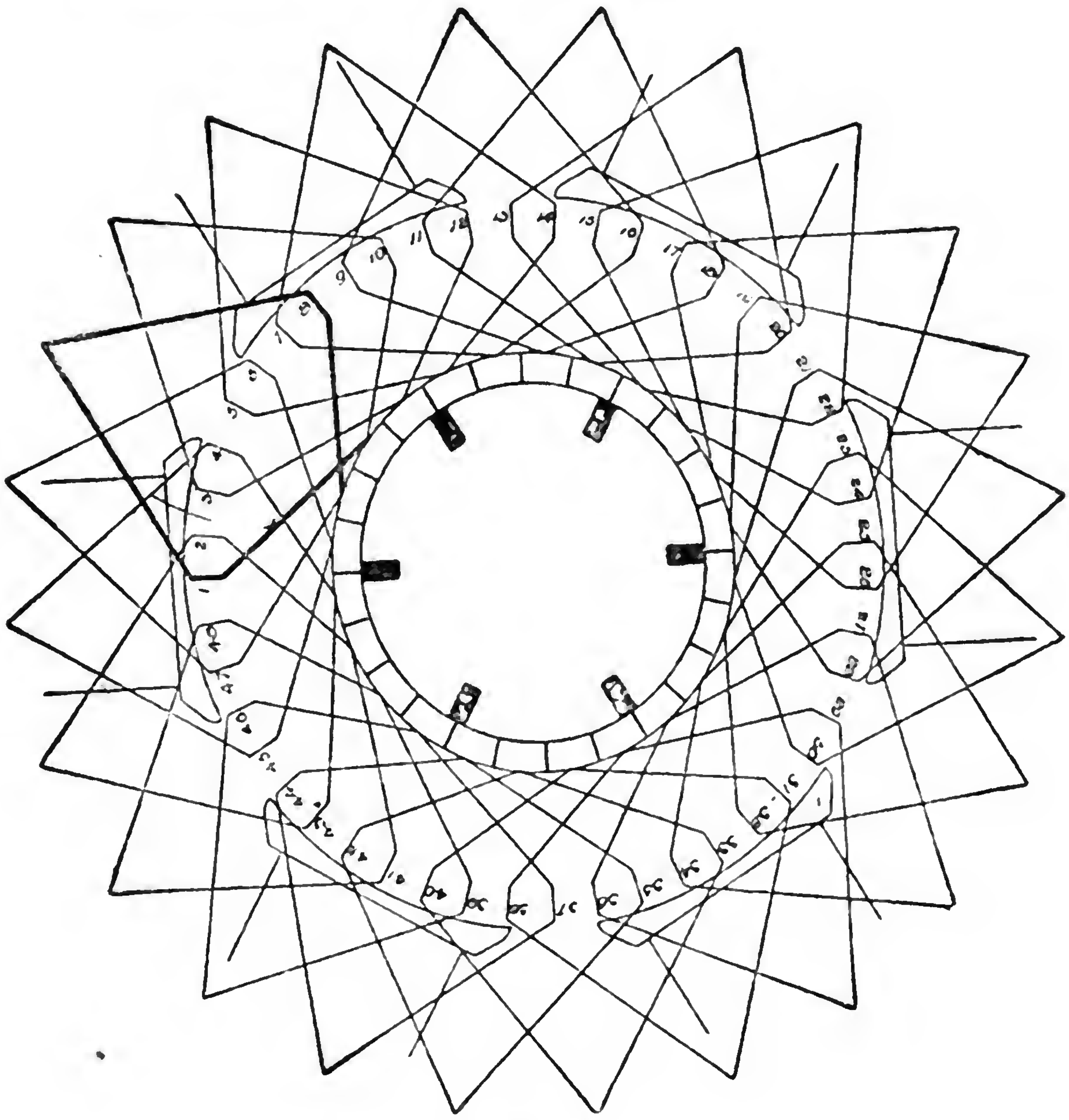


Fig. 11

Φ = flux per pole

p = number of poles

Z = total armature conductors

a = number of armature parallel paths

n = r. p. m. of armature

d = diameter of armature drum (cm)

l = length of armature drum = active length of conductor (cm).

The average flux density in air-gap is

$$B = \frac{\textit{total flux in air-gap}}{\textit{total area of air gap}} = \frac{\Phi p}{\pi d l}$$

The velocity v per second is

$$v = \pi d \frac{n}{60}$$

Hence induced e m. f. per conductor is

$$E_c = \left(\frac{\Phi p}{\pi d l}\right) \times l \times \left(\frac{\pi d n}{60}\right) \times 10^{-8} \text{ volts.}$$

The total e. m. f. of the armature is that which is induced in one parallel path. The conductors in one parallel path $= \frac{Z}{a}$. Therefore the total induced e. m. f. is

$$E = \left(\frac{\Phi p}{\pi d l}\right) \times l \times \left(\frac{\pi d n}{60}\right) \times \frac{Z}{a} \times 10^{-8} \text{ volts.}$$

Cancelling and arranging

$$E = \Phi \times \frac{p}{a} \times Z \times \frac{n}{60} \times 10^{-8} \text{ volts.} \quad \dots \quad \dots \quad (1)$$

Eq. (1) is of great importance and is also applicable to determine the *back e. m. f.* or *induced* e. m. f. of a d. c. motor, and plays an important part in the study of motor characteristics.

Note that for a particular machine p and $\frac{Z}{a}$ are constant therefore

$$E \propto \Phi n \quad \dots \quad \dots \quad \dots \quad \dots \quad (2)$$

Eq. (2) shows that the e. m. f. or voltage of a generator can be varied either by varying Φ or n. But most prime movers are usually run at a fairly constant speed. Hence it is the usual practice to change Φ for a change in the generator voltage. This is done by controlling the current in the field windings by a ***field regulator.***

Example: An 8-pole lap-wound armature has 640 conductors, a flux of 4 megalines per pole and a speed of 600 r. p. m. Calculate the induced e. m. f.

Solution: $E = \Phi \times \frac{p}{a} \times Z \times \frac{n}{60} \times 10^{-8}$ volts

$$= 4 \times 10^6 \times \frac{8}{8} \times 640 \times \frac{600}{60} \times 10^{-8} = 256 \text{ volts.}$$

5. Losses and Efficiency; A d. c. generator receives mechanical power from its prime mover and converts this power into electrical form. Certain losses occur which are dissipated in the form of heat. The ability of a machine to withstand a certain temperature

rise depends entirely upon the type of insulating materials used. It is not safe to reach a temperature of more than 100° C because at this temperature the insulating properties of materials deteriorate rapidly. Thus the kW rating of a machine depends upon the maximum allowable temperature.

The losses of a machine may be grouped under two headings;

(1) Copper Losses,

(2) Iron and Friction Losses.

Copper Losses:

(*a*) Armature copper loss $= I_a^2 R_a$ watts

(*b*) Field copper loss includes losses in

(i) shunt winding $= I_{sh}^2 R_{sh}$ watts

(ii) series winding $= I_{se}^2 R_{se}$ watts

(iii) interpole winding $= I_a^2 R_i$ watts

(*c*) Loss due to brush contact resistance.

Iron Losses:

(*d*) Hysteresis loss in armature (loss $\propto f n B^{1\cdot6}{}_{max}$)

(*e*) Eddy current loss in armature and pole-shoes. This loss varies as $f^2 n^2 B^2{}_{max}$, where n = speed, B = flux density, and f = frequency.

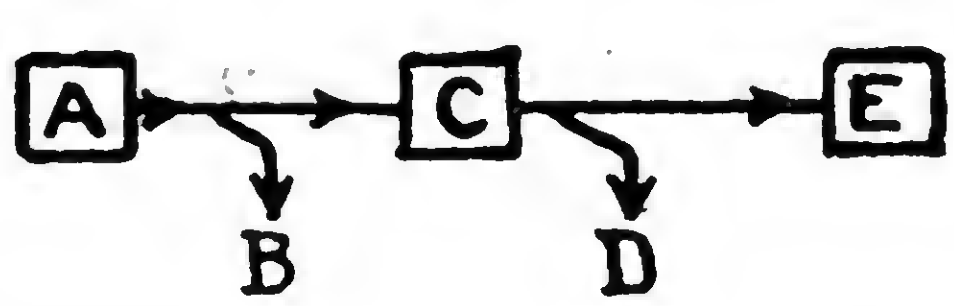
Friction losses are

(*f*) Brush friction loss,

(*g*) Wind resistance to rotating armature.

(*h*) Bearing friction loss.

The input, losses and the output of a generator are shown diagrammatically below—



A = mechanical input = b.h.p. × 746 watts

B = iron and friction losses = A − C.

C = electrical power developed

= induced *e. m.f.* $\times I_a$ watts.

D = copper losses = C − E

E = electrical output

= *terminal volts* $\times I_l$ watts;

where I_a is the armature current,

I_l is the load or external current.

$$\text{Mechanical efficiency} = \frac{C}{A}$$

$$\text{Electrical efficiency} = \frac{E}{C}$$

$$\text{Overall or commerical efficiency} = \frac{E}{A}$$

$$\eta = \frac{E}{A} = \frac{\text{output}}{\text{input}} \text{ or } = \frac{\text{output}}{\text{output} + \text{losses}} \text{ or } = \frac{\text{input} - \text{losses}}{\text{input}} \quad \dots (3)$$

Constant and Variable Losses: In the case of shunt and compound machines the constant losses are—

(*a*) iron and friction losses

(*b*) shunt field losses, $I_{sh}^2 R_{sh}$;

and the variable losses are—

(*c*) copper loss in armature, $I_a^2 R_a$ watts

(*d*) copper loss in series winding, $I_{se}^2 R_{se}$ watts

(*e*) copper loss in interpole winding, $I_a^2 R_i$ watts,

where R_i = resistance of interpole winding,

R_{se} = ,, ,, series ,,

R_{sh} = ,, ,, shunt ,,

R_a = ,, ,, armature.

For a given machine maximum efficiency occurs at that load when *constant losses = variable losses.*

Summarising—

(1) Electric power developed $= E \times I_a$ watts ... ... (4)

(2) Input to generator = b. h. p. of prime mover × 746 watts... (5)

(3) Armature copper loss $= I_a^2 R_a$ watts ... ... (6)

(4) Shunt field copper loss $= I_{sh}^2 R_{sh}$ watts ... ... (7)

(5) Series field copper loss $= I_{se}^2 R_{se}$ watts ... ... (8)

(6) Interpole copper loss $= I_a^2 R_i$ watts ... ... (9)

(7) Iron and friction loss = input − elec. power developed ... (10)

(8) Output of generator = terminal volts × I_l watts ... (11)

Example : A shunt generator supplies 72 amperes at 220 volts. The shunt field and armature resistances are 55 and 0·15 ohm respectively. If the b. h. p. of the prime mover is 25, calculate (*a*) iron and friction losses, (*b*) total copper losses and (*c*) overall efficiency.

Solution : Input watts = 25 × 746 = 18650 W.

Output watts = 72 × 220 = 15840 W.

$$\therefore \quad \text{overall efficiency} = \frac{15840}{18650} \times 100 = \mathit{85\%}.$$

$$\text{shunt field current} = \frac{220}{55} = 4 \text{ A}$$

∴ armature current = 72 + 4 = 76 A.

volt drop in armature = 76 × 0·15 = 11·4 volts.

∴ e. m. f. induced in armature = 220 + 11·4 = 231·4 V

electrical power developed = 76 × 231·4

= 17586·4 W

∴ iron and friction losses = 18650 − 17586·4

= *1063·6 W*

copper losses = 17586·4 − 15840 = *1746·4 W.*

6. **Armature Reaction** means distortion and weakening of the main magnetic field in a machine. This is due to the effects of magnetic field produced by currents in armature conductors. This reduces the voltage and the electrical power developed in the armature of a generator.

For simplifying our discussion a 2-pole armature is considered and the results are equally applicable to multipolar machines. Under no-load condition, i. e. when there is no current in the armature conductors, the flux present is due to the field windings and the flux density in the air-gap under the poles is uniform. The brushes are on the magnetic neutral plane (M.N.P.) which coincides with the geometric neutral plane (G. N. P.), and shown in Fig. 12 as *ab*. The magnetic axis is *cd*.

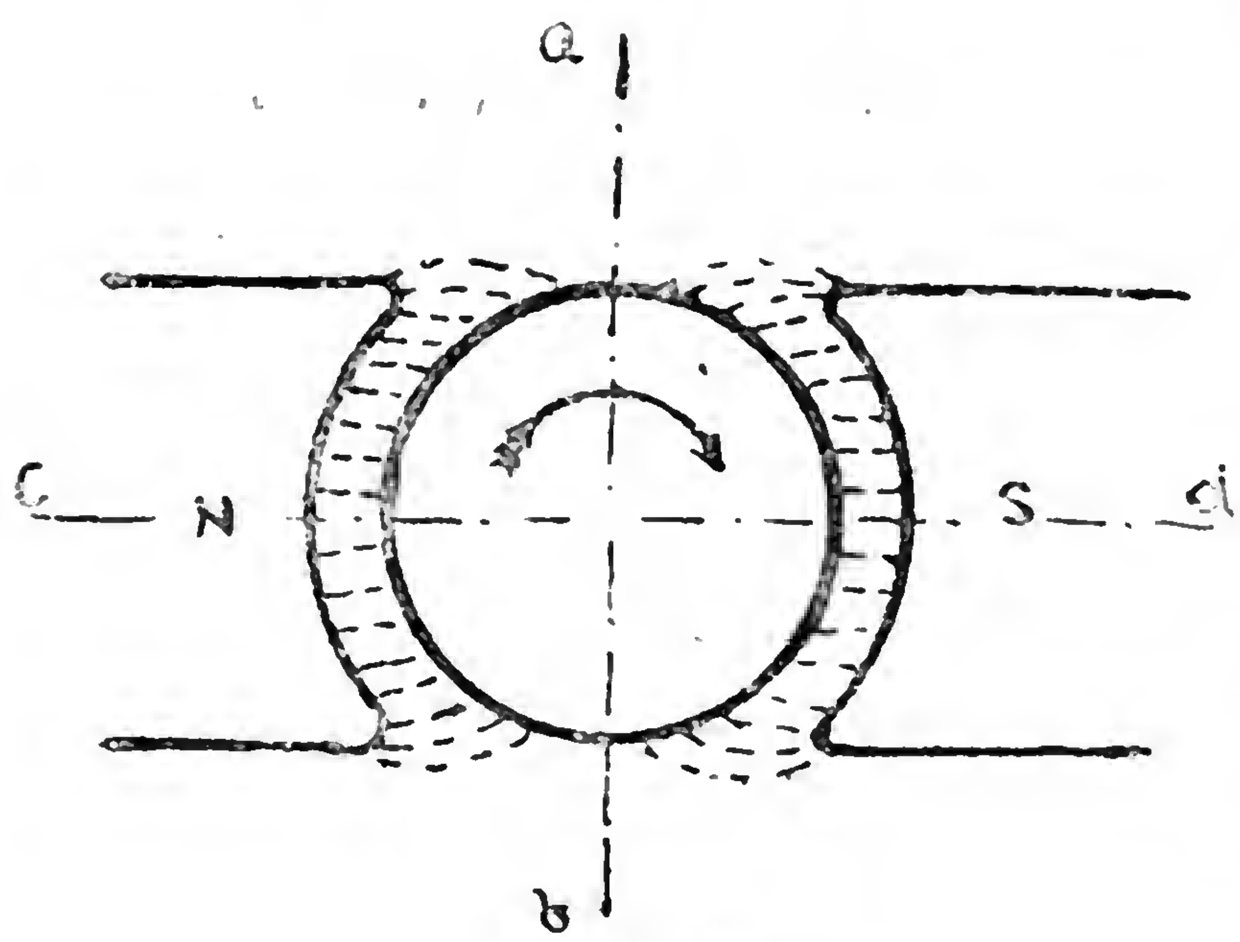


Fig. 12

When current flows in armature conductors a magnetic field is created, its average path and direction is shown in Fig. 13. For clarity the main field is not shown, and only very few armature conductors are shown with direction of current in each one. The resultant of the two fluxes now lies along a line which is θ^o from *cd in the direction of rotation.* Therefore the brushes must be shifted to the new M. N. P. which is θ^o from the G. N. P. in the direction of rotation. The neutral magnetic axis (i. e. M. N. P.) is called the *axis of commutation.*

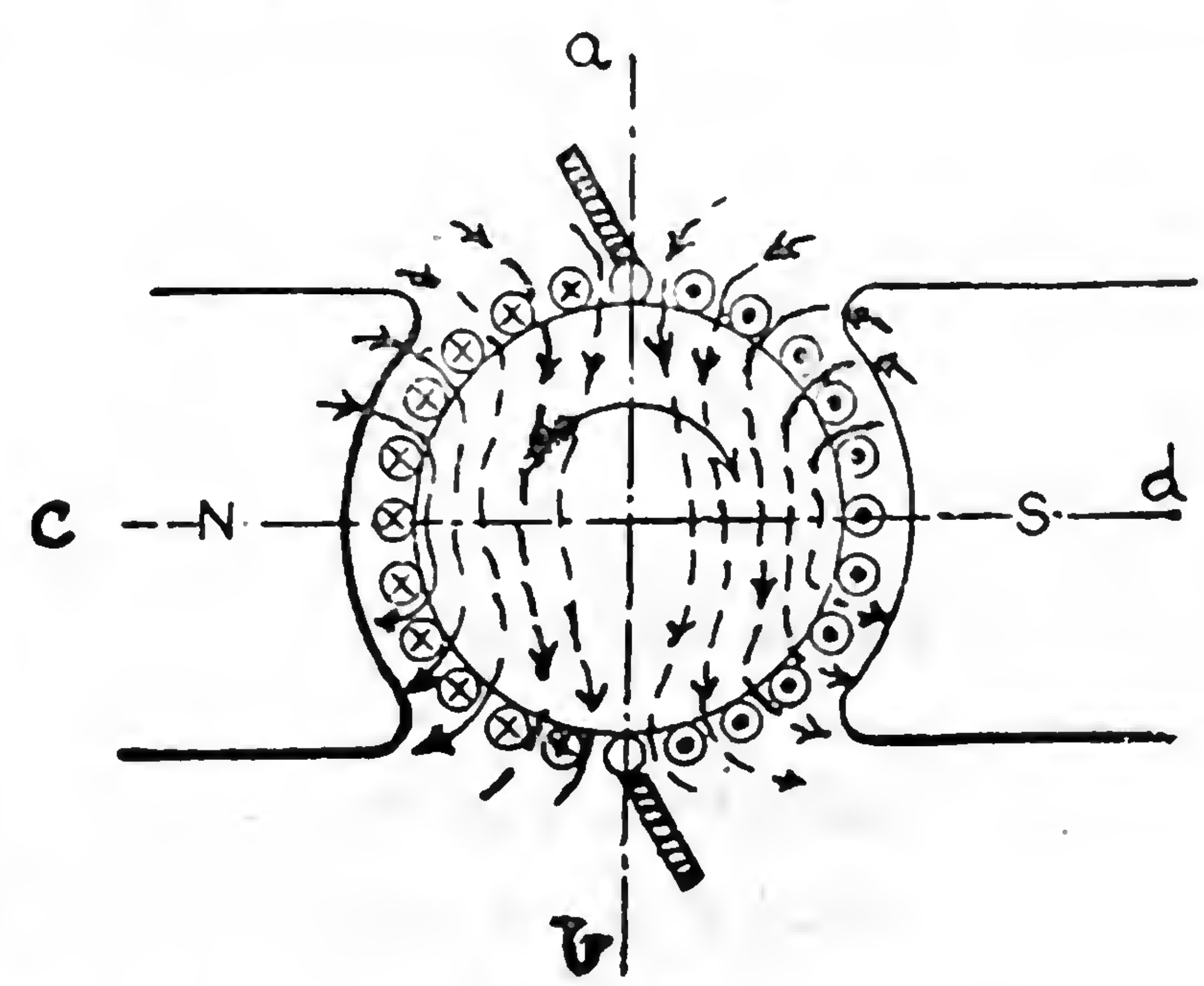


Fig. 13

Fig. 14 shows the original and the new M. N. P. The brushes are shifted to the new neutral axis *a′b′*. The resultant flux axis is at right angles to *a′ b′*.

In the vector diagram of Fig. 14,

$$OA = \text{main flux } \Phi_F$$

$$AB = \text{armature flux } \Phi_A$$

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$$\left.\begin{aligned} AC &= \Phi_c \\ CB &= \Phi_d \end{aligned}\right\} \text{two component of } \Phi_A \text{ at } 90^\circ.$$

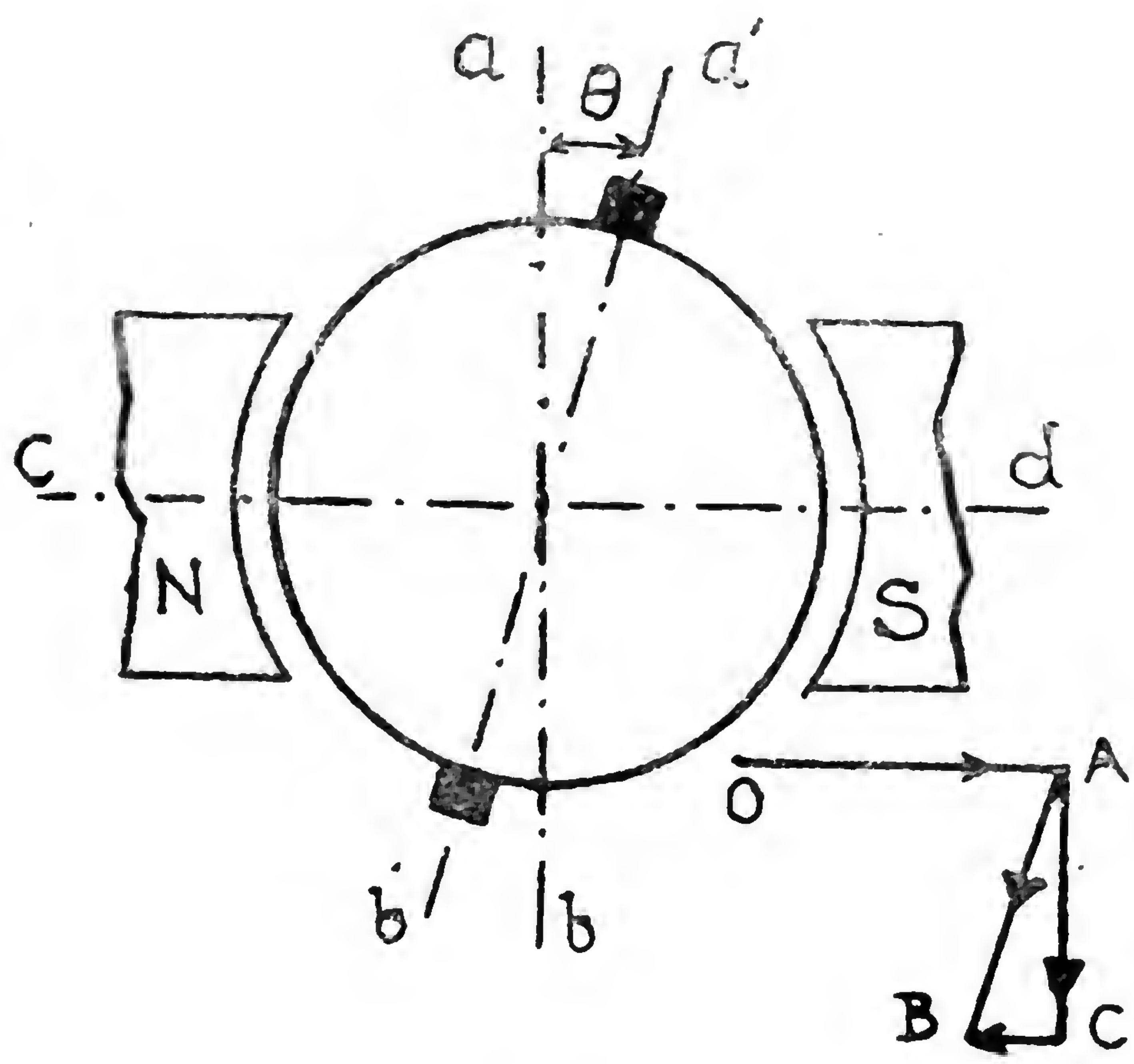


Fig. 14

The direction of Φ_d is opposite to that of Φ_F. Hence it is called the *demagnetising component*; and Φ_c being at right angles to Φ_F is called *cross-magnetising component.* This clearly shows the weakening of main flux. As the armature current increases these effects are more pronounced.

Instead of fluxes it is better to use the quantity known as "ampere-turns", since $\Phi \propto$ amp-turns. So that

OA = main field ampere-turns per pole

AB = armature ,, ,, ,,

AC = cross-magnetising ,, ,, ,,

CB = demagnetising ,, ,, ,,

In order to prevent too much difference of flux densities at the tips of a pole, the pole-shoes are bevelled at the ends. This makes the machine to have a stiff magnetic field.

A better method is to completely neutralise Φ_A. This can be done by providing a winding in slots specially made in the pole-shoes.

Since the load current passes through this winding its flux opposes and neutralises the armature flux in the interpolar zone at all loads. This winding is called *compensating winding.*

7. Commutation: Whenever an armature coil passes under a brush, the direction of current in that coil changes. This change of direction of current in coils is called *commutation.* When the *time/current* graph is a straight line commutation is said to be *ideal* as shown by graph *a* in Fig. 15. In ideal commutation the coil

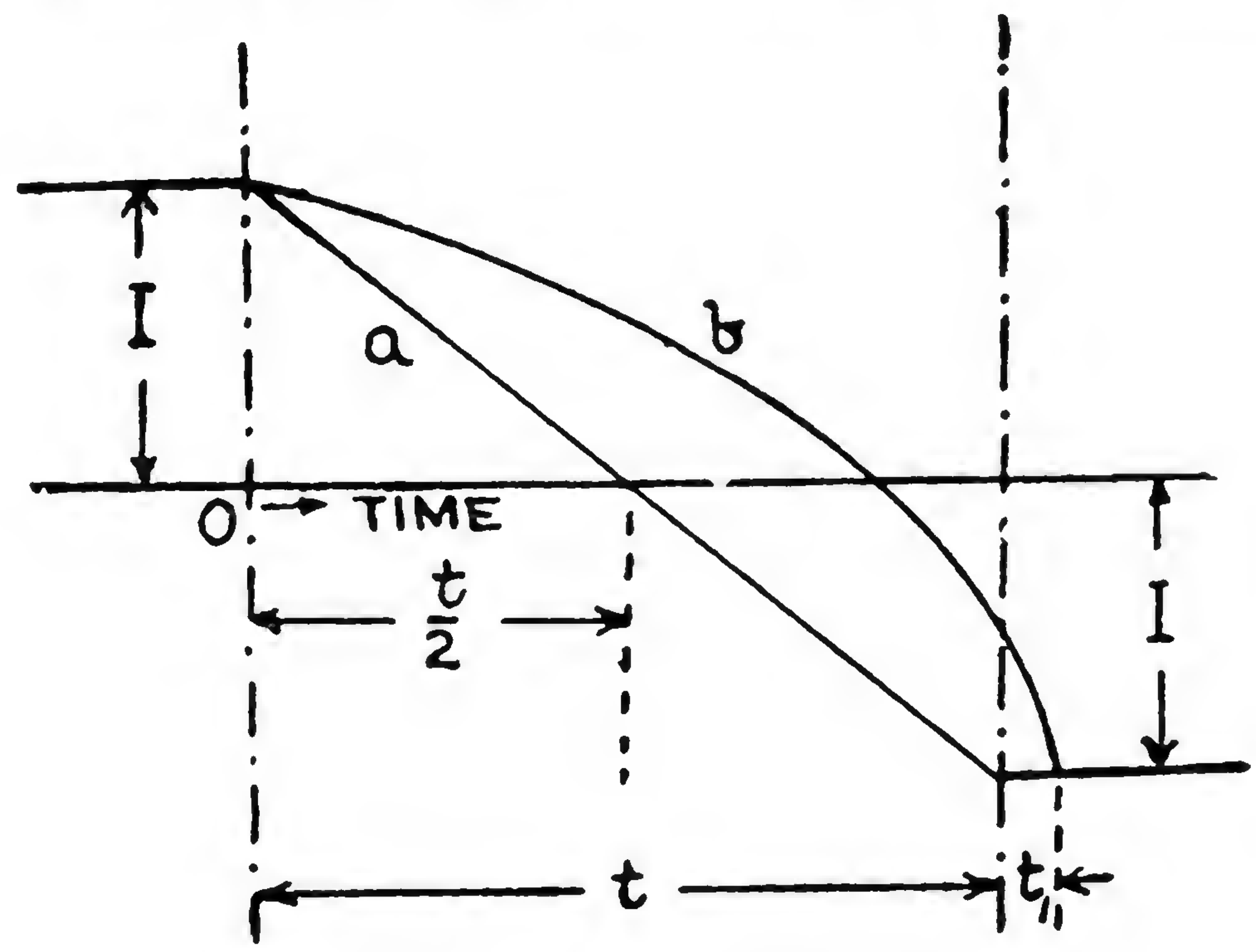


Fig. 15

current falls from I to zero in time $t/2$, where t is the period of commutation. In the remaining half time $t/2$, the coil current rises from zero to I, the direction of flow of current being reversed.

In practice, however, the current does not become zero in half the periodic time of commutation because coils possess inductance (L), and so when the current in the coil changes there is induced in it an e. m. f. of self-induction called reactance voltage $\left(L \dfrac{di}{dt} \right)$. The result is that the current takes a longer time to reach zero value. The *time/current* graph assumes a shape shown by the curve *b* in Fig. 15. Here the zero value is reached at a much later stage. The higher the value of the initial current, the later is the zero value reached, so that in the little time that is left the current in the coil does not reach the value I as it should, and when the trailing end of the brush leaves segments the current broken is less than I. The reactance voltage at this instant is much higher than the average and the current follows in the form of an arc for a short time t_1. This then is the cause of sparking at the brushes.

Therefore some steps must be taken to assist speedy reversal of current. Two main methods are employed. They are

(1) *resistance commutation,* and

(2) *e. m. f. commutation.*

Resistance commutation is the method of variation of *brush contact resistance,* and e. m. f. commutation is the method of providing positive reversing e. m. f. induced in the coil by a suitable *counter* or *commutating flux.* Both methods are adopted simultaneously to ensure sparkless commutation.

(1) To understand the need of application of the two methods, Fig. 16 is drawn to study the stages of reversal of current in a coil undergoing commutation.

Fig. 16 (a) shows coil B about to enter the period of commutation. The direction of motion of coils and segments is from left to right. The brushes are stationary. Direction of current in coils A and B is clockwise, and that of coil C is anti-clockwise. The current in each coil is I and the brush current is $2I$ which flows through one segment only.

Fig. 16 (b) shows that the brush has just short-circuited coil B and therefore it is undergoing commutation. The current in it has fallen from I to a smaller value x, but its direction is as before. The brush now gets its current of $2I$, from segment P the current is $(I-x)$ and from segment Q $(I+x)$.

Fig. 16 (c) shows the condition much later than time $\frac{t}{2}$. The current in coil B did not become zero at time $\frac{t}{2}$ but at a later period. This figure shows that the current has just reversed in direction, its value is very small at present compared to the final value I. The time available for the current to reach full value of I is therefore very short. The current I of coil C can pass by two paths to the brush, one path is directly through segment Q and the other path is through coil B and segment P. But the current divides in these paths inversely as the resistances of these paths.

By employing carbon brushes, instead of copper brushes, the contact resistance (i. e. the resistance between the surfaces of the brush and segments) is much increased. Hence a major portion of I

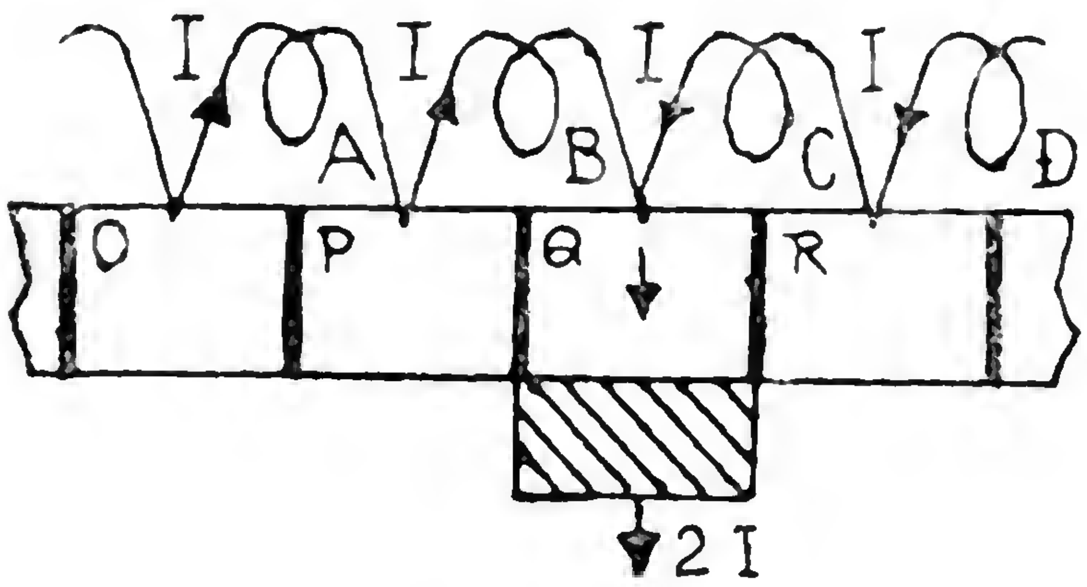


Fig. 16 (*a*)

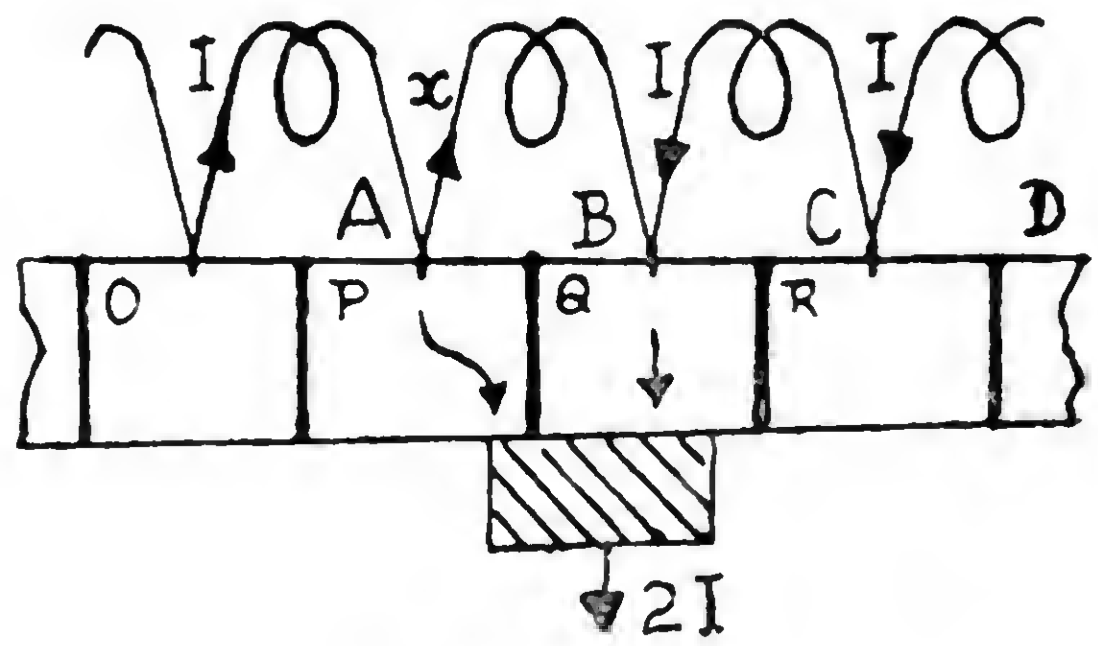


Fig. 16 (*b*)

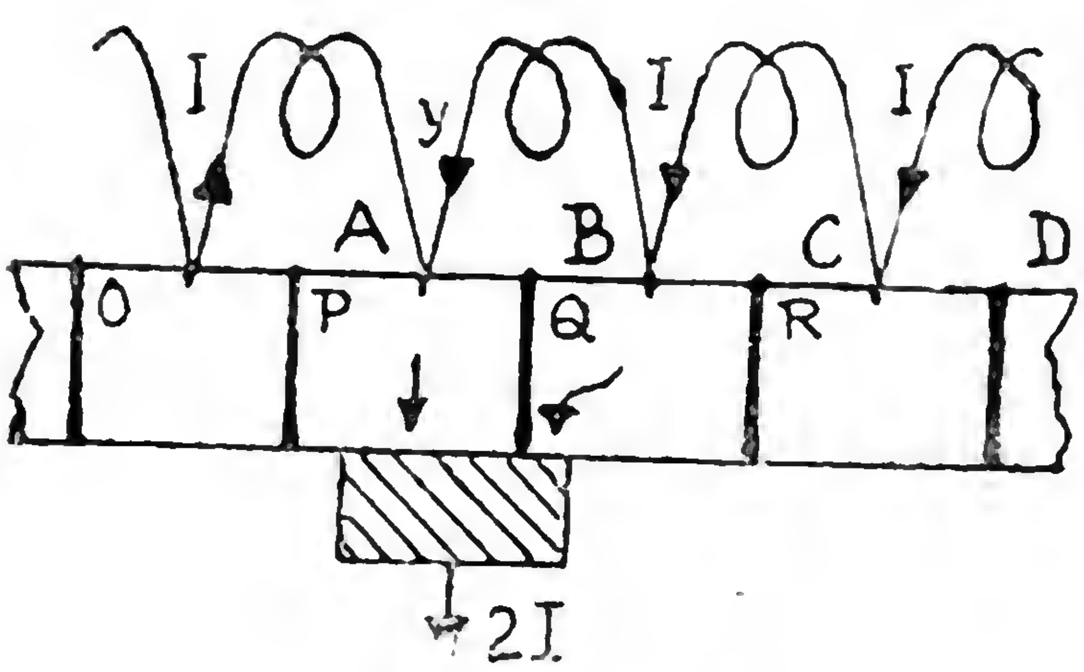


Fig. 16 (*c*)

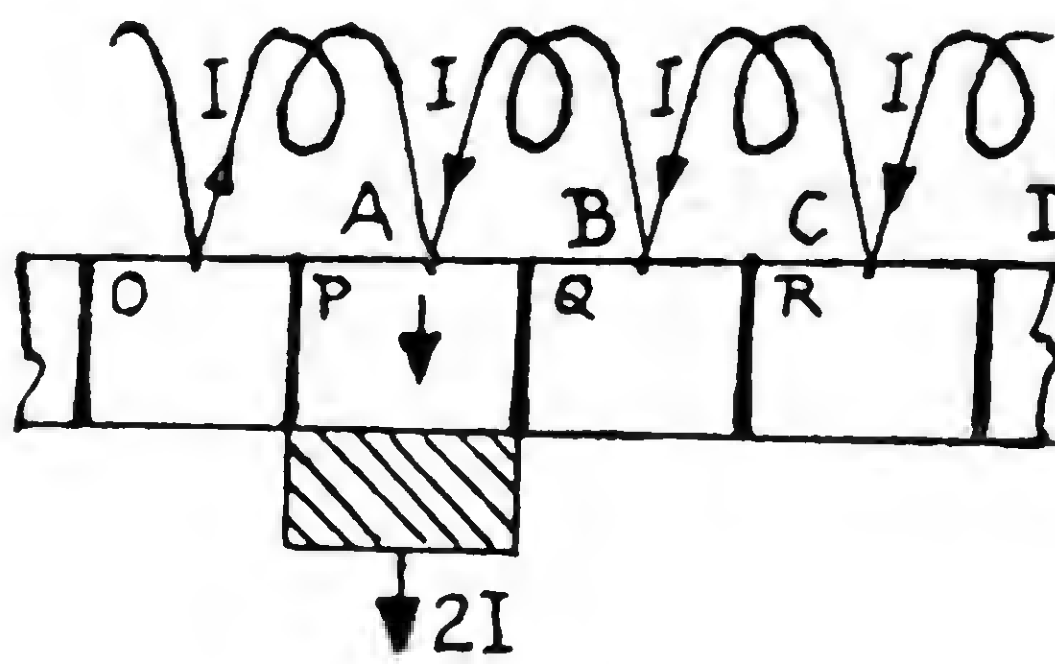


Fig. 16 (*d*)

from coil C will flow through coil B. Segment Q delivers $(I - y)$ and segment P $(I + y)$. This helps somewhat to increase current in B. This is resistance commutation.

Fig. 16 (d) show the end of commutation period of coil B. The brush is now in contact with segment P alone. In spite of resistance commutation the current in coil B does not attain the full value I but is less. The current broken is not zero, as it should be, but $(I - y)$, where y is now slightly less than I. Thus there will be a spark between the trailing end of brush and the segment which it has just eft. In ideal commutation the current broken is zero.

(2) If coils undergoing commutation cut a small flux in a direction such as to assist reversal of current in them, a positive (or reversing) e. m. f. will be generated to overcome the reactance voltage of the coils. In the case of generators by giving a lead of $\theta°$ to the brushes in the direction of rotation, the coils undergoing commutation are brought under the influence of field of the next pole which provides the necessary kind of flux. This is one type of e. m. f. commutation.

One drawback of this method is that the brush position must be shifted at every change of load, and the other disadvantage is that brush shifting creates demagnetising armature ampere-turns which weaken the main magnetic field as stated in the last Section.

The most logical method then is to provide special field poles at the neutral axis. These poles are known as *interpoles* or *commutating poles*. These are now fitted on all machines (generators and motors) except those of very small sizes or those running at very low speeds. Since the interpoles are excited by the armature current they provide for all conditions of load, and overcome the effects of armature reaction upon commutation. This method of e. m. f. commutation is far superior.

Thus the brush position is fixed, i. e. there is no need to shift the brushes, not only at any load but also when the direction of armature current reverses. For generators the polarity of an interpole is that as the next main pole in the direction of rotation. In the case of motors the polarity is opposite. The air-gap under these poles is much longer than that under the main poles. This makes the interpole flux vary directly as the armature current at all loads.

Interpoles do not correct armature reaction though they do eliminate the demagnetising effect, since no brush lead is necessary for interpolar machines. The distortion of the resultant field is not eliminated it is still present. The interpoles merely give a local correction to the magnetic field in the neutral zone.

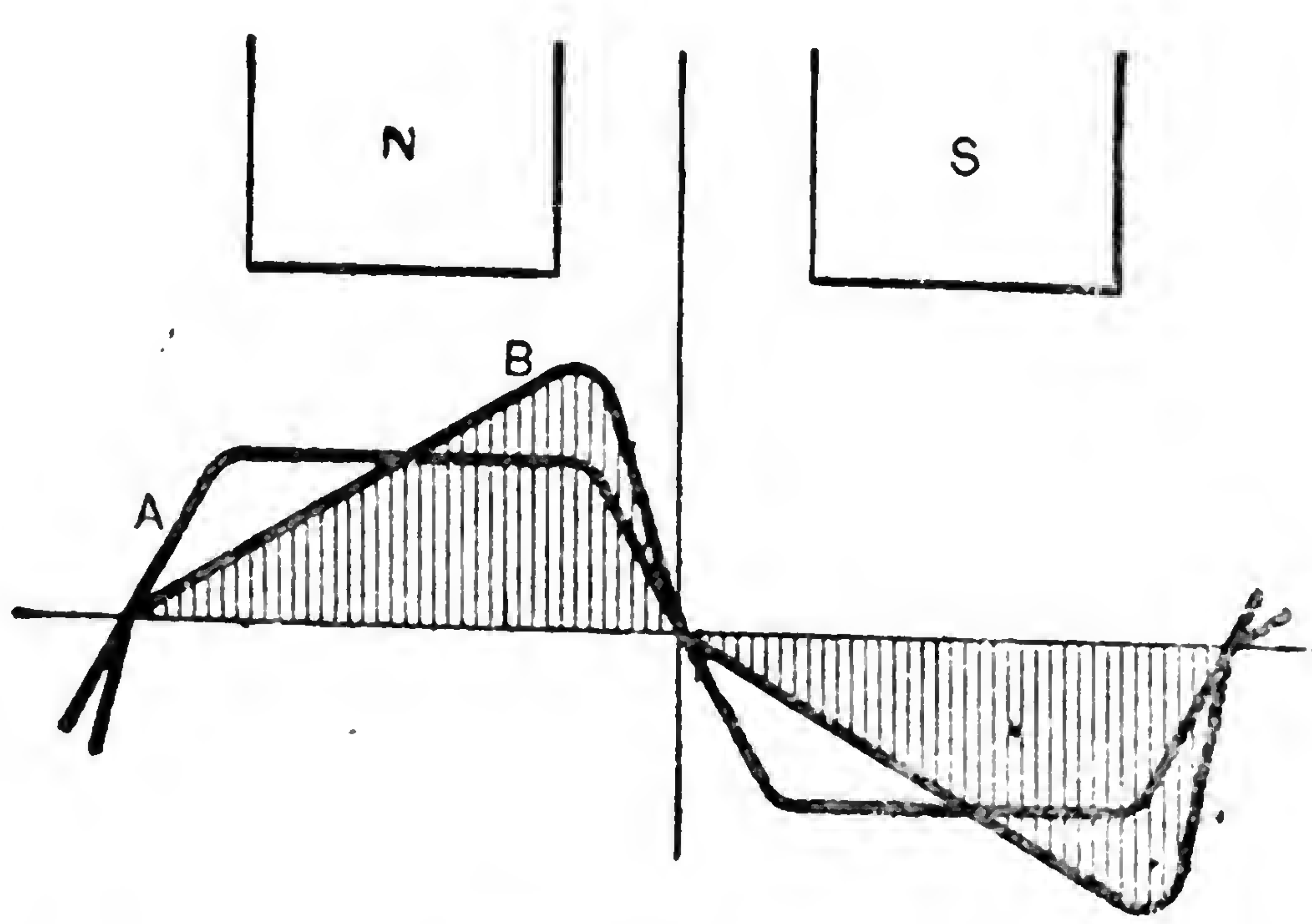


Fig. 16 (*e*)

The effect of this is seen by the e. m. f. curves shown in Fig. 16 (e) and (f). graph A of Fig. 16 (e) is the e. m. f. curve under no-load and B is that under load for a non-interpolar machine. Fig. 16 (f) shows the e. m. f. curve for an interpolar machine under load conditions. These graphs may also be considered to be those of flux distribution under the poles.

The use of interpoles has greatly increased the output capacity of d. c. machines. The student is advised to observe carefully and investigate the location, size, length of air-gap of interpoles on a machine. Shims are usually provided to alter the air-gap under these poles.

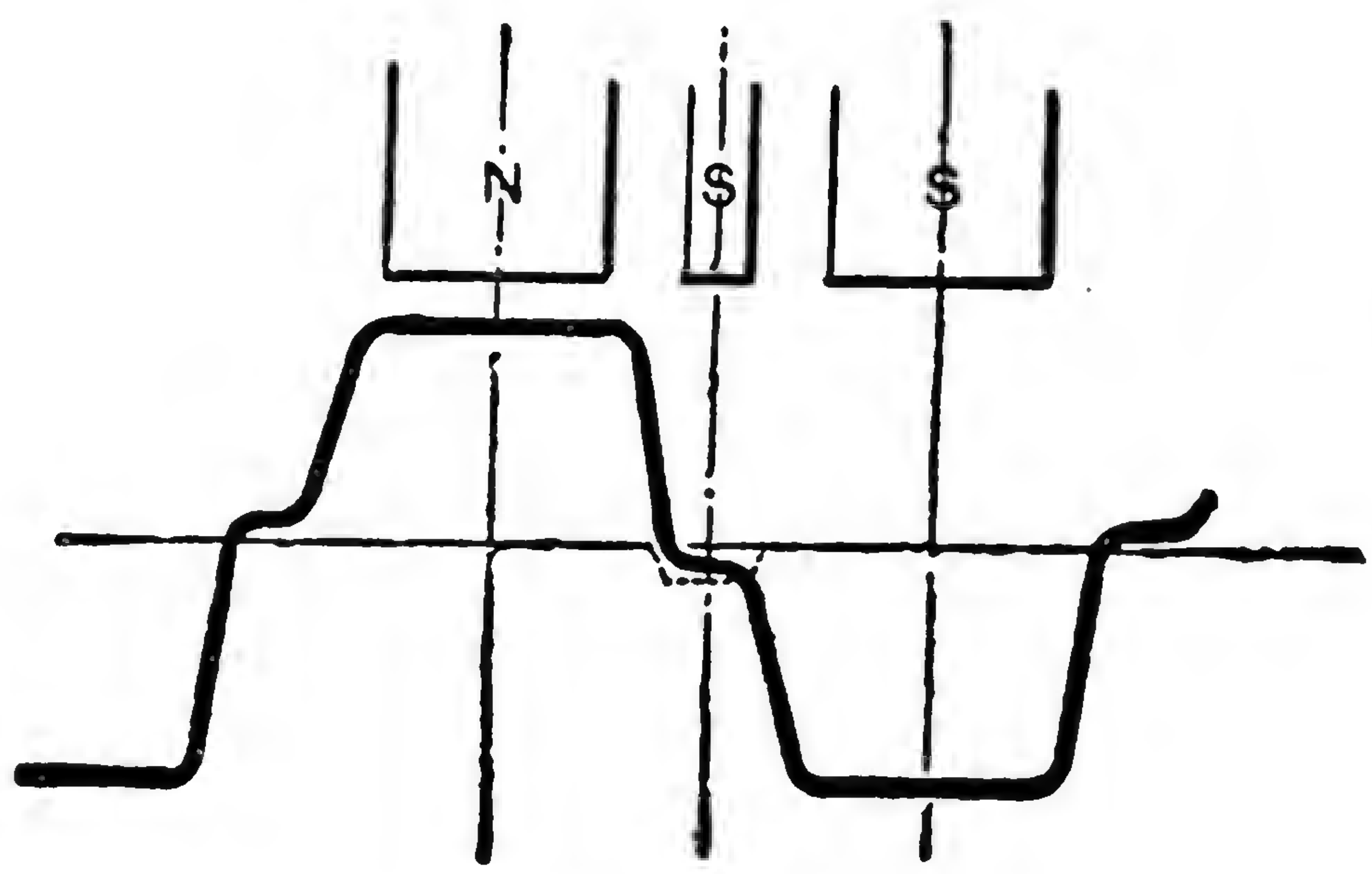


Fig. 16 (f)

Further, compensating winding or interpoles are external devices to help commutation. The armature winding may be so designed that the reactance voltage of a coil undergoing commutation can be reduced considerably by adopting (*a*) fewer turns per coil, since reactance varies as the square of the number of turns, and (b) by the use of "short-chord" winding.

In short chorded windings the back span y_B of the winding pitch is made less than the pole-pitch. If a brush spans more than two commutator segments (as usual), the conductors undergoing commutation simultaneously do not lie side by side but are separated by few conductors which carry the full current in the opposite direction.

8. Characteristic Curves : There are three important characteristic curves of a d. c. generator. They are :—

1. The *open circuit characteristic* (o. c. c.) or *magnetisation curve* or *no-load characteristic.* This is a graph showing the

relationship between the e. m. f. generated (E) and the field current (I_f) at the normal speed of the machine.

2. The *external characteristic* is a graph showing the relationship between the terminal voltage (V) and the load current (I_l).

3. The *total characteristic* is a graph showing the relationship between e. m. f. (E) and the armature current (I_a).

I. The Magnetisation Curve.

The O. C. C. is determined experimentally as follows :—

The shunt field (in the case of a shunt dynamo) is excited from an independent source of supply. Potentiometer control is used to obtain a wide range of values of I_f. See Fig. 17 which is the wiring diagram of connections for the experiment.

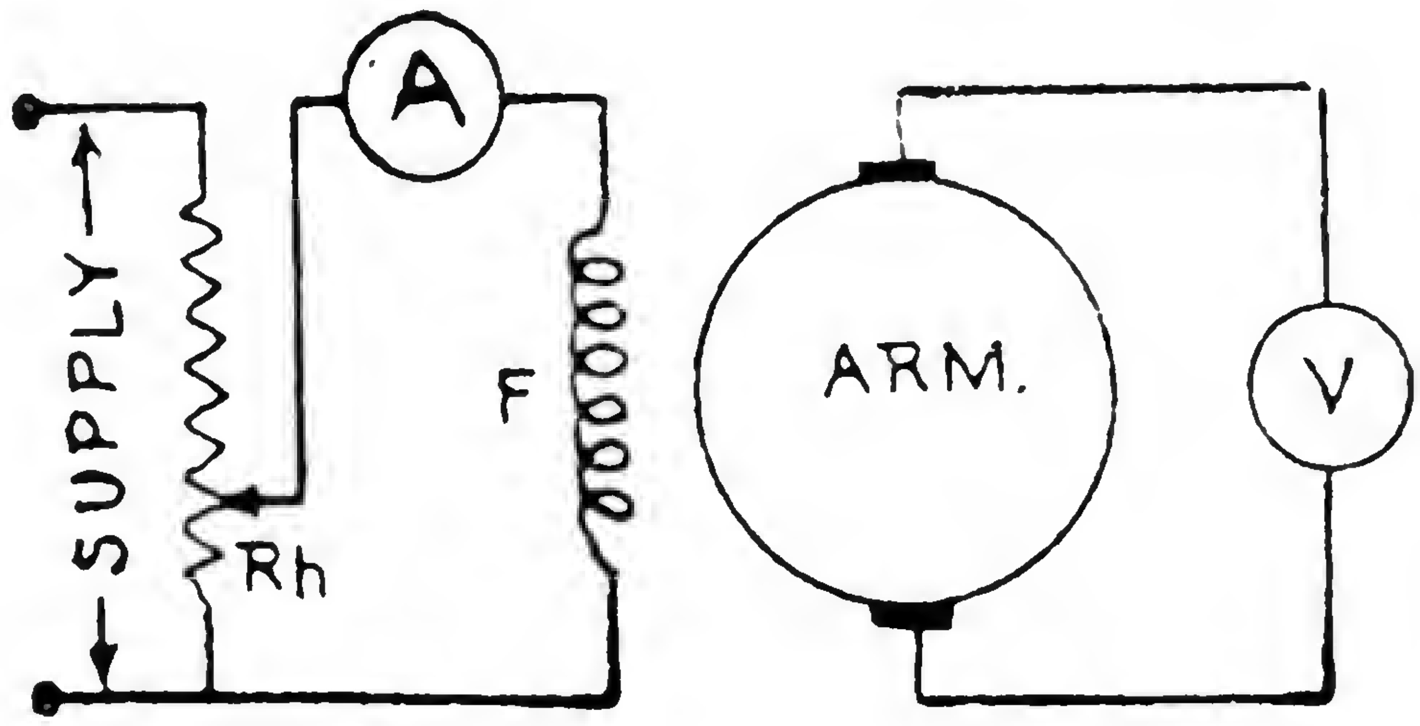


Fig. 17

The machine should be driven at its normal speed throughout the experiment. Due to residual magnetism in the poles a small e. m. f. is recorded on the voltmeter when $I_f = 0$. I_f is increased in suitable steps and the corresponding values of E as shown on the voltmeter are recorded and the curve is then plotted as shown in Fig. 18.

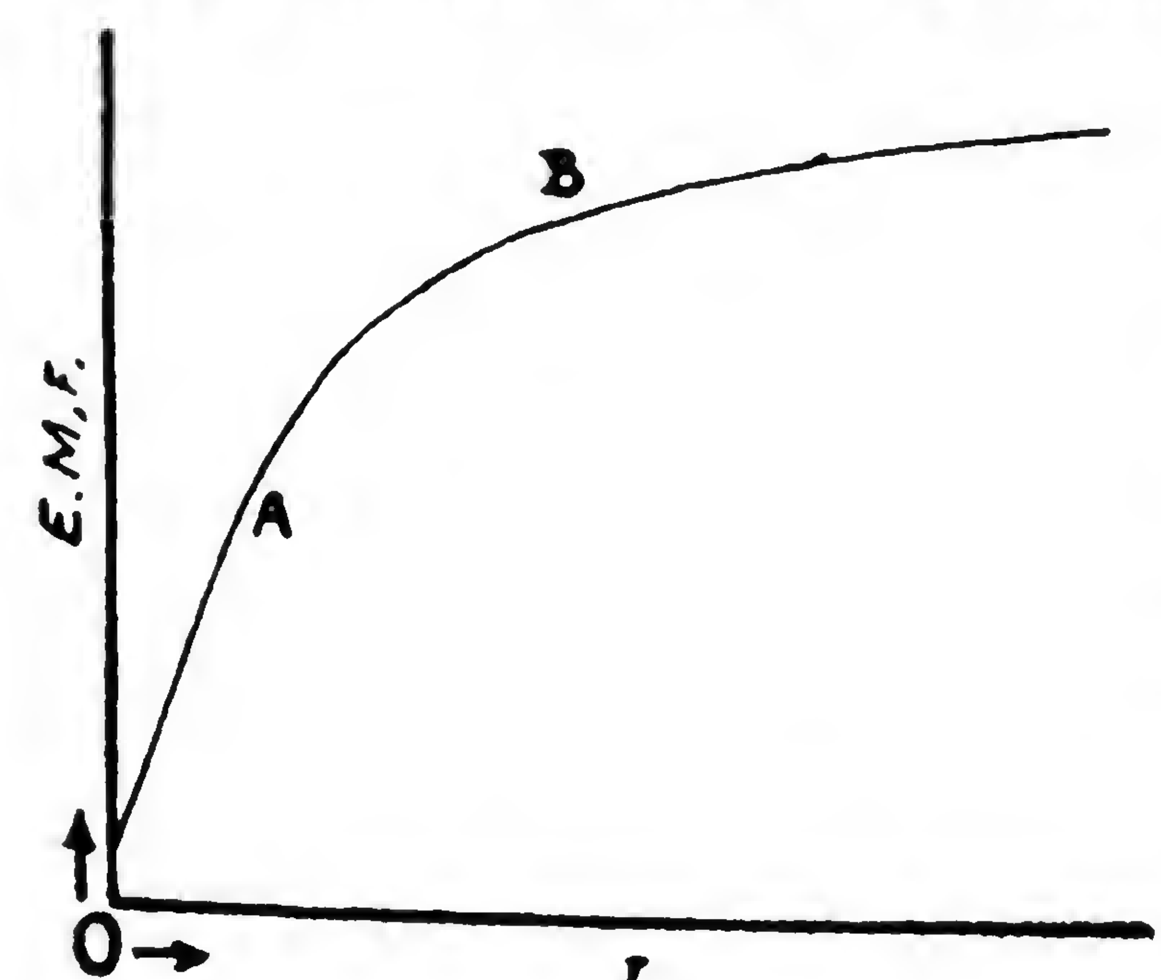


Fig. 18

The lower portion of the curve upto A is almost straight. Between A and B the curve

bends. This is the "knee" of the curve and the iron path gets saturated After B when the iron parts are fairly highly saturated the increment in E goes on at a much reduced rate.

To find the O. C. C. at any other speed when the O. C. C. at normal speed is given :

Procedure : Since $E \propto$ *speed* when the flux is constant, it is true that

$$\frac{E \text{ at normal speed } n}{E_1 \text{ at speed } n_1} = \frac{\text{normal speed } (n)}{\text{new speed } (n_1)}$$

$$\therefore \quad E_1 = E\frac{n_1}{n}.$$

Thus drawing a vertical line PQ cutting the O. C. C. at normal speed at L, see Fig. 19,

$$E_1 = QL \times \frac{n_1}{n} = QM.$$

Thus M is one of the points on the O.C C. at speed n_1 which is lower than n. A number of other points on the curve may be obtained as above.

If the speed is higher than normal the curve is situated higher than the O.C.C. at normal speed.

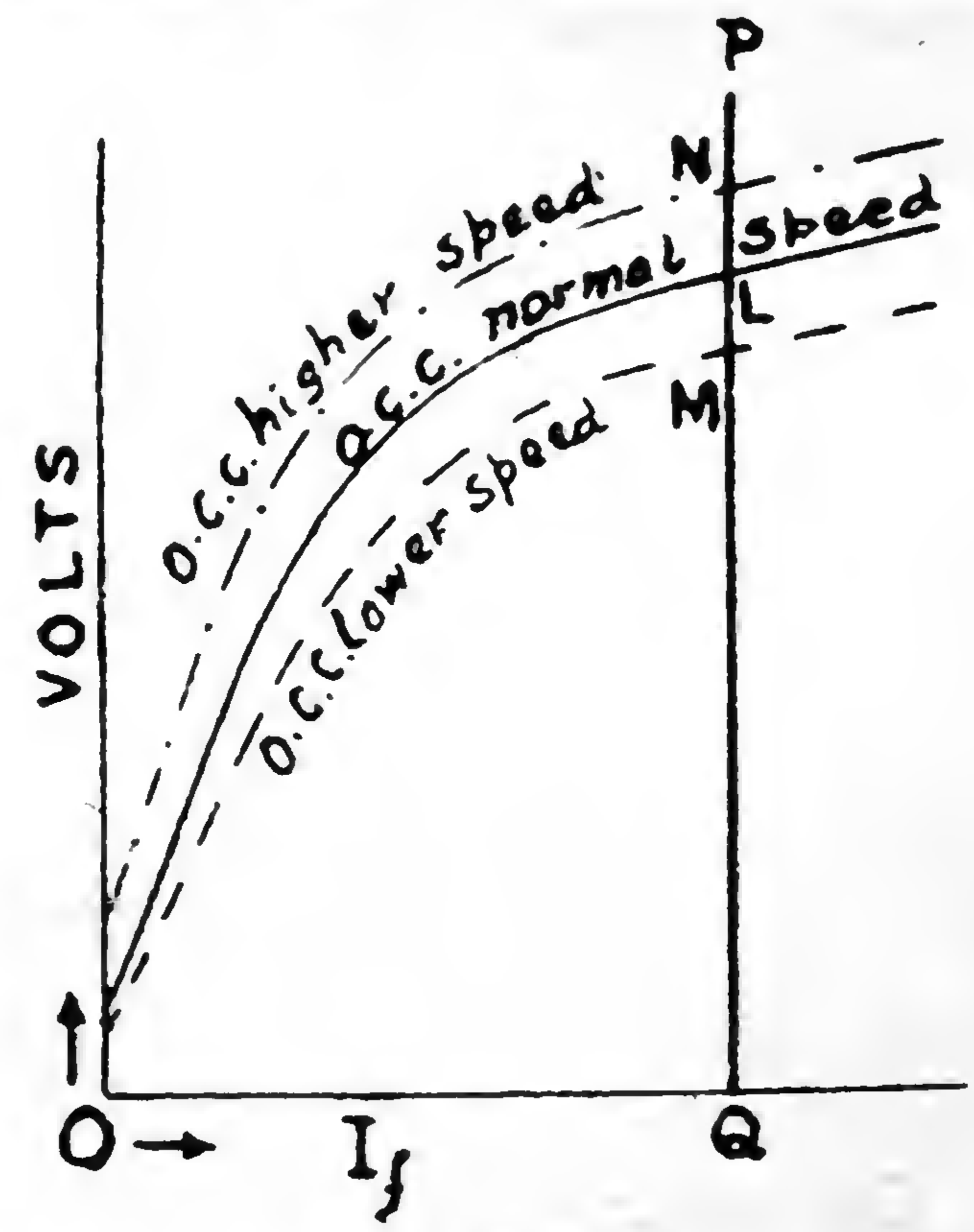


Fig. 19

Let the machine. now be self-excited, and let the field resistance including its regulator be R_{sh}. Draw the *resistance line* OA such that any point on this line gives R_{sh}, i. e. if x and y are the co-ordinates of the point, then $\frac{y}{x} = R_{sh}$. See Fig. 20 where the O. C. C. curve is already given.

If this line OA cuts the O. C. C. at a point P, the machine will build up its open circuit voltage equal to OE, the field current being equal to OB. This is readily seen by considering any lower value of field current such as OC. Erect a perpendicular from C to cut the resistance line at D and the O. C. C. curve at F. Here

$$CF = \text{e. m. f. generated}$$

CD $= R_{sh} \times I_f$, the drop in field system i. e. CD is the voltage required to drive the current through R_{sh}. But since CF > CD, F is an unstable point on the O. C. C.

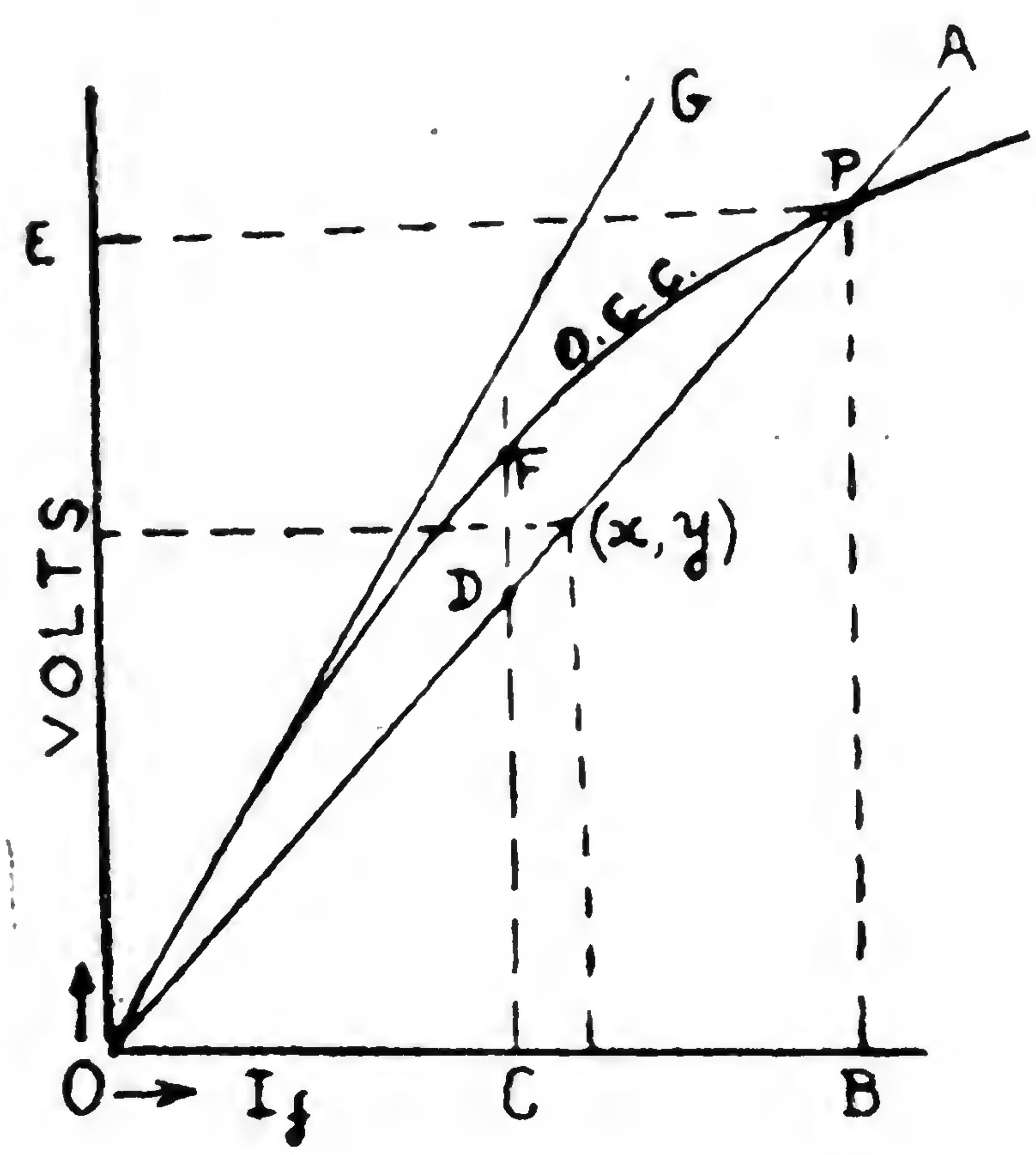


Fig. 20

So that at F the field current rises and goes on rising until the stable point P is reached. Here at P the e. m. f. generated $= I_f \times R_{sh}$. A resistance line which cuts the O. C. C. curve at a point, that point is the stable point. If the field circuit resistance is reduced the stable point moves further up the O. C. C. and the induced voltage increases. If the field circuit resistance is increased the stable point moves down the O. C. C. giving decreased induced e. m. f. Hence there is only one value of resistance whose slope will be a tangent to the initial portion of the O.C.C. and it will not cut, so to say, the curve. This resistance is called the *critical resistance of the shunt circuit for a given speed.* The critical resistance line is shown as OG, in Fig. 20, when the speed is normal. This is then the reason why all resistance from the regulator must be removed when starting a shunt generator.

Since there is a critical resistance for a given speed there is a *critical speed* for a given field resistance. For during starting period the machine does not begin to build up voltage until its speed has reached such a value that the resistance of the shunt field becomes the critical value. This speed is called the *critical speed.* A worked example below illustrates all that has been discussed so far.

Example : A 4-pole d. c. shunt generator at normal speed of 800 r. p. m. has an O. C. C. given by the following readings:

Field amperes:	1	2	3	4	5	6
Induced volts:	82·5	180	225	255	273	282

The resistance of the field windings is 50 ohms and that of the field regulator 25 ohms. Calculate

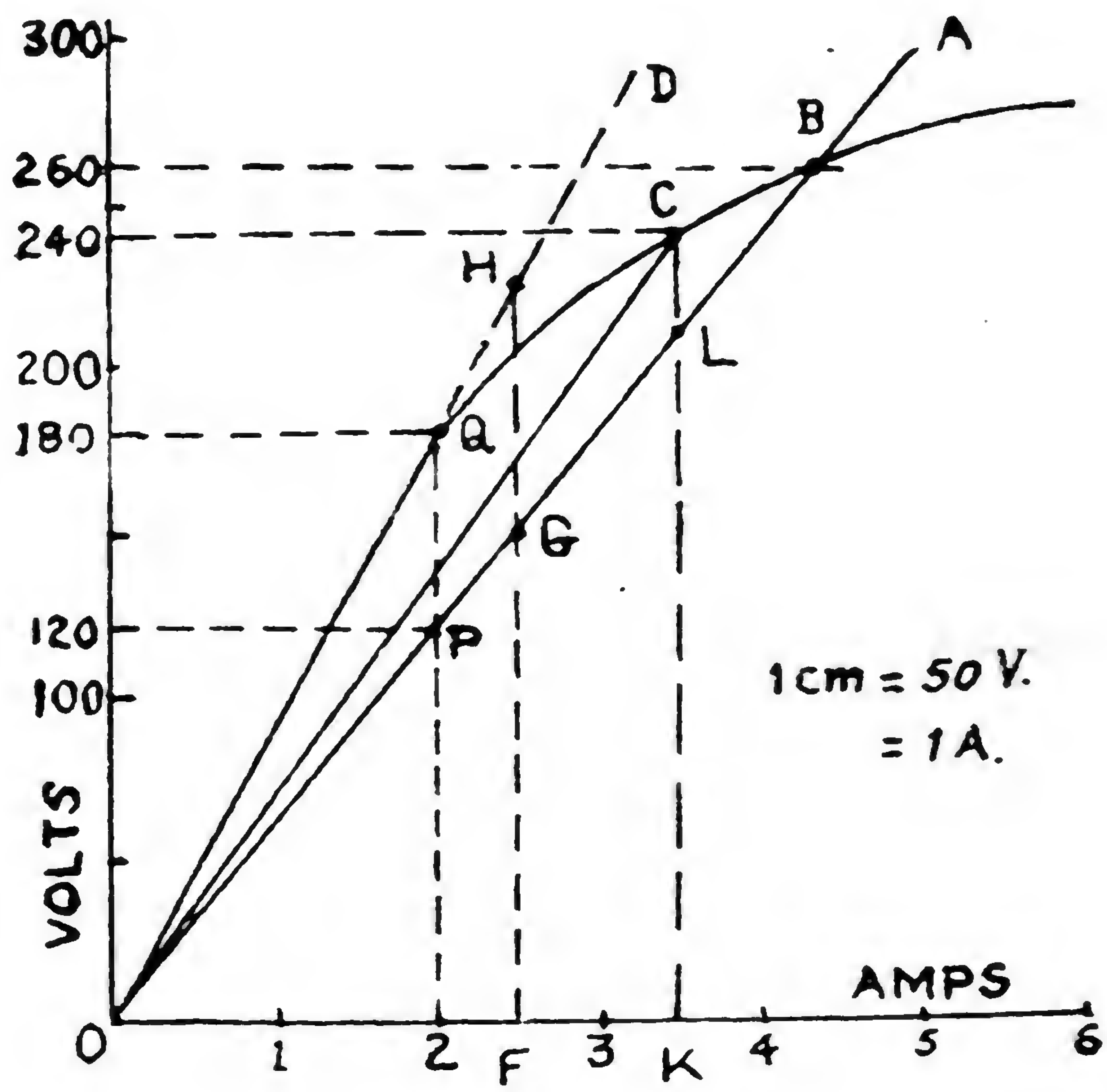
(*a*) the induced e. m. f. when $R_{sh} = 60$ ohms.

(*b*) the value of R_{sh} to reduce the induced e. m. f to 240 volts

(*c*) the critical value of shunt field resistance

(*d*) the critical speed when $R_{sh} = 60$ ohms

(*e*) the lowest speed at which an e. m. f. of 240 volts can be obtained when $R_{sh} = 60$ ohms.



FIELD AMPERES
Fig. 21

Solution; (*a*) The O. C. C. curve is plotted from data. The resistance line of 60 ohms is drawn by taking a point P whose co-ordinates are $x = 2$ amps and $y = 120$ V. $\frac{y}{x} = \frac{120}{2} = 60$ ohms. The line is OA passing through origin and P cutting the O. C. C. at B. This gives the induced voltage of *260 volts* when measured on the ordinates.

(*b*) A line from 240 volt marking cuts the O. C. C, at C. Join C to O. This is the slope of the required resistance.

$$R_{sh} = \frac{240}{3{\cdot}54} = 68{\cdot}5 \text{ ohms.}$$

(*c*) Draw OD tangent to the O. C. C. as shown. This is the critical resistance line. A point Q on this line has coordinates

180 volts and 2 amperes. Hence

$$\textit{critical resistance} = \frac{180}{2} = \textit{90 ohms.}$$

(d) Taking any convenient point on OD, such as H, drop a perpendicular on the x-axis at F, cutting OA at G. By measurements FG = 150 volts and FH = 225 volts. Hence

$$\frac{FG}{FH} = \frac{\text{critical speed } n_c}{\text{normal speed } n}$$

$$\therefore \quad n_c = \frac{150}{225} \times 800 = \textit{533 r. p. m.}$$

(e) From C drop a perpendicular on the x-axis at K cutting OA at L. By measurement KL = 210 volts and KC = 240 volt. Hence

$$\frac{KL}{KC} = \frac{n_2}{800} = \frac{210}{240}$$

$$\therefore \quad n_2 = 800 \times \frac{210}{240} = \textit{700 r. p. m.}$$

II External and Total Characteristics :

A. *Shunt Machines* :

A load test is performed on a shunt generator and readings of terminal voltage and load current are recorded at regular intervals from no load to 120 % full load. The speed is kept constant and the field circuit resistance is not disturbed during the test. At the end of the test when the machine is hot, measurements are made of field and armature resistances (field resistance in this case includes the resistance of its regulator). Load amperes plotted against terminal volts gives the external characteristic. Curve (i) Fig. 22. The reading are :

Load current (amps.) :	0	5	10	15	20	25	30
Terminal voltage (volts) :	120	118	115	111	106	99	90

armature resistance = 0·3 ohm.

field circuit resistance = 50 ohms.

Curve (i) is the external characteristic. OA is the armature resistance line and OB is the field circuit resistance line. Choosing any point P on curve (i), we proceed to find a corresponding point P′ on the total characteristic curve.

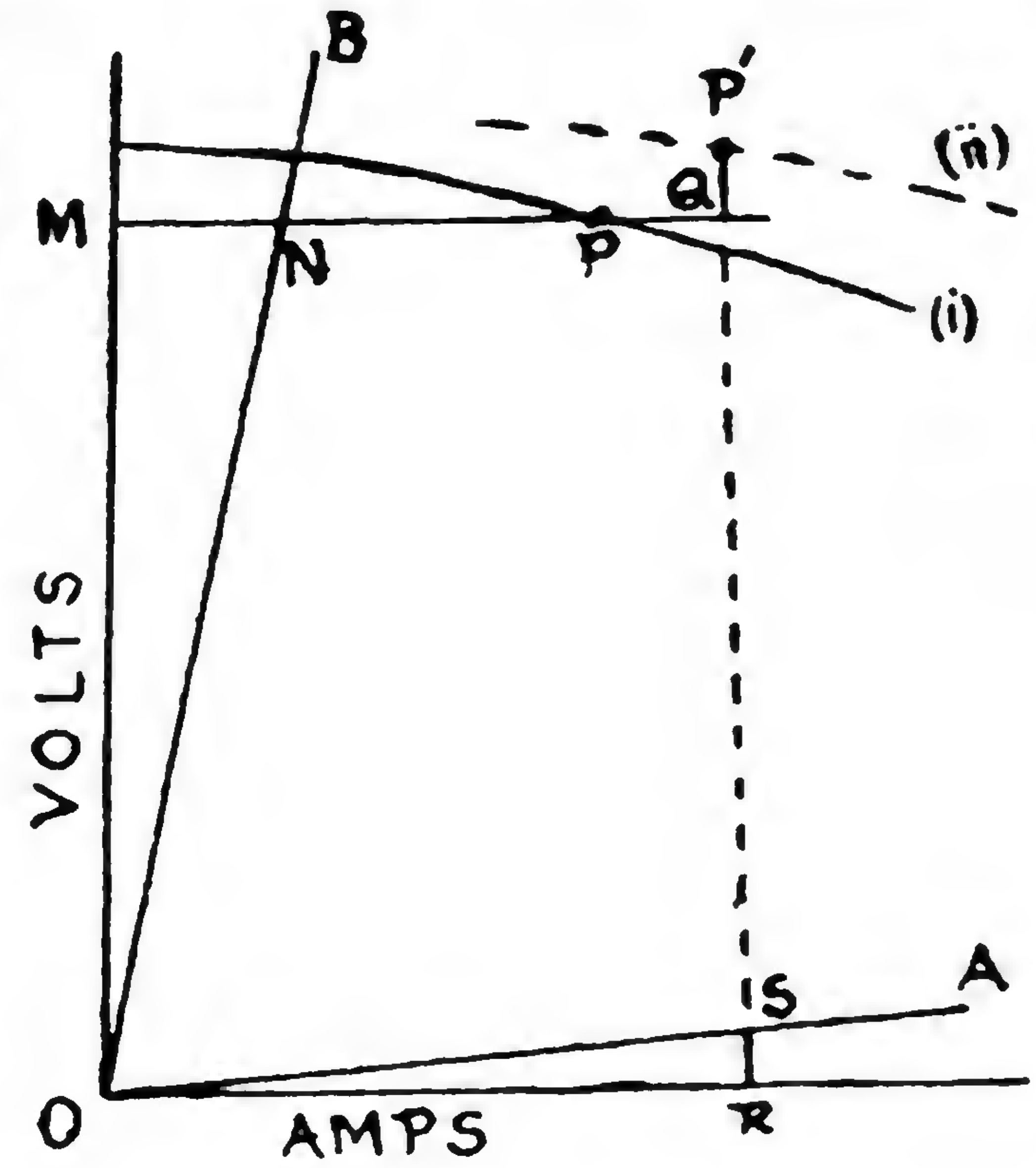


Fig. 22

Draw PM, cutting OB at N. MN is the field current. Produce MP to Q such that PQ = MN. Drop a perpendicular QR from Q cutting OA at S. Then SR = armature drop. Extend RQ to P′ making QP′ = SR. Then OR is the armature current and P′R is the induced e. m. f. Thus P′ is one point on the total characteristic curve. Similarly finding a few more points gives the total characteristic curve. This method ignores the effect of armature reaction.

B. *Series Machines:* The O. C. C. of a series generator is obtained by separately exciting the series field from a source of low voltage, since the series resistance is very low, less than 0·5 ohm. The speed is kept constant.

When a load test is performed on series machines, self-excited of course, the curve obtained is shown by curve (ii) in Fig. 23. This is the external characteristic. n is the short circuit point.

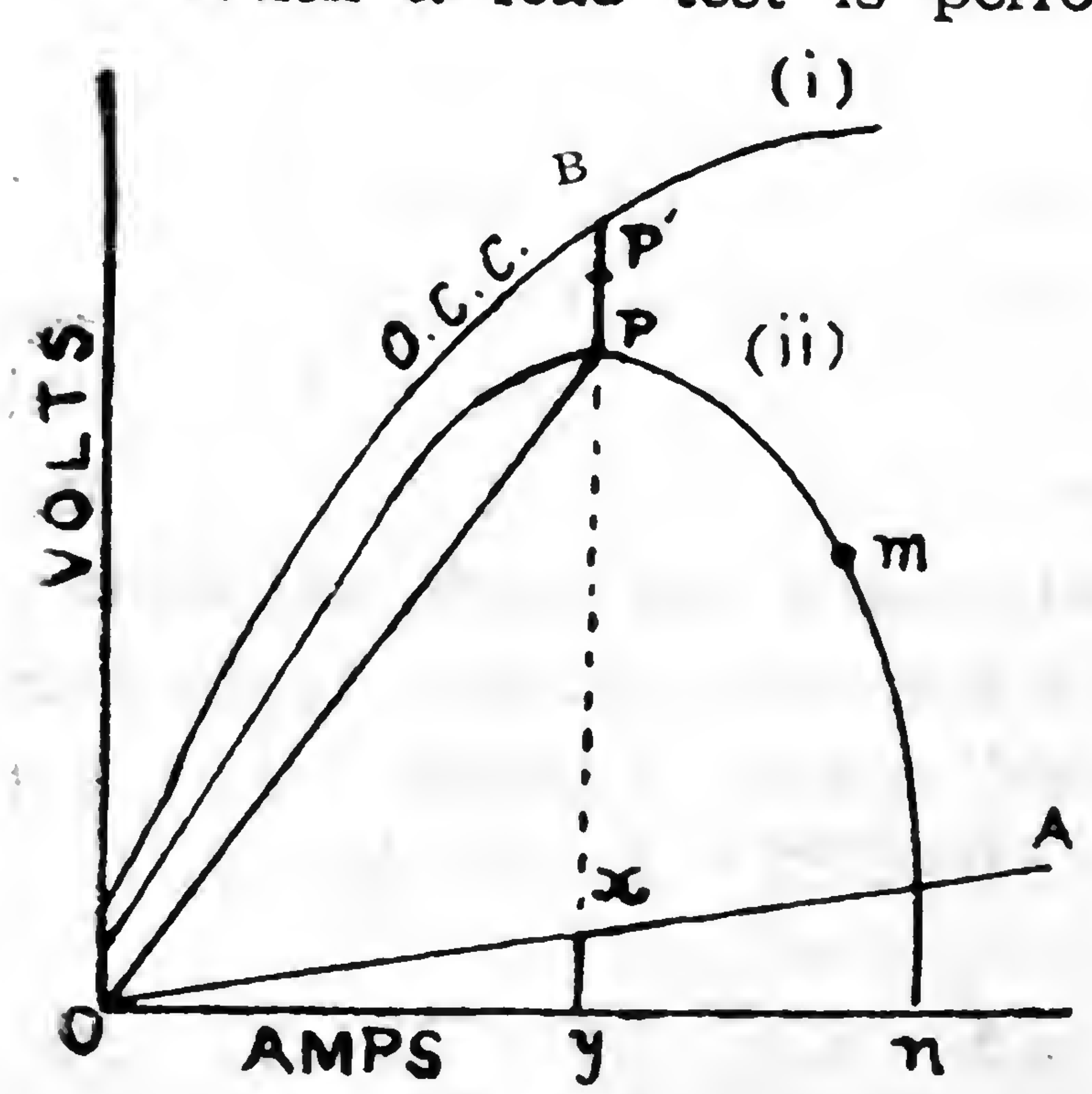


Fig. 23

Taking any point P on the external characteristic, the corresponding point P′ on the total characteristic is found as follows:

Draw OA which represents a line whose resistance = $(R_a + R_{se})$. Drop a perpendicular from P on to the x-axis at y cutting OA at x. Then xy is the volts drop in the armature and the series field. Add this drop to yP produced to P′ so that PP′ = xy. Then P′ is the corresponding point on the total characteristic. If yP is still produce

further to meet the O. C. C. at B, then BP′ is the volts lost due to armature reaction.

OP is the slope of the load resistance $= \frac{Py}{Oy} = \frac{\text{volts}}{\text{amps}}$. If the machine is worked on the drooping portion of the curve, and if so designed, the series generator will give approximately a constant current, irrespective of the resistance value of the load. Series generators of this type are not used to-day, but many years ago they were used for lighting arc lamps in series for street lighting.

C. *Compound Machines*: The resultant e. m. f. (or the terminal voltage) of these machines depends upon the relative strength of the flux due to series winding with that of the shunt field flux. But since both have drooping characteristic the resultant also has a drooping characteristic.

In Fig, 24 PQ is the shunt field characteristic. OM is that due to the series field and ON is also that due to a series field but in this case the field is stronger. The resultant of PQ and OM is the curve PR and the resultant of PQ and ON is the curve PS. AB is the full load line, i. e. OB is the full load current of the machine, and the no load voltage is equal to OP.

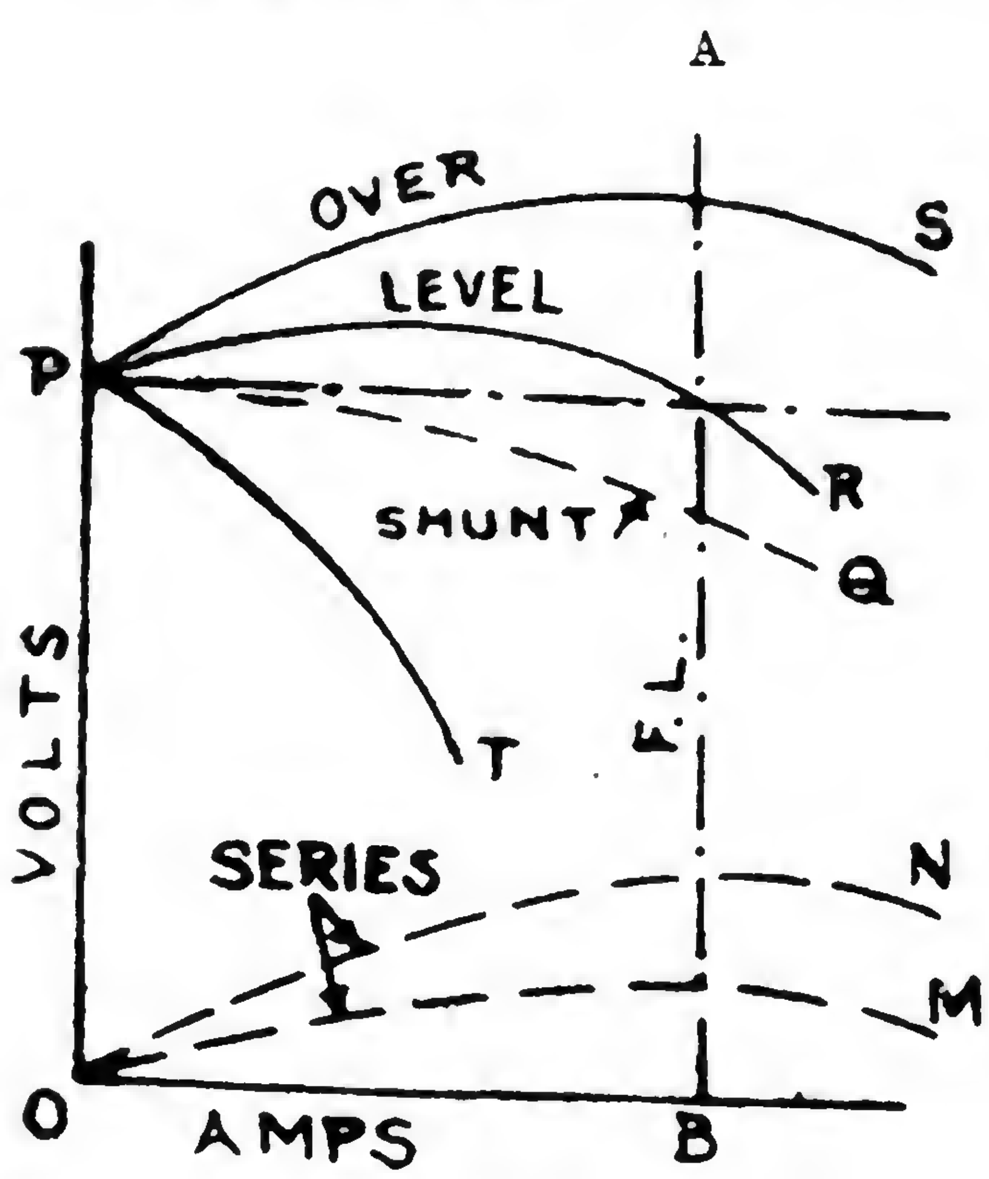


Fig. 24

(*a*) When a compound generator gives on full load the same terminal voltage as on no load, the machine is said to be *level compounded*, curve PR.

(*b*) When a compound generator gives on full load a greater terminal voltage than on no load, the machine is said to be *over compounded*, curve PS.

(*c*) In generators used for arc welding, the series field m. m. f. opposes those of the shunt field. The machine is then said to be *differentially* compounded. To strike the arc, voltage required is higher than that required to maintain the arc. Curve PT is the characteristic curve of such machines.

9. Effects of Reversal of Rotation: For building up a voltage it is necessary that all generators should have residual magnetism in their field poles. This is particularly so in self-excited machines. Since the direction of this initial flux is fixed, the polarities of the terminals of the d. c. machine is also fixed for a particular direction of rotation.

If the direction of rotation is reversed, the residual magnetism of poles is destroyed and the machine will not "build up" any voltage. Hence if the direction of rotation is to be changed, the field winding connection must be so changed that the field current instead of dying out will build up, i. e. the direction of current in the field windings should not change.

If, however, the residual magnetism in field poles is destroyed, a current is passed through the field coils for a very short time. This is called *flashing*. At the time of flashing, the machine should be at rest and the armature circuit open.

10. Parallel Operation of Generators: Power Supply Undertakings are expected to give power to its consumers at all hours of the day and night. Moreover, during certain hours the load reaches peak values and it is then necessary to use more than one generator. These generators must run in parallel. It is also necessary to shut down a machine after it has been running, say, for about 10 hours, so as to enable the staff to inspect it and to do cleaning and minor repairs if necessary.

Any abrupt change in the load should not upset the stability of parallel operation, i. e. the machines should possess an inherent ability to automatically adjust themselves to variations of load, be they gradual and small or sudden and large. The parallel operation of series machines will be taken up first, and later on those of shunt and compound machines.

A—Series Generators in Parallel: As a rule, series generators are hardly used by Supply Undertakings in their Power Houses. But in electric traction work these are used as self-excited generators for "electric braking" when the train is going down an incline. Tram-cars and electric locomotives have series wound motors. They are always used in pairs, and on a locomotive identical motors are used, hence their characteristics are also identical.

The series generators are inherently unstable when operating in parallel. Therefore, for satisfactory operation an *equaliser connection* is necessary. This connection puts the series fields of the two generators in parallel so that the load current divides itself equally in each series field winding.

Consider the two series generators working in parallel as shown in Fig. (25a). Without the equaliser connection, the flux of each machine will vary as its armature current. When the two machines are working at a point on the rising portion of their characteristics, any increase in armature current will produce a corresponding increase in their terminal voltages. If now due to some accidental cause, an increase of current in machine I will result in machine II giving up its load by an equal amount, so that the terminal voltage of machine II falls. This will further reduce its armature current and so on. Once this action starts it will continue until machine I will take all the load and its armature may get overloaded. The armature current of machine II will not only become zero but will reverse in direction. This reversal will further aggravate the situation, for then the e. m. fs. of the two machines instead of being in opposition to each other will be additive, and cause a very heavy current to circulate through both the armatures. The equaliser connection prevents this state of affairs.

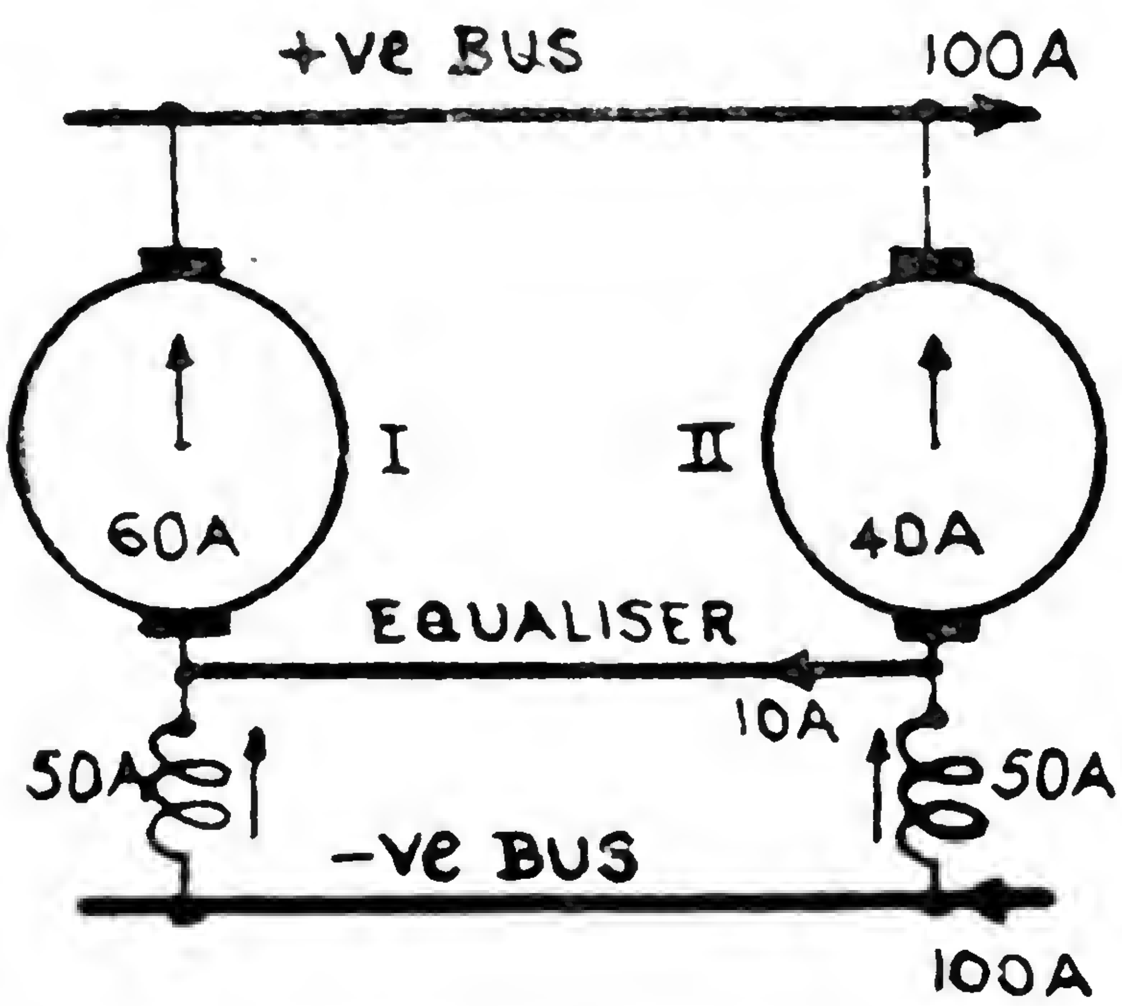


Fig. 25 (a) Parallel Operation

Should there be only two series generators running in parallel, a better practice would be to cross connect the two fields, as shown in Fig. 25 (b). Then, should the current in one machine fall, the field current, and consequently the flux of the other machine, automatically weakens. This ensures stable operation more positively.

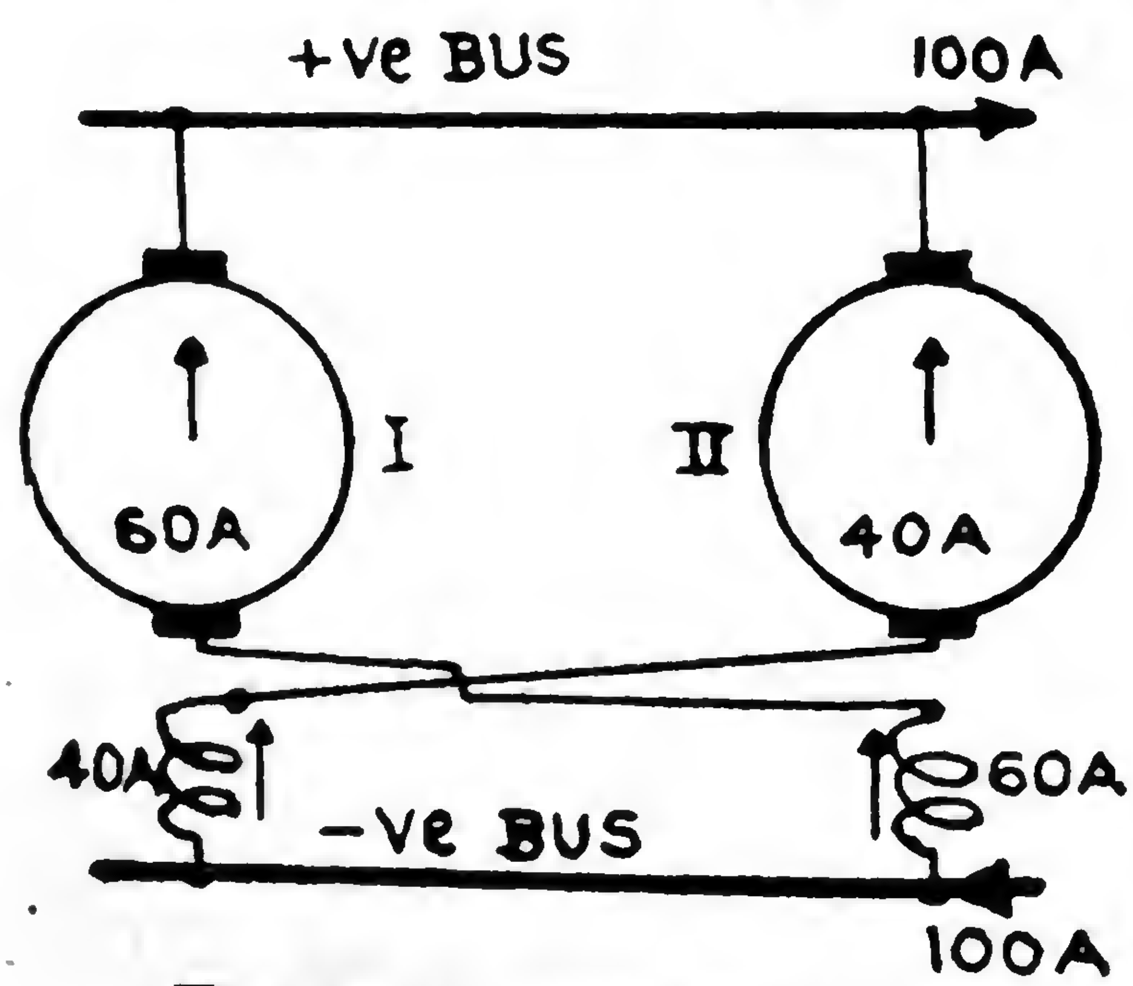


Fig. 25 (b) Series Generators

B–Shunt Generators in Parallel: Because of their drooping charateristics shunt generators behave extremely well when operating

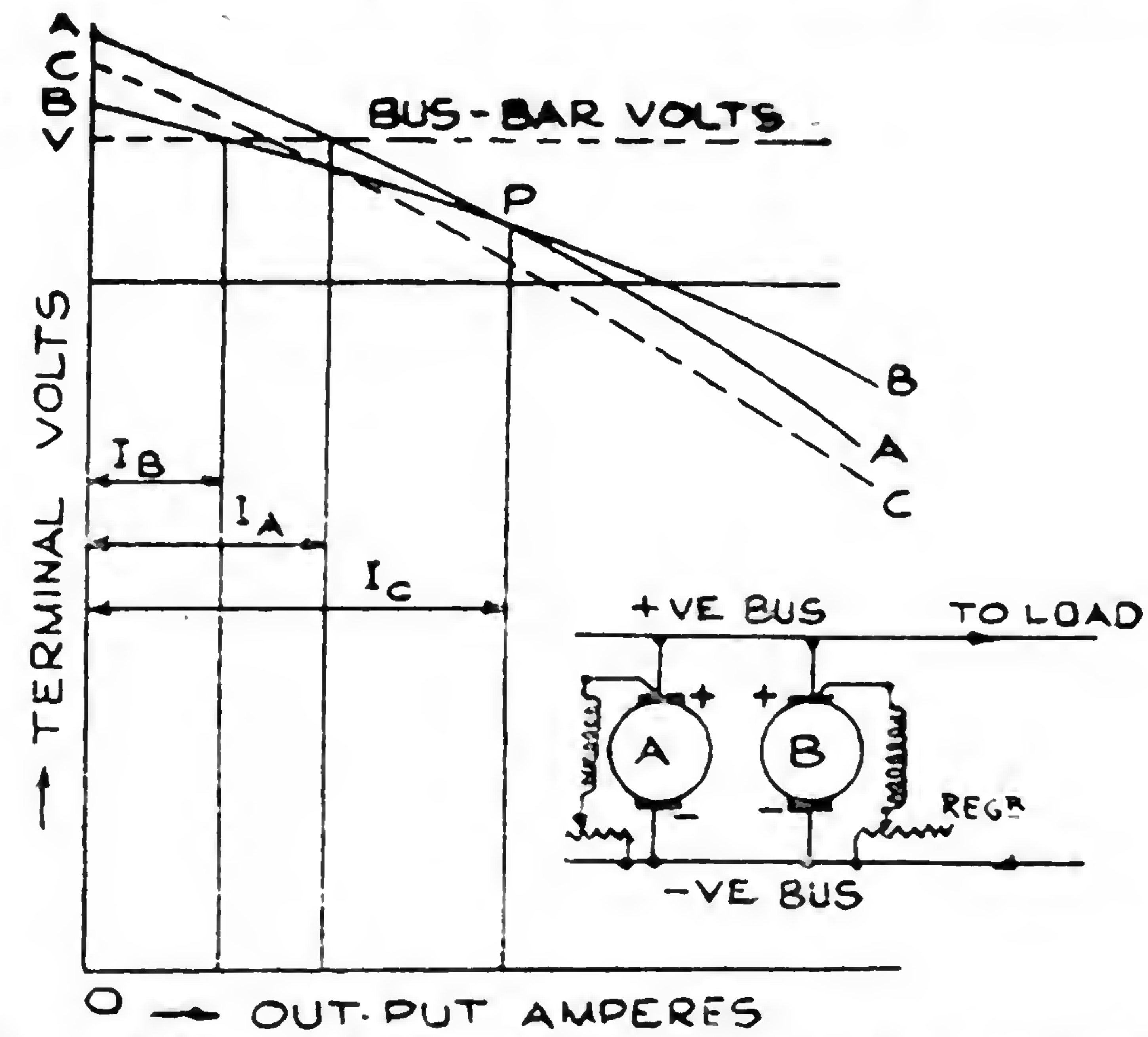


Fig. 26 Parallel Operation: Shunt Generators

in parallel. The parallel operation particularly becomes ideal if the characteristics of all the machines are similar, i. e. the voltage drops are the same at $\frac{1}{2}$, $\frac{3}{4}$, full load and $1\frac{1}{4}$ full load. This makes the load sharing equal for each machine. But even if the characteristics are not similar, the machines do share the load somehow and do not upset the working stability as will be seen from a consideration of conditions shown in Fig. 26.

Two shunt machines, *A* and *B* are running in parallel to supply a common load. A's characteristic is shown by the line *AA* and that of *B* by the line *BB*. The induced e. m. f. of *B* is less than that of *A*. If the bus-bar voltage is *OV* and the machines supply a certain load current, then *A*'s share is I_A amperes and *B*'s share is I_B amperes, I_A being greater than I_B. If the load is increasing *A* continues to share larger amount of load than *B* up to the point *P* which is the point of intersection of the two characteristic curves, *AA* and *BB*. At *P* the two machines share the load equally, the share of each being I_C and the total external load is $2I_C$ amperes. If the load increases still further *B* will take up a larger share of the load than *A*.

If due to some accidental cause, the speed of the prime mover of *A* falls slightly, its characteristic curve will assume the position shown by the dotted line *CC*. *A* will give up some of its load and *B* will carry a little more load. There will then take place a slight

change in the bus-bar voltage, but the two machines will continue to operate in parallel quite satisfactorily. The slight change in the bus-bar voltage may be taken care of by the adjustment of field regulators of *both* machines.

C–Compound Generators in Parallel: Since compound generators have series field winding, in addition to the shunt field

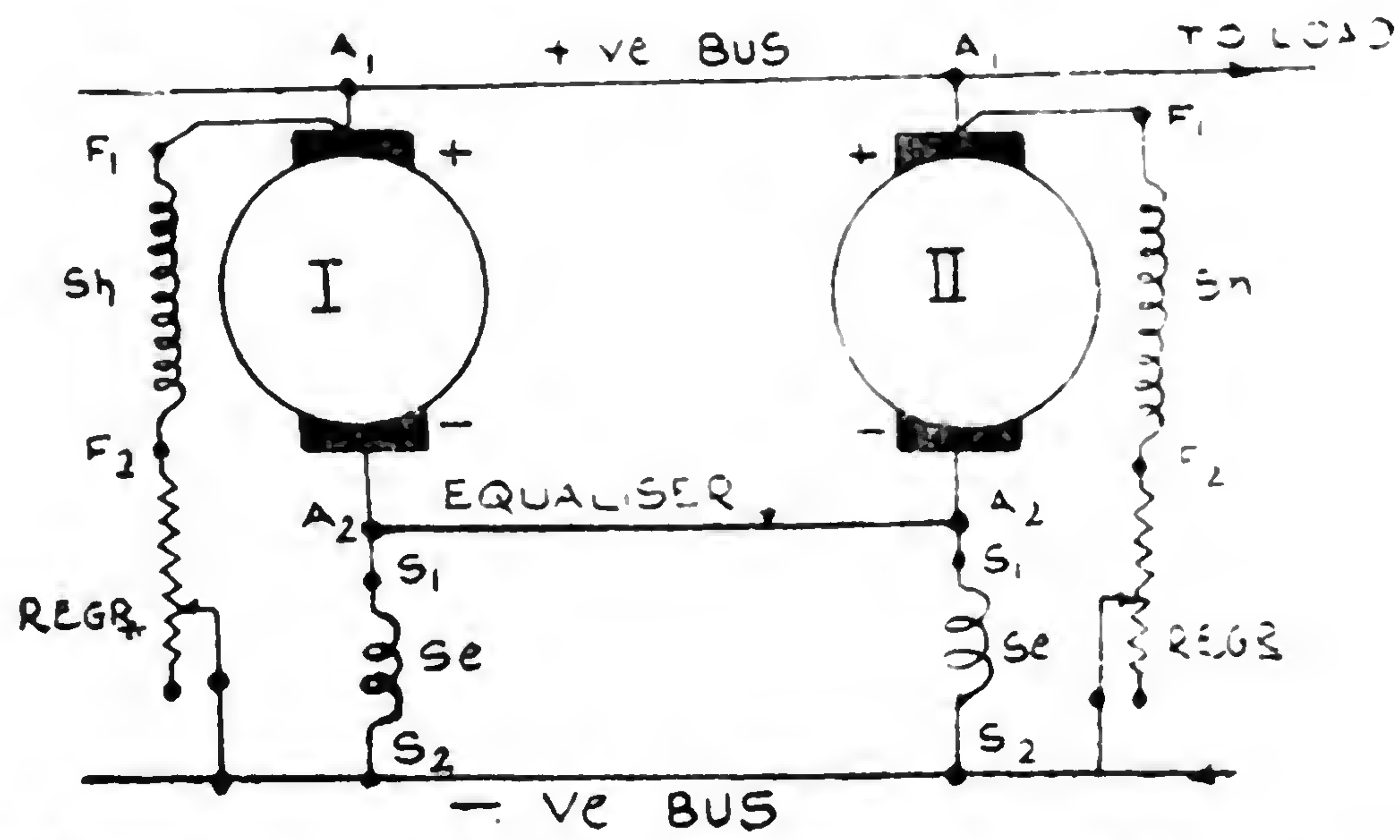


Fig. 27 Parallel Operation : Compound Generators.

winding, an equaliser connection is necessary. The method of connection and the purpose are the same as those for the series generators. See Fig. 27. The equaliser connection must be made by a thick copper bar whose resistance must be less than the resistance of the series fields. The student is referred to a very useful book by Dover, named "Power Wiring Diagrams," wherein complete standard connections are given for parallel operation of various types of generators.

Example: Two shunt generators, A and B, run in parallel and supply a total external load of 1200 amperes. The induced e. m. fs, of A and B are 475 and 480 volts respectively. Calculate the bus-bar voltage and the load shared by each.

The armature and field resistances of A are 0·04 and 25 ohms respectively and those of B are 0·045 and 30 ohms,

Solution: Let the bus-bar voltage be V volts and I_A and I_B the currents of A and B which supply the load

$$\therefore \quad I_A + I_B = 1200$$

$$\text{Field current of } A = \frac{V}{25}$$

$$\text{and field current of } B = \frac{V}{30}$$

Total armature current of A $= \left(I_A + \frac{V}{25}\right)$ amperes,

$\therefore$ total armature drop in $A = 0{\cdot}04\left(I_A + \frac{V}{25}\right)$ volts.

Similarly total armature drop in $B = 0{\cdot}045\left(I_B + \frac{V}{30}\right)$ volts.

$$\therefore \quad V = 475 - 0{\cdot}04\left(I_A + \frac{V}{25}\right) = 480 - 0{\cdot}045\left(I_B + \frac{V}{30}\right).$$

Taking the first relationship and solving for V

$$V = 475 - 0{\cdot}04 I_A - \frac{0{\cdot}04}{25} V$$

$$\text{i. e.} \quad V = \frac{11875 - I_A}{25{\cdot}04}$$

Substituting the value of V in the second relationship we have:

$$\frac{11875 - I_A}{25{\cdot}04} = 480 - 0{\cdot}045\left(I_B + \frac{V}{30}\right)$$

$$= 480 - 0{\cdot}045\, I_B - \frac{0{\cdot}045}{30}\left(\frac{11875 - I_A}{25{\cdot}04}\right)$$

multiplying throughout by $25{\cdot}04$ and substituting the value of $I_B = 1200 - I_A$

$$11875 - I_A = 480 \times 25{\cdot}05 - 0{\cdot}045 \times 25{\cdot}04\,(1200 - I_A)$$
$$- 0{\cdot}0015 \times 11875 + 0{\cdot}0015 I_A$$

Solving for I_A we have

$$2{\cdot}1253\, I_A = 1190{\cdot}15$$

$$\therefore \quad I_A = 560 \text{ amperes}$$

$$\therefore \quad I_B = 1200 - 560 = 640 \text{ amperes}$$

And the bus-bar voltage is

$$V = \frac{11875 - 560}{25{\cdot}04} = 451{\cdot}88 \text{ volts.}$$

21. Fields of Application: Separate excitation requires an independent source of supply and is more expensive than self-excitation. For this reason it is only used where self-excitation will give unsatisfactory service, as in the case of Ward Leonard System. (See next Chapter). Prompt response to changes of field regulators is obtained when the field is separately excited. A generator whose

sole function is to supply the field current of other machines is called an ***exciter***.

Shunt and compound generators are ordinarily used for supplying lighting and power loads. But where the variation of load is abrupt and large, compound generators are exclusively used. Overcompounded generators are used where feeders are long, so that the drop in feeders is compensated by the rise in voltage as the load current increases.

Series generators have a very limited field of utility. They are used for arc welding and for arc lighting. They are giving very satisfactory service in the Thury high voltage constant current system. This system is used in France, the maximum voltage is 60000 volts and the current is constant at 75 amperes.

CHAPTER V

D. C. MOTORS

1. Motor Action: When a current is passed in a d. c. armature lying in a magnetic field, the armature conductors experience a force which tends to rotate the armature. Each conductor which lies across the magnetic field contributes to the force. The magnitude of the force experienced by each conductor is

$$\text{force} = \frac{B.\,l.\,I}{10} \text{ dynes}$$

where B = flux density in gauss, l = active length of conductor and I is the current in the conductor in amperes.

The direction of force is given by Flemings Left-hand Rule. Fig. 1 shows a portion of an armature. Conductors under N-poles carry current in one direction and those under S-poles in the opposite direction. By applying Fleming's Left-hand Rule, it is seen from the figure that the direction of force on all the conductors is the same. If the resisting torque is less than the combined torque due to all the conductors the armature will rotate.

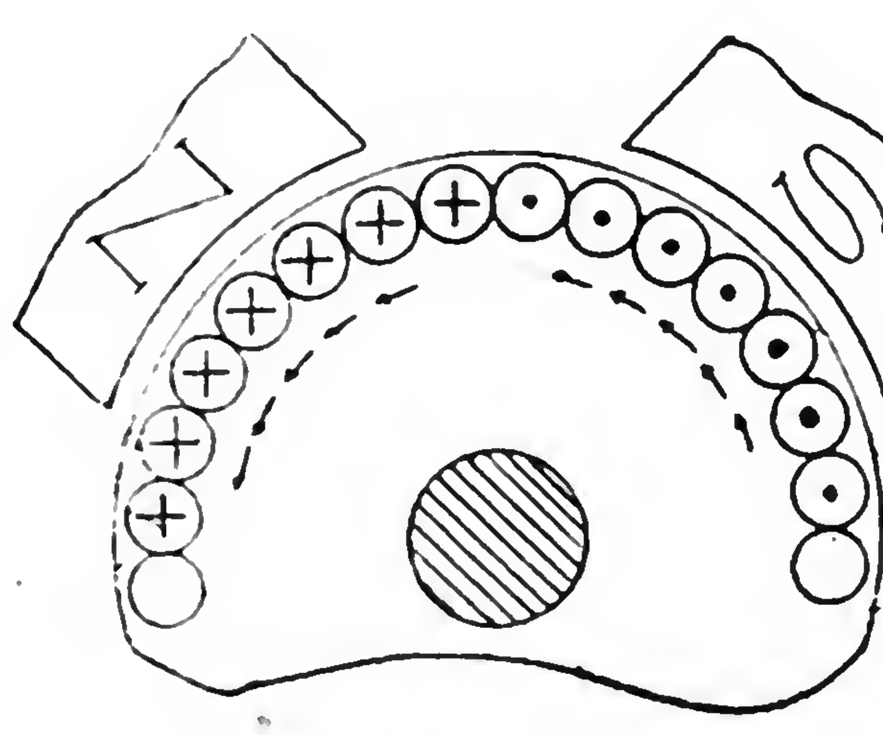


Fig. 1.

Torque is a term expressing the *turning* or *twisting moment* of a force about an axis. It is a measure given by the product of force multiplied by the radius at which the force acts. The c. g. s. unit is the dyne-centimeter and the f. p. s. unit is the pound-foot (lb.-ft.).

2. Back E. M. F. or Counter E. M. F.: When the armature of a d. c. motor rotates, an e. m. f. is induced in the armature conductors, just as in the armature of a generator. The direction of this induced e. m. f. is in opposition to the applied voltage. Hence it is called ***back e. m. f.*** or ***counter e. m. f.*** The symbol used is E_b. The armature current I_a is due to the difference in these two voltages and E_b is always less than V, the applied voltage. If R_a is the total

resistance of the armature, we have

$$I_a = \frac{V - E_b}{R_a} \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

solving for E_b

$$E_b = V - I_a R_a \ldots \quad \ldots \quad \ldots \quad (2)$$

Multiplying both sides of Eq. (2) by I_a, we have

$$E_b I_a = V I_a - I_a^2 R_a \quad \ldots \quad \ldots \quad (3)$$

This is a power equation, where

$V I_a$ = power supplied to the armature in watts,

$I_a^2 R_a$ = power lost in the armature (copper loss) in watts.

Therefore $E_b I_a$ must be the *mechanical power developed* in the armature in watts. All this power is not available at the shaft of a motor, a part of it being required to overcome the iron and friction losses.

3. Torque: In Fig. 2, a force F is acting on the rim of a cylinder of r radius and the drum makes n revolutions per minute. If F is in lb. and r in feet, *work done per minute*

$$= 2\pi n F r \text{ ft-lb.}$$

and *torque* $= F r$ lb.-ft.

$\therefore$ *Work done per minute* $= 2\pi n T$ ft-lbs. since 1 h. p. per minute = 33000 ft-lbs. per minute

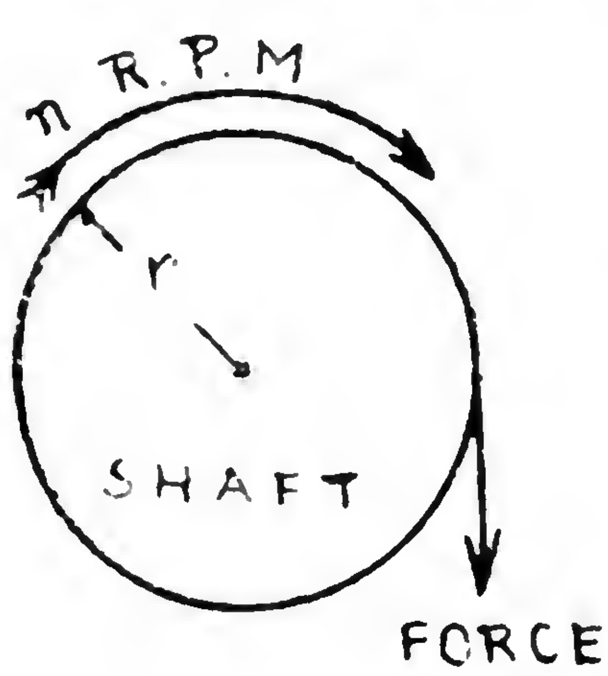


Fig. 2

$$h.p. = \frac{2\pi n T}{33000} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

Now 1 h. p. = 746 watts, therefore

$$\frac{E_b I_a}{746} = \frac{2\pi n T}{33000}$$

$$\therefore \quad T = \frac{33000}{2\pi 746} \times \frac{E_b I_a}{n}$$

$$T = 7{\cdot}04 \frac{E_b I_a}{n} \text{ lb.-ft.} \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

The *useful torque* (T_u) available at the rim of the pulley of a motor is less than the total torque. The horse power obtained by

using T_u is called *brake horse power* (b. h. p.) Thus

$$b.\,h.\,p. = \frac{2\pi n T_u}{33000}$$

$$\text{motor output} = \text{b. h. p.} \times 746 = \frac{2\pi n T_u}{33000} \times 746 \text{ watts,}$$

$$= \frac{1}{7{\cdot}04} \times n T_u \text{ watts}$$

and $\quad$ *lost torque* $= T - T_u = 7{\cdot}04 \times \dfrac{\text{iron and friction losses}}{n}$

$$\therefore \quad \textit{iron and friction losses} = \frac{n\,(T - T_u)}{7{\cdot}04} \text{ watts} \quad \ldots \quad (5)$$

where $\quad n$ = revolutions per minute

T and T_u are in lb.-ft.

Since the expression for induced voltage in an armature is given by Eq. (1) Chap. IV we write

$$E_b = \Phi \times \frac{p}{a} \times Z \times \frac{n}{60} \times 10^{-8} \text{ volts} \quad \ldots \quad \ldots \quad (7)$$

Substituting this value of E_b in Eq. (5) above

$$T = 7{\cdot}04 \times \frac{I_a}{n} \times \left[\Phi \times \frac{p}{a} \times Z \times \frac{n}{60} \times 10^{-8} \right]$$

$$\therefore \quad \textit{total torque } T = 0{\cdot}1173 \times \Phi I_a \times \left(\frac{p}{a} \times Z \right) \times 10^{-8} \text{ lb.-ft.} \quad (8)$$

For any machine $\left(\dfrac{p}{a} \times Z \right)$ is a constant quantity

$$\therefore \quad T \propto \Phi I_a \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

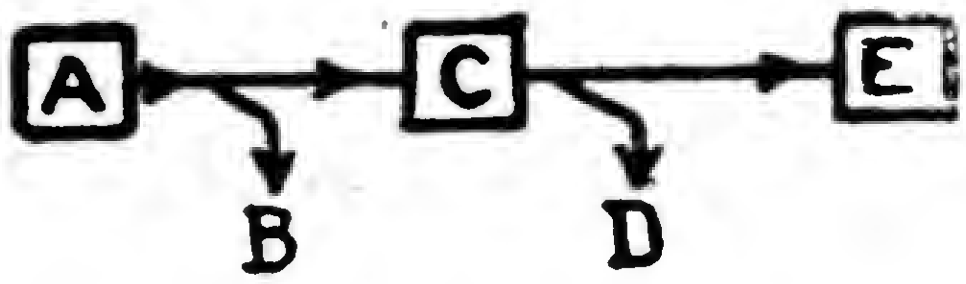
Similarly for any given machine

$$\frac{E_{b1}}{E_{b2}} = \frac{n_1 \Phi_1}{n_2 \Phi_2} \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

$$\frac{T_1}{T_2} = \frac{I_{a1} \Phi_1}{I_{a2} \Phi_2} \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

4. Losses and Efficiency: The losses that occur in a motor are the same as those which occur in a generator, namely (1) copper losses and (2) iron and friction losses. See Chap. IV, Section 5.

The input, losses and the output of a motor are shown diagrammatically below :—



A = electrical input = VI watts

B = copper losses = A − C

C = mechanical power developed

= $E_b I_a$ watts

D = iron and friction losses = C − E

E = mechanical output

$$= \frac{n \times T_u}{7{\cdot}04} \text{ watts}$$

where V = applied voltage

I = line current to the motor

I_a = armature current

E_b = back e. m. f.

n = revolutions per minute

T_u = useful torque in lb.-ft.

Hence

(i) electrical efficiency $= \dfrac{C}{A}$

(ii) mechanical efficiency $= \dfrac{E}{C}$

(iii) overall or commercial efficiency $= \dfrac{E}{A}$.

Example : A 220-volt, 4-pole shunt motor has 540 lap-wound conductors. Its armature resistance is 0·9 ohm, field winding resistance 220 ohms and the flux per pole is 3 megalines. When the line current is 32 amperes the output is 7·5 b. h. p. Calculate

(a) the total copper losses,

(b) iron and friction loss,

(c) the speed,

(*d*) the total torque developed, and

(*e*) the overall efficiency.

Solution: Line current = armature current + field current. See Fig. 3. Field current

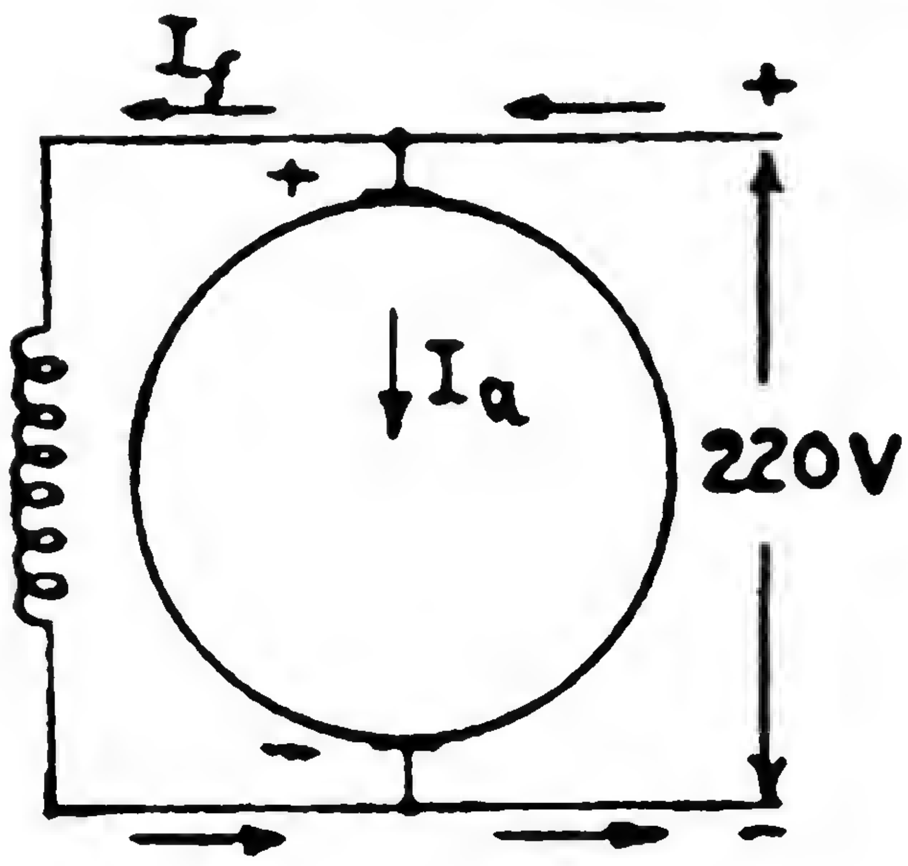


Fig. 3

$$I_f = \frac{220}{220} = 1A.$$

$\therefore$ armature current $= 32 - 1 = 31$ A.

Field copper loss $= 220 \times 1$
$= 220$ watts

Armature copper loss
$= 31^2 \times 0{\cdot}9 = 865$ watts

$\therefore$ total copper losses = *1085 watts*

Input to motor $= V \times I = 220 \times 32$
$= 7040$ watts

$\therefore$ mechanical power doveloped $= 7040 - 1085$
$= 5955$ watts.

$\therefore$ $E_b I_a = 5955$, but $I_a = 31$ A:

$$\therefore \quad E_b = \frac{5955}{31} = 192\text{V. } OR\ E_b = 220 - 0{\cdot}9 \times 31 = 192{\cdot}1\text{V.}$$

$$E_b = \Phi \times \frac{p}{a} \times Z \times \frac{n}{60} \times 10^{-8} \text{ volts}$$

Substituting the known values and solving for n,

$$192 = (3 \times 10^6) \times \frac{4}{4} \times 540 \times \frac{n}{60} \times 10^{-8}$$

$$\therefore \quad n = 711 \ r.\ p.\ m.$$

$$\therefore \quad \text{torque developed} = 7{\cdot}04 \frac{E_b I_a}{n} = 7{\cdot}04 \times \frac{5955}{711} = 59 \text{ lb.-ft.}$$

Output $= 7{\cdot}5 \times 746 = 5595$ watts.

$\therefore$ iron and friction losses $= 5955 - 5595 =$ *360 watts.*

$$\% \text{ efficiency} = \frac{\text{output}}{\text{input}} \times 100 = \frac{5595}{7040} \times 100$$

% efficiency = 79·5 %

The efficiency curve of a d. c. motor is shown in Fig. 4. Usually maximum efficiency occurs at about 75 % full load. But if the machine load happens to be steady at full load, then the machine is so designed that the maximum efficiency occurs at full load. The idea is that the machine, during its working period, should have an average load which lies somewhere near the load at which maximum efficiency occurs.

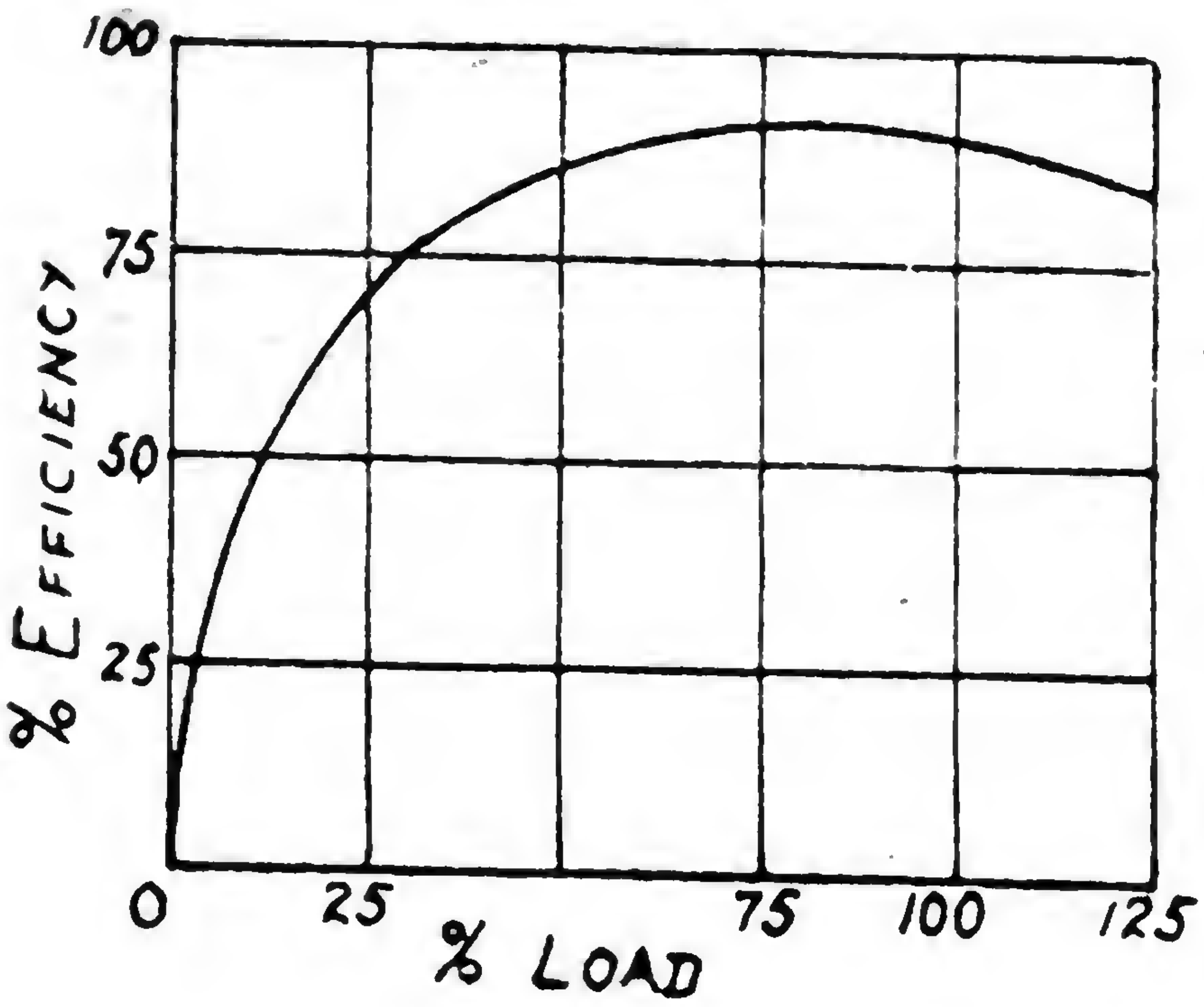


Fig. 4

5. Dependance of Speed on Voltage and Φ: From Eq. (7) the speed of a motor is

$$n = \frac{E_b}{\Phi} \times \left(\frac{60 \times 10^8}{Z} \times \frac{a}{p}\right) \text{ r. p. m.}$$

For a particular motor the terms in the bracket are constant. Putting

$$k = \left(\frac{60 \times 10^8}{Z} \times \frac{a}{p}\right) \text{ and } E_b = V - I_a R_a, \text{ (Eq. 2)}$$

$$n = k\,\frac{V - I_a R_a}{\Phi} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (12)$$

Now $I_a R_a$ is the voltage drop in the armature and is a small percentage of V, the applied voltage, the variation from no load to full load being about 4 to 5% of V. Hence $V - I_a R_a$ may be considered as almost a constant quantity, and equal to V. Therefore, ignoring the armature drop and effects of armature reaction the speed of a motor varies directly as the applied voltage and inversely as Φ.

6. Characteristic Curves of Motors: Like generators the motors are classified according to the method of excitation. Therefore there are three types of d. c. motors; (a) shunt, (b) series and (c) compound.

A. *The Shunt Motor*: It is always assumed that the terminal voltage V is constant at all loads. Hence in a shunt motor I_{sh}, he field current, is constant. Therefore Φ is constant, if the resis-

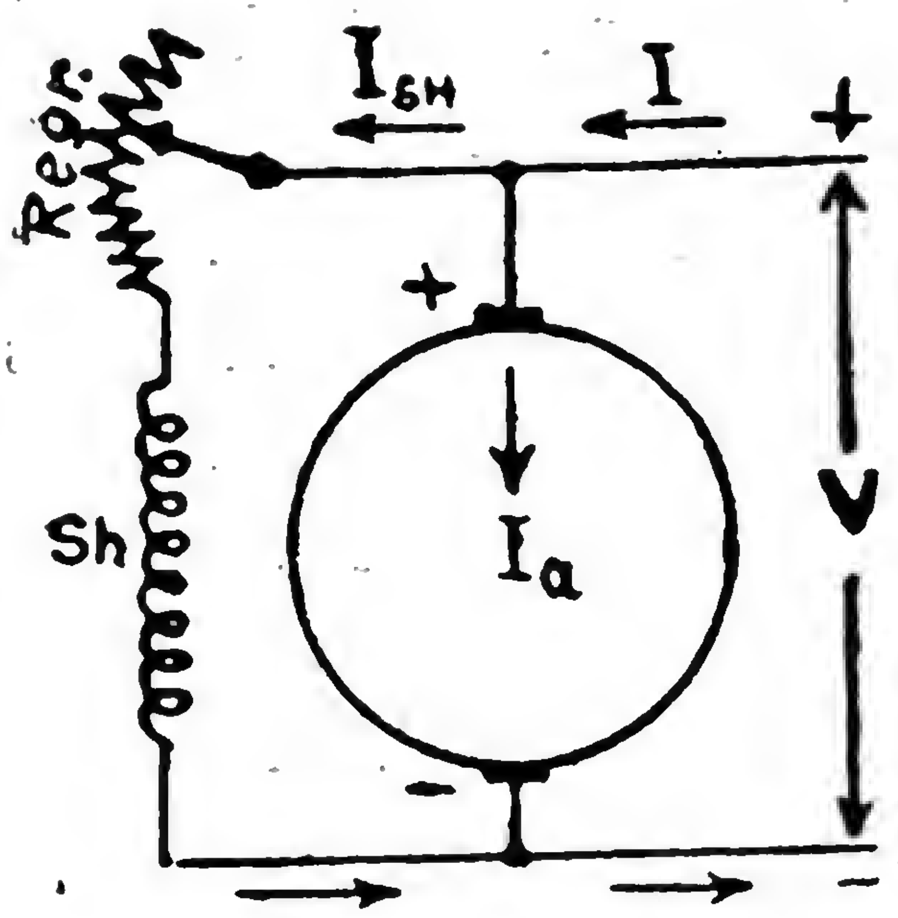


Fig. 5

tance of the field circuit is assumed constant, and armature reaction is ignored.

Therefore in a shunt motor, by Eq. (9) torque $T \propto \Phi I_a \propto I_a$, since Φ is assumed constant and since

$$E_b \propto n\Phi, \quad n \propto E_b$$

$$\therefore \quad \text{speed } n = k\,\frac{V - I_a R_a}{\Phi}\,.$$

After the motor attains its normal working temperature, the field current of a shunt motor remains constant. But the flux Φ, due to armature reaction, is slightly reduced from its no load value. This slight reduction in Φ, to some extent, compensates the drop in speed due to $I_a R_a$ drop in the armature, as per Eq. (12).

Fig. 6 shows the variation of torque, Φ, speed and E_b against armature current, the field current remaining constant. At any other value of field current the set of curves will have different values.

Where constant speed is required the shunt motor is ideal. However the speed of a shunt motor can be changed over a wide range by adjusting the field rheostat in the field circuit and thereby the current in the field windings.

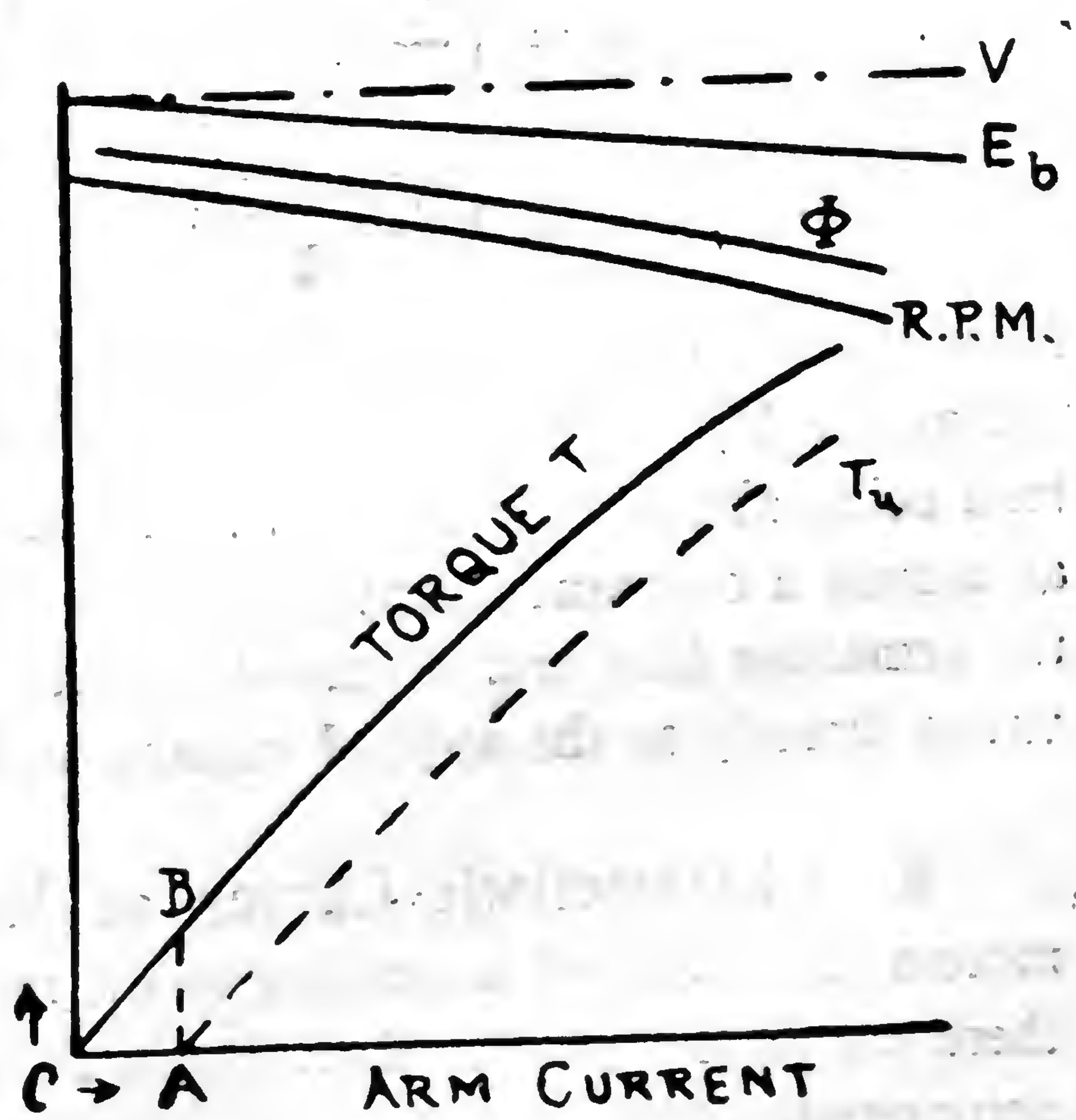


Fig. 6

I_a varies as load torque. When the load on the shaft of a motor increases I_a increases and the speed will fall slightly. But if Φ decreases due to armature reaction, the speed may remain constant as I_a varies. But if by any chance the weakening of Φ is such that the speed increases slightly the operating condition becomes unstable. In such cases a series

winding with very few turns must be provided on the motor poles as a compensating winding. But generally the shunt motor speed characteristic is drooping.

If Φ is constant, the torque characteristic is a straight line through the origin. On no load the armature takes a small current OA and the corresponding torque AB is the lost torque to overcome the iron and friction losses. The curve of useful torque T_u is obtained by subtracting, this constant torque AB from the total torque developed.

Suppose that the starting current of a shunt motor is 150% greater than full load current (1·5 times full load), the starting torque is 1·5 times full load torque, since Φ is constant.

B. *The Series Motor*: Since the field current of a series motor is also its armature current, Φ varies as load current assuming that there is no armature reaction. The following relationships hold for the series motor ;—

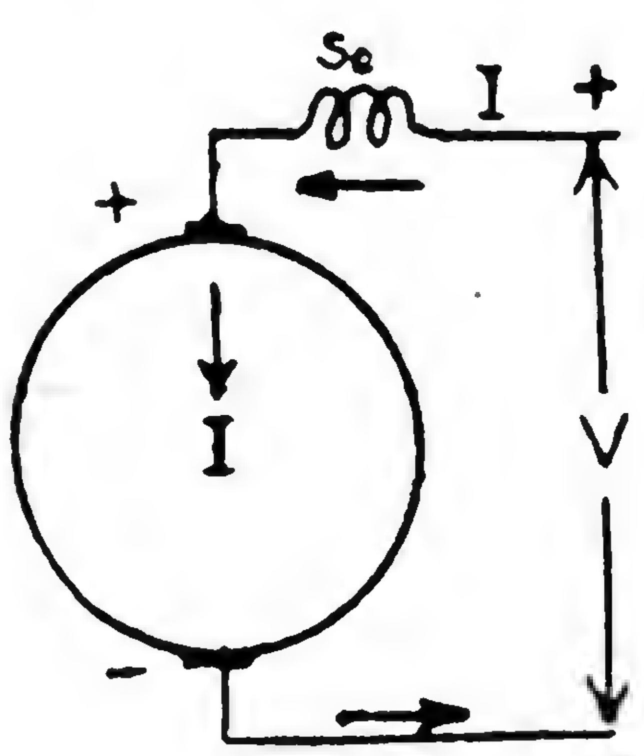


Fig. 7

I_a (armature current) = field current, I_{se}

I_a (armature current) = line current, I.

Resistance of motor = resistances of armature + field resistance.

i. e. $R_m = R_a + R_{se}$ and $I_a = I_{se} = I$

Hence $E_b = V - I(R_a + R_{se})$

and speed $n = k \dfrac{V - I(R_a + R_{se})}{\Phi}$.

At light load the torque is small and the speed is high, because both I_a and Φ are small. Therefore series motors must be used only in cases where a load always exists on its shaft. Hence on heavy loads the current is high, Φ is high in value and consequently the speed is low. In the case of series motors if the poles do not get saturated, working on the straight line portion of the O. C. C., the torque $\propto I_a^2 \propto \Phi^2$. Hence initial portion of torque characteristic is a parabola. At heavy current, i. e. when the iron is becoming saturated, the torque characteristic assumes a straight line.

Suppose that the starting current of a series motor is 150% greater than full load current (i. e. 1·5 times full load) and assuming this current increases Φ 120% (i. e. 1·2 times the flux at full load), the starting torque of a series motor will be

Starting torque = $1{\cdot}5 \times 1{\cdot}2 = 1{\cdot}8$ times the full load torque.

Further, there are no constant losses in the case of a series motor since both speed and Φ vary. Fig. 8 shows the variation of torque, Φ, speed and E_b against current, and Fig. 9 shows the relation between torque and speed.

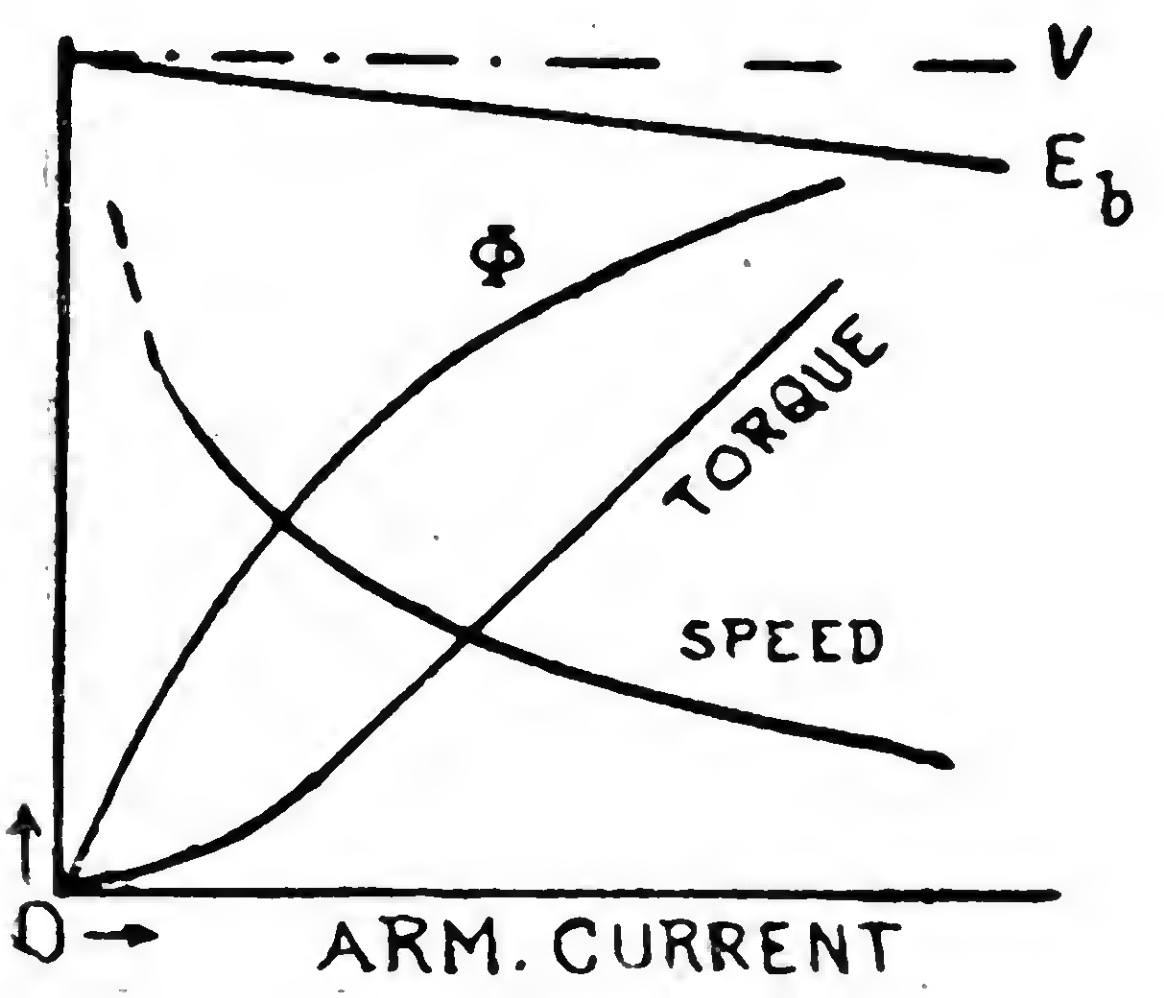


Fig. 8

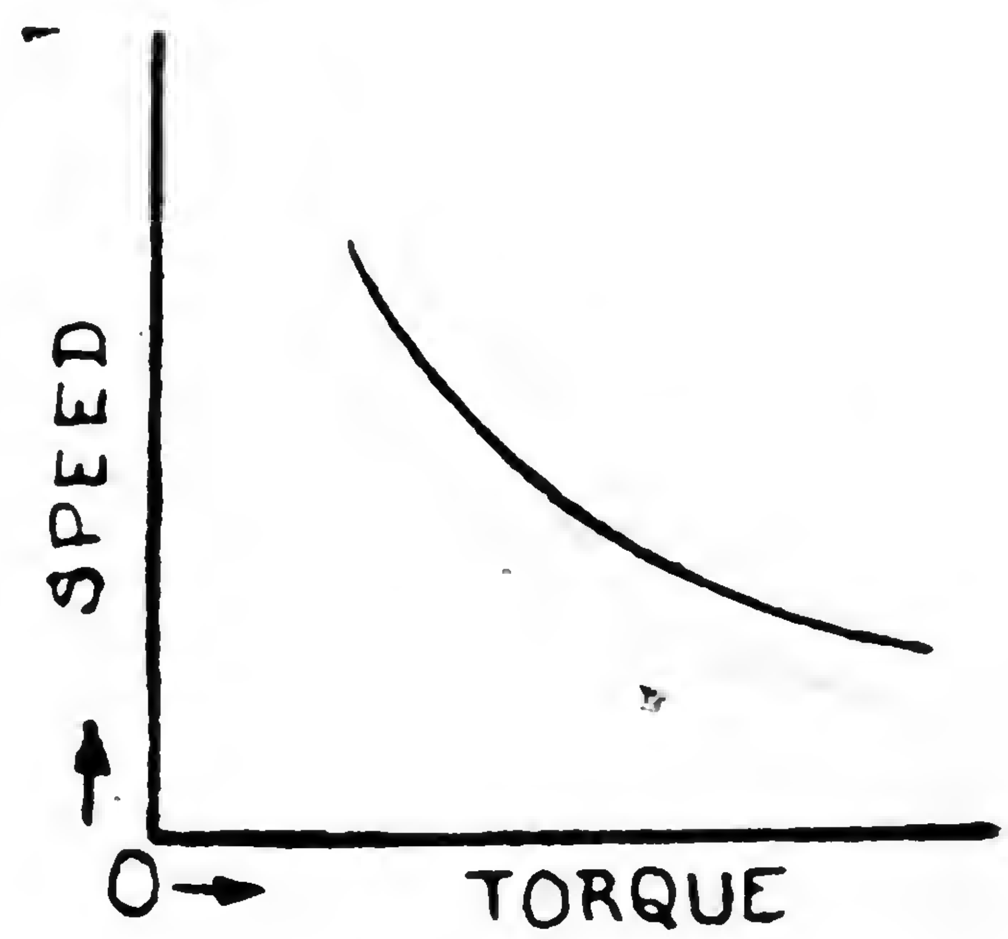


Fig. 9

C. *Compound Motors*: A compound wound motor may be given a characteristic lying anywhere between the extremes of the shunt motor and the series motor by suitably designing or manipulating the relative values of the series and shunt winding resistances.

When the series field m. m. f. assists the shunt field m. m. f. the machine is said to be *cumulative compound*. This type of compound motor is extensively used with Cranes, Lifts and Winches. One advantage this motor possesses over the plain series motor is that though it possesses the series characteristic, its no load speed is not high. When used in conjunction with a flywheel, the motor is very suitable for Rolling Mills, the flywheel acting as a load equaliser during heavy sudden loads and light loads.

When the series field m. m. f. opposses the shunt field m. m. f. the machine is said to be *differential compound*. It is possible to so adjust the two m. m. fs. that the percentage variation in speed from

no load to full load is very small, i. e. the motor is almost a constant speed motor. But this type of motor has two disadvantages—

(1) At starting the series field winding must be short-circuited. This is done because the series Φ_{se} gets more quickly established than that due to the shunt winding, the result is that the motor starts in the wrong direction.

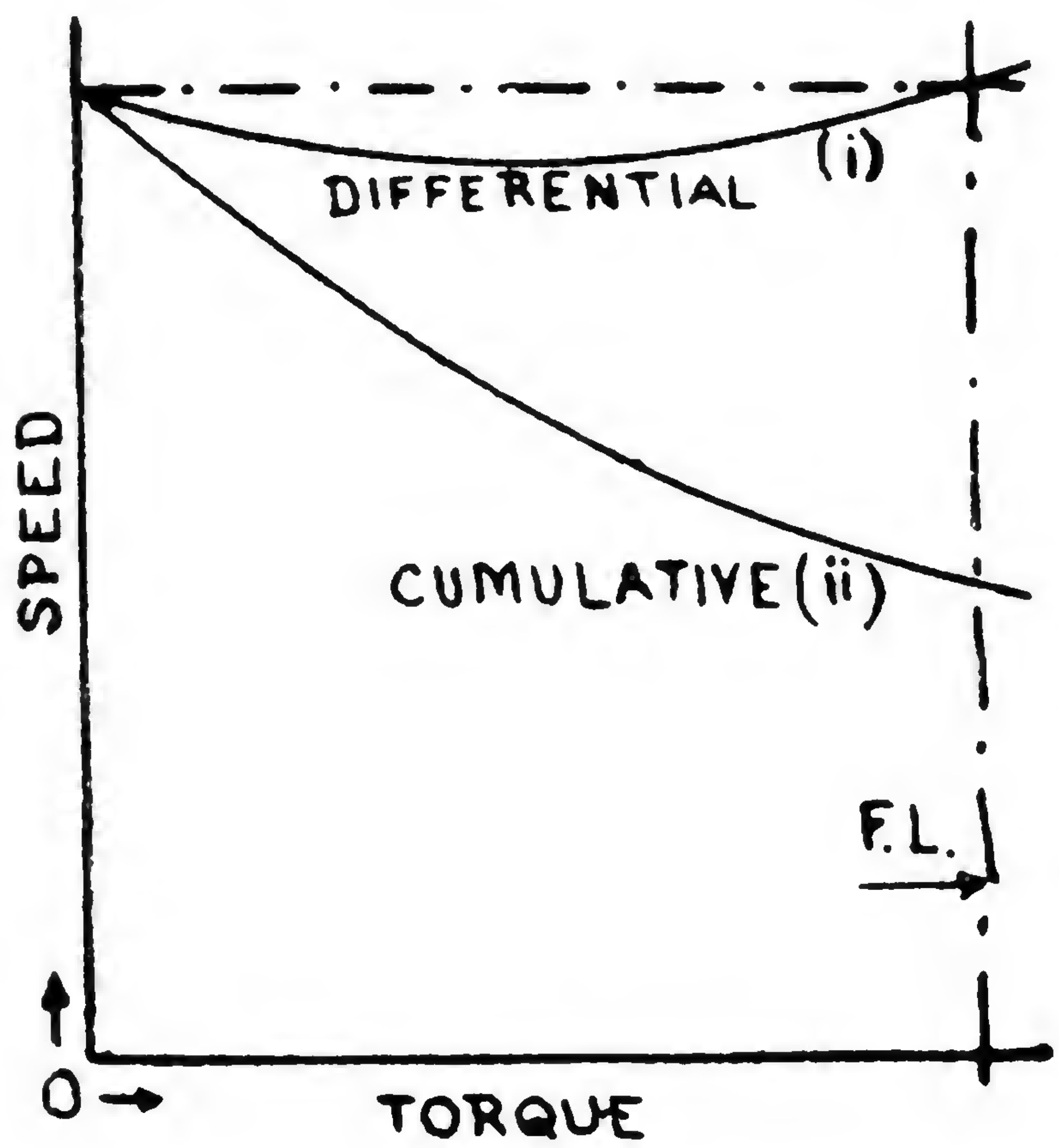


Fig. 10

(2) This type of motor gives trouble on overloads. Because at heavy loads Φ_{se} (flux due to series winding) weakens Φ_{sh} (flux due to shunt winding) to such an extent that the resultant flux Φ is not sufficient to develop a driving torque greater than the resisting torque. The result is the motor stops and goes on taking heavy current. For these reasons this type of motor is seldom used, since the shunt motor is sufficiently a constant speed motor. Fig. 10 shows the characteristic curves of the two types.

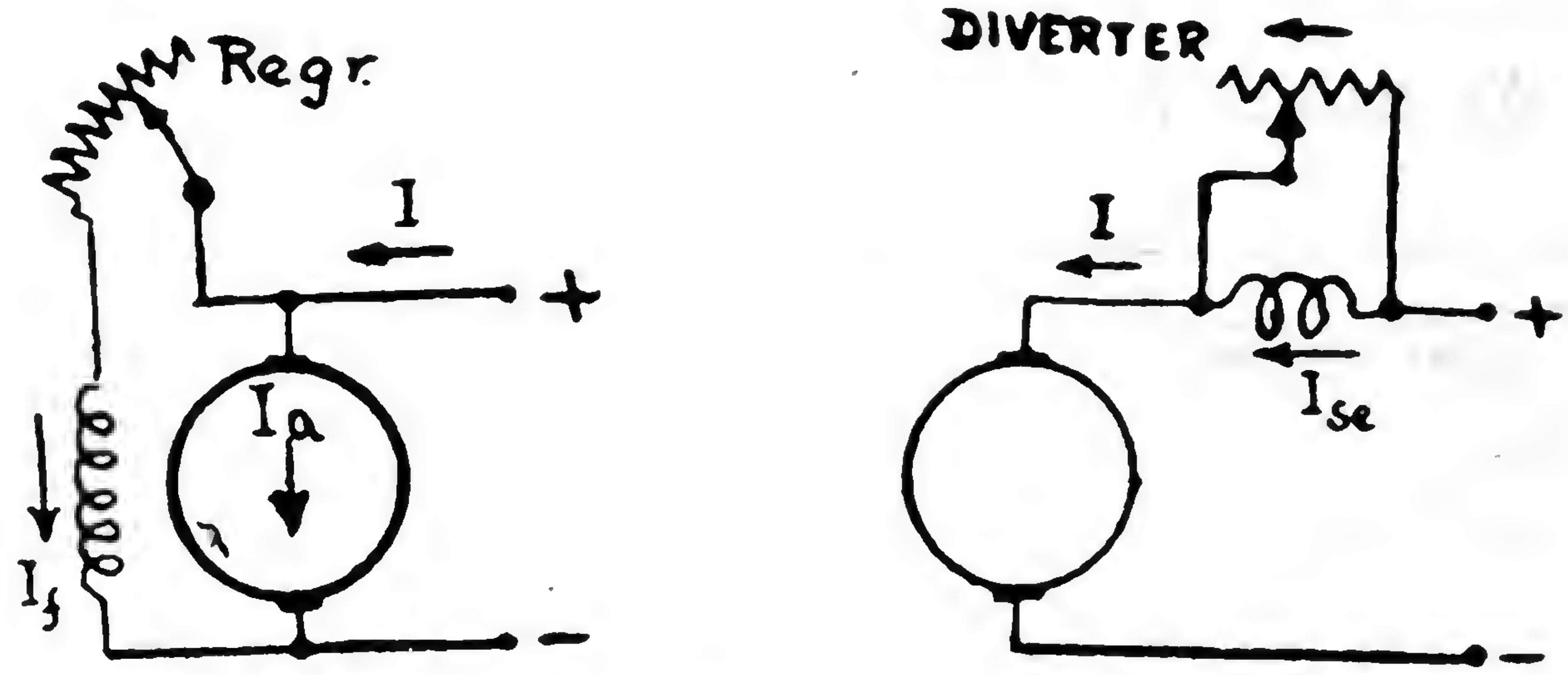
7. **Speed Control:** From Eq. 12

$$\text{speed} \propto \frac{V - I_a R_a}{\Phi} \propto \frac{E_b}{\Phi}.$$

Therefore also, speed $\propto \dfrac{V}{\Phi}$ approximately. Thus there are two principal methods of varying the speed of a d. c. motor. These are (1) by varying Φ for getting speeds higher than no load speed and (2) by varying the armature drop for getting lower speeds.

(1) Φ can be varied by placing a resistance in series with the field winding of a shunt motor. This resistance is called *shunt field regulator.* In the case of series motors a resistance is placed *in parallel* with the field winding. Resistance used thus is called a *diverter,* because it diverts a portion of the current from the field coils, and so the flux is reduced or weakened. The two connections

are shown in Fig. 11 and the effect of field current upon Φ and speed is shown in Fig. 12.



(a) Fig. 11 (b)

Sometimes regrouping of field coils in a series motor is adopted to give different speeds.

(2) Since the applied voltage V is usually constant, the drop of volts in the armature can be varied only by connecting a resistance in series with the armature. Fig. 13 (a) shows the connection for a shunt (or compound) motor where the voltage across the shunt field is constant. Fig. 13 (b) shows the connection for a series motor.

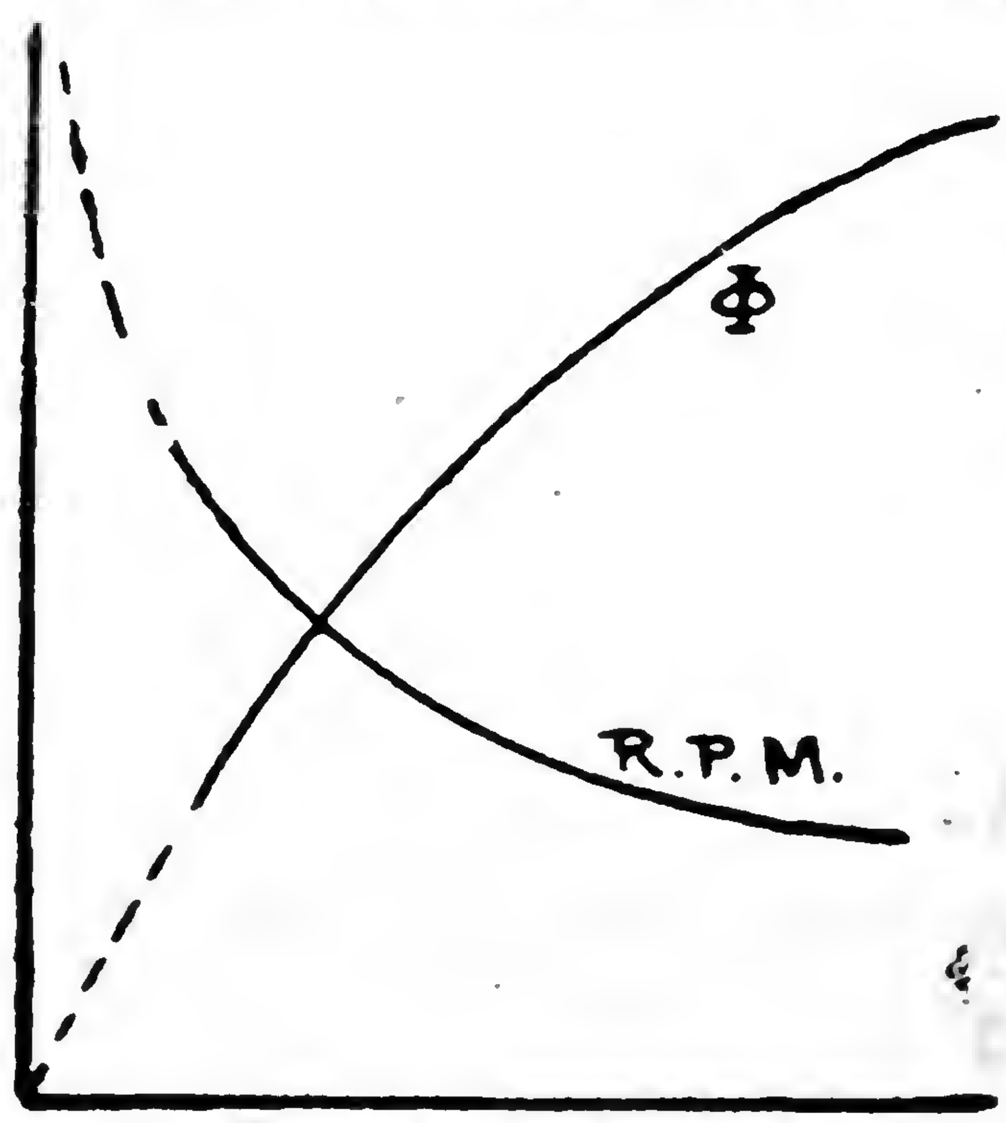
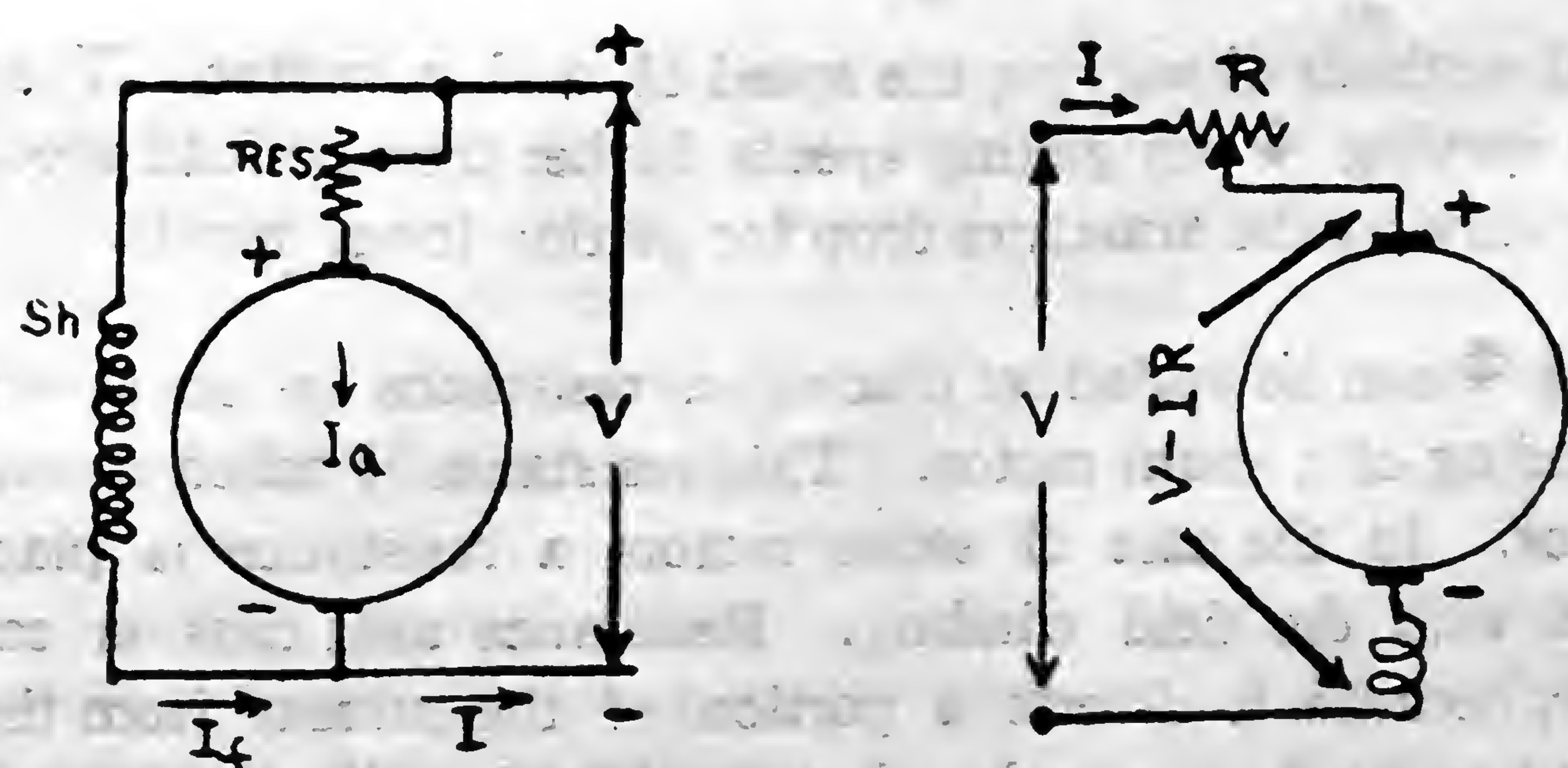


Fig. 12

As the value of R increases the voltage across the armature falls. This reduces the speed until the back e. m. f. decreases sufficiently to allow the current to produce the required torque. This method is very inefficient since the total I^2R losses are



(a) Fig 13 (b)

high. A lot of power is wasted in R alone. The speed variation of a shunt motor is shown in Fig. 14 where curve (i) is the graph of speed without any external resistance and curve (ii) is the graph of speed with an external resistance.

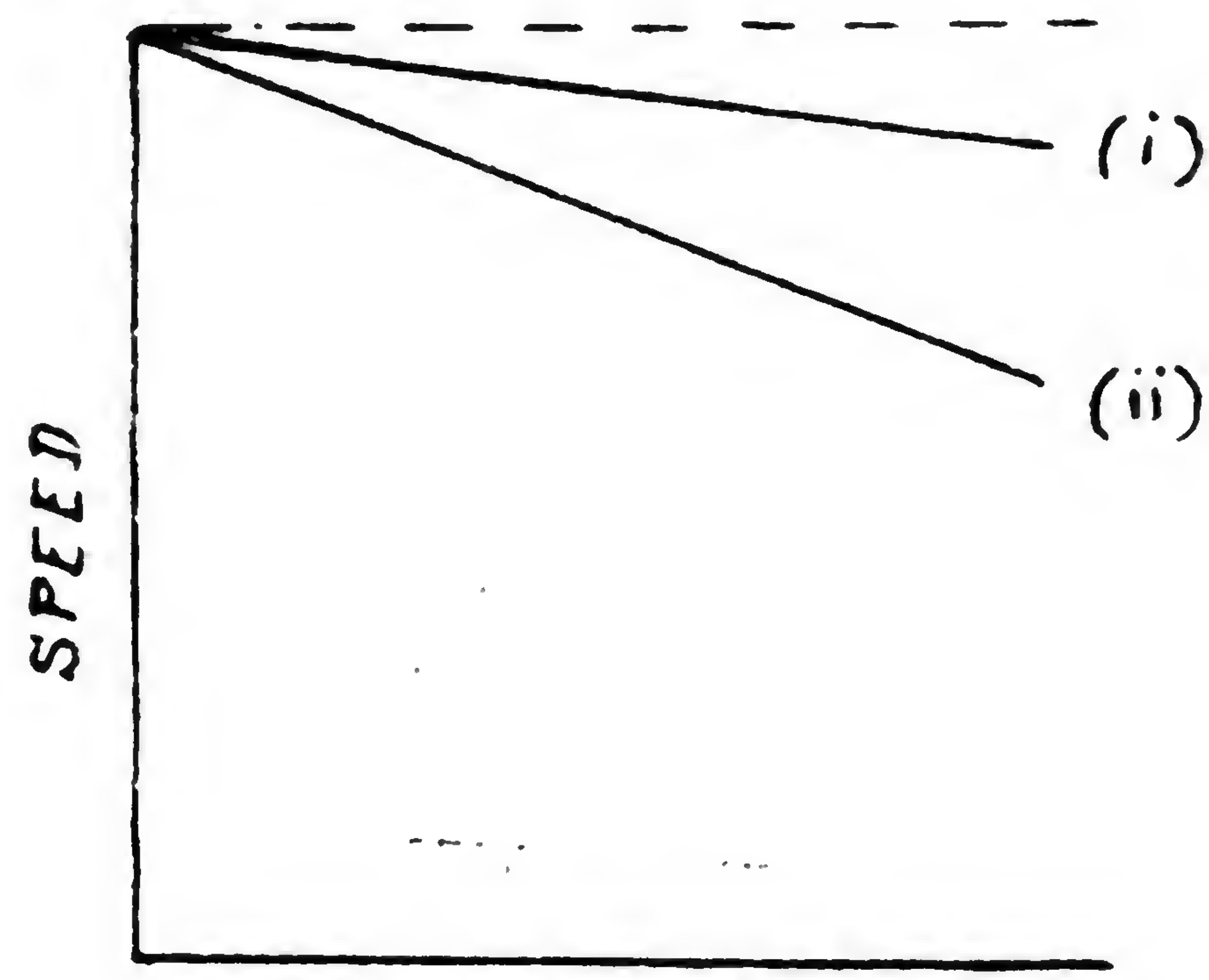


Fig. 14

Example: A 460–volt shunt motor takes a current of 85A and runs at 600 r. p. m. at full load. Its shunt field and total armature resistances are 92 ohms and 0·2 ohm respectively. Assume that the motor works on the straight line portion of the magnetisation curve and that the load torque varies directly as speed.

Calculate (a) the resistance that must be put in series with the armature to lower the speed to 500 r. p. m.; and (b) the resistance that must be put in series with the shunt field winding to raise the speed from 600 to 700 r. p. m.

Solution : (a) $\dfrac{E_{b2}}{E_{b1}} = \dfrac{n_2}{n_1} = \dfrac{500}{600}$. ... (i)

The shunt field current $= \dfrac{460}{92} = 5\text{A}$

$\therefore$ armature current $I_{a1} = 85 - 5 = 80$ A.

and $E_{b1} = 460 - 80 \times 0{\cdot}2 = 444$ V.

$\therefore \quad E_{b2} = 444 \times \dfrac{500}{600} = 370\text{V}$

$\therefore \quad I_a R$ drop $= 460 - 370 = 90$ V

Since torque varies as speed, the necessary current I_{a2} in the armature can be deduced from the relation

$$\frac{n_2}{n_1} = \frac{T_2}{T_1} = \frac{\Phi_2 I_{a2}}{\Phi_1 I_{a1}}$$

but $\Phi_2 = \Phi_1$, since flux is constant

$$\therefore \quad \frac{T_2}{T_1} = \frac{500}{600} = \frac{I_{a2}}{I_{a1}} \qquad \therefore \quad I_{a2} = 80 \times \frac{500}{600} = 66{\cdot}67 \text{ A.}$$

Let R be the total resistance of armature including the external resistance. Hence

$$66{\cdot}67 \times R = 90 \qquad \therefore \quad R = 1{\cdot}35 \text{ ohm.}$$

$$\therefore \quad \text{external resistance} = 1{\cdot}35 - 0{\cdot}2 = \textit{1·15 ohm.}$$

(b) Since the motor works on the straight line portion of the magnetisation curve $\Phi \propto$ field current I_f.

$$\text{Speed } n \propto \frac{E_b}{\Phi} \qquad \therefore \quad \frac{n_2}{n_1} = \frac{E_{b2}}{\Phi_2} \times \frac{\Phi_1}{E_{b1}}$$

$$= \frac{E_{b2} I_{f1}}{E_{b1} I_{f2}} = \frac{700}{600} = \frac{T_2}{T_1} = \frac{I_{f2} I_{a2}}{I_{f1} I_{a1}}$$

from (a) $E_{b1} = 444$ V, $I_{f1} = 5$A; $I_{a1} = 80$ A

$$\therefore \quad \frac{700}{600} = \frac{I_{f2} I_{a2}}{I_{f1} I_{a1}} \qquad \therefore \quad I_{f2} I_{a2} = \frac{700}{600} \times 5 \times 80$$

$$\therefore \quad I_{a2} = \frac{1400}{3} \times \frac{1}{I_{f2}} \quad \text{... (ii)}$$

$$\frac{E_{b2} I_{f1}}{E_{b1} I_{f2}} = \frac{I_{f2} I_{a2}}{I_{f1} I_{a1}}. \qquad E_{b2} = (460 - I_{a2} \times 0{\cdot}2)$$

Substituting the known values

$$\frac{(460 - I_{a2} \times 0{\cdot}2) \times 5}{444 \times I_{f2}} = \frac{I_{f2} \times I_{a2}}{5 \times 80}$$

Substituting the value of I_{a2} in terms of (ii) and solving for I_{f2}.

$$460 - \frac{280}{3 I_{f2}} = \frac{444 \times 7}{30} \times I_{f2}$$

which reduces to $I^2_{f2} - 4{\cdot}44 I_{f2} + 0{\cdot}9 = 0$

$$\therefore \quad I_{f2} = 4{\cdot}22 \text{ A.}$$

Let R be the total field resistance including the external resistance

$$\therefore \quad R = \frac{460}{4 \cdot 22} = 109 \text{ ohms.}$$

$\therefore$ external resistance in series with the field winding is

$$109 - 92 = 17 \textit{ ohms.}$$

Example: A 4-pole, 460-volt series motor drives a fan at 800 r. p. m. taking 12 amperes from the supply mains. Its armature resistance is ohm and that of the series field is 0·2 ohm per pole winding. Assume that the magnetisation curve is a straight line and that the torque varies directly as the square of the speed.

Neglecting effects of armature reaction, calculate the speed and the current taken by the motor when the four field coils are correctly arranged in two parallel groups.

Solution: In the first case the four field coils are all in series, which is the usual practice.

$\therefore$ total armature resistance $= 1 + 4 \times 0 \cdot 2 = 1 \cdot 8$ ohm

$\therefore$ drop of volts in armature $= 12 \times 1 \cdot 8 = 21 \cdot 6$ V,

and the back e. m. f. $E_{b1} = 460 - 21 \cdot 6 = 438 \cdot 4$ V.

When the four field coils are in two parallel groups, their combined resistance $= \dfrac{2 \times 0 \cdot 2}{2} = 0 \cdot 2$ ohm.

$\therefore$ total armature resistance $= 1 + 0 \cdot 2 = 1 \cdot 2$ ohm.

Let I_2 and n_2 be the new current and speed.

$\therefore$ drop in armature $= 1 \cdot 2\ I_2$

and $E_{b2} = (460 - 1 \cdot 2\ I_2)$ V.

The machine flux Φ_2 per pole is proportional to $\dfrac{I_2}{2}$ in the second case, because the current per field coil is $\dfrac{I_2}{2}$.

Now $E_{b2} \propto \Phi_2 n_2$ and $T \propto \Phi I_a$ where $I_a =$ arm. current; also $T \propto n^2$ by data and $\Phi \propto I_f$ where I_f is the field current.

$$\frac{n_2}{n_1} = \frac{E_{b2}}{\Phi_2} \times \frac{\Phi_1}{E_{b1}} = \frac{(460 - 1 \cdot 2\ I_2)\ I_1}{\frac{I_2}{2} \times 438 \cdot 4} = \frac{(460 - 1 \cdot 2\ I_2)\ 12}{I_2 \times 438 \cdot 4} \times 2 \cdots \text{(i)}$$

$$\text{and} \quad \frac{T_2}{T_1} = \frac{n_2^2}{n_1^2} = \frac{\Phi_2 I_{a2}}{\Phi_1 I_{a1}} = \frac{\frac{I_2}{2} \times I_2}{I_1 \times I_1} = \frac{I_2^2}{2 I_1^2}$$

$$\therefore \quad \frac{n_2}{n_1} = \frac{I_2}{\sqrt{2}\, I_1} \quad \therefore\ n_2 = 800 \times \frac{I_2}{\sqrt{2} \times 12} \quad \ldots\ldots\ldots \text{(iii)}$$

Substituting the value of n_2 in (i)

$$\frac{800 \times I_2}{\sqrt{2} \times 12} \times \frac{1}{800} = \frac{(460 - 1{\cdot}2\, I_2)\, 12 \times 2}{I_2 \times 438{\cdot}4}$$

Rearranging

$$438{\cdot}4\, I_2^2 = \sqrt{2} \times 288\, (460 - 1{\cdot}2\, I_2)$$

$$1{\cdot}08\, I_2^2 = 460 - 1{\cdot}2\, I_2$$

$$\therefore \quad I_2^2 + 1{\cdot}114\, I_2 - 426 = 0$$

Solving the quadratic $I_2 = 20{\cdot}1$ A.

Substituting in (iii) the value of I_2

$$n_2 = \frac{800 \times 20{\cdot}1}{\sqrt{2} \times 12} = 950 \text{ r. p. m.}$$

The Ward Leonard System of Speed Control: A set of 3 machines, mechanically coupled together is required to regulate the speed of the main motor. See Fig. 15 (*a*). The set consists of (1) a d. c. generator, (2) an exciter and (3) an a. c. or d. c. motor of constant speed to drive the set. Both the generator and the main motor get their field winding current from the exciter, [which is really a small d. c. generator. Its function is to provide exciting current to other machines. Hence the name *exciter*]. The location of the set need not be near the main motor.

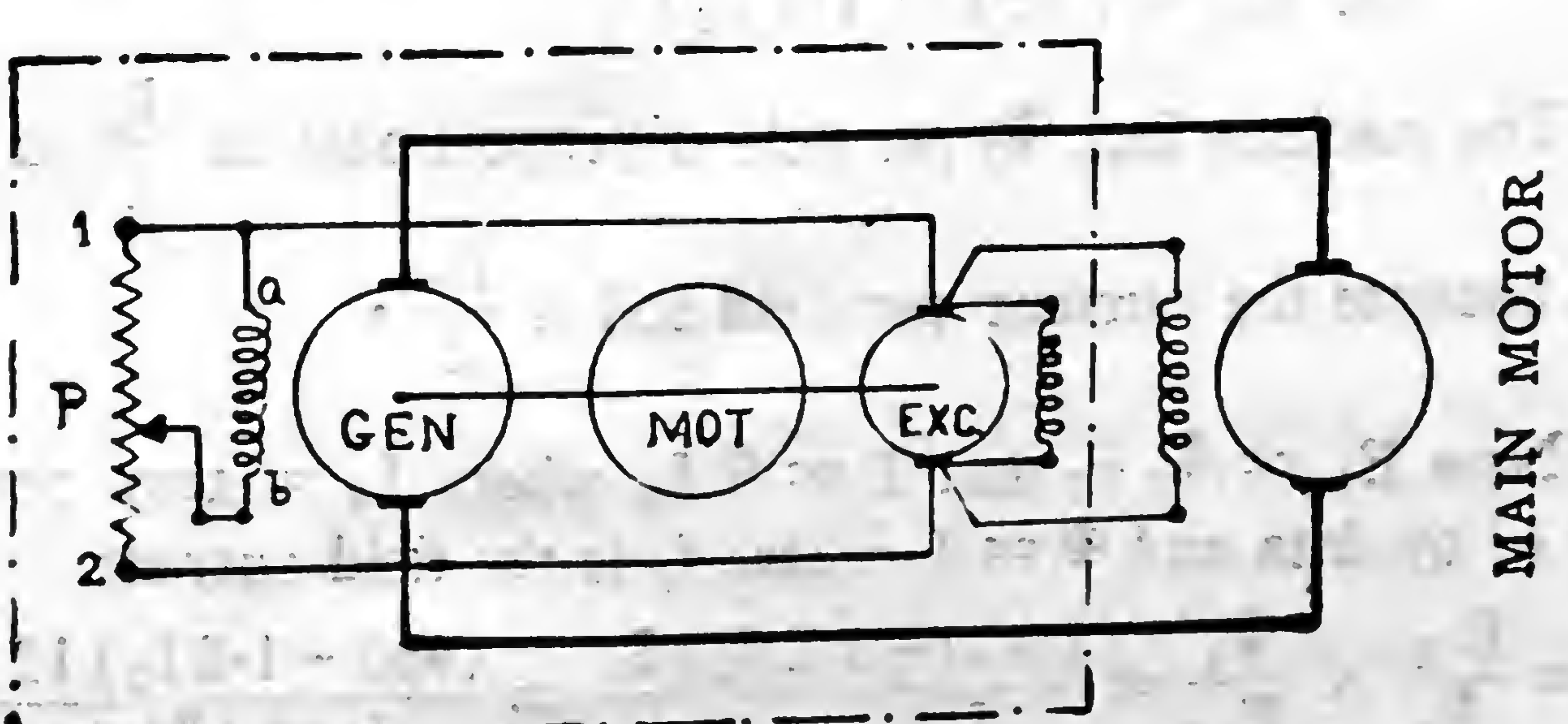


Fig. 15 (a)

In the figure shown, P is the *potentiometer regulator* for the field of the generator. Thus the voltage of the generator is varied over a wide range. The field current of the main motor is constant, but its armature voltage is variable. To start the main motor, the slider of P is at 1 and is gradually moved towards 2. Thus at start the main motor gets low voltage, and as the motor speeds up the generator field current is increased by moving the slider of P. The main motor accelarates rapidly even when heavily loaded. A setting of the slider will give a definite speed to the main motor. If the motor works in both directions the regulator P is connected as shown in Fig. 15 (*b*). 3 is the mid-point of P.

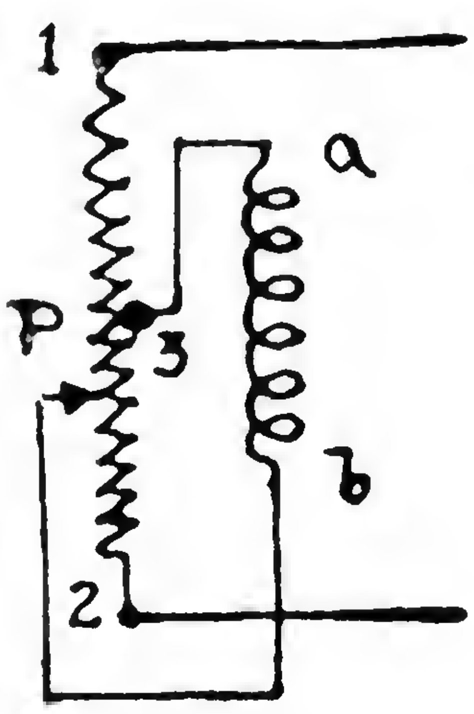


Fig. 16 (b)

This system is very useful for such works as Mine Hoists, Rolling Mills, Elevators, Shears, Gun-turrets on warships, Electric shovels, etc. If the fluctuations of load are violent a flywheel is used as a load equaliser. The speed variation is very smooth, but the initial cost is high.

8. Motor Starters and Controllers: If a d. c. motor is con-

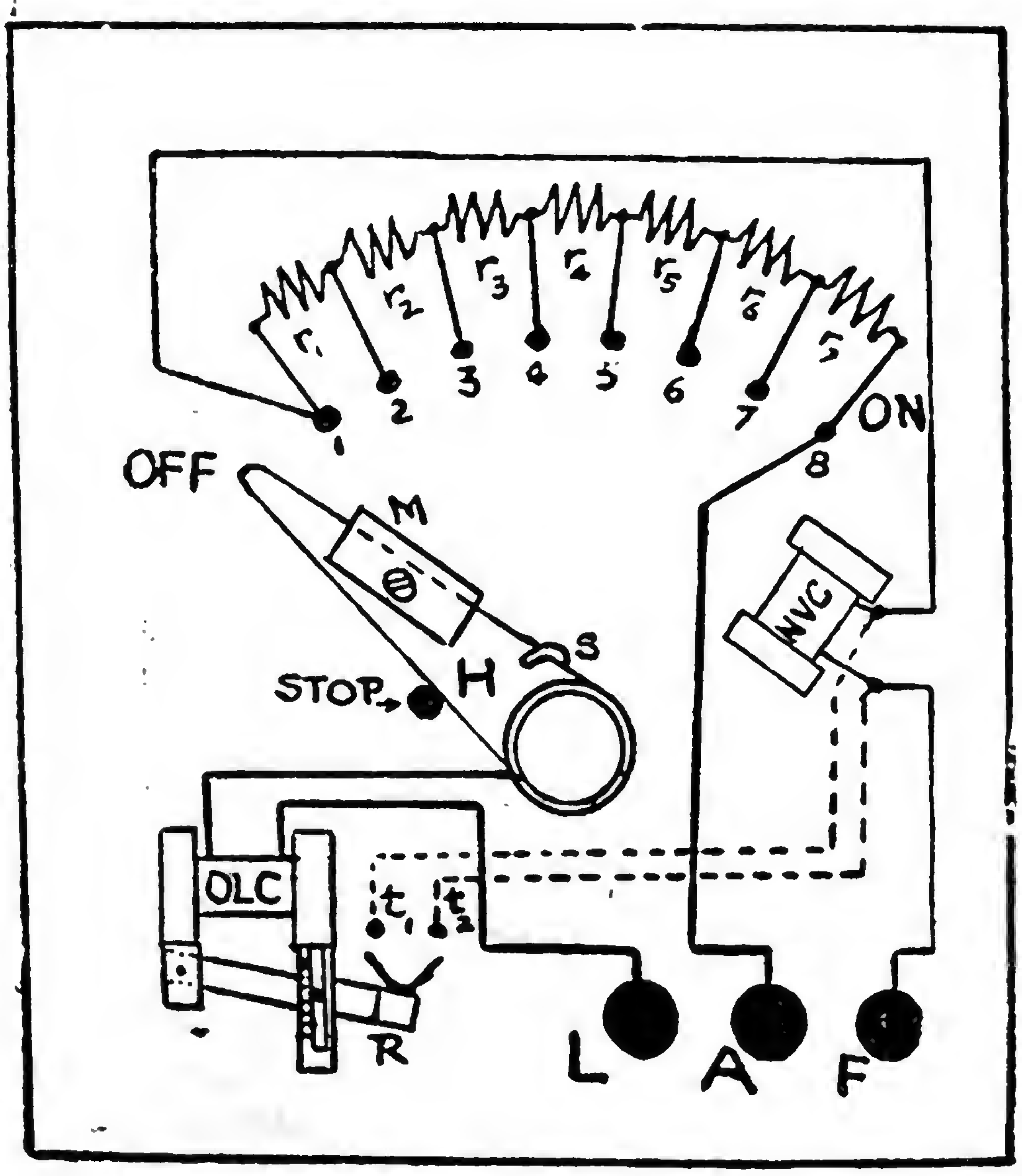


Fig. 16. Shunt Motor Starter

nected directly across the supply mains of rated voltage a dangerously high current will flow through the motor armature. This is due to (1) low value of armature circuit resistance and (2) absence of the back e. m. f. at starting time. Therefore to limit the high initial current a ***starting resistance*** is placed in series with the armature. This resistance is variable, and as the back e. m. f. rises due to increase in speed it is gradually cut out and when the motor attains full speed it is out altogether.

Fig. 16 shows a face-plate starter for a shunt motor. The handle is moved over the studs against the tension of a strong spring. There is a soft iron piece on the handle which is held by an electromagnet N. V. C. when the handle reaches the last stud, the "ON" position. The current in this electomagnet is the field current. If the voltage falls below a certain limit the electromagnet is not strong enough to hold the handle against the tension of the spring. So the handle returns to the "OFF" position. This is a safety device.

The full armature current passes through another electromagnet shown in the figure as O. L. C. If the current exceeds a predermined value, this electromagnet becomes strong and lifts up the pivoted armature R which short-circuits the two terminals t_1 and t_2, which in turn short-circuits the N. V. C. (*No-Volt-Coil*) and the handle returns to the "OFF" position. This is a safety device against over loads. Fig. 17 shows a schematic diagram of a Controller for a series wound Crane Motor. A Controller besides being a starter, controls the speed of a motor in both the directions of rotation.

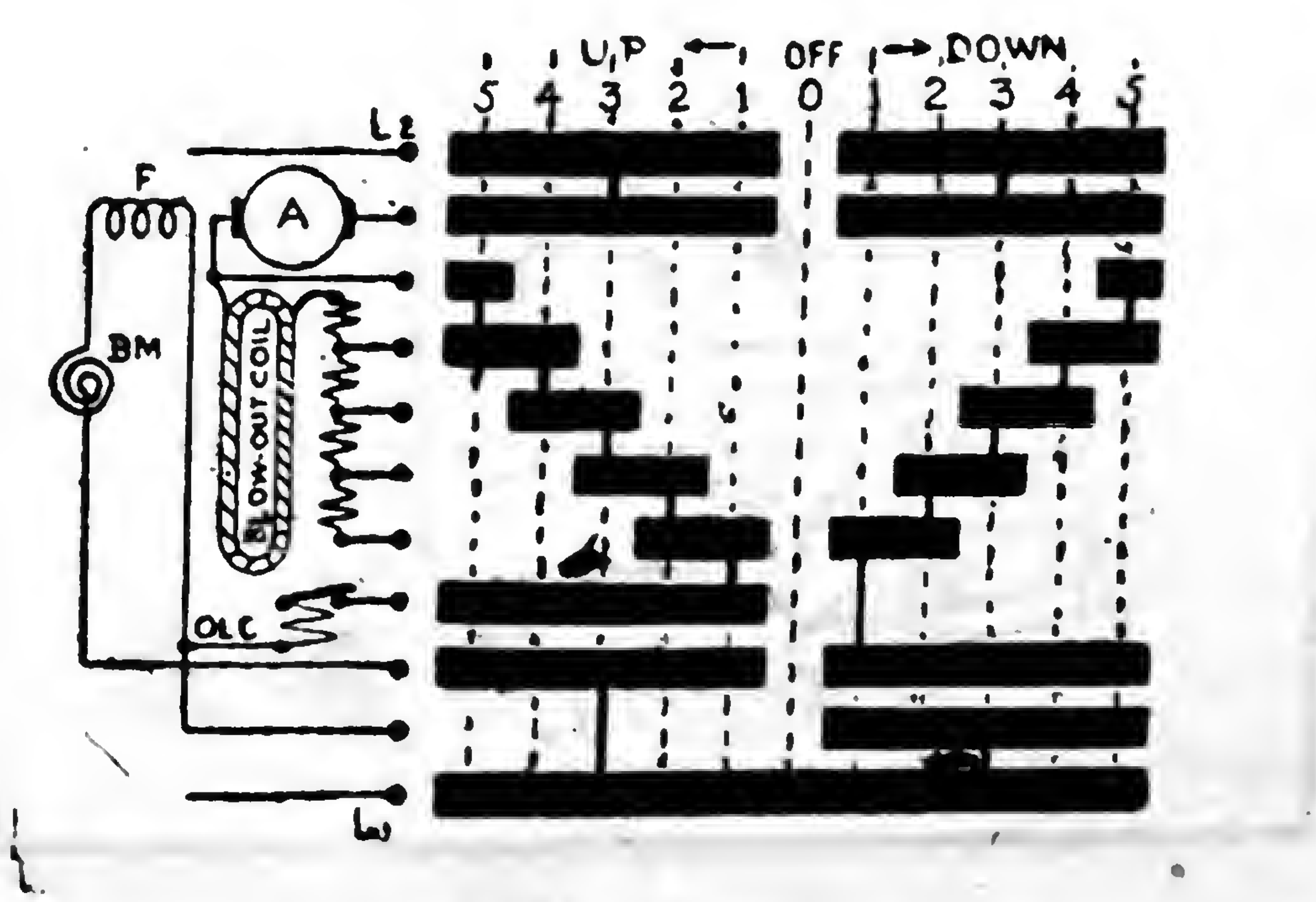


Fig. 17

In the figure shown, when the Controller handle is brought from the OFF position to position 1 the motor is connected to the Supply Mains in series with all the starting resistance units. At position 5 all the starting resistance is out of circuit and the motor runs at full speed. For lowest speed the handle is kept on position 1.

BM is the brake-magnet and OLC is the over-current protection provided when the motor is lifting a load, i. e. in the UP direction. The Blow-out Coil prevents heavy sparking at the contacts by providing a magnetic flux across the path of arc which is liable to form between each pair of FINGERS and the rotating segments. At the time of breaking a contact the arc is blown vertically up by mechanical force due to the magnetic flux.

A Push-Button Starter is shown in Fig. 18. This type of starter is used for large motors which have to perform a definite duty cycle, i. e. which has to start and stop frequently. By means of electromagnets, which are made to operate in a certain fixed sequence, all the switching operations are performed automatically.

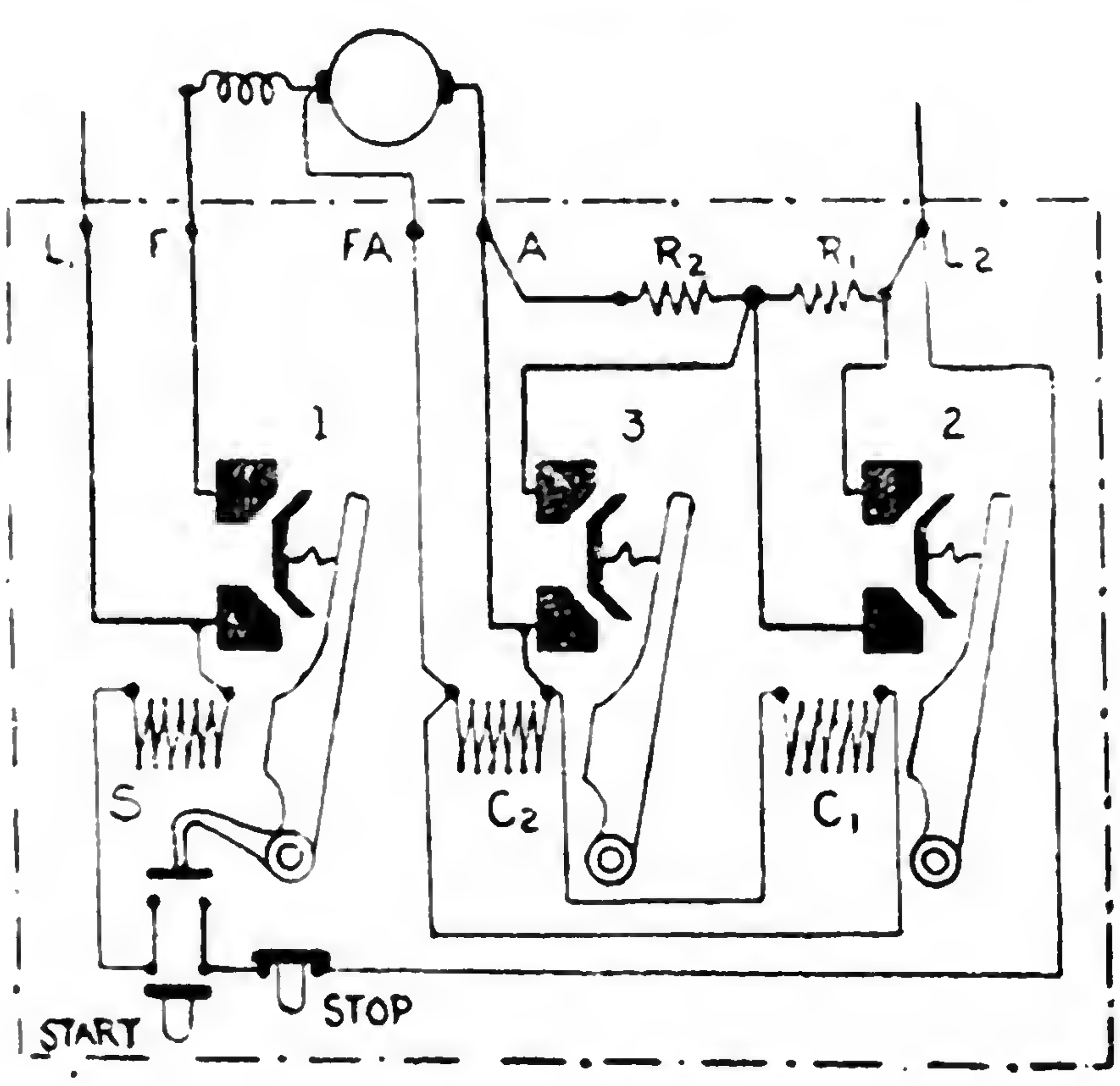


Fig. 18

The starter of Fig. 18 has three *contactors* which operate when their individual operating coils get energised. When the "start" push-button is pressed S gets energised and pulls the lever arm of contactor 1 against the tension of a spring. This connects the motor to the line L_1 and L_2 in series with starting resistance units R_1 and R_2, and at the same time provides a second path through which

the circuit of S is completed. This is done by short-circuiting the terminals of the "start" push-button. The motor now speeds up and its back e. m. f. increases. C_1 is so designed that it operates when the motor back e. m. f. is about 50%. This pulls the lever arm of Contactor 2 thereby short-circuiting R_1. When the motor back e. m. f. is about 80% C_2 gets fully energised and the contactor 3 short-circuits R_2.

To stop the motor the "stop" push-button is pressed. This opens the circuit of coil S. The lever arm of contactor is pulled back by the spring thus disconnecting the motor from the supply.

9. **Grading of Starting Resistance:** Fig. 19 gives a schematic diagram of a shunt motor with its starting resistance and a number of studs. The total starting resistance is made up of several resistance units marked r_1, r_2, r_3 etc. The number of studs is one more than the number of resistance units.

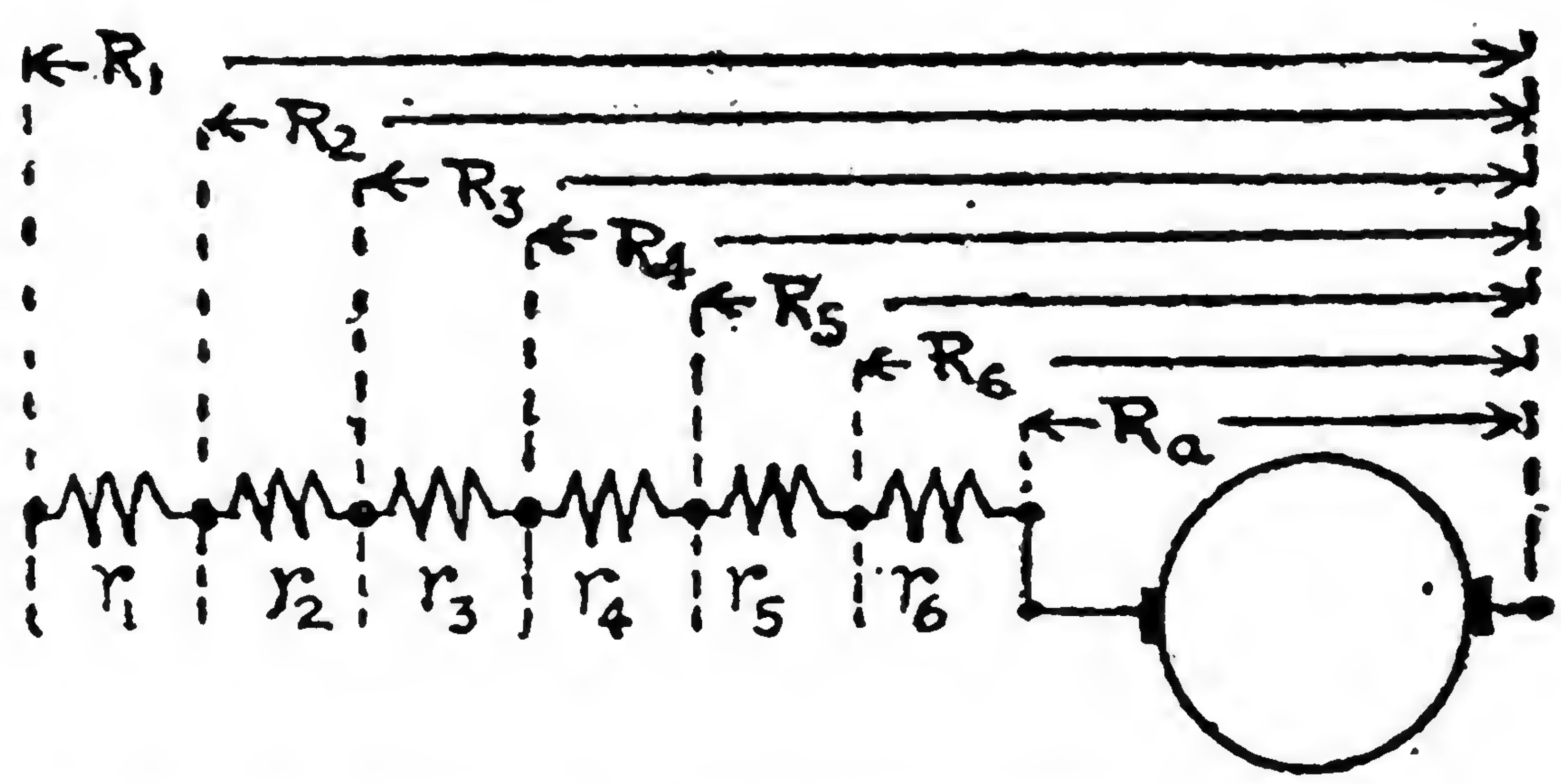


Fig. 19. Starting Resistance Steps: Sh. Motor.

It is assumed that the current falls from I_1 to I_2 when the starter handle is on any stud. Values of I_1 and I_2 are predetermined or assumed, I_1 is usually taken as full load current value and I_2 is taken as 75% of I_1 or 2/3 of I_1. The resistance R_a is the resistance of the armature winding *plus* the commutating pole winding resistance *plus* the brush contact drop resistance. The last one is omitted on account of its variable nature and its small value as compared with the other two.

At the instant the handle is placed on the 1st stud the speed is zero and therefore the back e. m. f. is zero. So that

$$I_1 = \frac{V}{R_1} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (i)$$

When the motor starts to rotate, it builds up a back e. m. f. E_1 and a steady condition will occur when there is no change in speed. The current now will be $I_2 = \dfrac{V - E_1}{R_1}$ ... ... (ii)

Moving the handle on the 2nd stud, the current will jump to the value of I_1 immediately, assuming that the speed at the instant is the same. Therefore $I_1 = \dfrac{V - E_1}{R_2}$ ... ... (iii)

The speed will now increase to another steady value, and the motor back e. m. f. will be then E_2. The current will fall from I_1 to I_2. Therefore $I_2 = \dfrac{V - E_2}{R_2}$ ... ... (iv)

When the handle is moved on to the 3rd stud, the current again jumps to the value of I_1, its equation being

$$I_1 = \frac{V - E_2}{R_3} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (\text{ v })$$

and after the speed is higher and steady, its value is

$$I_2 = \frac{V - E_3}{R_3} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (\text{ vi })$$

This process continues until the handle is on the last stud when

$$I_1 = \frac{V - E_n}{R_{n+1}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (\text{vii})$$

$$\text{and} \quad I_2 = \frac{V - E_{n+1}}{R_{n+1}} \quad \ldots \quad \ldots \quad \ldots \quad (\text{viii})$$

Dividing (iii) by (ii), (v) by (iv) etc., we get

$$\frac{I_1}{I_2} = \frac{R_1}{R_2}; \quad \frac{I_1}{I_2} = \frac{R_2}{R_3} = \frac{R_3}{R_4} = \ldots \quad \ldots = \frac{R_n}{R_{n+1}}$$

If there are n number of units, the number of studs are $(n + 1)$. Hence R_a, the armature resistance, becomes the $(n + 1)$th resistance.

If the ratio $\dfrac{I_1}{I_2} = k$, then

$$k = \frac{R_1}{R_2} = \frac{R_2}{R_3} = \ldots\ldots\ldots = \frac{R_n}{R_{n+1}} \quad \ldots \quad \ldots \quad (\text{ ix })$$

Hence $\frac{R_1}{R_2} \times \frac{R_2}{R_3} \times \ldots\ldots \times \frac{R_n}{R_{n+1}} = k^n = \frac{R_1}{R_{n+1}} = \frac{R_1}{R_a}$.

Or $k = \sqrt[n]{\frac{R_1}{R_a}}$ ... ... (x)

It will be noticed that R_1, R_2, R_3 etc. are in geometric progression. It can be shown that r_1, r_2, r_3 etc. are also in geometric progression.

Writing equation (ix), $k = \frac{R_1}{R_2} = \frac{R_2}{R_3} = \cdots$ etc.

then $\frac{R_1}{R_2} = \frac{R_2}{R_3} = k$. Subtracting 1 from each side,

$$\frac{R_1}{R_2} - 1 = \frac{R_2}{R_3} - 1 \qquad \therefore \quad \frac{R_1 - R_2}{R_2} = \frac{R_1 - R_3}{R_3}.$$

Transposing, $\frac{R_1 - R_2}{R_2 - R_3} = \frac{R_2}{R_3}$ which is equal to k.

Similarly, $\frac{R_1 - R_2}{R_2 - R_3} = \frac{R_2 - R_3}{R_3 - R_4} = \frac{R_3 - R_4}{R_4 - R_5} = k$ ... (xi)

But $R_1 - R_2 = r_1$; $R_2 - R_3 = r_2$; $R_3 - R_4 = r_3$ and so on, hence, substituting in (xi)

$$\frac{r_1}{r_2} = \frac{r_2}{r_3} = \frac{r_3}{r_4} = \cdots\cdots = \frac{r_n}{r_{n+1}} = k \quad \ldots \quad \ldots \quad \text{(xii)}$$

To calculate r_1, r_2, r_3 etc. either the ratio $\frac{I_1}{I_2} = k$ should be known or the number of resistance units n must be known. $R_a = r_{n+1}$ is always known, and hence the calculations become simple. For instance, if k is known

$r_n = k R_a$; $r_{n-1} = k r_n$; $r_{n-2} = k r_{n-1}$ etc., and one setting on the *Slide Rule* will give all the readings.

Again. $r_1 = R_1 - R_2$, and $k = \frac{R_1}{R_2}$,

$$\therefore \quad r_1 = R_1 - \frac{R_1}{k}, \quad \text{or} \quad r_1 = R_1 \left(\frac{k-1}{k}\right) \quad \ldots \quad \ldots \quad \text{(xiii)}$$

The value of R_1 is obtained from $I_1 = \frac{V}{R_1}$.

Example : Calculate the values of resistance units of a 7-stud starter for a shunt motor working on 460 volt supply and having an armature resistance of 0·15 ohm. The starting current is not to exceed 100 amperes.

Solution :

Since $I_1 = 100$ and $V = 460$; $R_1 = \dfrac{460}{100} = 4{\cdot}6$ ohms.

$$k^n = \frac{R_1}{R_a}, \qquad \therefore \quad k = \sqrt[n]{\frac{R_1}{R_a}}$$

$n = 6$ units from the data, $R_1 = 4{\cdot}6$ and $R_a = 0{\cdot}15$.

$$\therefore \quad k = \sqrt[6]{\frac{4{\cdot}6}{0{\cdot}15}} = 1{\cdot}768.$$

$$\text{Since } k = \frac{R_1}{R_2} = \frac{R_2}{R_3} = \frac{R_3}{R_4} \text{ etc. [Eq (ix)]}$$

$$R_2 = \frac{R_1}{1{\cdot}768} = \frac{4{\cdot}6}{1{\cdot}768} = 2{\cdot}6 \quad \therefore \quad r_1 = R_1 - R_2 = 2{\cdot}0 \text{ ohms;}$$

$$R_3 = \frac{R_2}{1{\cdot}768} = \frac{2{\cdot}6}{1{\cdot}768} = 1{\cdot}47 \quad \therefore \quad r_2 = R_2 - R_3 = 1{\cdot}13 \text{ ohms;}$$

$$R_4 = \frac{1{\cdot}47}{1{\cdot}768} = 0{\cdot}83 \quad \therefore \quad r_3 = R_3 - R_4 = 0{\cdot}64 \text{ ohm;}$$

$$R_5 = \frac{0{\cdot}83}{1{\cdot}768} = 0{\cdot}468 \quad \therefore \quad r_4 = R_4 - R_5 = 0{\cdot}362 \text{ ohm;}$$

$$R_6 = \frac{0{\cdot}468}{1{\cdot}768} = 0{\cdot}264 \quad \therefore \quad r_5 = R_5 - R_6 = 0{\cdot}204 \text{ ohm;}$$

$$R_7 = R_a = 0{\cdot}15 \quad \therefore \quad r_6 = R_6 - R_7 = 0{\cdot}114 \text{ ohm.}$$

The values of R_2, R_3 etc. are calculated from one setting of the Slide-Rule. [Since $\dfrac{1}{1{\cdot}768} = 0{\cdot}566$, set the "10" of the top scale on 0·566 then moving the cursor all the values are obtained]

Alternatively, using Eq. (xiii),

$$r_1 = R_1\left(\frac{k-1}{k}\right) = 4{\cdot}6\left(\frac{1{\cdot}768 - 1}{1{\cdot}768}\right) = 4{\cdot}6 \times 0{\cdot}435 = 2 \text{ ohms.}$$

$r_2 = r_1 \times 0{\cdot}566$ [Eq. (xii)]

$= 2 \times 0{\cdot}566 = 1{\cdot}13$ ohm, and so on. The values obtained by this way are

$r_3 = 0{\cdot}64$; $r_4 = 0{\cdot}36$; $r_5 = 0{\cdot}205$; $r_6 = 0{\cdot}114$ and these check very well with the above values.

The same problem can be solved graphically by the construction of Fig. 20.

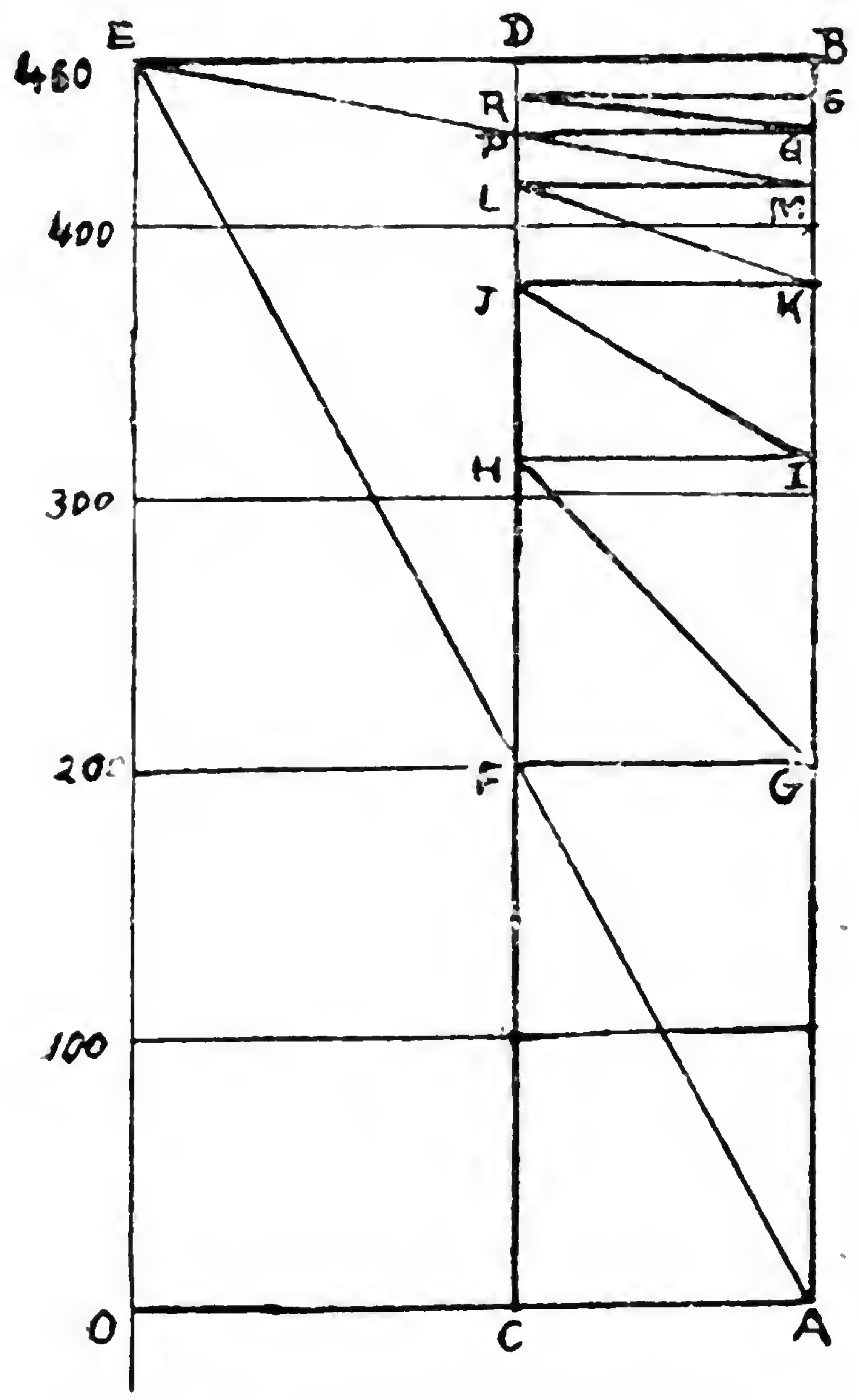


Fig. 20 Graphical Method

Draw to a suitable scale the maximum current I_1 shown by the line OA. Also draw AB at right angles to OA and to a suitable scale of resistance, making AB equal to R_1.

Now find the value k from equation (x). k then is 1·768

$$\therefore \quad I_2 = \frac{I_1}{1{\cdot}768} = 56{\cdot}6$$

amperes. Mark off OC equal to 56·6 and draw a perpendicular from C on OA. Draw OE equal and parallel to AB and join E to B.

To determine the values of resistance steps proceed as follows :—

(i) Join EA cutting CD at F. From F drop a perpendicular on AB at G. Then AG $= r_1$ to the same scale of resistance.

(ii) Join G and E cutting CD at H. Dropping a perpendicular from H on to AB at I, gives r_2 i. e. GI $= r_2$.

(iii) Join I and E cutting CD at J. Drop a perpendicular from J on to AB at K. Then IK $= r_3$. Proceed in this manner until six values are obtained, the last value BS is the armature resistance R_a.

OE, in the diagram to a certain scale, is the value of the assumed constant flux Φ, at which the motor works. In the case of variable flux motors, particularly the series motor, OE will assume the shape of the magnetisation curve, since $\Phi \propto I_a$, the armature current.

Observe also that if CD were made to shift to the right by reducing the value of k, the number of resistance units is increased.

10. Temperature Rise in Electrical Machines: The maximum allowable working temperature for any electrical machine is determined by the type of insulation used. Thus the output of a machine, or better the rating, depends not on the mechanical stresses but on the type of insulating materials used in the machine. The classification of these materials is given below :—

Class A consists of organic substances such as cotton, silk, paper etc. impregnated or immersed in oil.

Class B consists of inorganic materials such as mica, asbestos, glass etc. in built-up form with a binding substance.

Class C is composed of materials such as pure mica, porcelain, quartz, etc.

The impregnating compound is made from asphaltum or paraffin base mixed in a thinning compound, and this must not be subjected to temperatures above 100° C. The breakdown temperature of cotton is only 120° C.

Machines which are subjected to occasional overloads must not have a temperature rise of more than 40° C. Others which are not overloaded may be allowed to operate with a temperature rise of not more than 55° C.

11. Determination of Efficiency of Machines: The best method to determine the efficiency of a motor (or generator) is to load it and measure accurately the input and the output power. The usual practice is to measure the efficiency at $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ and full load. In testing a motor, the output or b. h p. is measured by applying a friction brake to the rim of a rotating pulley on the shaft of the motor and measuring the retarding force exerted by the brake. This is the *direct method* and is applicable for machines whose output is not more than 3 b. h. p. This is due to the limitations of a friction brake.

When the rating of a motor is high, it cannot be fully loaded by a friction belt, nor is there a method of any form of direct loading which does not entail a waste of power. Hence the *indirect method* is to determine the losses of a machine, as accurately as possible, from no load to full load. Once the losses at any load are known it is an easy matter to calculate the efficiency of the machine at that load. The efficiency measured by indirect method is only of approximate value depending upon the care taken in determining the losses.

A. *The Brake Test.* One form of friction brake is shown in Fig. 21. P is the motor pulley making n revolutions per minute and having a radius of r feet. The retarding force is measured by two spring balances their respective readings being W_1 and W_2 lb. Thus the output of the motor is

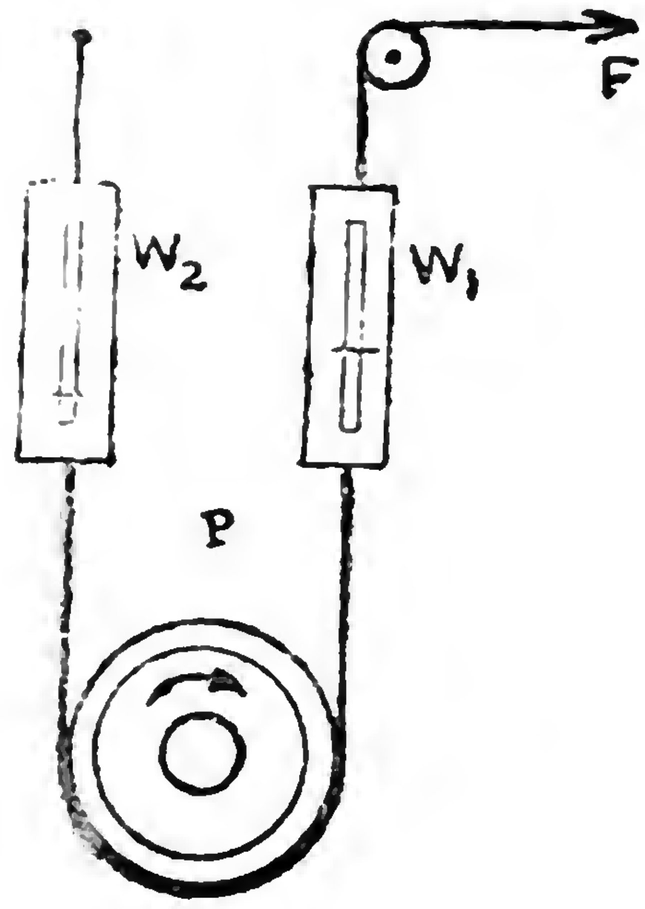


Fig. 21

$$\frac{2\pi n T}{33000} \times 746 \text{ watts}$$

$$\text{and} \quad T = (W_1 - W_2)\, r \text{ lb. ft.}$$

If V is the supply voltage measured at the terminals of the motor and I is the total current taken by the motor, the input $= VI$ watts. Therefore the percentage efficiency is

$$\%\ \text{efficiency} = \frac{2\pi n (W_1 - W_2)\, r}{33000} \times \frac{746}{VI} \times 100.$$

B. *The Swinburne Method:* The machine is run as an unloaded motor, the voltage and speed being normal. The total input is measured. This input is used up in (i) shunt field loss, (ii) iron, friction and windage loss and (iii) armature copper loss + series and interpole field losses, if the machine has a series field and interpoles. Input to armature is also measured.

All the field winding resistances and armature resistance are carefully measured. These resistance should be converted to the values of working temperature assuming the rise in temperature to be 40°C.

Knowing which losses are approximately constant and which vary with the load, the efficiency of the machine can be calculated at any load.

Example: In a brake test on a shunt motor the following readings were obtained:—

Supply voltage ... 220 V; Load on one band ... 45 lb.

Line current ... 12·5 A: Load on other band ... 13 lb.

R. P. M. ... ... 1000; Radius of pulley ... ... 6 inches.

Calculate, the torque, *b. h p.* and the efficiency.

Solution: The torque T is $(45 - 13) \times \frac{6}{12}$ lb. ft.

$$\therefore \quad T = 16 \text{ lb. ft.}$$

$$B.h.p. = \frac{2\pi nT}{33000} = \frac{2\pi \times 1000 \times 16}{33000}$$

$$\doteq 3{\cdot}05 \ b.h.p.$$

output in watts $= 3{\cdot}05 \times 746 = 2275$ W.

input in watts $= 220 \times 12{\cdot}5 = 2750$ W.

$$\therefore \quad \% \text{ efficiency} = \frac{2275}{2750} \times 100 = 82{\cdot}8\%.$$

Another Example: A 460 volt long-shunt compound motor takes 9 A from the supply mains on no load and runs at its rated speed. The resistance of all its windings when taken gave the following results:—

Armature and series field winding $= 0{\cdot}13$ ohm

Shunt field winding $= 85$ ohms

Interpole winding $= 0{\cdot}05$ ohm

Assuming an average drop of 1 volt per brush arm, calculate the b. h. p. and the efficiency when the total current taken by the motor is 185 A.

Solution: Field current in the shunt winding is

$$I_{sh} = \frac{460}{85} = 5{\cdot}41 \text{ A.}$$

$\therefore$ armature, series field and interpole current on no load is

$$9 - 5{\cdot}41 = 3{\cdot}59 \text{ A.}$$

Shunt field loss $= 460 \times 5{\cdot}41 = 2489$ W

Iron, windage and friction losses $= 460 \times 3{\cdot}59$
$= 1651$ W

These are constant losses $= 2489 + 1651 = 4140$ W

When loaded and the line current is 185 A

armature current $= 185 - 5{\cdot}41 = 179{\cdot}59$ A.

$\therefore$ armature, series field and interpole loss

$$= (179{\cdot}59)^2 \times (0{\cdot}13 + 0{\cdot}05) = 5800 \text{ W.}$$

Brush contact loss $= 2 \times 179{\cdot}59 = 359$ W.

$$\text{Total losses} = 5800 + 359 + 4140 = 10299 \text{ W.}$$

$$\text{Input} = 460 \times 185 = 85100 \text{ W.}$$

$$\therefore \quad \% \text{ efficiency} = \frac{85100 - 10299}{85100} \times 100 = \mathit{86{\cdot}9\,\%}.$$

$$\text{B. h. p.} = \frac{85100 - 10299}{746} = \mathit{100\ h.p.}$$

C. *Hopkinson's Method.* The method is to load two identical dynamos "back-to-back", i. e. one machine acts as a motor driving the other as a generator, which in turn supplies electrical power to the motor. The two machines are thus coupled mechanically and electrically. The set is connected to supply mains having the same voltage as that of the machines. The losses of the two machines are supplied by the mains. Thus for a small expenditure of power machines of large capacity can be fully loaded.

The diagram of connections is shown in Fig. 22. Before starting

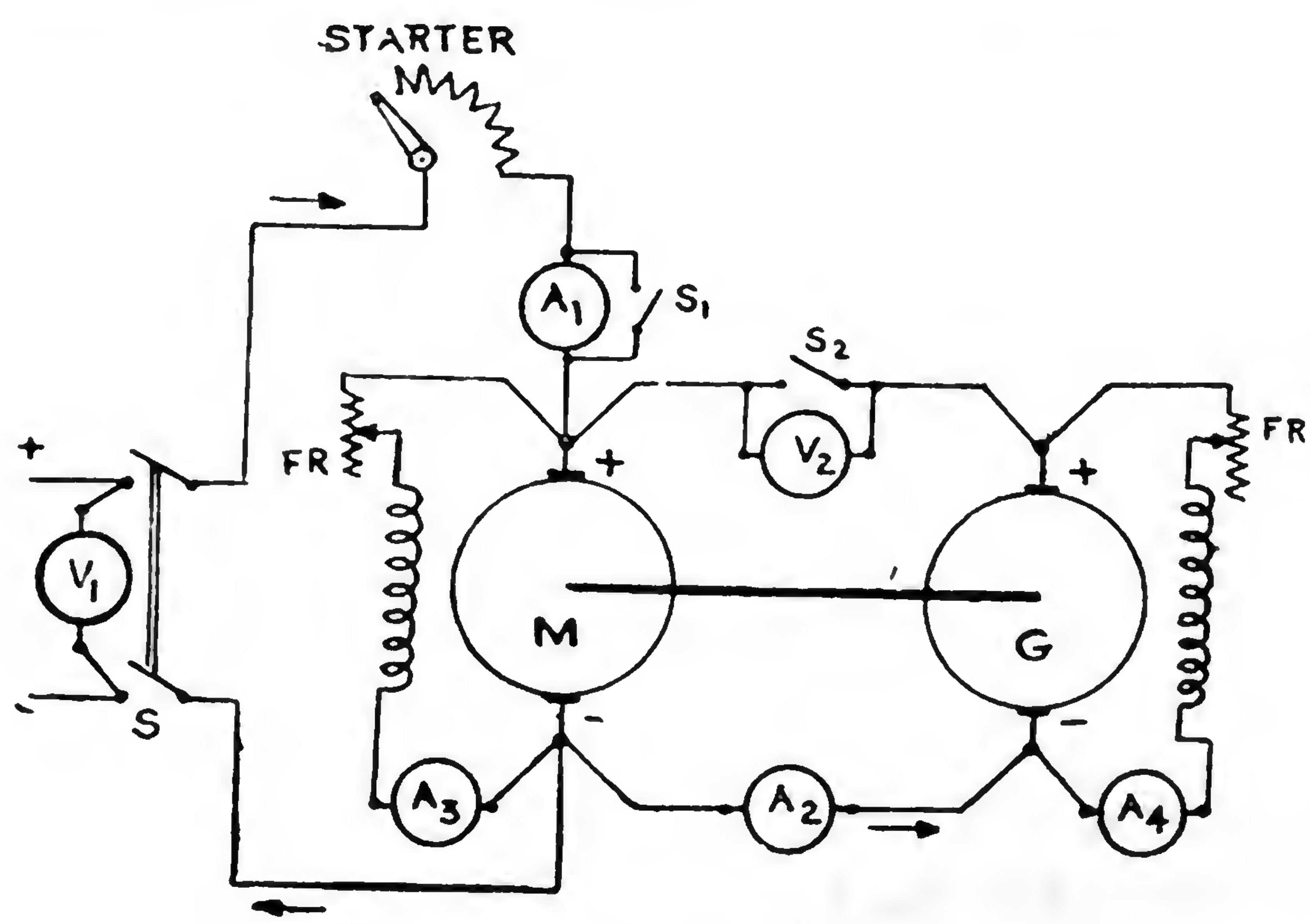


Fig. 22

the motor M, S_2 must be open and S_1 closed. The field rheostat FR of the motor must be out of circuit. Since A_1 is a low reading ammeter, it must be short-circuited by switch S_1 so that the initial heavy current at the instant of starting the motor will not damage it. S_2 is necessary to safeguard the machines if their polarities are not correct.

When the motor is started, the field regulator of the motor is adjusted to bring the set to normal speed and the field regulator of the

generator G is adjusted so that its field current as shown by A_4 is normal. If the voltmeter V_2 shows very little voltage, the polarities of the two machines are correct for the purpose of paralleling G with the supply mains. If V_2 shows almost double the voltage of supply mains, the armature terminals of G must be interchanged. This must be done after stopping the set.

When V_2 indicates zero voltage, switch S_2 is closed and S_1 is opened. At this stage G does not give out any electrical power. For G to be loaded the field rheostats of both machines are so adjusted that A_2 shows increasing current while the speed is kept constant by the field regulator of M. Thus the machines can be loaded to any desired extent.

A_1 shows the current from the supply mains and it includes the currents of the two shunt field windings.

A_2 shows the armature current of G.

The armature current of M is obtained by adding the readings of A_1 and A_2 and subtracting the two field currents as shown by A_3 and A_4.

V_1 indicates the supply voltage, the terminal voltage of M and the terminal voltage of G.

For the purpose of greater accuracy the armature resistance must be known. In this test the two machines are not equally loaded, the motor being the more heavily loaded machine. Hence it is better to calculate the individual efficiency, because in the motor the armature copper loss is greater and shunt field loss lesser than those in the generator.

CHAPTER VI

CELLS AND BATTERY OF CELLS

7. Introduction: There are two types of cells, the primary and the secondary. In primary cells the parts which react chemically require to be renewed, while in the secondary cells the reaction is reversible, i. e. by passing a current, from an external source, in a reverse direction the original chemical conditions are restored.

Each cell has poles, positive and negative. These poles act as terminals to which an external circuit is connected. The terminal from which a current enters the external circuit is the positive pole or terminal. The other is the negative pole. The poles consist of plates or rods. They are surrounded by chemicals called *electrolytes*. The total electric potential between the poles with respect to the electrolyte is the total e. m. f. of a cell.

This e. m. f. is not available at the terminals of a cell when it supplies current to an external circuit. This is due to the internal resistance drop and due to *polarization.* The internal resistance drop is the product of the current *multiplied by* the internal resistance of the cell. Polarization is a counter e. m. f. caused mostly by formation of hydrogen at the positive plate and transfer of metal from the negative to the positive plate or weakening of the electrolyte.

One method to depolarize a cell is to surround the positive pole by a salt of itself which has the same acid radical as the salt surrounding the plate; and the second method is to use an oxidizing agent either incorporated in the positive plate or surrounding the positive plate.

Some cells develop local or internal short-circuit due to faulty electrode. This is called *local action* and which causes a complete discharge of the cell even when it is on open circuit or the electrode may waste away.

The positive electrode of a cell is called the *cathode* and the negative electrode is called the *anode.*

2. Primary Cells: Following is the classification of these cells:

(1) *Wet cells,*

(a) single fluid and (b) two fluid cells.

(2) *Dry cells.*

Primary cells usually consist of a containing jar, two electrodes and an electrolyte in the form of a chemical compound. During the discharge of the cell the anode wastes away by going into solution as positively charged ions which travel towards the cathode, the positive electrode.

A. *The Leclanche's Cell.* This is a single fluid wet cell, the electrolyte is a solution of ammonium chloride, NH_4Cl, in which the negative electrode, zinc, is placed. The positive electrode is carbon rod or plate placed in a porous pot in which is packed a mixture of manganese dioxide and carbon. This prevents polarization. But the process of depolarization is slow. Therefore this cell is only suitable for intermittent service, such as used for bell ringing. Fig. 1 shows the cell.

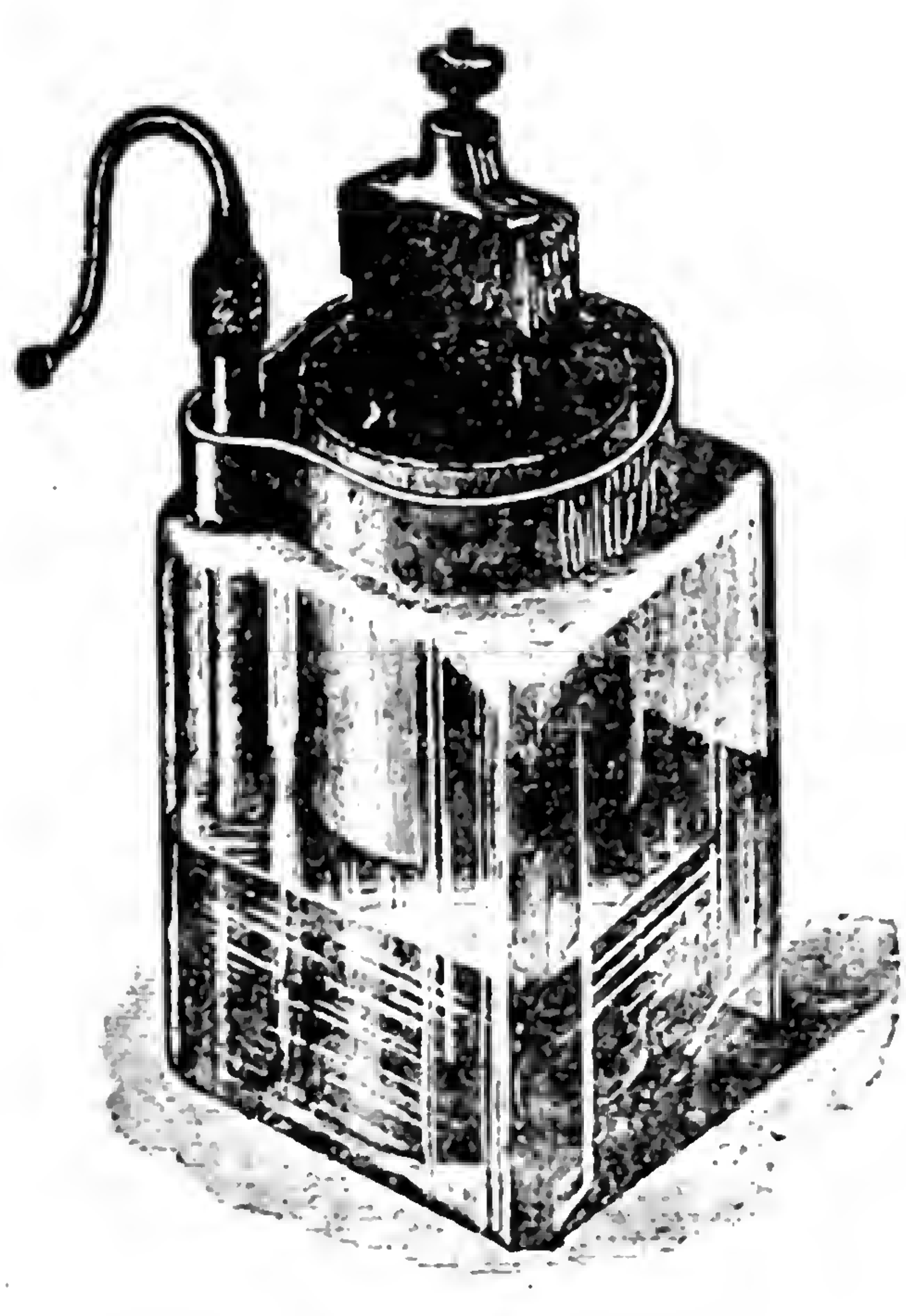


Fig. 1

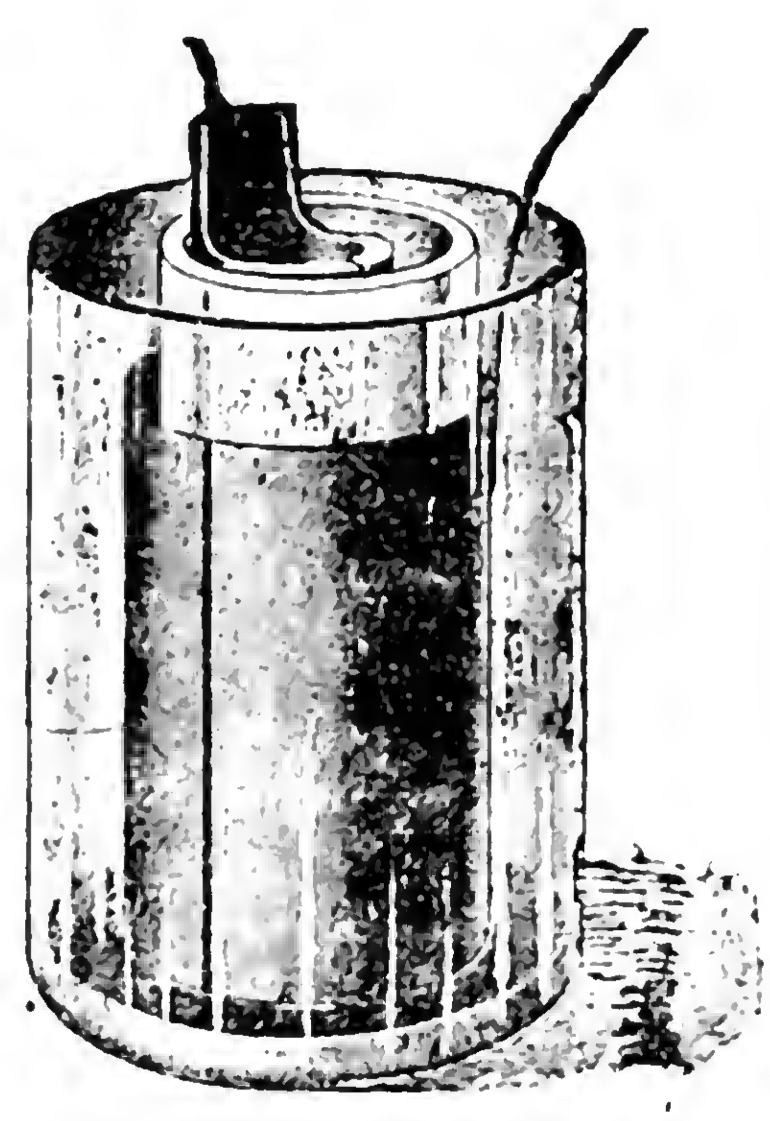


Fig. 2

Its voltage is about 1·5 volt. Zinc chloride, $ZnCl$, is formed when the cell supplies current. The NH_4 ions travel to the carbon plate, where they are split up in ammonia and hydrogen gas. The manganese dioxide reduces hydrogen to water.

B. *The Daniell Cell*: This is a two fluid wet cell. The two electrolytes are copper sulphate and dilute sulphuric acid. The negative electrode, zinc, is placed in the sulphuric acid and the positive electrode, copper, is immersed in copper sulphate solution. There is a porous pot separating the two electrolytes. The zinc rod and the sulphuric acid are in the porous pot. See Fig. 2.

This cell has a steady voltage of 1·07 volt. When the cell is discharging, the current flow inside the cell is from zinc to copper. Hydrogen, which is liberated, travels to the copper sulphate solution through the porous pot, liberates copper and forms sulphuric acid. The SO_4 ions travel to the zinc rod and form zinc sulphate. Thus there is no polarization. This cell can be used on continuous service.

C. *Dry Cells*. The most common dry cell consists of a zinc jar which is the negative terminal. The jar or container is coated inside with either a thin paste of starch or a paste board impregnated with a solution of zinc chloride and sal ammoniac. The positive electrode is a carbon rod placed centrally in the jar, surrounded by manganese dioxide mixed with powdered coke and graphite and sal ammoniac solution. The initial voltage is about 1·5 volt.

The two most common sizes are: $2\frac{1}{2}$ inches in diameter by 6 inches in height with a capacity of about 30 ampere-hours; and the other is a flash-light cell having a diameter of $1\frac{1}{4}$ inch and a height of $2\frac{1}{4}$ inches, the capacity being 3 amp-hr. Besides these two sizes there are others to suit particular situations such as used by medical men in some of their instruments, cells used for hearing aids, illuminometers etc.

3. Secondary Cells:

A. *The Lead-Sulphuric Acid Cell*: The positive plate of this cell is composed of lead grid, in the spaces of which is a paste of lead peroxide (PbO_2). The negative plate also consists of lead grid the spaces of which are filled with a paste of pure (spongy) lead.

In order to increase the current capacity and reduce the internal resistance of a cell, several plates are connected in parallel, and these are so assembled that the plates of opposite polarity are interleaved very closely. To prevent these plates from touching each other, *separators*, made of wood or glass rods are used.

The electrolyte is dilute sulphuric acid. Its specific gravity in a new cell is between 1·215 and 1·195. The hydrogen gas, which is given off due to electrolysis, is highly explosive in nature. Hence no naked flame should be brought near a battery while it is being charged.

The chemical actions during charge and discharge are complex in nature, but they can be represented approximately by the following equations :—

At negative plate : $Pb + H_2 SO \rightleftarrows PbSO_4 + 2H$

At positive plate : $PbO_2 + H_2 SO_4 \rightleftarrows PbSO_4 + H_2O + O$

During discharge read the equations from left to right and for reactions during charge read from right to left.

It will be noticed that during discharge water is formed, thereby reducing the specific gravity of the electrolyte, and that since all the hydrogen is utilised *there is no polarisation.*

And during charging more acid is formed. Hence the specific gravity of the electrolyte increases. When the cell is fully charged, the sp. gravity should be 1·21, and when fully discharged its value should be 1·17. Thus the value of the sp. gravity may be used as an indicator of the condition of a cell.

Similarly, when the cell is fully discharged, its e. m. f. should be 1·8 volts, but never less than this. When it is put on charge, the voltage gradually rises, until when fully charged at the end of normal period, its value is about 2·4 volts. At this stage the cell should "gas" freely. The reason for gassing is that there is no further chemical transformation possible and therefore hydrogen and oxygen are free to pass off into the atmosphere.

Fig. 3 shows the relation of the cell e. m. f. and the sp. gravity of the electrolyte with respect to time during normal rate of charge and discharge periods. It will be observed that the voltage of a cell remains round about 2 volts during a greater part of discharge time *when the rate of discharge is normal.* The normal time is mostly 8–hour period with rated normal current.

CARE should be taken not to discharge the cell any further when its voltage is 1·8 V. If further discharge takes place, the lead sulphate changes into an insoluble salt which decreases the capacity of a cell

and increases its internal resistance. Further, the cell that has been fully discharged must be charged within a day or two. There should always be a checking of the sp. gravity of the electrolyte both before

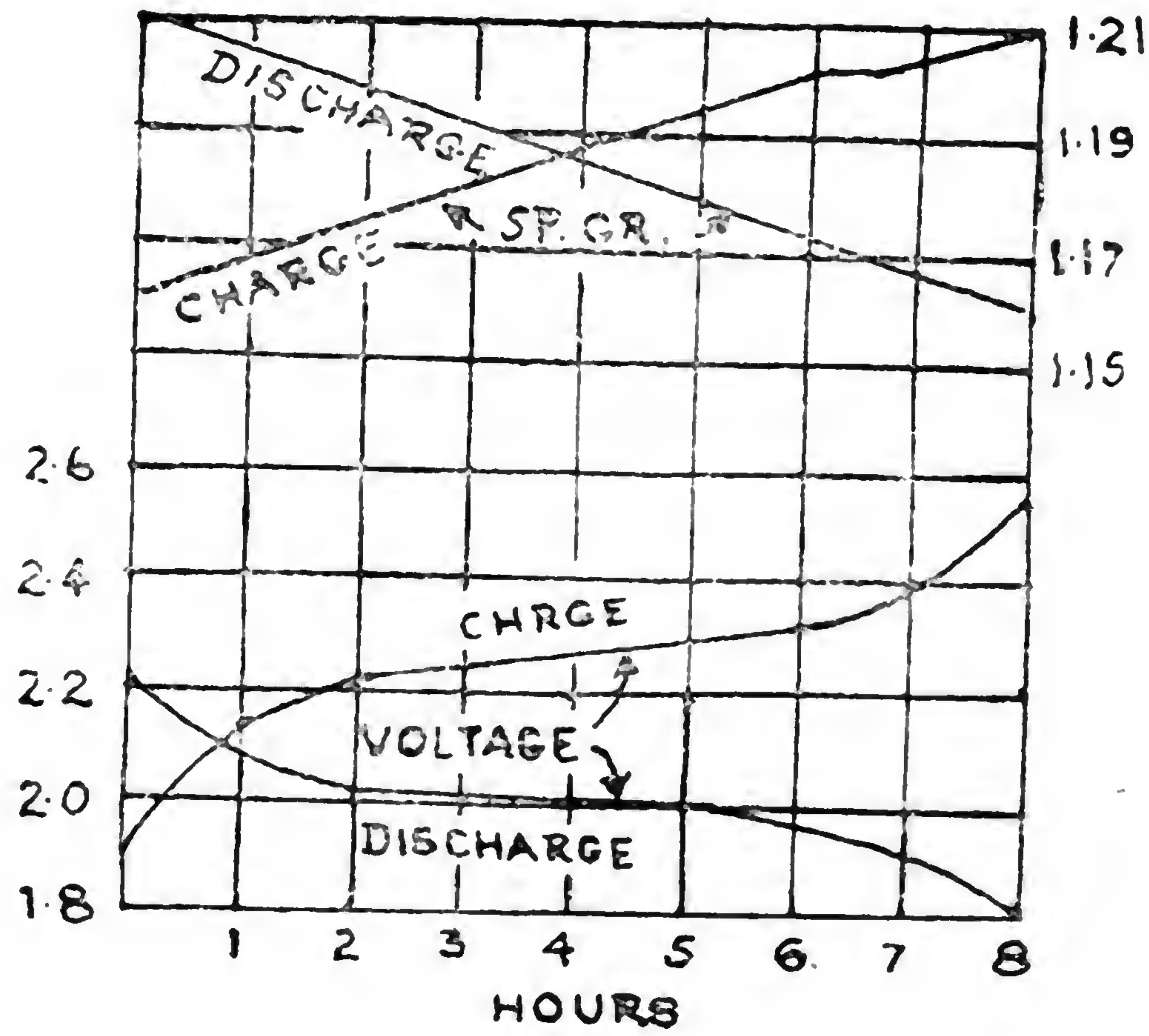


Fig. 3 Charge and Discharge Curves: Lead Cell

and after charging. The plates of the cell must be completely submerged in the electrolyte at all times. In fact, the level of the electrolyte should be at least half an inch above the top of the plates. Distilled water is added from time to time to make up the loss due to evaporation. The terminals of the lead cell usually get eaten up by electrolysis. This can be prevented by keeping the terminals well greased.

During charging, when the cell "gasses", particles of acid do creep over the surface, and atmosphere surrounding the cell is also full of particles of these gases. Hence it is very desirable that lead cells be housed in a separate room all by themselves, and the room ventilation should be such as to allow the harmful gases to escape into open atmosphere where they will do no harm.

The Electrolyte: The mixture is prepared from pure sulphuric acid and water. *The acid should be poured into a vessel containing water and not vice-versa.* An electrolyte having a lower sp. gravity produces less *local action* and therefore lengthens the life of a cell. On the other hand, a high sp. gravity increases the output

of a cell and reduces the size of a container. Hence, when the cell is fully charged the sp. gravity should not exceed 1·280. for at higher values *local action* increases greatly. Similarly, when the cell is discharged, the sp. gravity should not fall below 1·100, for at lower values the e. m. f. of the cell drops very rapidly.

The sp. gravity of the solution is measured by a *hydrometer.* It consists of a glass container having a narrow tube at one end and a rubber bulb at the other end. Inside this is a glass piece having a weighted bulb at the bottom and a graduated narrow stem at the top. When the acid is sucked in by the rubber bulb the weighted glass piece begins to float in the acid and the sp. gravity is read off from the graduated stem. This is known as the *syringe hydrometer.* In stationary cells, such as those used in power houses or telephone exchanges, the syringe part, is omitted and the weighted bulb is kept permanently floating in a pilot cell. The operator can read the sp. gravity any time he wishes.

B. **The Nickel-Iron-Alkaline Cell:** This cell was invented by Edison. As compared with the lead cell, this one is lighter in weight and has a longer life. The positive plate consists of a nickeled steel grid which holds nickeled-steel tubes containing the positive active material, nickel oxide (NiO).

The negative plate construction is similar to that of the positive plate but the active material is iron oxide (FeO). The electrolyte is an alkaline solution made up of potassium hydroxide to which is added a small quantity of lithium hydroxide.

On charge, the negative plate is the cathode and the positive is the anode. The iron oxide (FeO) at the negative plate is reduced to iron (Fe) and the nickel oxide (NiO) at the positive plate is oxidised

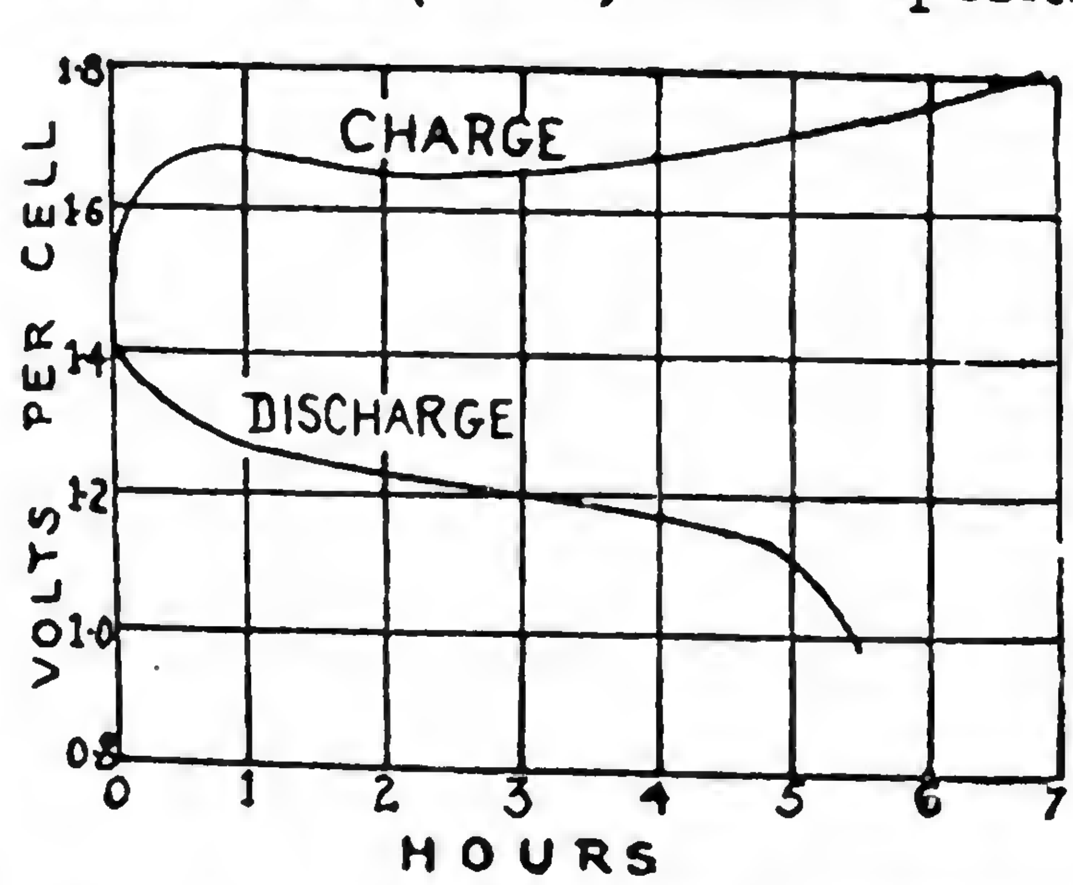


Fig. 4 Charge and Discharge Curves—Ni-Fe Cell.

to nickel dioxide (NiO_2). There is no change in the electrolyte chemically nor in its sp. gravity. *On discharge*, the positive plate is reduced to nickel oxide and the negative plate is oxidised to iron oxide. Hence the plates return to their original conditions. During discharge, the electrolyte again remains in the same condition, a great advantage over the lead cell. The charge and discharge curves of this cell are shown in Fig. 4. The cell can give satisfactory service for about five hours with normal rate of discharge. The cell is considered as fully charged when the voltage ceases to rise over a period of half an hour when a normal current has been flowing through it.

The normal working voltage of these cells varies between 1·4 and 1·2 volts. The sp. gravity of the electrolyte, when new, should be 1·2, but when it drops to about 1·16, after a number of years of service, new electrolyte must be poured into the cell. The container is made of nickled steel.

The escaping gases of this cell are not harmful, hence there is no need of housing these cells in a special separate room.

Following are the advantages of Ni–Fe cell over the Lead cell :—

(1) There is no buckling or warping of plates.

(2) There are no corrosive acid fumes.

(3) It may be over-charged or over-discharged.

(4) Short-circuit has little effect on its condition.

(5) It may remain idle or in a discharged condition for an indefinite period.

(6) It is very robust and shock-proof.

(7) Lighter in weight per watt-hour capacity.

4. **Number of Cells in a Battery**: The most usual practice is to connect cells in simple series to obtain the required terminal voltage. Suppose, for instance a battery of lead cells is required to supply power at 120 volts to a bus-bar. The method of calculating the total number of cells is as follows :—

Since the maximum and minimum voltages of a lead cell are 2·4 and 1·8 volts respectively, the total number of cells required in

series at the end of a discharge period will be

$$\frac{120}{1{\cdot}8} = 66{\cdot}7, \text{ i. e. } 67 \text{ cells.}$$

But when the cells are fully charged there need be only.

$$\frac{120}{2{\cdot}4} = 50 \text{ cells in series.}$$

The difference between 67 and 50 is **17.** These 17 cells are called *end-cells*, and they are so connected to an *end-cell regulator* that any number can be removed or added from the main battery as shown in Fig. 5.

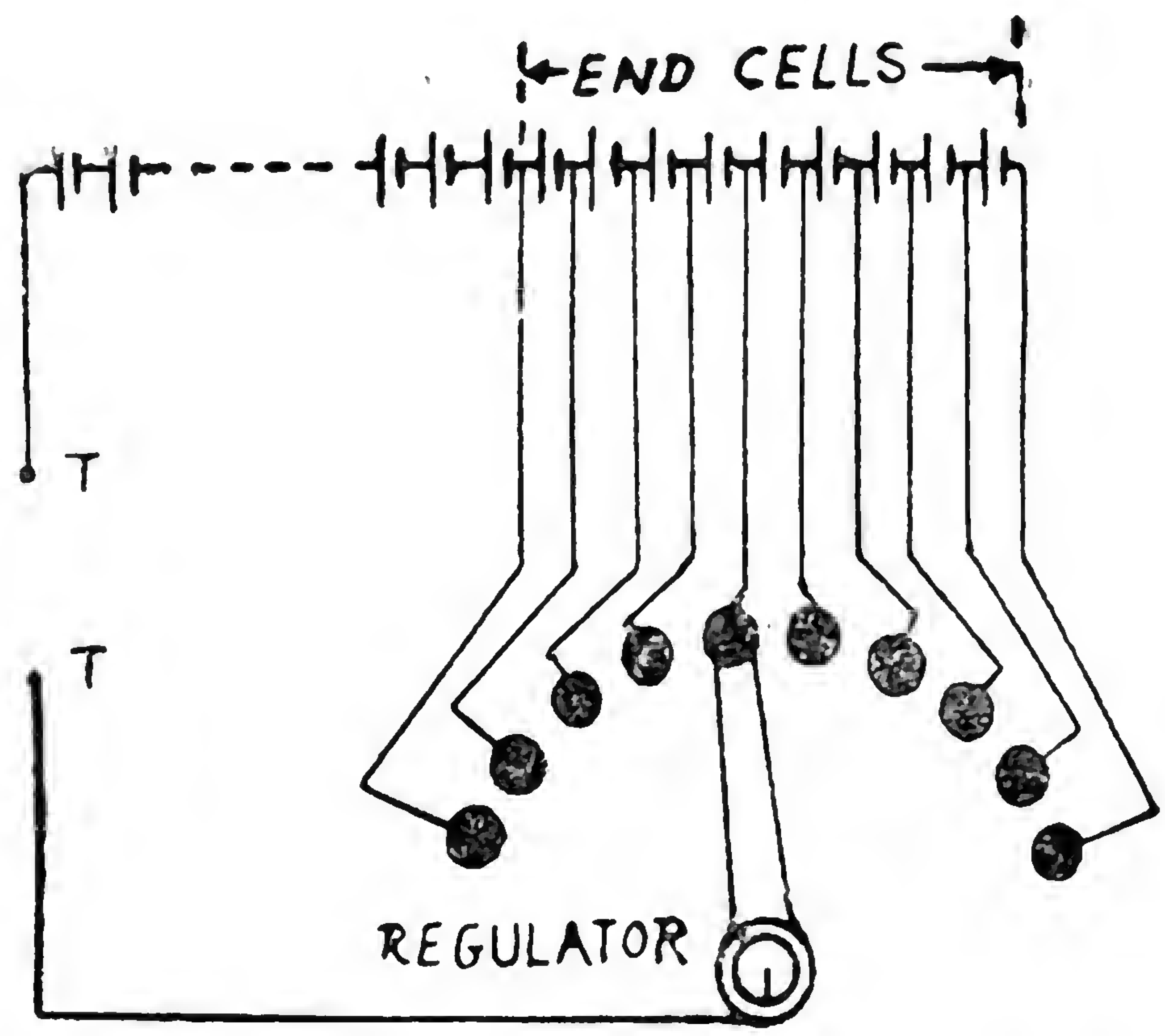


Fig. 5 End-cell Regulator.

When a number of similar cells are connected in series-parallel combination the current in the external resistance is given by

$$I = \frac{E}{\dfrac{R}{n} + \dfrac{r}{m}}$$

where E = e. m. f. of one cell ;

n = number cells in series per group ;

m = number of groups in parallel ;

r = internal resistance of each cell ;

R = external or load resistance.

The condition for maximum current is when $\frac{R}{n} = \frac{r}{m}$, so that

the expression for maximum current becomes

$$I_{max} = \frac{mE}{2r}\,; \quad \text{or} \quad I_{max} = \frac{nE}{2R}\,.$$

5. **Methods of Charging Batteries:** Small car-batteries may be charged from a D. C. house-supply mains by using a lamp load to drop the voltage to 6 or 12 volts as the case may be, but this method is not economical. There is a lot of wastage of energy in the lamp bank.

If the supply is A. C. it can be done economically by a rectifying valve arrangement. In fact, commercial rectifiers are available, such as *Tungar* etc., which do the charging at an economical rate.

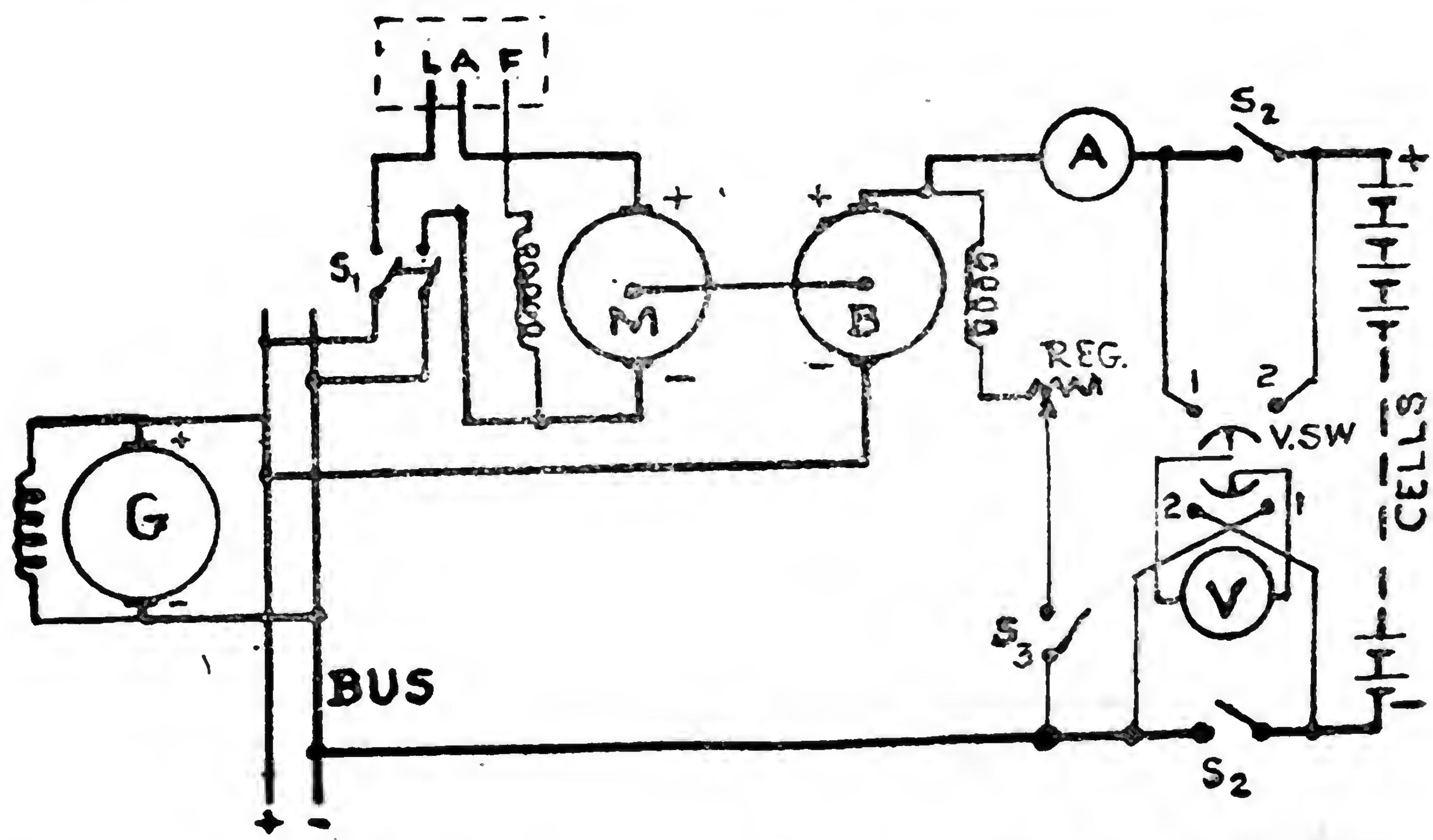


Fig. 6 Charging Circuit Diagram.

Where a battery of 120 volts is concerned, a motor-generator set is necessary. If the battery is made to "float" on the bus-bars during certain periods of light load, and to supply energy at peak loads then charging arrangement requires a booster to charge it from the bus-bars. Because when the cell voltage attains its maximum value of 2·4 volts, then for battery of the size mentioned in the last Section, the bus-bar voltage, at the time of full charge, will have to be more than

$$2{\cdot}4 \times 67 = 160{\cdot}8 \text{ volts.}$$

This is not possible, if the bus-bar supplies other loads rated at 120 volts. Hence the need of a booster. The charging arrangement is shown in Fig. 6 where a booster, with its motor, is included.

6. The Minimum Current Cut-Out: During the charging period, i. e. when the battery is being charged, the direction of current should be always from the generator, or bus-bars, to the battery and never in the reverse direction. The voltage of the battery rises during the charging period. If due to some cause, the voltage of the generator or the bus-bars falls suddenly, there is a flow of current in the reverse direction and the battery will be discharging unnecessarily. Moreover the difference between the two voltages may be such as to make the battery discharge at a higher rate than the normal. All this is harmful and wasteful.

If there is an operator present when the generator voltage falls, he may notice the ammeter in the charging circuit and switch off the current. Thus an operator must be present watching instruments throughout the charging period.

To avoid the current flowing from the battery to the generator automatically, use is made of the minimum current cut-out. This is a solenoid switch which operates the switch contacts of the main circuit. The contacts are closed when generator voltage is higher than the battery voltage by about, say, 4 volts. To pass the full normal charging current, the generator voltage has to be higher than the battery voltage by about 20 to 30 volts, depending upon the internal resistance of the battery and the connecting wires, and the value of the charging current. When the battery voltage, rises as it is being charged, the difference between the two voltages decreases. The minimum current cut-out will open the main circuit if this difference reaches 4 volts. Hence there is no chance ever for the current to flow in the wrong direction.

The switch has two coils on a common spool. One coil is made

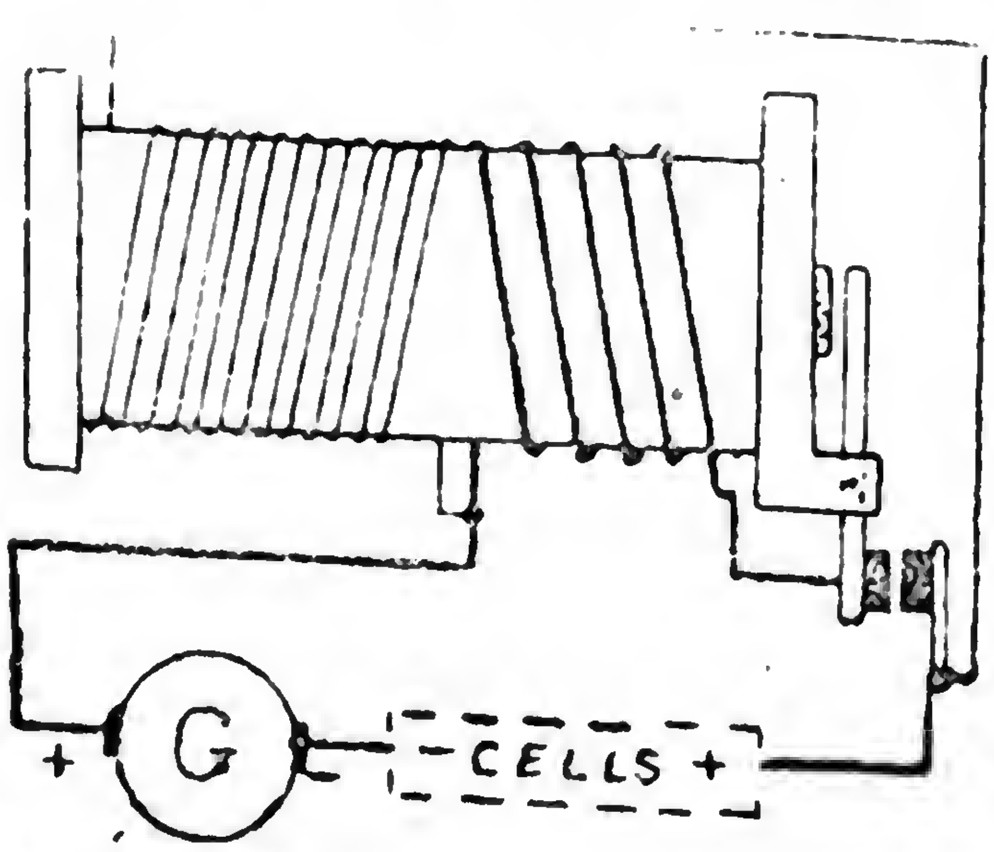


Fig. 7 Minimum Current Cut-out.

up of thin wire having many turns and high resistance. It is called the *differential coil.* The other coil is made up of thick wire having few turns and of low resistance. The solenoid has an armature at one end which makes a contact, when attracted, against the tension of a spring. The armature of the solenoid is pulled

up only when the current in the differential coil flows in one particular direction i. e. from the generator to the battery. This current flows from the positive of the generator to the positive of the battery. When the armature is pulled up it closes the circuit of the main coil and the bulk of the current, i. e., the charging current, flows through it. This current still further strengthens the solenoid and the contacts are held tightly.

There are various types of these cut-outs on the market. Every automobile is fitted with one. Diagram of Fig. 7 is now easy to understand.

7. **Rating of Batteries**: Lead-Acid batteries have a *nominal rate* based on *8-hour rate* of discharge. This means that the cell, or the battery, can give a certain specified amount of current continuously for 8 hours at the end of which the cell voltage will have dropped to 1·8 volts. Of course, the battery, or the cell. can be discharged within a shorter or longer period than 8 hours depending upon whether the current is greater or less than the specified normal current.

The unit of capacity is the *ampere-hour.* Thus we may have a battery rated at 120 ampere-hours. This means that the battery will give for eight hours a current of $\frac{120}{8} = 15$ amperes without being unduly discharged.

But a heavier discharge current will not give 120 ampere-hours. i. e. suppose the discharge current is increased from 15 to 20 amperes, the battery voltage per cell will drop to 1·8 volts much before 6 hours. Therefore at a heavier discharge current the discharge capacity is reduced.

The efficiency of a battery can be determined as

$$\eta = \frac{\text{ampere-hours output (discharge)}}{\text{ampere-hours input (charge)}}.$$

But this is not the true efficiency, since output and input involve energy or power. Therefore true efficiency can be stated as

$$\eta = \frac{\text{ampere-hours discharge} \times \text{average volts during discharge}}{\text{ampere-hour charge} \times \text{average volts during charge}}.$$

Calculation of average volts can best be done by noting the voltage every $\frac{1}{2}$ hour during charge and discharge periods.

If a *Watt-hour meter* is placed both in the charge and discharge circuits, then it becomes a simple matter of calculating the efficiency. For in this case

$$\eta = \frac{\text{watt-hours during discharge}}{\text{watt-hours during charge}}.$$

CHAPTER VII

D. C. DISTRIBUTION AND NETWORK

1. Introductory: It is not economical to transmit electric power by direct current over long or even medium distances owing to the limitations of high voltage d. c. generation. So that there is no long distance transmission of d. c. power, and one finds local d. c. distribution networks now-a-days in some towns and cities. Even these d. c. networks have diminished in numbers.

At the present time long distance transmission of power is by alternating current systems. The alternator is more suitable than a d. c. generator for generation of high voltage. Moreover, the a. c. voltage can be either raised or lowered very efficiently by means of static transformer. Theoretically there is no limit to the ratio of voltage transformation in a. c. This is then the reason why a. c. system is universally adopted for both generation and transmission.

Moreover the 3-phase induction motor for power drives is so cheap, simple and robust that it is now used by majority of industries. So that one finds a. c. distribution network in large cities and towns. However there are still some localities where d. c. distribution exists.

D. C. power can be generated in many ways. A few methods are listed below :—

(*a*) A d. c. generator coupled to a steam engine, or an oil engine, or an a. c. motor etc.

(*b*) A rotary converter, which transforms a. c. power into d. c. power very efficiently. It is a rotating machine.

(*c*) A mercury arc rectifier, which has no rotating parts. A. C. power is converted into d. c. power with high efficiency. Moreover it is possible to obtain high d. c. voltage from this device.

2. 2-Wire and 3-Wire Systems: The d. c. standard voltages in this country are 230 V, 460 V, 500 V and 1500 V.

1500 V is used for railway electrification;

500 V is used for electric tramways;

460 V is used for power drives ;

230 V is used for residential lighting and small h. p. motors and domestic appliances.

Since there is an increase in efficiency by using a higher voltage, the universal practice is to use 3-wire system rather than the 2-wire system. Besides, the 3-wire system gives both the voltages 230 and 460 V. Thus from the same system power is supplied to residential loads and power loads.

In the 2-wire system, shown in Fig. 1, there are two conductors, positive and negative, and in the 3-wire system, shown in Fig. 2,

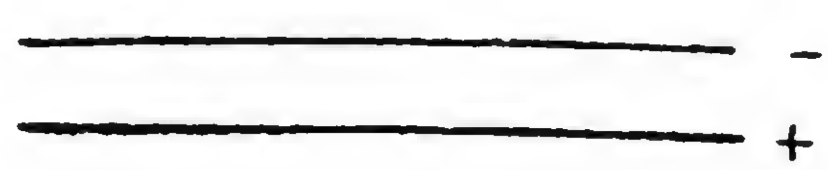


Fig. 1

Fig. 2

there are three conductors, positive, negative and neutral. Usually the neutral conductor is at earth potential, and its cross-sectional area is half of that of the positive conductor. Both positive and negative conductors have the same cross-sectional area. If the voltage between the positive and negative conductors is V, the voltage between the neutral and any one of the other two conductors of a 3-wire system is $\frac{1}{2} V$. The positive and negative conductors of a 3-wire system are called the "outers". In a 460/230 volt, 3-wire system the outers supply power to motors whose working voltage is 460 volts. The neutral and the positive, and the neutral and the negative supply residential lighting loads and motors whose working voltage is 230 volts.

3. The Network: The arrangement of feeders and distributors is called a network. There is no standard pattern for network. A network takes it shape solely due to local conditions and requirements. However certain standard practice is followed to make power distribution efficient and economical.

Fig. 3 shows a portion of a city's network system. It is a one-line diagram. The source of power is either a Generating Station or a Sub-Station. Feeders are connected to the Station bus-bars and they radiate over the entire area. Each portion of a city is served by a distributor which now-a-days is in the form of a closed path. Hence

it is called a Ring-Main Distributor. The distributors usually run along streets of the city. The feeders terminate at some convenient points on the Ring-Main Distributor. These points are shown as F_1 and F_2 in the figure. Sub-Distributors are tapped from the Ring-Main Distributor and taken along cross-roads. Service Mains are tapped either from Main Distributors or from Sub-Distributors. Each consumer should have his own Service Mains.

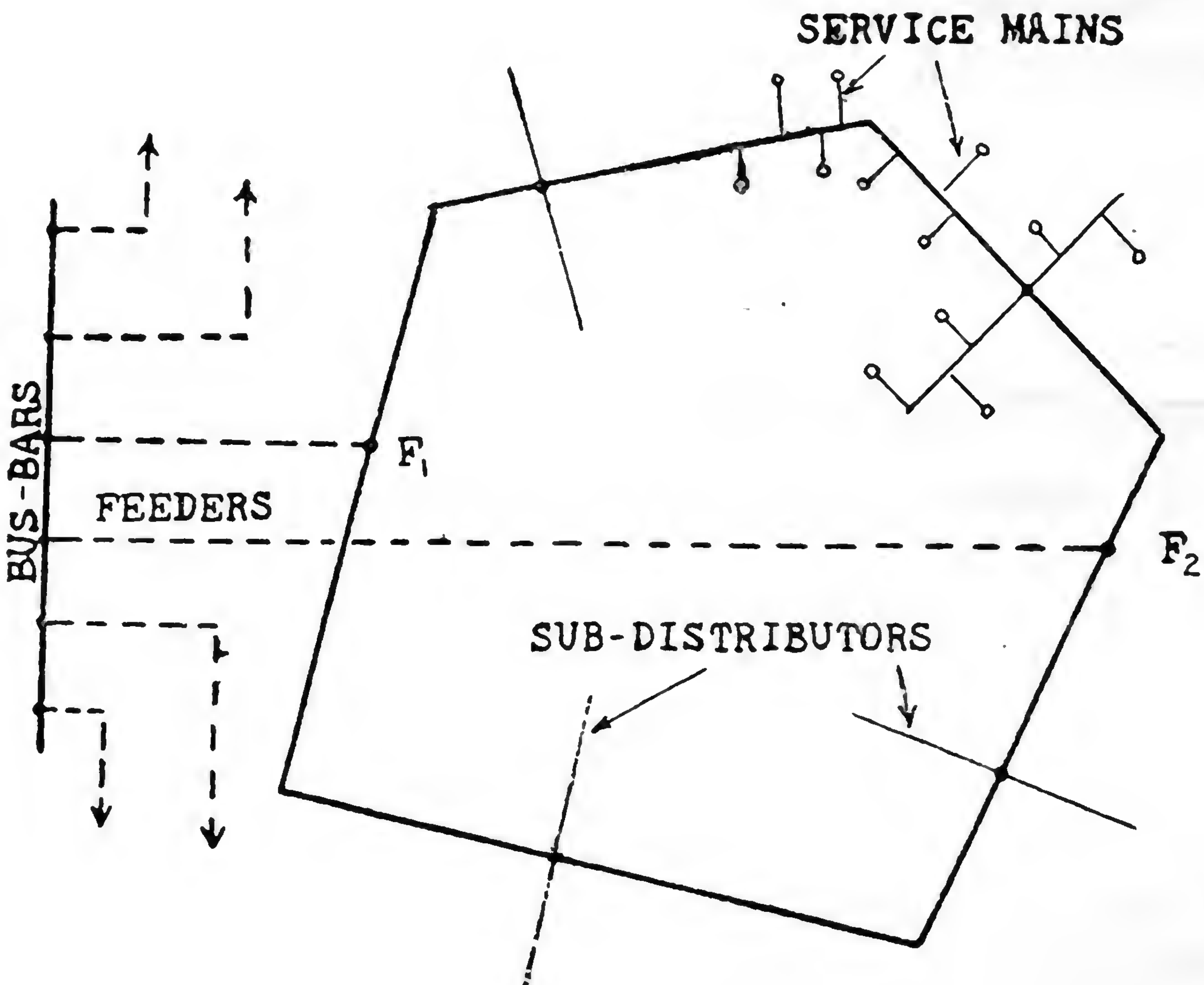


Fig. 3

In most of the cities and towns in this country the practice is to have overhead network, the conductors are bare copper wire supported on poles, cross-arms and insulators. In very large cities, like Bombay, the network system consists of underground cables. The Service Mains are also cables and they are connected to Pillar Boxes provided at regular and convenient points in the network. In Fig. 3 the Ring Main is shown with heavy lines. Power is supplied by two feeders at two points, F_1 and F_2, which are called *feeding points.* Only a portion of the Ring Main is shown tapped by Sub-Distributors and Service Mains.

In d. c. distribution, the consumers require power at 230 volts or 460 volts. Hence the Station bus-bars must have a higher voltage to compensate for the *voltage drops* in feeders, distributors and service mains. The variation of voltage at the load point of any consumer

must not exceed ± 6 % of the "declared" voltage. This is a Government stipulation and is binding on all Power Supply Undertaking Authorities. Thus it is important to know how to calculate the voltage drops, so that the Station bus-bar voltage may be kept at such a value that, after the loss in volts in the feeders and distributors, the consumers will be provided with power at the declared voltage.

4. Calculation of Voltage Drop : In feeders the current is the same at both its ends. If R ohms is its *total* resistance (i. e. resistance of both conductors $= R$), the voltage drop in the feeder is IR volts, where I is the current in the feeder. Hence, if the bus-bar voltage is V volts, the voltage at the end of the feeder, i. e. at the feeding point, will be ($V - IR$) volts.

In the case of distributors, which are tapped at several places, the current goes on reducing from the feeding point upto the point of minimum voltage. Hence the total voltage drop in the distributor is equal to the sum of the voltage drops in all its Sections.

Example : A 2-wire distributor AB, 400 yd. long, has a total resistance of 0·2 ohm and is loaded as shown in Fig. 4. The feeding point A is at 240 volts. The distances of C, D, and E from the feeding point are 40, 200 and 300 yd. Calculate the voltages at each load point.

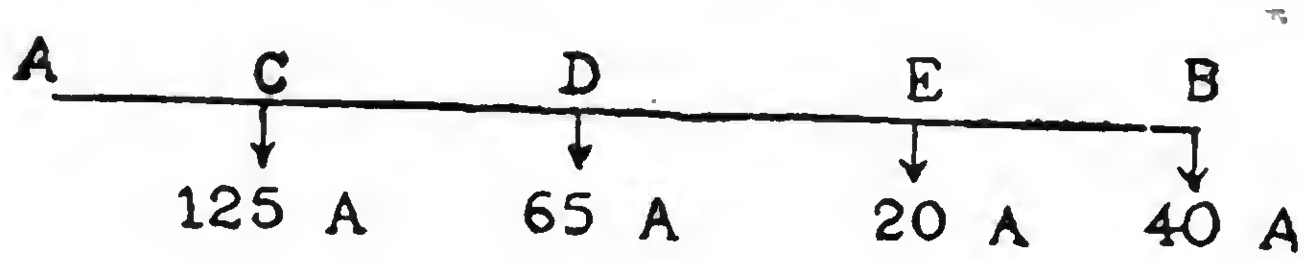


Fig. 4

Solution : The current flows from A to B. The current in EB = 40A; in DE = 60A; in CD = 125A and in AC = 250A. The section resistances are EB = 0·05 ohm; DE = 0·05 chm; CD = 0·08 ohm and AC = 0·02 ohm.

Therefore voltage drop in AC $= 250 \times 0{\cdot}02 = 5$ volts
" " " CD $= 125 \times 0{\cdot}08 = 10$ volts
DE $= 60 \times 0{\cdot}05 = 3$ volts
EB $= 40 \times 0{\cdot}05 = 2$ volts.

Hence voltage at C $= 240 - 5 = 235$ volts
" " D $= 235 - 10 = 225$ "
" " E $= 225 - 3 = 222$ "
" " B $= 222 - 2 = 220$ "

Thus the total voltage drop in the distributor is

$$\text{total drop} = 5 + 10 + 3 + 2 = 20 \text{ volts.}$$

The power lost in the distributor is calculated as follows :—

Power lost in Section AC = $(I^2 R) = 250^2 \times 0{\cdot}02 = 1250$ watts

" " " " CD = $125^2 \times 0{\cdot}08 = 1250$ watts

" " " " DE = $60^2 \times 0{\cdot}05 = 180$ watts

" " " " EB = $40^2 \times 0{\cdot}05 = 80$ watts

Total power loss = $1250 + 1250 + 180 + 80 = 2760$ watts

Power supplied = $240 \times 250 = 60000$ watts

$$\text{Therefore efficiency} = \frac{60000 - 2760}{60000} \times 100 = 95{\cdot}4\%$$

There are different methods of feeding a distributor, such as :—

(a) feeding at one end;

(b) feeding at both ends with equal voltages

(c) feeding at some intermediate point

(d) feeding at both ends with unequal voltages.

Besides these, the nature of loading is also different, namely :—

(e) concentrated loading

(f) uniform loading

(g) combination of (e) and (f).

(i) Uniform loading means that the distributor has a load point at equal distances, the load, in amperes, being the same at each point. For instance, in Fig. 5, a distributor is loaded uniformly at

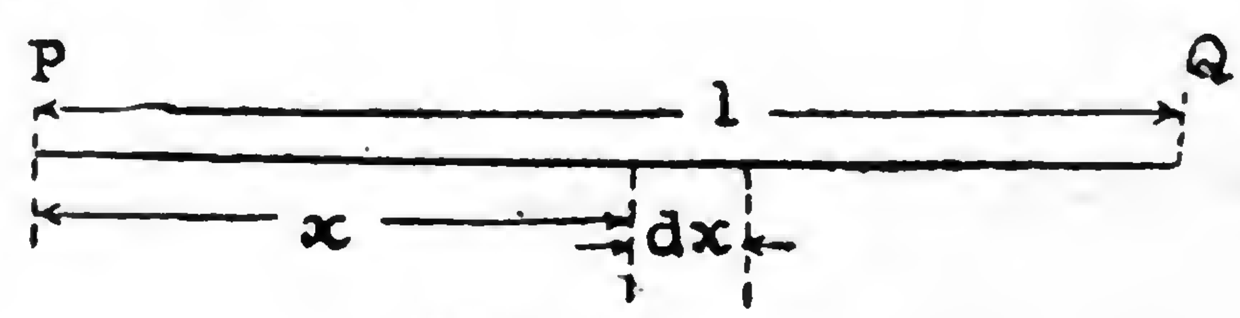


Fig. 5

I amperes per unit length. If the resistance of the unit length is r ohms the voltage drop at any point from P, which is the feeding point, is calculated as follows :—

Consider the distance x from P, where the current is $I(l - x)$ amperes. The resistance of a small section dx is rdx therefore the drop in dx is

$$dv = I(l - x)r\,dx$$

Hence the total drop upto x from P is

$$V = Ir\left(lx - \frac{x^2}{2}\right) \text{ volts.} \qquad \ldots \qquad \ldots \qquad (1)$$

When $x = l$, i. e. the voltage drop at Q is

$$v = Ir\left(l^2 - \frac{l^2}{2}\right) = \frac{Irl^2}{2}$$

Now $\dfrac{Irl^2}{2} = \dfrac{(I \times l)(r \times l)}{2}$

$(I \times l)$ = the total current entering at P and

$(r \times l)$ = R = the total resistance of the distributor PQ.

$\therefore$ total drop in PQ = $\frac{1}{2}$ × total resistance × total current.

The unit length may be a foot, a yard or 10 yds.

Example: A 400 yd distributor is loaded uniformly at the rate of 2 amperes per 5 yd. It is fed at one end with a voltage of 240 volts and its total resistance is 0·2 ohm. Calculate the voltage at the mid-point and at the other end of the feeder.

Solution: Since unit length is 5 yd. $l = \dfrac{400}{5} = 80$

Resistance of unit length $= r = \dfrac{0{\cdot}2 \times 5}{400} = \dfrac{1}{400}$.

At mid-point $x = \dfrac{l}{2} = 40$ unit lengths.

$\therefore$ voltage drop $= 2 \times \dfrac{1}{400}\left(80 \times 40 - \dfrac{40^2}{2}\right) = 12$ volts.

$\therefore$ voltage at mid-point = 240 − 12 = *228 volts.*

voltage drop at end $= 2 \times \dfrac{1}{400} \times \dfrac{80^2}{2} = 16$ volts

$\therefore$ voltage at the other end = 240 − 16 = *224 volts.*

(ii) **Feeding at both ends** is really a case of a Ring-Main Distributor. If the Ring-Main shown in Fig. 6 is split at F the two ends will have the same feeding point voltage as shown in Fig. 7.

Example: Find the minimum point of voltage and the value of this voltage, if the feeding point voltage F of Fig. 6 is 230 volts. The distances are:—

FA=50 yd.; AB=90 yd.; BC=100 yd.; CD=40 yd.; DF=30 yd.

The resistance of each distributor wire is 0·1 ohm per 200 yd.

Solution: When the Ring-Main is split at F and is assumed to be straight, Fig. 6 is converted into Fig. 7. The resistance of the sections are :—

FA = 0·05; AB = 0·09; BC = 0·1; CD = 0·04; DF = 0·03 ohm.

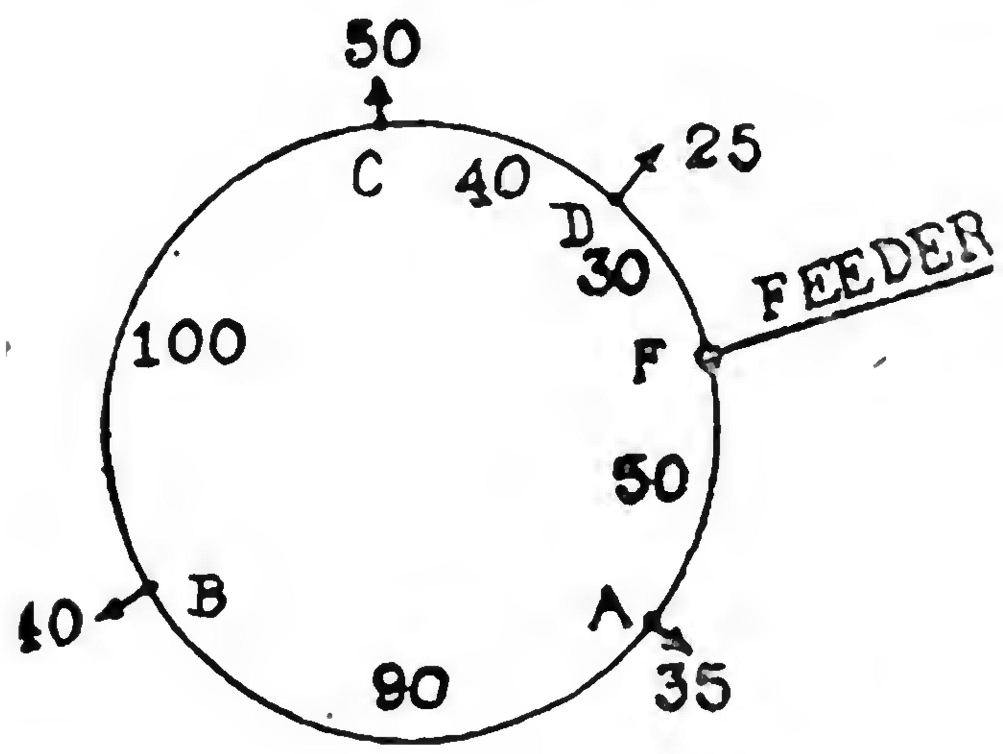


Fig. 6

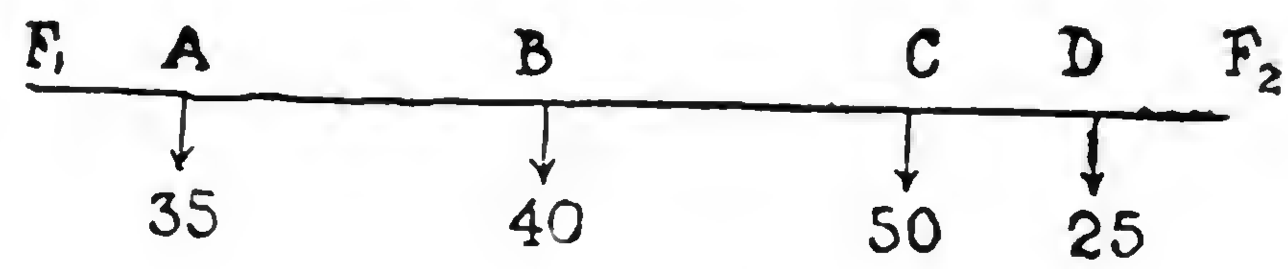


Fig. 7

The total current supplied to the distributor is 150 A, but at what point the currents from F_1 and F_2 meet is a guess work in the beginning. It is either B or C. Let us assume it is C, and let the current from B to C be i amps. Fig 8 shows the currents and the section resistances.

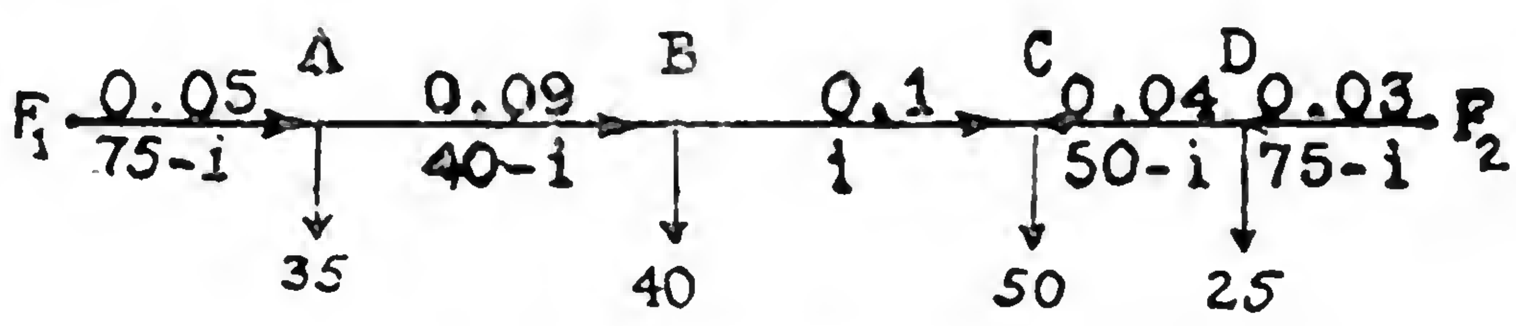


Fig. 8

Since the voltage at both ends is the same, the sum of voltage drops on the left of C must be equal to the sum of voltage drops on the right of C. Equating these drops

$0{\cdot}05\,(75+i) + 0{\cdot}09\,(40+i) + 0{\cdot}1i = 0{\cdot}04\,(50-i) + 0{\cdot}03\,(75-i)$

Solving the above, $i = -10$ amps.

This shows that B and not C is the point of minimum voltage. The currents in the sections now can be written as :—

$F_1A = 65$ amps, $AB = 30$ amps.

Hence drop in $F_1A = 0{\cdot}05 \times 65 = 3{\cdot}25$ volts,

drop in $AB = 0{\cdot}09 \times 30 = 2{\cdot}7$ volts

Total drop $= 5{\cdot}95$ volts.

∴ Voltage at B $= 230 - 5{\cdot}95 =$ *224·05* volts

Or we can proceed from the other end and calculate the total drop upto B :—

Current in BC $= 10$ amps; CD $= 60$ amps; $DF_2 = 85$ amps.

Drop in $DF_2 = 0{\cdot}03 \times 85 = 2{\cdot}55$ volts

„ „ CD $= 0{\cdot}04 \times 60 = 2{\cdot}4$ volts

„ „ BC $= 0{\cdot}1 \times 10 = 1{\cdot}0$

total drop $5{\cdot}95$ volts as before.

$\therefore$ Voltage at B $= 230 - 5{\cdot}95 = \mathit{224{\cdot}05}$ volts.

But now suppose the distributor, shown in Fig. 9 with its loads and section distances, is fed at F_1 with a voltage of 240 volts and at F_2 with a voltage of 237 volts. It is required to determine the load voltages and the currents fed at F_1 and F_2. The total resistance of the distributor is $0{\cdot}2$ ohm. Accordingly the section resistances are :—

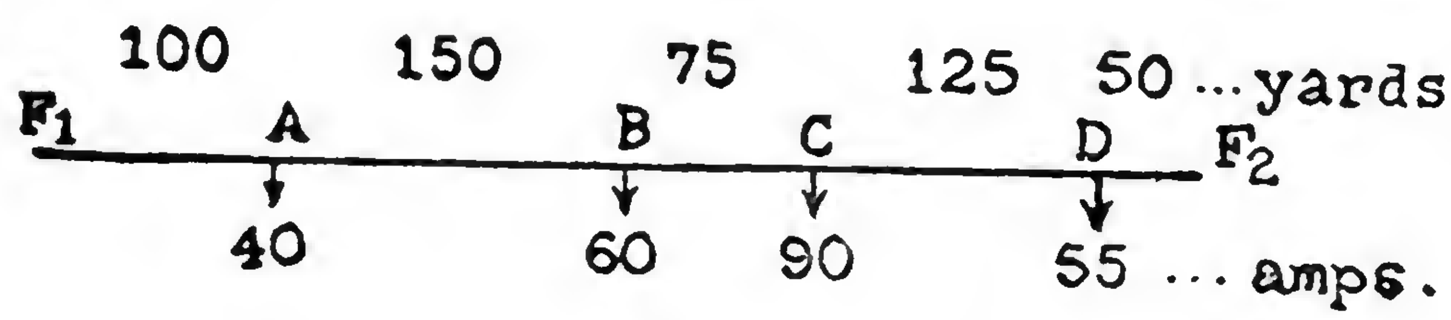


Fig. 9

$F_1A = 0{\cdot}04$: $AB = 0{\cdot}06$; $BC = 0{\cdot}03$; $CD = 0{\cdot}05$ and $DF_2 = 0{\cdot}02$ ohm.

If C is assumed to be the point of minimum voltage, then the sum of voltage drops to the left of C is equal to the sum of drops to the right of C *plus* 3. Because F_1 is at a higher voltage by 3 volts than the voltage at F_2. Proceeding as in the previous case,

$$0{\cdot}03\,i + 0{\cdot}06\,(60+i) + 0{\cdot}04\,(100+i) = 0{\cdot}05\,(90-i) + 0{\cdot}02\,(145-i) + 3$$

$$0{\cdot}2\,i = 2{\cdot}8 \qquad \therefore \quad i = 14 \text{ amps.}$$

Thus the current at F_1 is 114 amps and at $F_2 = 131$ amps.

Drop in $F_1A = 0{\cdot}04 \times 114 = 4{\cdot}56$ volts

„ „ $AB = 0{\cdot}06 \times 74 = 4{\cdot}44$ „

„ „ $BC = 0{\cdot}03 \times 14 = 0{\cdot}42$ „

$9{\cdot}42$ „

$\therefore$ voltage at C $= 240 - 9{\cdot}42 = 230{\cdot}58$ volts.

(iii) In 3-wire systems the calculation of voltage drop is somewhat similar. The current in the neutral wire has to be calculated

carefully. An example worked out below will clearly show the method of calculating the current in each section.

Fig. 10 shows the loads and distances in yards. The neutral has half the cross-sectional area of the outers. The voltages between T_1 and T_2, and T_2 and T_3 are constant at 230 volts. M_1 M_2 is a motor load taking 50 amps. and connected between the outers. The resistance of each outer conductor is 0·02 per 100 yds. Hence the resistance of neutral becomes 0·04 ohm per 100 yds.

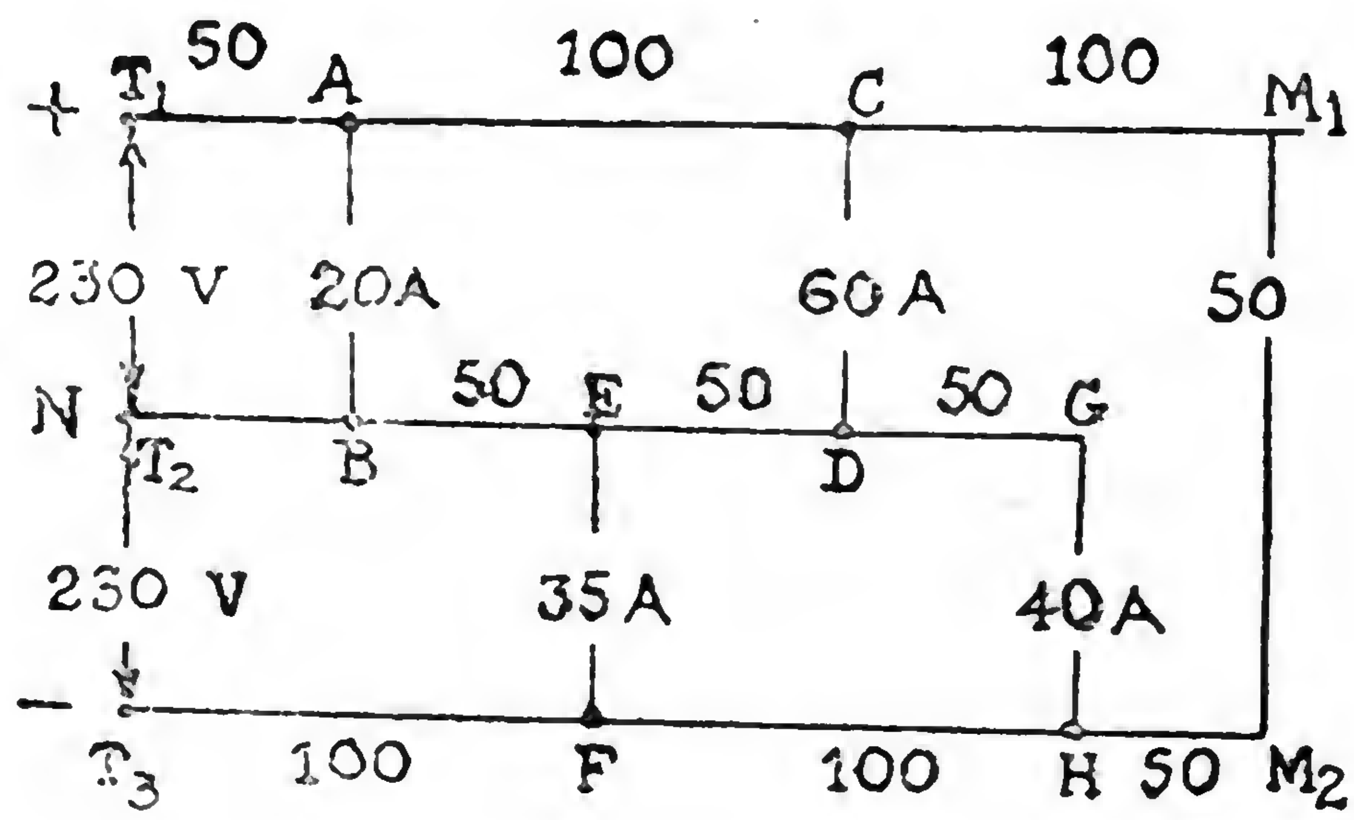


Fig. 10

The direction and magnitude of current in each section of the neutral wire is calculated first and marked in Fig. 11. One check is to see that as many amperes go out as enter the network, i. e. at T_1

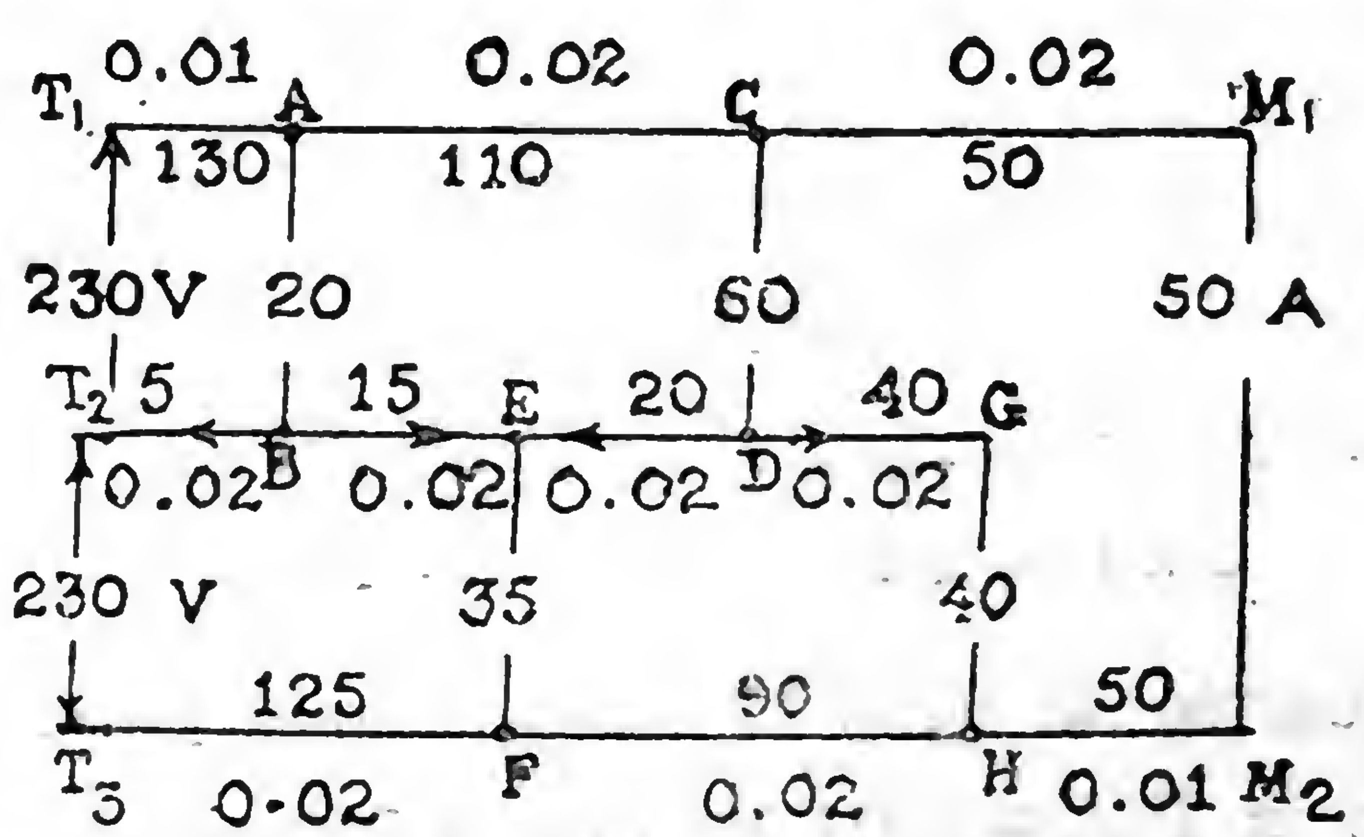


Fig. 11

130 amps enter and at T_2 and T_3 130 amps leave the network. Having calculated the resistance of each conductor section, we proceed to calculate the voltages at each load point thus :—

(a) Drop in $T_1A = 0{\cdot}01 \times 130 = 1{\cdot}3$ V

Drop in $T_2B = 0{\cdot}02 \times 5 = 0{\cdot}1$ V

Proceeding from T_1 to A, B and T_2 the direction of current is also the same hence the two voltage drops in T_1A and T_2B are of negative sign. Therefore the voltage at load point AB is

$$230 - (1\cdot3 + 0\cdot1) = 228\cdot6 \text{ V}$$

(b) Drop in AC $= 0\cdot02 \times 110 = 2\cdot2$ V;

Drop in DE $= 0\cdot02 \times 20 = 0\cdot4$ V;

Drop in EB $= 0\cdot02 \times 15 = 0\cdot3$ V.

Proceeding from A to C, D, E and B we find that the voltage drop in EB has a positive sign since the current has a reverse direction. Therefore the voltage at load points CD is

$$228\cdot6 - (2\cdot2 + 0\cdot4) + 0\cdot3 = 226\cdot3 \text{ V.}$$

(c) Similarly, the voltage at the load point EF is

$$230 + 0\cdot02 \times 5 - 0\cdot02 \times 15 - 0\cdot02 \times 125$$
$$230 + 0\cdot1 - 0\cdot3 - 2\cdot5 = 227\cdot3 \text{ V}$$

(d) Voltage at GH

$$227\cdot3 + 0\cdot02 \times 20 - 0\cdot02 \times 40 - 0\cdot02 \times 90$$
$$227\cdot3 + 0\cdot4 - 0\cdot8 - 1\cdot8 = 225\cdot1 \text{ V}$$

(e) Voltage at $M_1 M_2$

$$460 - 0\cdot01 \times 130 - 0\cdot02 \times 110 - 0\cdot02 \times 50$$
$$- 0\cdot01 \times 50 - 0\cdot02 \times 90 - 0\cdot02 \times 125$$
$$460 - 1\cdot3 - 2\cdot2 - 1\cdot0 - 0\cdot5 - 1\cdot8 - 2\cdot5$$
$$460 - 9\cdot3 = 450\cdot7 \text{ V.}$$

5. Efficiency of Transmission at Higher Voltages: Let us determine the efficiency of transmission at two different voltages when

(a) Load at receiving end = 100 kW,

(b) distance of transmission = 1 mile,

(c) current density in feeder conductors = 1000 A/□″.

Case I: Load point voltage = 100 volts

Case II: „ „ „ = 400 volts

	Case I : 100 volts	*Case II* : 400 volts
Current in feeder	$\frac{100\times1000}{100}=1000$ A	$\frac{100\times1000}{400}=250$ A
Area of each conductor...	1 sq. inch	0·25 sq. inch
Resis. of each conductor	$\frac{0{\cdot}67}{10^6}\times\frac{1760\times36}{1}=0{\cdot}042$	$=0{\cdot}168$ ohm
Voltage drop in feeder	$2\times1000\times0{\cdot}042=84$ V	$2\times250\times0{\cdot}168=84$ V
Sending-end voltage	$100+84=184$ V	$400+84=484$ V
%efficiency $\left(=\frac{\text{output}}{\text{input}}\right)$	$\frac{100\times1000}{184\times1000}\times100=$ **54·4%**	$\frac{400\times250}{484\times250}\times100=$ **82·7%**

This clearly shows that it is very economical to use high voltage for transmission of power. The longer the distance and greater the power the higher should be the transmission voltage. But in direct current system it is not the practice to transmit over long distances power at high voltage at the present time. The universal practice is to generate and transmit electric power by using alternating current.

6. Balancers: The 3-wire d. c. feeders get their current from ordinary d. c. generators which are all connected in parallel across the outer conductors. In order to fix the potential of the middle wire (i. e. the neutral), two identical shunt machines, mechanically coupled and having their armatures connected in series, are connected across the outer. The middle wire is connected to the junction of the two armatures and the point is earthed. The field windings of the two machines are also in series and connected across the outers. See Fig. 12. There is no current in the neutral wire if the load on the positive side is equal to that on the negative side. But if the loads are not "balanced", a current flows in the neutral wire. This current

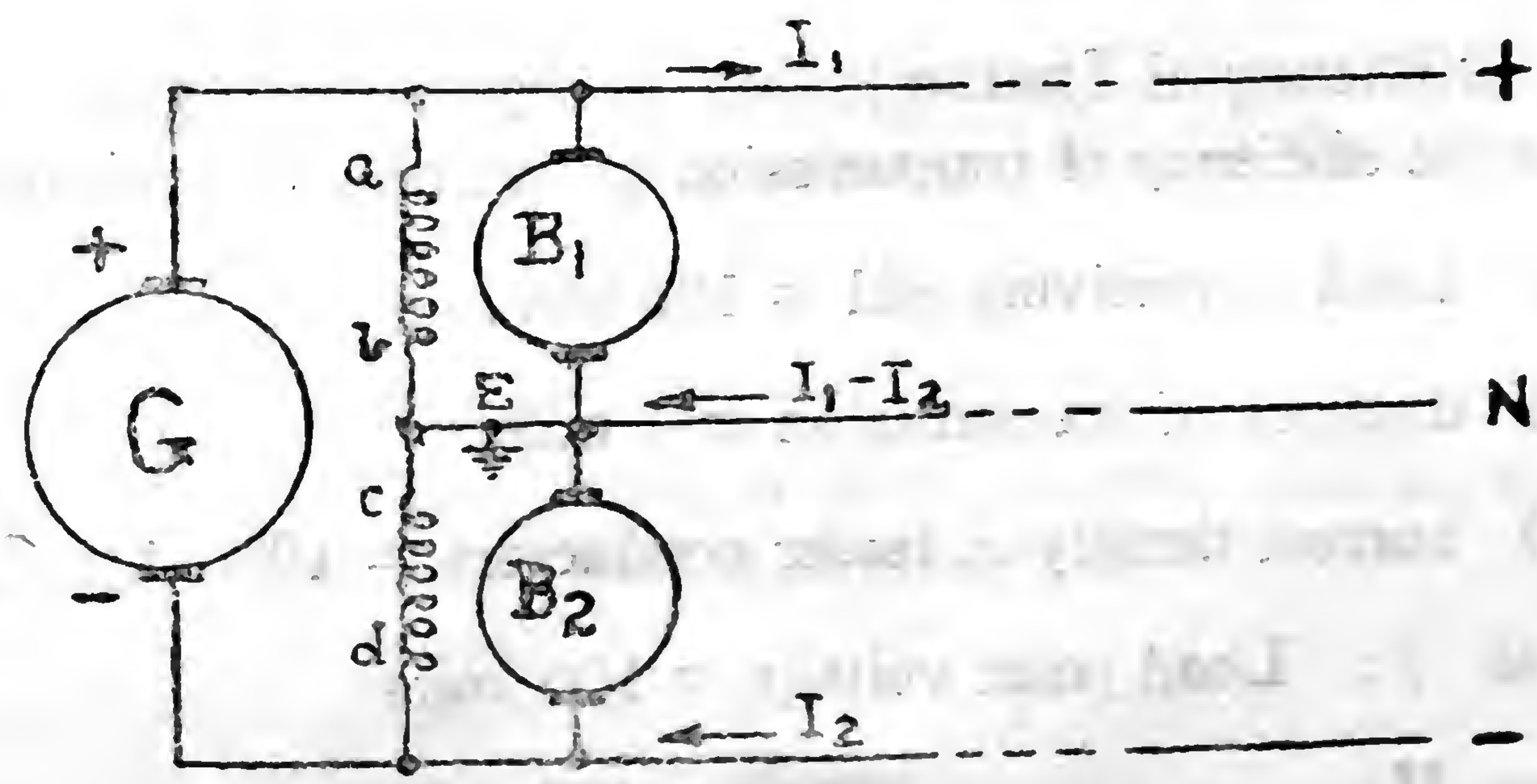


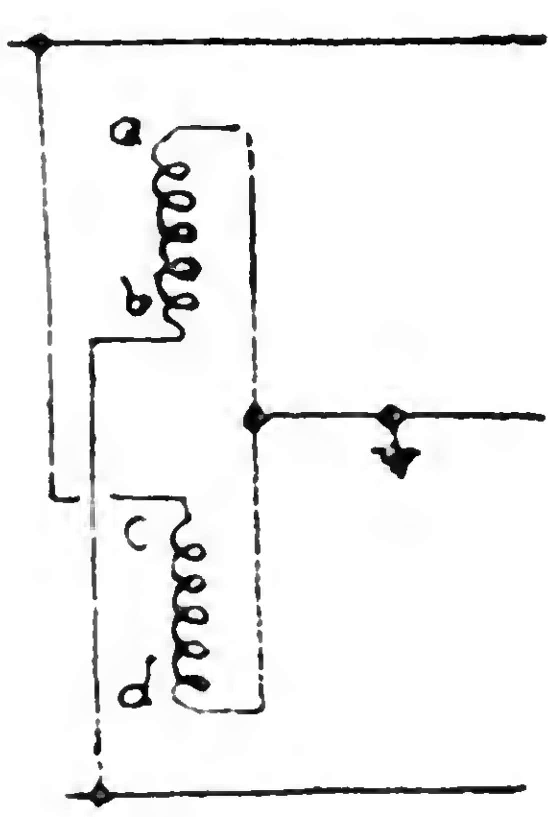
Fig. 12

is called *out-of-balance current.* It is the arithmetic difference between the currents in the outer conductors.

If the balancers are not used, the difference between the voltages of the two sides vary greatly when the loads are not balanced. For example, on a 230/230 V feeder 3-wire system if 10 lamps are connected on the positive side and 9 similar lamps on the negative, the voltages on the positive and the negative sides will be in the ratio of 9 : 10, i. e. the positive side voltage will be 218 V and negative side 242 volts. This unbalance of voltage is reduced by the balancer set.

When the loads are balanced, the two machines of the set run as unloaded motors. When one side gets more loaded than the other, the machine on the heavily loaded side acts as a generator being driven by the other machine acting as a motor. So that the generating machine injects into the heavily loaded side a voltage to make up partly the deficit.

If the field windings of the two machines are "crossed" as shown in Fig. 13, the difference in the voltages of the two sides is still further reduced because the motoring machine gets less field current and runs faster and the generating machine e. m. f. is higher because (a) it runs faster and (b) its field current is greater. A starter is necessary to start the balancer set because the two machines in the initial stage operate as unloaded motors.



Fig, 13

The two machines of the set have the usual losses. These losses are, of course, supplied by the main generators. So that the input to the motoring machine is equal to

the output of the generating machine + losses of both machines.

When the balancer fields are connected "straight" the e. m. f. and terminal voltages of the two machines have the following relationship :—

Let I_0 = no load current of the set,

I_1 = current in positive outer under load condition,

I_2 = current in negative outer under load condition,

R_a = resistance of armature of each machine,

$2V$ = voltage between the outers.

The back e. m. f. of machine

$$E_b = V - I_0 R_a \text{ (assumed constant)} \quad \ldots \quad \ldots \quad (2)$$

On unbalanced load, the terminal voltage of the motoring machine is

$$V_m = E_b + I_m R_a \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

and the terminal voltage of the generating machine is

$$V_g = E_b - I_g R_a \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

where I_m and I_g are the currents in the armatures of the two machines.

$$\therefore \quad V_m - V_g = (I_m + I_g) R_a \quad \ldots \quad \ldots \quad (5)$$

This shows that in order to have a small difference in voltage on the two sides, R_a must be low.

$(I_m + I_g)$ is the current in the neutral wire $= I_N$.

The current in the neutral is the difference of the currents in the outers.

$$\therefore \quad I_m = \frac{I_N}{2} + I_0 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

$$I_g = \frac{I_N}{2} - I_0 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

Example : A 3-wire feeder supplies power to a distribution network. The bus-bar voltage is kept constant at 460 volts and the balancer machine fields are connected "straight". The armature resistance of each balancer machine is 0·08 ohm, and the no load currents of the set is 10 amperes. Neglecting the feeder conductor resistances, calculate (a) the terminal voltages across each balancer machine and (b) the current in each balancer machine, when the load on positive outer is 600 A and on the negative outer 800 A.

Solution : $I_N = 800 - 600 = 200$ A.

$V_m - V_g = 200 \times 0{\cdot}08 = 16$ volts. But $V_m + V_g = 460$ volts.

(a) $230 + 8 = V_m = 238$ volts,

$230 - 8 = V_g = 222$ volts.

(b) $I_m = \dfrac{200}{2} + 10 = 110$ A

$I_g = \dfrac{200}{2} - 10 = 90$ A.

This can be checked by

$$V_m I_m = V_g I_g + I_m^2 \times 0{\cdot}08 + I_g^2 \times 0{\cdot}08 + 4600$$

$$238 \times 110 = 222 \times 90 + 12100 \times 0{\cdot}08 + 8100 \times 0{\cdot}08 + 4600$$

$$26180 = 19980 + 968 + 648 + 4600 = 26196 \text{ W}.$$

Let us see the effect of disconnecting the *neutral* from the balancer set :—

The resistance of the lightly loaded side is approximately $R_+ = \dfrac{230}{600}$ and $R_- = \dfrac{230}{800}$ ohm on the heavily loaded side, i. e. in the ratio of $\dfrac{R_+}{R_-} = \dfrac{4}{3}$. Therefore when neutral is disconnected R_+ and R_- are in series and the current is common to both. Hence the voltage drop across the lightly loaded side in this case is

$$V_+ = 460 \times \frac{4}{7} = 262{\cdot}86 \text{ volts}$$

$$V_- = 460 \times \frac{3}{7} = 197{\cdot}14 \text{ volts.}$$

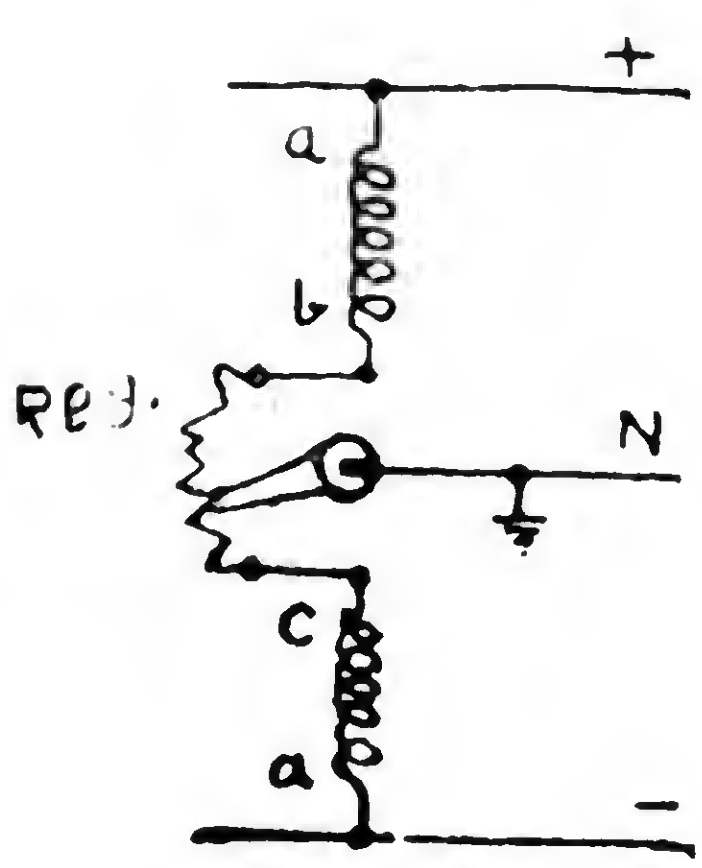


Fig. 14

This illustration clearly shows why *a fuse must not be inserted in the neutral wire.* A fuse when it blows disconnects the circuit, and may cause undesirable voltages across the loads. In the case of lamp loads, all the lamps on the lightly loaded side are liable to be burnt out.

If a regulator in the field circuit of the balancer set is used, see Fig. 14, the voltages on the two sides can be controlled at will.

7. Feeder Boosters : Boosters are used where a particular feeder has a large and fluctuating voltage drop. Since there are usually a number of feeders on the same bus-bars it is not practicable to adjust the voltage of bus-bars for the sake of one feeder. The voltage drop can be reduced in the particular feeder by increasing the cross-sectional area. This is done when the loading is heavy, but when the voltage drop is large due to the feeder length increasing, the cross-sectional area of the feeder is uneconomical. Instead a booster set is employed.

Fig. 15 shows a feeder booster. It is a motor-generator set. The generator is series wound and is worked on the straight portion of its magnetisation curve. So that its terminal voltage is directly proportional to the current in the feeder. And since the drop in the feeder is proportional to the current, the booster compensates the voltage drop at all loads. Thus the customers on that feeder get proper voltage. The cost of the booster set is comparatively much less than the cost of increasing the cross-sectional area of the feeder.

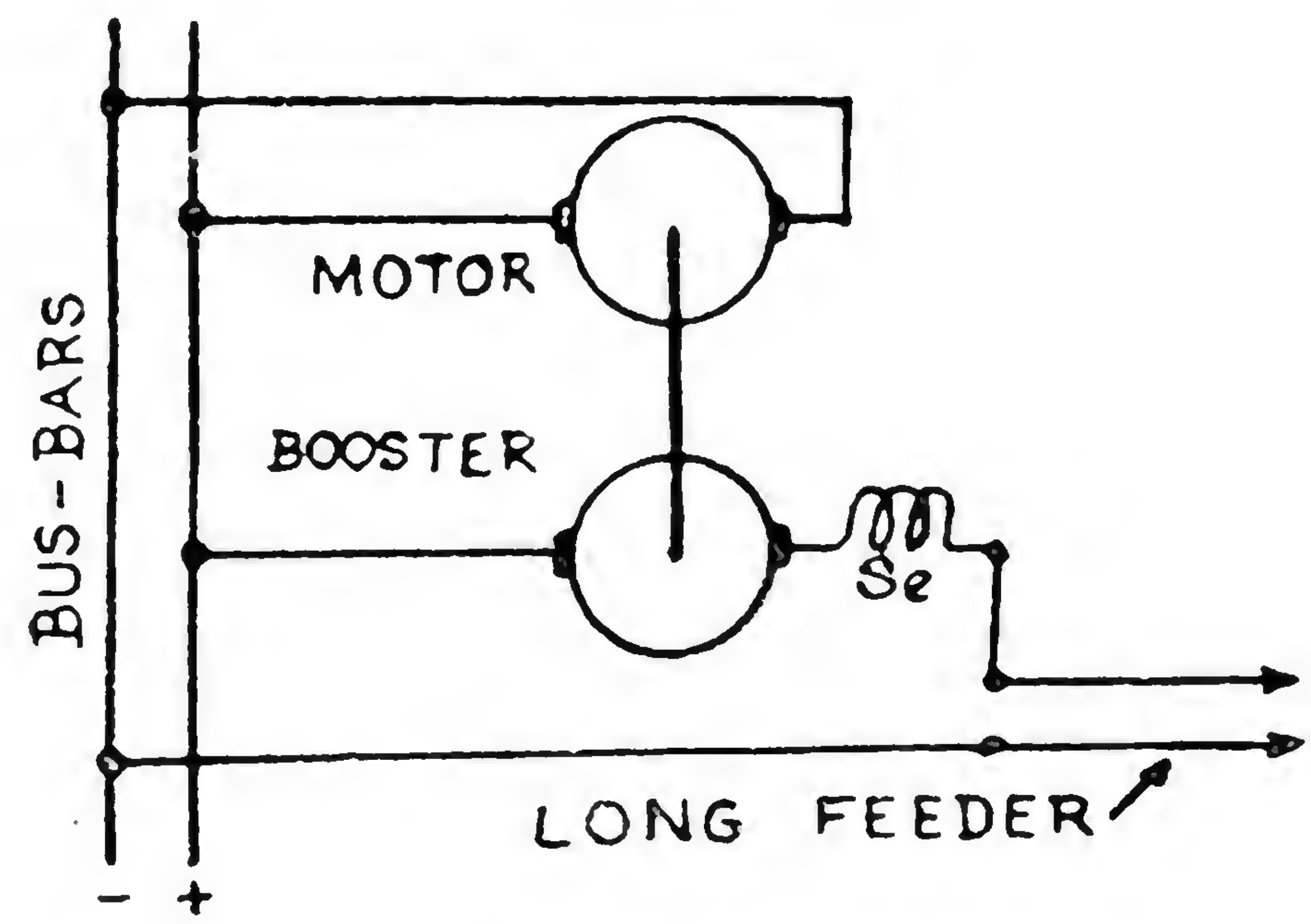


Fig. 15

8. Kelvin's Rule Applied to Feeders: Kelvin's Rule says that the most economical size for a feeder is that size for which the annual cost of energy lost in the feeder equals the annual cost of interest and depreciation on the capital cost of the conductor material.

For any particular case, where the voltage and the distance of transmission are fixed, let

R = total resistance of the feeder,

l = the total length of the feeder,

t = time in hours of working period during a year,

I = the average value of current during time t,

and X = the cost of 1 B. O. T. unit in fraction of a rupee.

The total energy lost during the year in the feeder is $\frac{I^2 Rt}{1000}$ B. O. T. units or kW-hrs. and the annual cost of energy will be $\frac{I^2RtX}{1000}$ rupees, but $R = \rho \frac{l}{A}$, where A = area of conductor

$$\therefore \quad annual\ cost = \text{Rs}\ \frac{I^2 tX}{1000}\ \rho\ \frac{l}{A} = \text{Rs}\ \frac{a}{A},$$ where a is a constant and is equal to $\rho\ \dfrac{I^2 l t X}{1000}$.

Secondly, let the cost of the conductor material be Rs. y per unit volume. The volume of the feeder is $(A \times l)$. Hence the cost of the feeder material will be, $cost = yAl$ rupees.

If the rate of interest and depreciation is taken as z per cent, the annual cost of interest and depreciation for the conductor material will be

$$annual\ cost = \frac{1}{100} (yzAl) \text{ rupees.}$$

Since y, z and l are constant for a particular case the above cost reduces to

$$annual\ cost = bA \text{ rupees, where } b = \frac{1}{100} yzl.$$

Adding the two costs,

$$total\ cost = \frac{a}{A} + bA$$

For the total cost to be a minimum

$$\frac{d\,(\text{total cost})}{dA} = 0 = -\frac{a}{A^2} + b \quad \text{i. e.} \quad A = \sqrt{\frac{a}{b}}. \quad \ldots \quad (5)$$

The most economical size will be when the conductor area A is equal to $\sqrt{\dfrac{a}{b}}$.

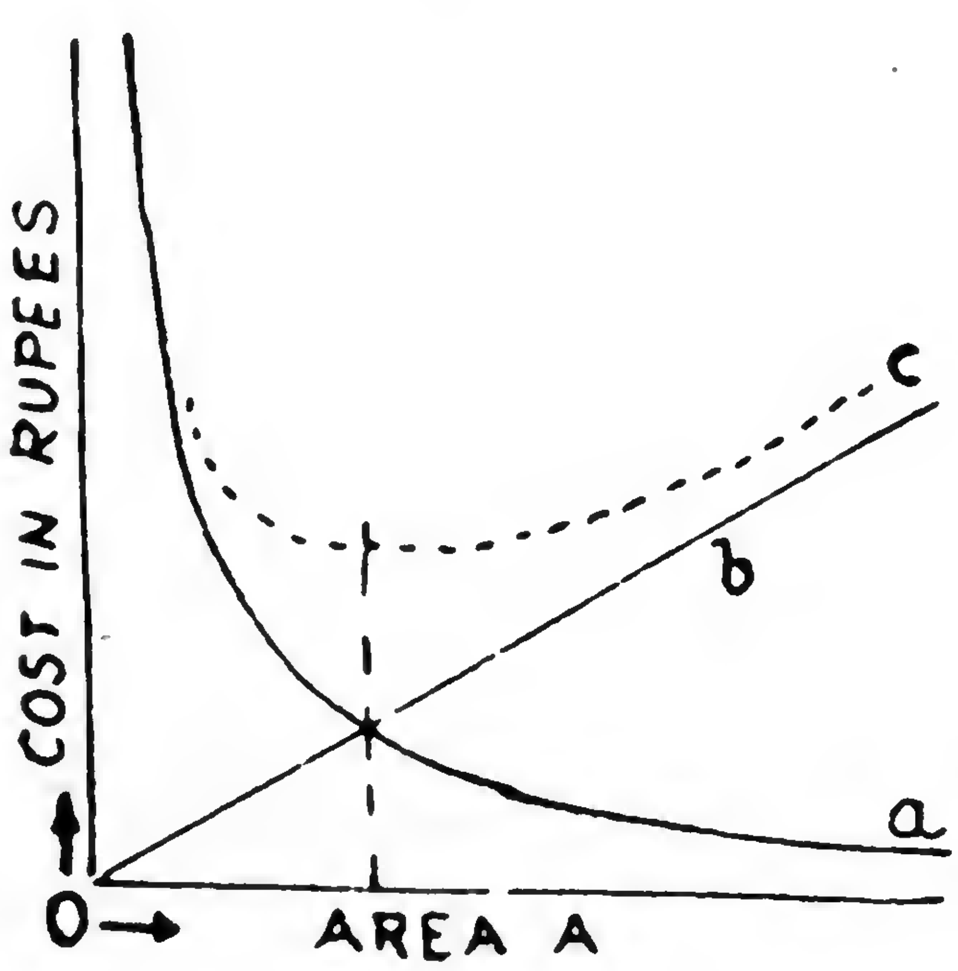


Fig. 16

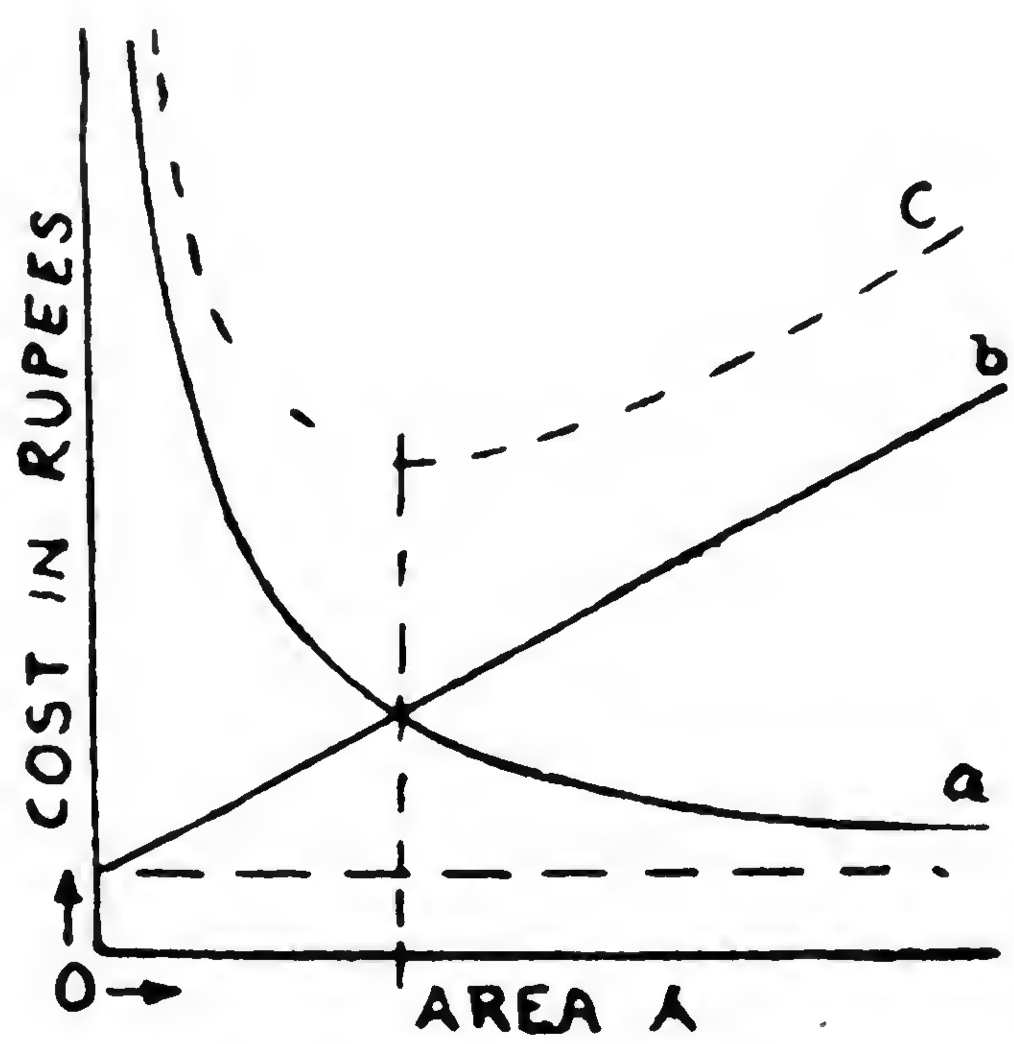


Fig. 17

When the feeder is an underground cable the cost of the cable consists of two parts, one is proportional to the area of the conductor and the other is due to the insulation and armouring of the cable and

is independent of the cross-sectional area of the conductor. Similarly, for an overhead line the charges, or cost of poles, cross-arms, insulators etc. are also independent of the conductor area.

Hence it is the usual practice to state the cost of a feeder in the form of Rs. $(mA + n)$ per unit length, where m and n are constants, and the unit of length may be a foot, a yard or a mile. This is the modified form of Kelvin's Rule.

Plotting the curves for the two costs, one is a straight line and the other a rectangular hyperbola. See Figs. 16 and 17.

The curves marked (a) are the graphs of annual cost of energy lost in the feeder, curves (b), the annual cost of interest and depreciation on the cost of the cable etc., and curves (c) are the graphs of total cost. The point where the two curves (a) and (b) meet gives the cross-section for which the total annual cost will be a minimum. This is true for both the cases.

9. Insulation Resistance of Live Mains: A: 2–Wire System. To determine the insulation resistance of a 2-wire system a high resistance voltmeter is employed. Two readings are taken, (a) between the *positive main* and *Earth*, and (b) between the *negative main* and *Earth*

Let V be the voltage between the Mains;
r the resistance of the voltmeter used;
R_1 the insulation resistance of the positive main to Earth;
R_2 the insulation resistance of the negative main to Earth;
V_1 reading of the voltmeter in (a);
V_2 reading of the voltmeter in (b).

Fig. 18 (a) and (b) show the schematic diagram of the circuit.

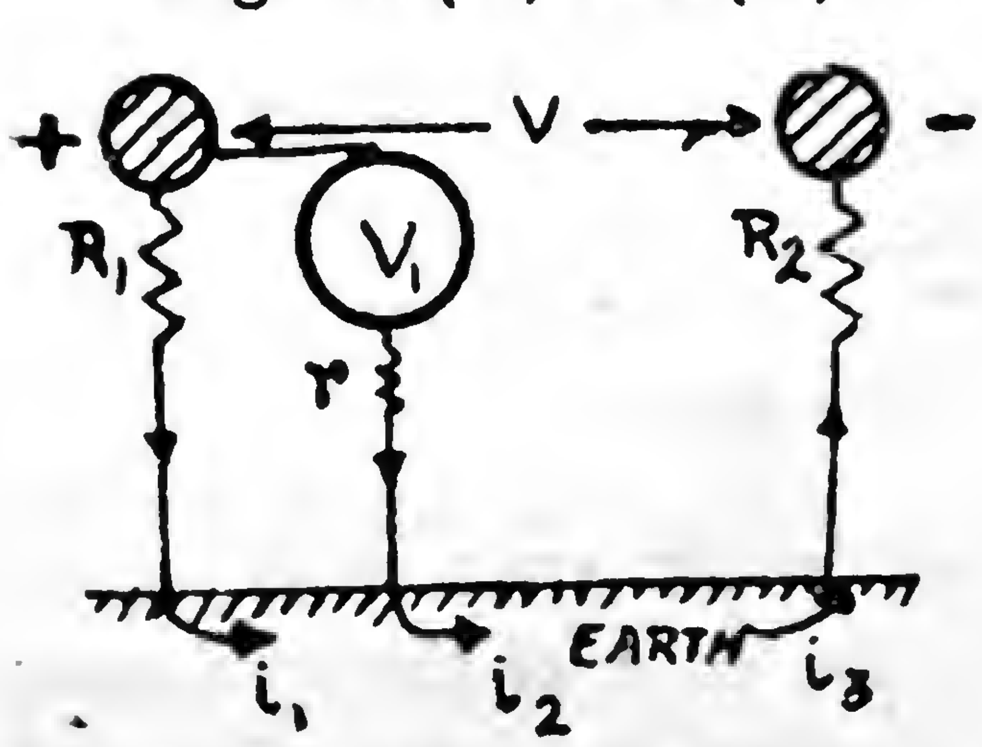


Fig. 18 (a)

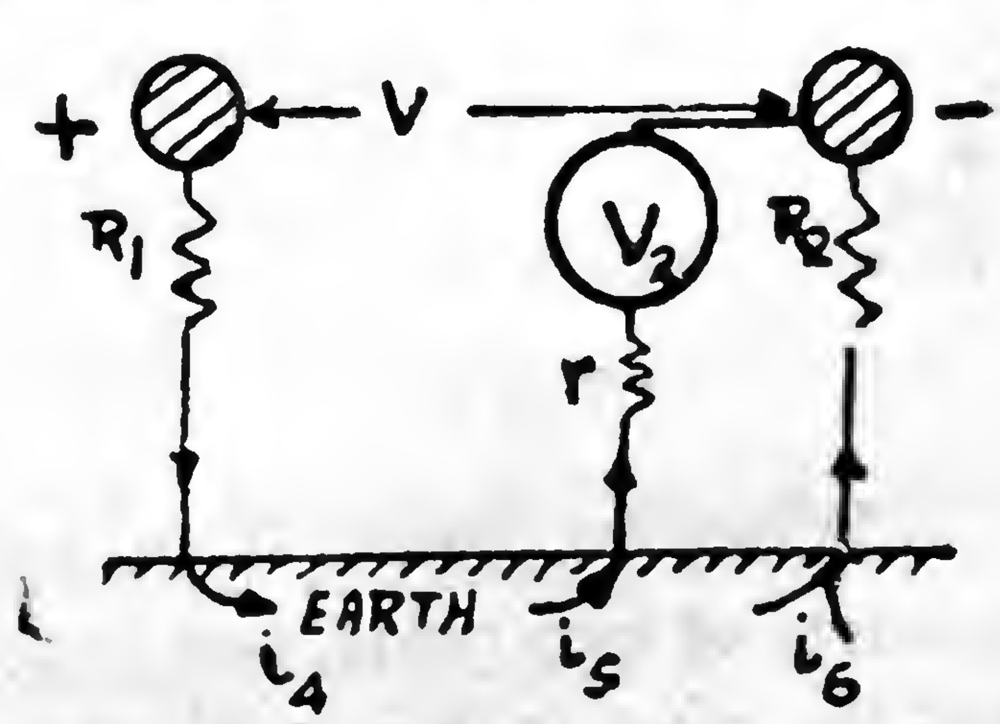


Fig. 18 (b)

Let the *leakage currents* of (a) be i_1, i_2 and i_3. Then

$$i_1 + i_2 = i_3 \qquad \dots \qquad \dots \qquad \dots \qquad \dots \quad \text{(i)}$$

Similarly for (*b*), $i_5 + i_6 = i_4 \qquad \dots \qquad \dots \qquad \dots \qquad \dots \quad \text{(ii)}$

Substituting the values of all the currents in terms of voltages and resistances, we get

$$\frac{V_1}{r} + \frac{V_1}{R_1} = \frac{V - V_1}{R_2} \qquad \dots \qquad \dots \qquad \dots \qquad \dots \quad \text{(iii)}$$

and $$\frac{V_2}{r} + \frac{V_2}{R_2} = \frac{V - V_2}{R_1} \qquad \dots \qquad \dots \qquad \dots \qquad \dots \quad \text{(iv)}$$

Transposing and arranging,

$$V_1\left(\frac{1}{r} + \frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{V}{R_2} \qquad \dots \qquad \dots \qquad \dots \quad \text{(v)}$$

and $$V_2\left(\frac{1}{r} + \frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{V}{R_1} \qquad \dots \qquad \dots \qquad \dots \quad \text{(vi)}$$

Dividing (v) by (vi)

$$\frac{V_1}{V_2} = \frac{R_1}{R_2} \qquad \dots \qquad \dots \qquad \dots \qquad \dots \qquad \dots \quad \text{(vii)}$$

Hence $R_2 = R_1 \dfrac{V_2}{V_1}$, and $R_1 = R_2 \dfrac{V_1}{V_2}$.

Substituting the value of R_2 in (iii),

$$\frac{V_1}{r} + \frac{V_1}{R_1} = \frac{V - V_1}{R_1} \times \left(\frac{V_1}{V_2}\right).$$

Solving for R_1,

$$R_1 = r\,\frac{V - V_1 - V_2}{V_2} \qquad \dots \qquad \dots \qquad \dots \quad (6)$$

Similarly, $$R_2 = r\,\frac{V - V_1 - V_2}{V_1} \qquad \dots \qquad \dots \qquad \dots \quad (7)$$

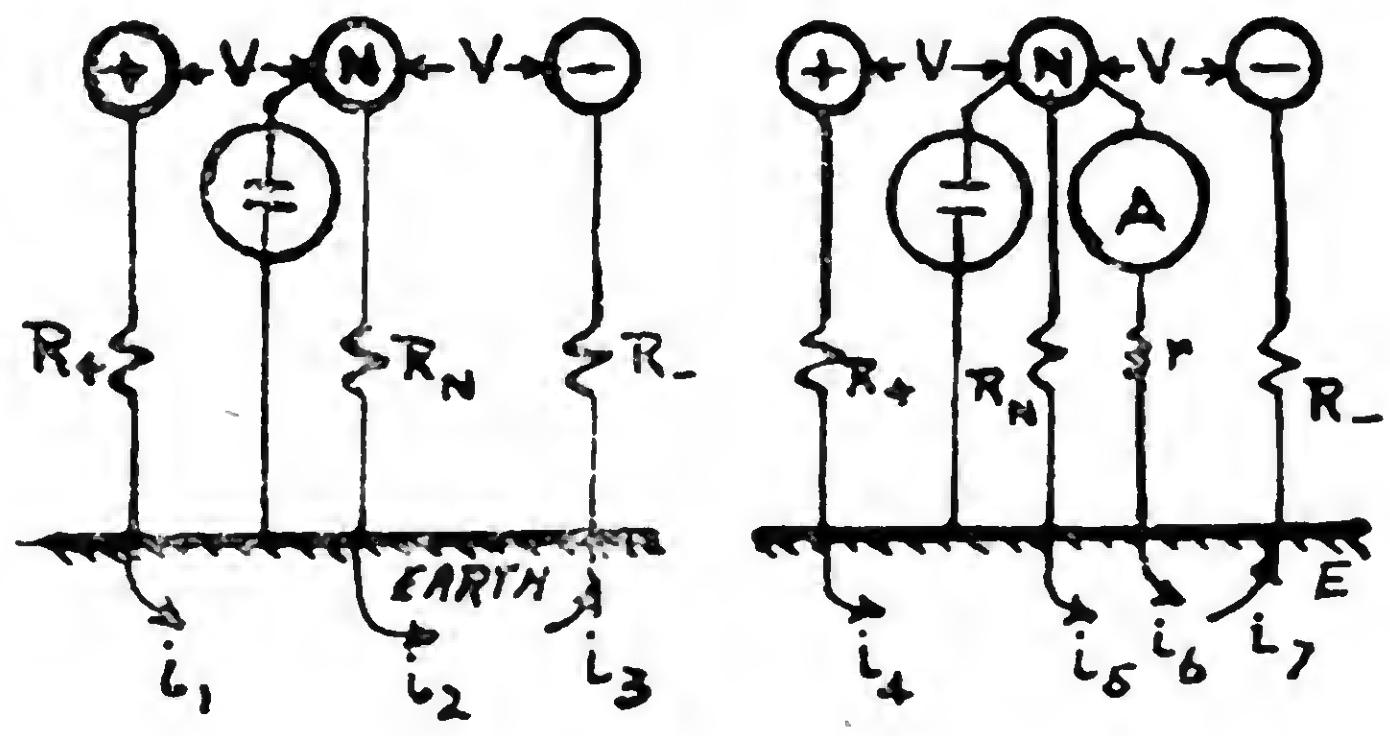
The total insulation resistance is the value obtained by assuming that the two Mains are bunched, and the resultant value then is the combined resistance of R_1 and R_2 in *parallel.*

Therefore the total insulation resistance of the Mains is

$$R = \frac{R_1 R_2}{R_1 + R_2} \qquad \dots \qquad \dots \qquad \dots \qquad \dots \quad (8)$$

B: 3-Wire Live Mains (Russell's Method): This test is performed by using an electrostatic voltmeter. Two sets of readings must be taken, as shown in Figs. 19 (a) and (b). In (b) an ammeter in series with a fairly high resistance (about 2000 ohms) is used, in addition to the electrostatic voltmeter.

Equations of *leakage currents* are obtained from Figs. 19 (a) and (b) as follows :—[V_1 and V_2 are the voltmeter readings].



(a) (b)

Fig. 19

$$\frac{V+V_1}{R_+}+\frac{V_1}{R_N}=\frac{V-V_1}{R_-} \quad \dots \text{ (i), from figure (a), and}$$

$$\frac{V+V_2}{R_+}+\frac{V_2}{R_N}+\frac{V_2}{r}=\frac{V-V_2}{R_-} \quad \dots \text{ (ii), from figure (b).}$$

Subtracting (ii) from (i), and arranging

$$(V_1-V_2)\left(\frac{1}{R_+}+\frac{1}{R_N}+\frac{1}{R_-}\right)-\frac{V_2}{r}=0.$$

Since the total insulation resistance R is

$$R=\frac{1}{\frac{1}{R_+}+\frac{1}{R_N}+\frac{1}{R_-}}$$

according to the definition, the above expression reduces to

$$\frac{(V_1-V_2)}{R}-\frac{V_2}{r}=0$$

putting $i=\frac{V_2}{r}$ $\quad R=\frac{(V_1-V_2)}{i} \quad \dots \quad \dots \quad \dots \quad \dots \quad (9)$

OR $\quad R=\frac{V_1}{i}-\frac{V_2}{i}=\frac{V_1}{i}-r \quad \dots \quad \dots \quad (10)$

In the above tests the *leakage current* is the current from the positive Main flowing *through* the insulating materials and also along the dust and dirt particles on the insulators to Earth, and from Earth over a similar path to the negative Main. The insulation resistance, therefore in the general sense, is the resistance of these paths. The insulation resistance of any system should be of such a magnitude that the leakage current shall not exceed $\frac{1}{2500}$th part of the full load current of the system.

The Earth is always considered to be at zero potential. The potential difference between the positive Main of a 2-wire system and Earth may not be equal to half the potential difference between the positive and negative Mains. But if the negative terminal of the generator is earthed the p. d. between the positive Main and Earth is equal to the p. d. between the two Mains.

CHAPTER VIII

A. C. CIRCUITS

1. Introductory : An electrical quantity, such as the voltage or the current, which changes its magnitude from instant to instant and its direction at fixed intervals and repeats faithfully these cyclic changes, is an alternating quantity. The source of an alternating voltage is a rotating machine called the alternator, whose coils rotate at a uniform angular velocity in a magnetic field of constant flux density. An elementary alternator (having only one coil) is shown in Fig. 1, and Fig. 2 shows the side view.

When the coil is along the x-axis it embraces maximum number

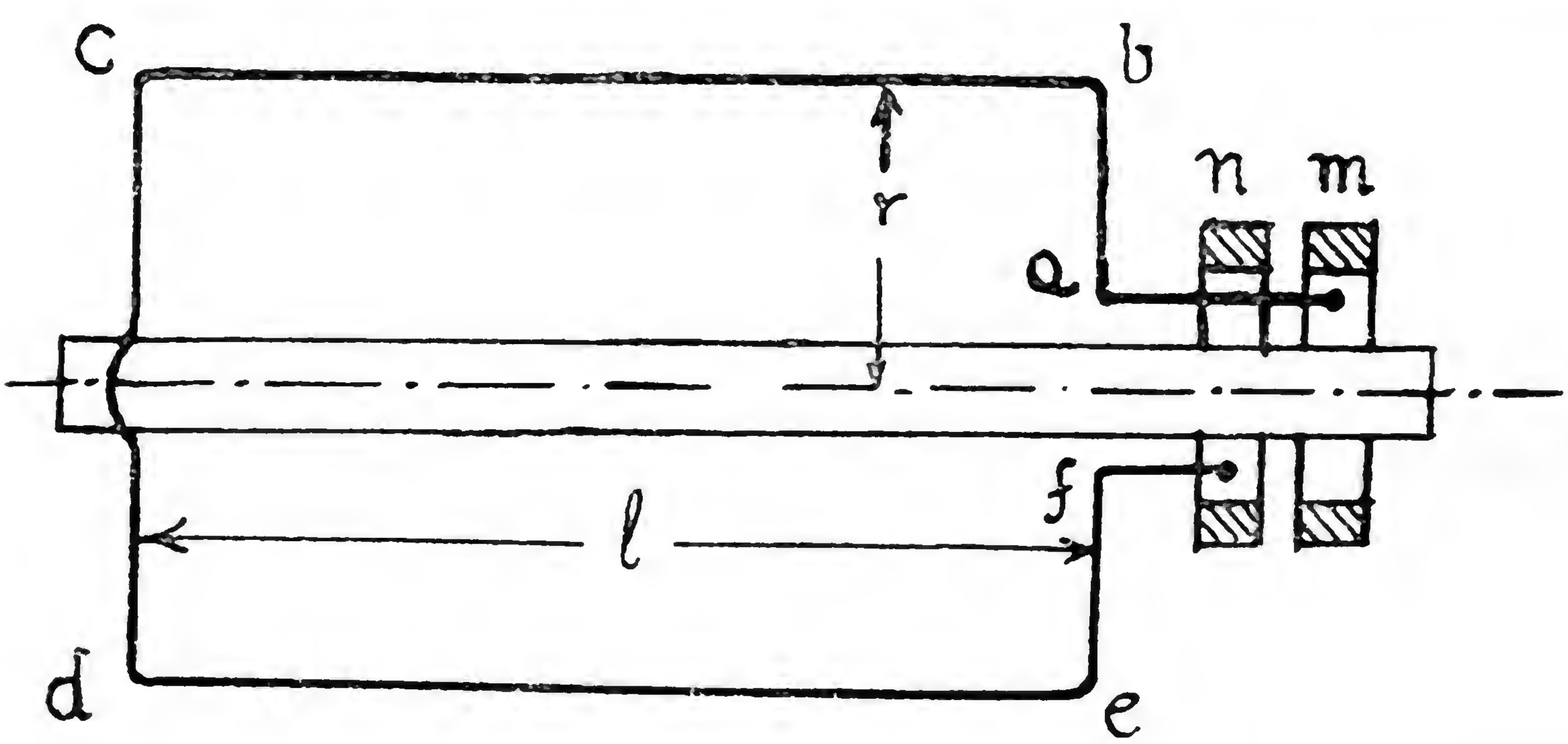


Fig. 1

of lines of force. Therefore if the linkages ($\Phi \times T$) are expressed as a function of time t, we get

$$\Phi T = \Phi_{max} T \cos \omega t$$

The induced e. m. f. e in the coil at any instant is

$$e = -\textit{ rate of change of linkages}$$

$$= -\frac{d}{dt}(\Phi T) = -\frac{d}{dt}(\Phi_{max} T \cos \omega t) \times 10^{-8} \text{ volts}$$

$$\therefore \quad e = \Phi_{max} T \omega \sin \omega t \times 10^{-8} \text{ volts} \quad \ldots \quad \ldots \quad (1)$$

where T = number of turns of the coil and ω = angular velocity of coil.

$\Phi_{max}\, T\, \omega$ is the maximum value, hence

$$e = E_{max} \sin \omega t \times 10^{-8} \text{ volts} \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

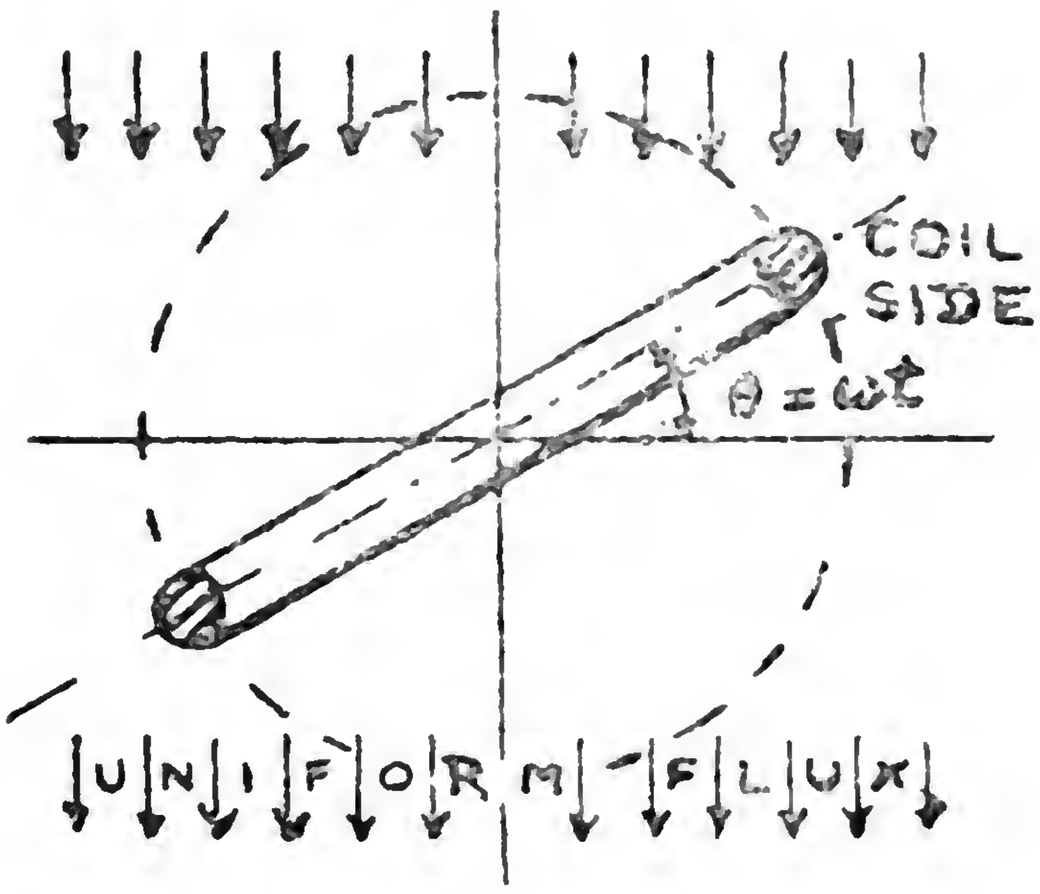


Fig. 2

Fig. 3 (a) shows the graph of instanteneous values of e for one complete cycle. The number of cycles per second is called the

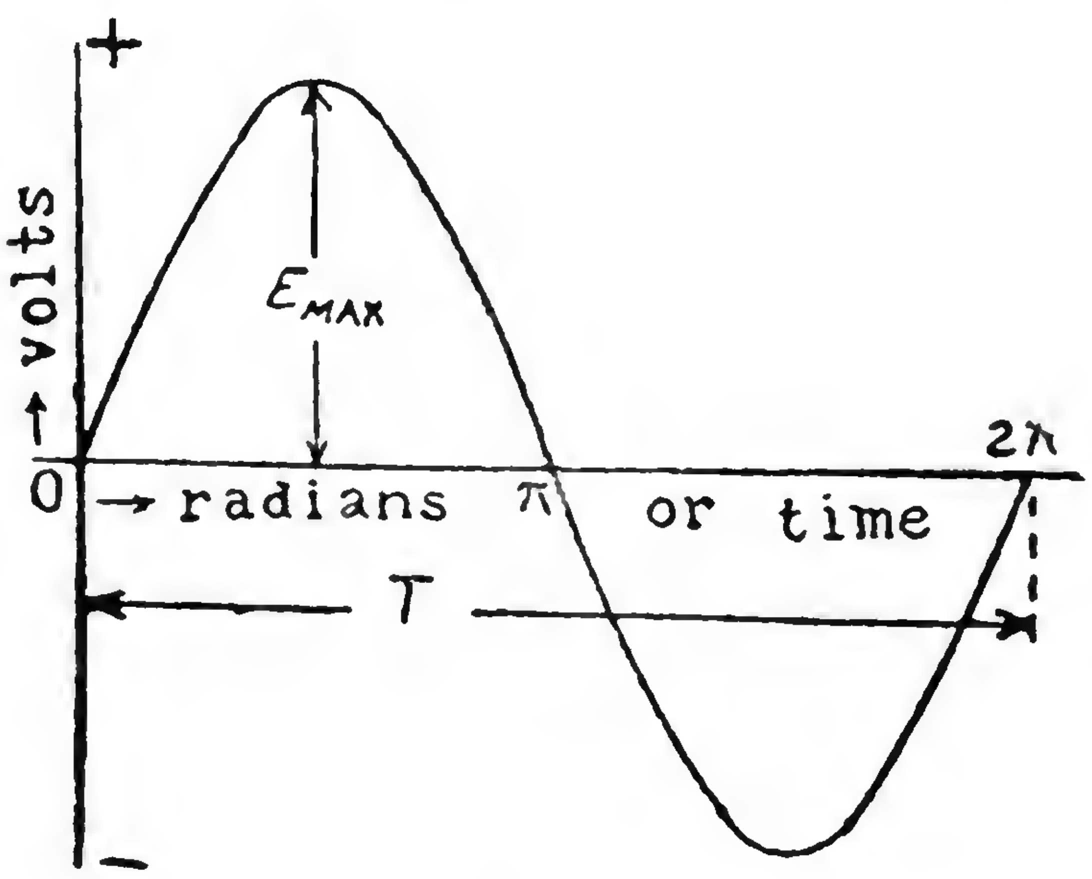


Fig. 3 (a)

frequency (f) and the time required to complete one cycle is called the periodic time (T). Hence

$$fT = 1$$

or $$f = \frac{1}{T} \text{ cycles per second} \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

The graph is a sine wave, and alternating voltages and currents are assumed to have a sine wave form unless specified otherwise. Such waves are called *sinusoidal* or *simple harmonic* waves.

The angle covered by one coil-side in one revolution is 2π radians or 360°. Hence since $\omega = 2\pi f$ the equation for e may be written as

$$e = E_{max} \sin 2\pi f t$$

What applies to voltage waves applies equally to current waves.

The sine wave can be obtained by the construction shown in Fig. 3 (b).

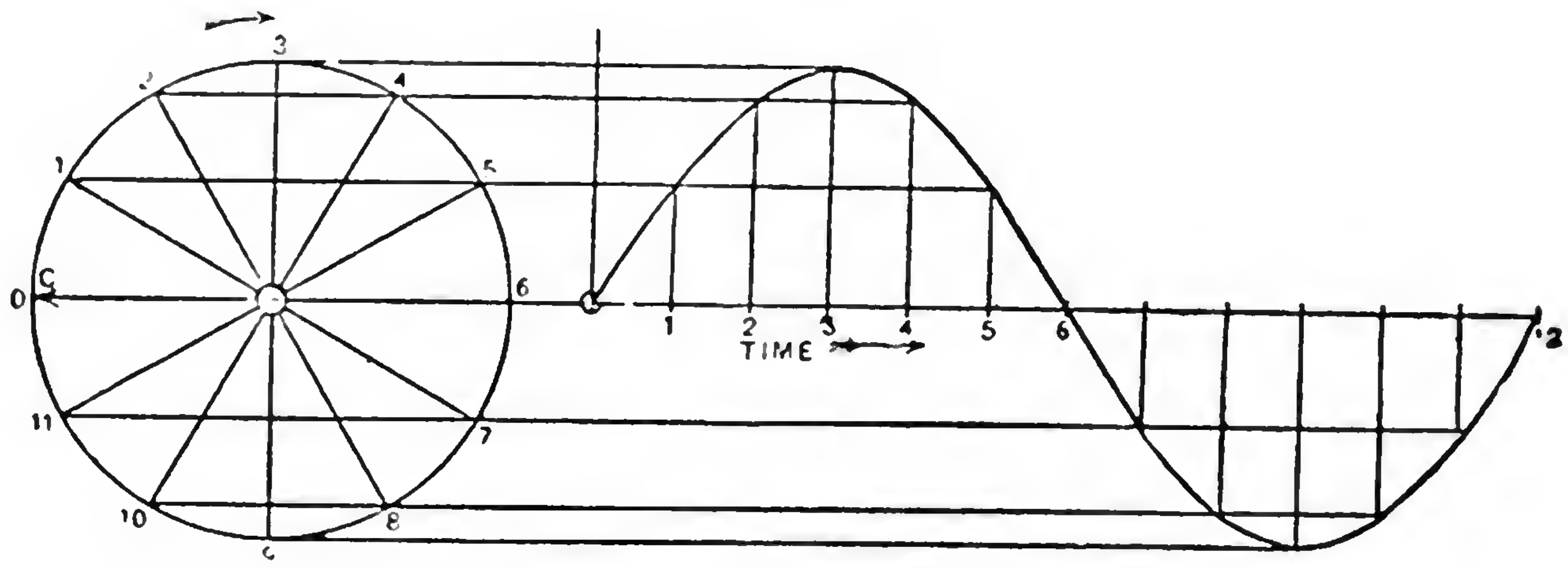


Fig. 3 (b)

2. Effective Value or Root-Mean-Square (R. M. S.) Value: Since alternating current changes its value from instant to instant its *effective value* must be such as to produce the same amount of work or heat as in direct current.

Now $I^2 Rt$ = *amount of heat produced*

where R = circuit resistance in ohms

t = time in seconds the current flows, and

I = value of direct current or alternating current.

Therefore the **mean value of i^2** must be taken to satisfy the equation. Hence

$$\textit{effective value} = \sqrt{\textit{mean value of } i^2}$$

This is obtained mathematically as follows :—

Let $$i = I_{max} \sin\theta$$

$$\textit{mean value of } i^2 = \frac{1}{2\pi}\int_0^{2\pi} I^2_{max} \sin^2\theta \, d\theta$$

$$= \frac{I^2_{max}}{2\pi}\int_0^{2\pi} (\tfrac{1}{2} - \cos 2\theta)\, d\theta$$

$$= \frac{I^2_{max}}{2\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{2\pi}$$

$$= \frac{I^2_{max}}{2\pi} \times \pi = \frac{I^2_{max}}{2}$$

$$\therefore \textit{ effective value} = \sqrt{\frac{I^2_{max}}{2}} = \frac{I_{max}}{\sqrt{2}} = 0{\cdot}707\, I_{max}$$

This effective value is also called the *root-mean-square value,* usually expressed as *r. m. s. value.*

The average value of an alternating current has no practical application except when the current wave is rectified and used for electro-chemical work. The average value then is represented by the mean height of the rectified wave. The average value of a sine wave over *half a cycle* is obtained as follows :—

Let the equation for current be

$$i = I_{max} \sin \theta$$

$$\textit{mean value of } i = \frac{1}{\pi} \int_0^{\pi} I_{max} \sin \theta \, d\theta$$

$$= \frac{I_{max}}{\pi} \Big[- \cos \theta \Big]_0^{\pi}$$

$$= \frac{2 I_{max}}{\pi} = 0{\cdot}636 \, I_{max}.$$

The ratio $\dfrac{\textit{r. m. s. value}}{\textit{average value}}$ is called the *form factor.* For a sine wave the form factor = 1·11.

$$\textit{Form factor} = \frac{\textit{r. m. s.}}{\textit{average}} = \frac{0{\cdot}707 \textit{ maximum value}}{0{\cdot}636 \textit{ maximum value}} = 1{\cdot}11.$$

3. Vectors and Vector Diagrams: Let a vector OA rotate about a fixed axis at O at f revolutions per second, as shown in Fig. 4. The angular velocity of the vector is $\omega = 2\pi f$. The time is measured from the instant when the vector lies along the x-axis. After an interval of t seconds, the vector moves through an angle $\theta = \omega t$. Its instantaneous value is obtained by the projection on the y-axis. Thus the instantaneous value of OA after t seconds is

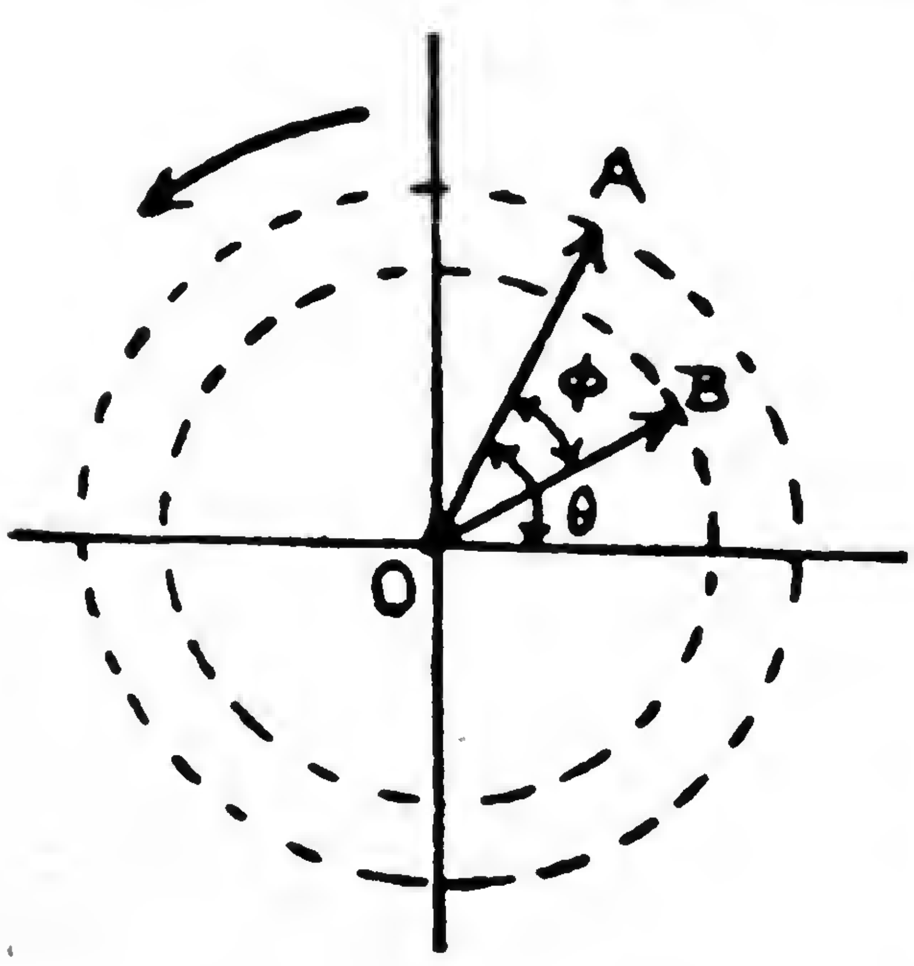


Fig. 4

$$OM = OA \sin \omega t.$$

Thus if OA is the maximum value, say of a voltage, then the instantaneous value is

$$e = E_{max} \sin \omega t.$$

If another vector OB rotating at the same frequency, but reaching zero value a little *later* than OA, is said to "lag" behind OA by an angle ϕ. Therefore OA "leads" OB. The angle ϕ is called the *angle of phase difference.* If OB represents current its equation is written as

$$i = I_{max} \sin(\omega t - \phi)$$

where $I_{max} = OB$.

Thus in Fig. 4 OA is a vector representing a voltage to a certain voltage scale and OB, representing current to a different scale. The lines OA and OB are called vectors and Fig. 4 is the vector diagram. The positive rotation of vectors is anti-clockwise.

In any vector diagram all quantities must rotate at the same frequency and their values, as shown by the vectors, must all be either r. m. s. or all maximum values. But when r. m. s. values are used, their projections on the y-axis do not give instantaneous values.

4. **Operator j:** Any vector can be split into its x- and y-components. These components are at right angles to each other. In Fig. 5 the vector OA is shown with its x-component OM and its y-component ON. If OA makes an angle θ with the x-axis, $OM = OA \cos\theta$ and $ON = OA \sin\theta$.

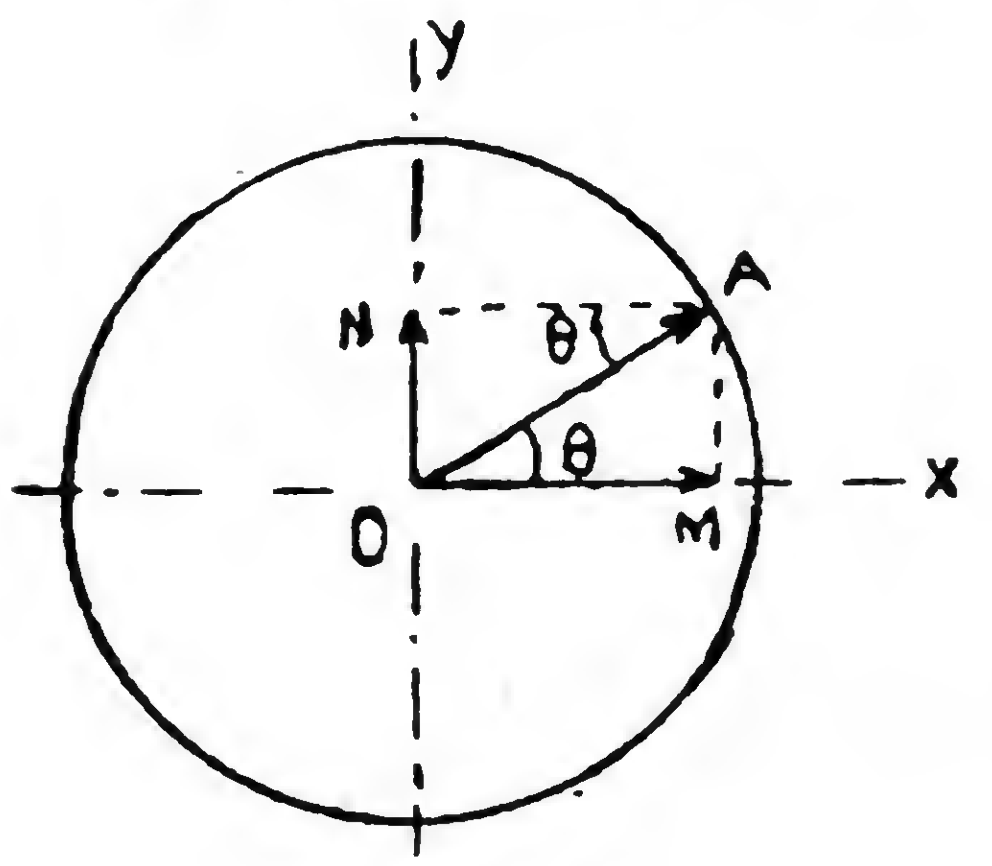


Fig. 5

By geometry,

$$OA^2 = OM^2 + ON^2$$

$$\therefore \quad OA = \sqrt{OA^2 \cos^2\theta + OA^2 \sin^2\theta}$$
$$= OA\sqrt{(\cos^2\theta + \sin^2\theta)} = OA.$$

In **vector** or **symbolic notation,**

$$OA = OM + j\,ON \qquad \ldots \qquad \ldots \qquad \ldots \quad (\text{i})$$

This means that OA consists of two components, OM along the x-axis and ON along the y-axis in the positive direction. The + sign before both the components indicates the positive directions. Eq. (i) is also written as

$$OA = OA\cos\theta + j\,OA\sin\theta$$
$$= OA(\cos\theta + j\sin\theta) \qquad \ldots \qquad \ldots \quad (\text{ii})$$

The letter j denotes a rotation of a vector by 90°, i. e. it changes the *direction* of a vector but not its magnitude. If j is *positive*, the rotation is in anti-clockwise direction and if j is *negative*, the rotation is in clockwise direction. Now if a vector is to be rotated by 180°, the operation is indicated by j^2 i. e. the vector is indicated as being "multiplied by j^2". If OM of Fig. 5 is rotated by 180° it will occupy the position OM′, which is really $-$OM. Therefore

$$j^2 \, \mathrm{OM} = - \, \mathrm{OM}$$

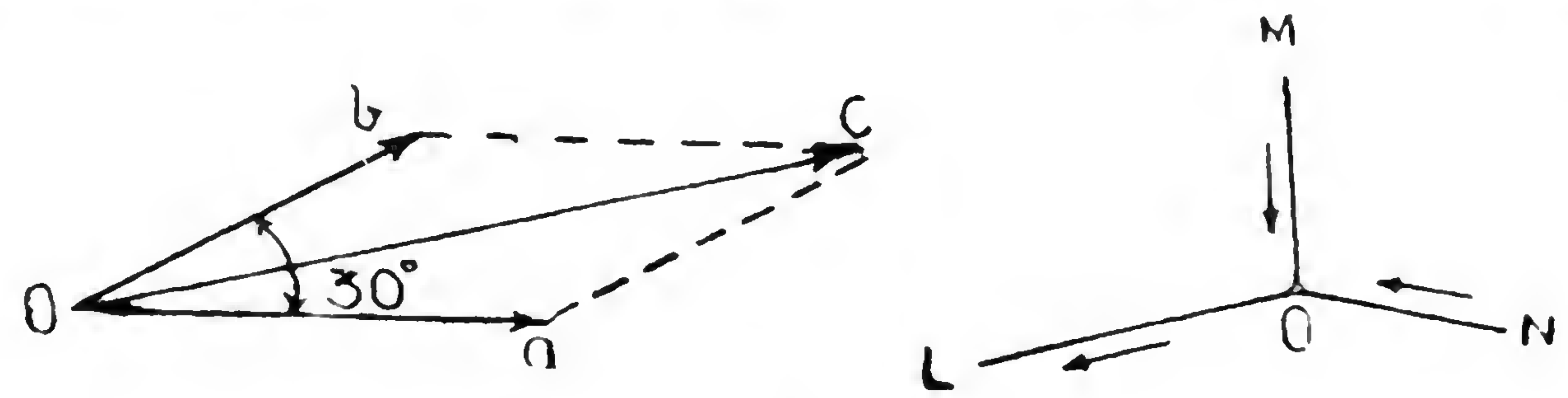
Thus j^2 merely changes the sign of OM. In other words

$$j^2 = -1$$

$$\therefore \quad j = \sqrt{-1}, \text{ an imaginary quantity.}$$

This is the basis of *Symbolic Notation.*

5. **Vector Addition and Subtraction :** Fig. 6 (a) shows two current vectors $\mathbf{O}a$ and $\mathbf{O}b$, $\mathbf{O}b$ leading $\mathbf{O}a$ by 30°. $\mathbf{O}a = 10$



(a) Fig. 6 (b)

amperes (r. m. s.) and $\mathbf{O}b$ = 8 amperes (r. m. s.). Now suppose, as shown in Fig. 6 (b), that 10 amperes flow in the wire marked **OM** and 8 amperes flow in the wire **ON**. From the junction **O** of these wires a third wire **OL** carries both the currents. It is required to find the value of current flowing in **OL.** The actual mechanical angles at which the wires meet have no bearing on the problem.

If there was no phase difference between the two currents, the wire **OL** would carry 18 amperes, the arithmetic sum as in a direct current system. But because of the phase angle between the two vectors, the following methods are useful in calculating the resultant current :—

(a) By constructing a parallelogram as shown in Fig. 6 (a) and measuring $\mathbf{O}c$:

(b) By the analytical method Fig. 6 (a) :—

$$\mathbf{O}c^2 = (\mathbf{O}a)^2 + (\mathbf{O}b)^2 + 2(\mathbf{O}a)(\mathbf{O}b)\cos\theta;$$

(c) By splitting the vectors into their x- and y- components separately, adding the two x- components x_1 and x_2 and the two y- components y_1 and y_2, and then taking the square root of the sum of the squares of $(x_1 + x_2)$ and $(y_1 + y_2)$, the resultant can be obtained easily. In other words, the vectorial addition is obtained by the hypotenuse of the right angled triangle, the sides being the sum of x- and y- components. Fig. 7 shows to scale, the resultant ***Oc***.

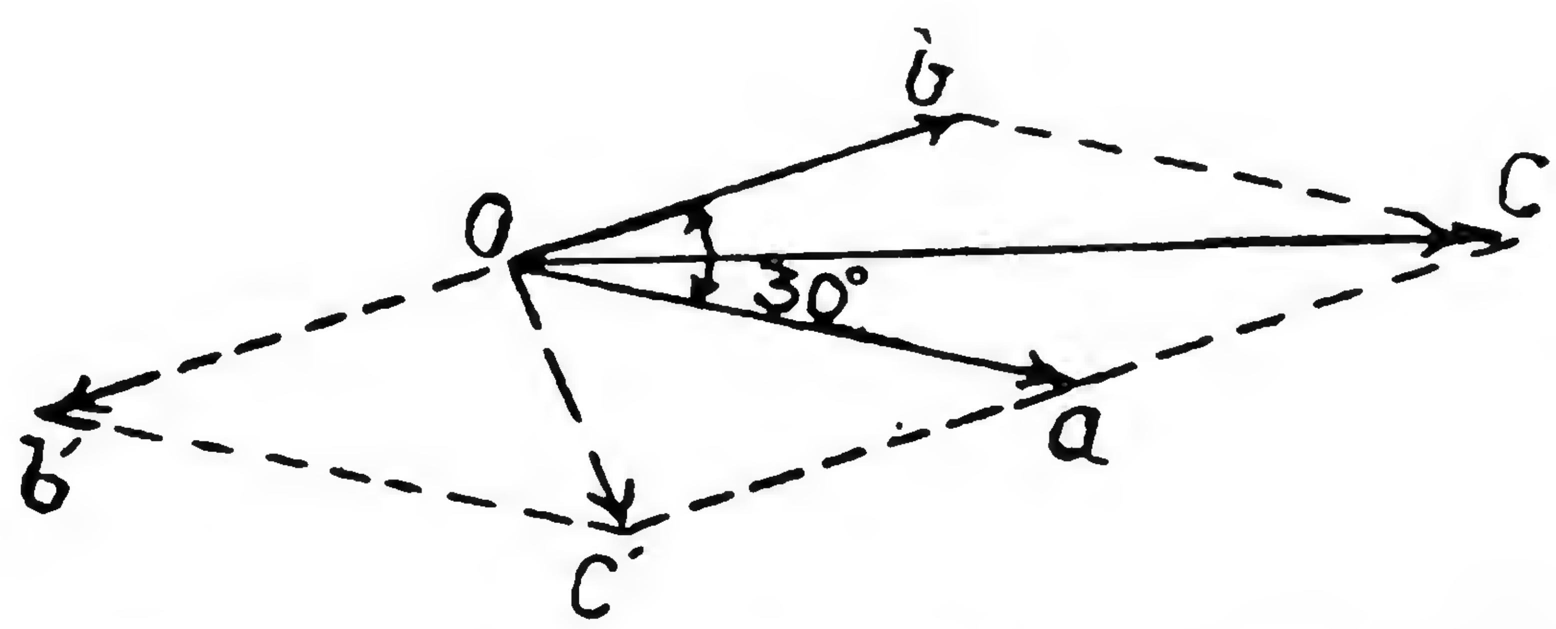


Fig. 7 Vector Addition and Subtraction.

The vector subtraction can be done by any one of the methods mentioned above, provided that the vector which bears the *minus* sign is rotated through 180°. For instance, it is required to find the resultant of ***Oa*** − ***Ob***. ***Ob*** in Fig. 7, has been turned by 180° and is shown as ***Ob'***. The parallelogram is completed, the sides being ***Oa*** and ***Ob'***. The diagonal ***Oc'*** is the resultant which, being in the 4th quadrant, is negative.

Taking the parallelogram ***Oabc***, the diagonal ***Oc*** gives the vector addition and the diagonal ***ab*** gives the vector subtraction.

Example : Calculate, using the three methods mentioned above, the vector addition of the two currents ***Oa*** and ***Ob*** of Fig. 7. ***Oa*** = 10 and ***Ob*** = 8 amperes and the phase angle $\theta = 30°$. Find also the vector difference of ***Oa*** and ***Ob***.

Solution : Taking a scale of 1" = 10 amperes, the parallelogram ***Oacb*** of Fig. 7 is constructed. The measurement of ***Oc*** is found to be 1·75". Therefore ***Oc*** = *17·5 amperes*. For subtraction the vector ***Ob*** is rotated through 180° to ***Ob'***, and another parallelogram ***Oac'b'*** is constructed. By measurement ***Oc'*** = 0·5". Therefore the vector difference is *5 amperes*.

By method (b) vector addition.

$$(Oc)^2 = (Oa)^2 + (Ob)^2 + 2\,(Oa)\,(Ob)\cos 30$$

[cos 30 = 0·866].

$\therefore \quad (\boldsymbol{Oc})^2 = 10^2 + 8^2 + 2 \times 10 \times 8 \times 0{\cdot}866 = 302{\cdot}56$

$\therefore \quad (\boldsymbol{Oc}) = \sqrt{302{\cdot}56} = 17{\cdot}4$ *amperes.*

Vector subtraction :

$(\boldsymbol{Oc'})^2 = (\boldsymbol{Oa})^2 + (\boldsymbol{Ob'})^2 + 2\,(\boldsymbol{Oa})\,(\boldsymbol{Ob'}) \cos (180^\circ - 30^\circ)$ $[\cos 150^\circ = -\,0{\cdot}866]$.

$\therefore \quad (\boldsymbol{Oc'})^2 = 10^2 + 8^2 + 2 \times 10 \times 8 \times (-\,0{\cdot}866)$

$\qquad = 164 - 138{\cdot}56 = 25{\cdot}44.$

$\therefore \quad (\boldsymbol{Oc'}) = 5{\cdot}04$ *amperes.*

By method (c) : Vector addition :

The x–component of $\boldsymbol{Oa} = 10$; its y–component is $= 0$

The x–component of $\boldsymbol{Ob} = 8 \cos 30^\circ = 6{\cdot}93$, and

The y–component of $\boldsymbol{Ob} = 8 \sin 30^\circ = 4$.

The sum of x–components is $10 + 6{\cdot}93, = 16{\cdot}93$, and the sum of y–components is $0 + 4 = 4$.

Hence the resultants is $\sqrt{[(16{\cdot}93)^2 + (4)^2]} = 17{\cdot}4$ *amperes.*

Vector Subtraction :

The x–and y–components of $\boldsymbol{Oa}$ are 10 and 0 as before.

The x–and y–components of $\boldsymbol{Ob'}$ are

$$(-\,8 \cos 60^\circ) \text{ and } (-\,8 \sin 60^\circ) = -\,6{\cdot}93 \text{ and } -\,4$$

Therefore the total of x–components is $(10 - 6{\cdot}93) = 3{\cdot}07$ and the total of y–components is $(0 - 4) = -4$. Hence the resultant becomes

$$\sqrt{[(3{\cdot}07)^2 + (4)^2]} = 5{\cdot}04 \textit{ amperes.}$$

In vector notation the two currents are written as

$$\text{OA} = (10 + j\,0), \text{ and}$$

$$\text{OB} = (8 \cos 30^\circ + 8j \sin 30^\circ) = 8\,(0{\cdot}866 + j\,0{\cdot}5)$$

$$= (6{\cdot}93 + j\,4)$$

The addition of these is

$$\text{OA} + \text{OB} = (10 + j\,0) + (6{\cdot}93 + j\,4) = (16{\cdot}93 + j\,4)$$

The numerical value is

$$\sqrt{(16{\cdot}93^2 + 4^2)} = 17{\cdot}4 \text{ amperes.}$$

Similarly, their subtraction is

$$OA-OB = (10+j\,0)-(6{\cdot}93+j\,4) = (3{\cdot}07-j\,4)$$

The numerical value is

$$\sqrt{(3{\cdot}07)^2+(4)^2} = 5{\cdot}04 \text{ amperes.}$$

6. Vector Multiplication and Division: The multiplication of vector quantities is as follows :—

$$\dot{I}_1 \times \dot{I}_2 = (10+j\,0)(6{\cdot}93+j\,4)$$
$$= (69{\cdot}3+j\,40) = 80{\cdot}15$$

The division is

$$\frac{\dot{I}_1}{\dot{I}_2} = \frac{10+j\,0}{6{\cdot}93+j\,4} \times \left[\frac{6{\cdot}93-j\,4}{6{\cdot}93-j\,4}\right] = \frac{69{\cdot}3-j\,40}{48{\cdot}03-j\,27{\cdot}72+j\,27{\cdot}72+16}.$$

[In order to get the result in the form of $(a \pm jb)$ the expression is rationalised by multiplying the numerator and the denominator by the denominator with the sign changed].

$$\frac{\dot{I}_1}{\dot{I}_2} = \frac{69{\cdot}3-j\,40}{48+16} = \frac{69{\cdot}3}{64} - j\,\frac{40}{64} \quad [\text{since } j^2 = -1]$$
$$= 1{\cdot}08 - j\,0{\cdot}6125.$$

A vector can also be expressed in *polar* notation. For instance, OA is written as $10\,\underline{|0^\circ}$, and OB as $8\,\underline{|30^\circ}$. If OP is in the 4th quadrant it is written in two ways

$$I'_2 = OB' = 8\,\underline{|-30^\circ} \text{ or } = 8\,\overline{|30^\circ}$$

Hence multiplication in polar notation is

$$\dot{I}_1 \times \dot{I}_2 = (10\,\underline{|0^\circ})(8\,\underline{|30^\circ}) = 10 \times 8\,\underline{|0^\circ+30^\circ} = 80\,\underline{|30^\circ}$$

and division,

$$\frac{\dot{I}_1}{\dot{I}_2} = \frac{10\,\underline{|0^\circ}}{8\,\underline{|30^\circ}} = \frac{10}{8}\,\underline{|0^\circ-30^\circ} = 1{\cdot}25\,\underline{|-30^\circ} = 1{\cdot}25\,\overline{|30^\circ}.$$

Powers and roots of vectors are given as

$$(I\,\underline{|\theta})^n = I^n\,\underline{|n\theta}$$

and $$\sqrt[n]{(I\,\underline{|\theta})} = \sqrt[n]{I}\,\underline{\Big|\frac{\theta}{n}}.$$

7. Phase Relation between Voltage and Current in A. C. Circuits:

A. *Circuit Containing Resistance Only.* Suppose an alternating voltage is applied to a non-inductive resistance of R ohms. At any instant the current is, when voltage (r. m. s.) $V = V_{max} \sin \omega t$.

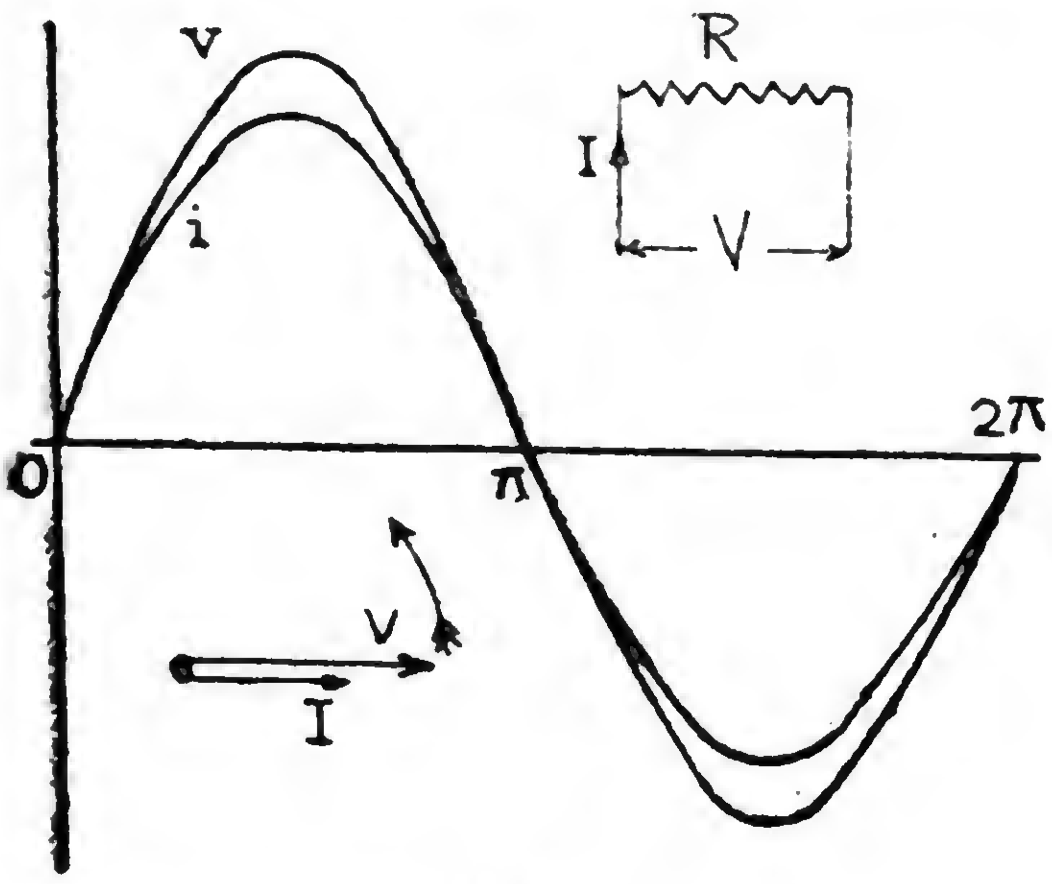


Fig. 8

$$i = \frac{V}{R} = \frac{V_{max}}{R} \sin \omega t$$

$$\therefore \quad i = I_{max} \sin \omega t$$

$$I_{max} = \frac{V_{max}}{R}.$$

Thus the current is a sine wave *in phase* with the voltage. Fig. 8 shows the voltage and current waves. They reach maximum values and zero values at the same instants.

B. *Circuit Containing Inductance only:* Let a voltage V be applied across an inductance of L henrys. The voltage induced at any instant across L is

$$e_L = -L \frac{di}{dt} \text{ volts.} \quad \ldots \quad \ldots \quad \ldots \quad \text{(i)}$$

The minus sign is given because it opposes the applied voltage at every instant, i. e. in a purely inductive circuit the induced e. m. f. e_L is equal and opposite in sign to v, the instantaneous value of applied voltage V.

Let the current flowing in the circuit have an equation

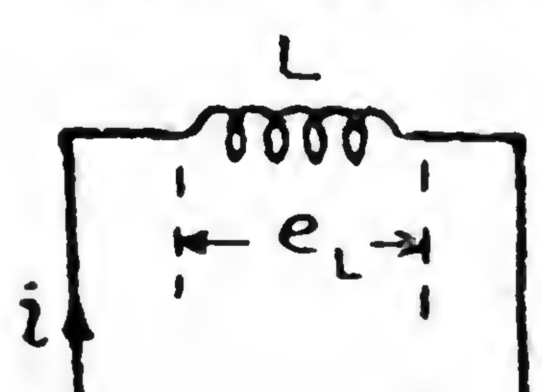


Fig. 9 (a)

$$i = I_{max} \sin \omega t$$

substituting the value of i in (i) above

$$e_L = -L I_{max} \frac{d(\sin \omega t)}{dt}$$

$$= -\omega L I_{max} \cos \omega t.$$

The maximum value of e_L is $\omega L I_{max}$ and the r. m. s. value of e_L is $\omega L I$, where I is the r. m. s. value of current. The applied voltage is therefore

$$v = +\omega L I_{max} \cos \omega t$$

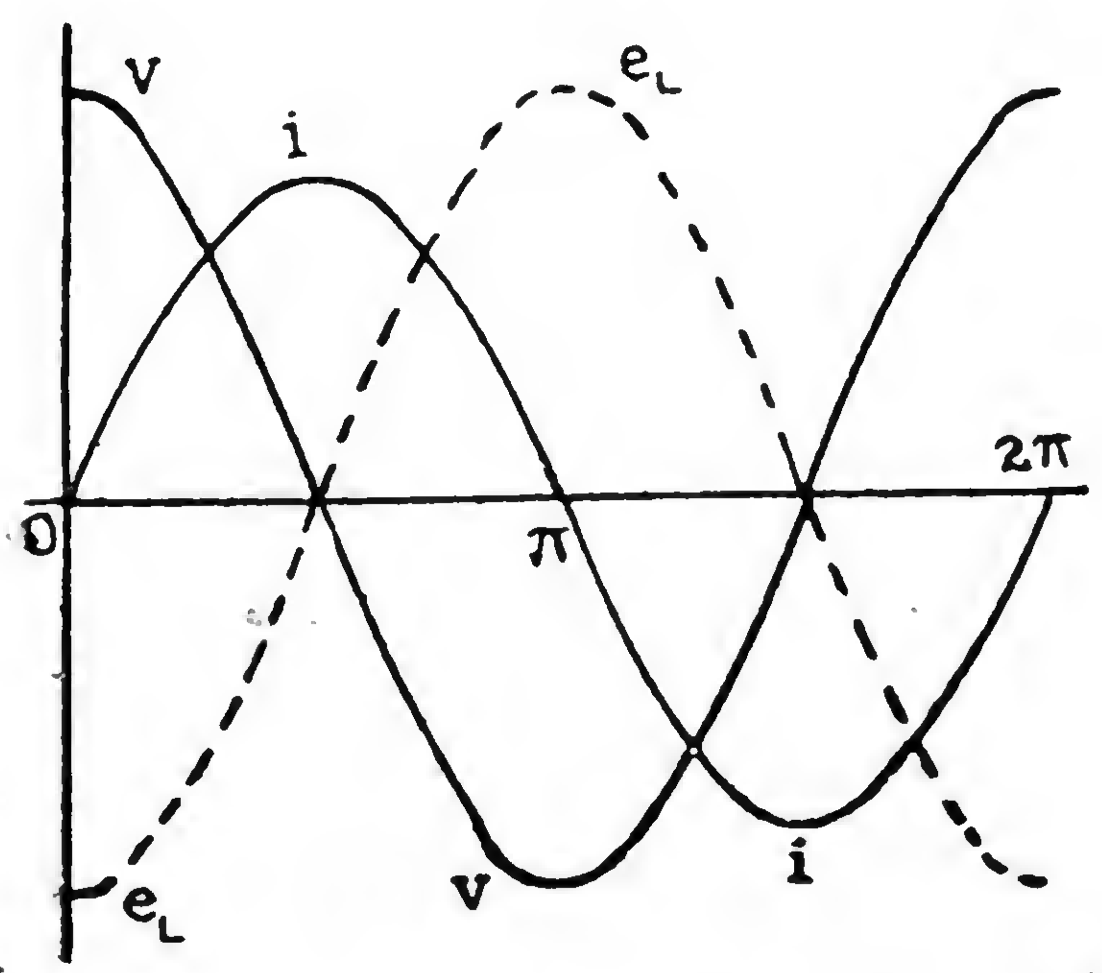
$$\therefore \quad v = +\omega L I_{max} \sin(\omega t + 90^\circ).$$

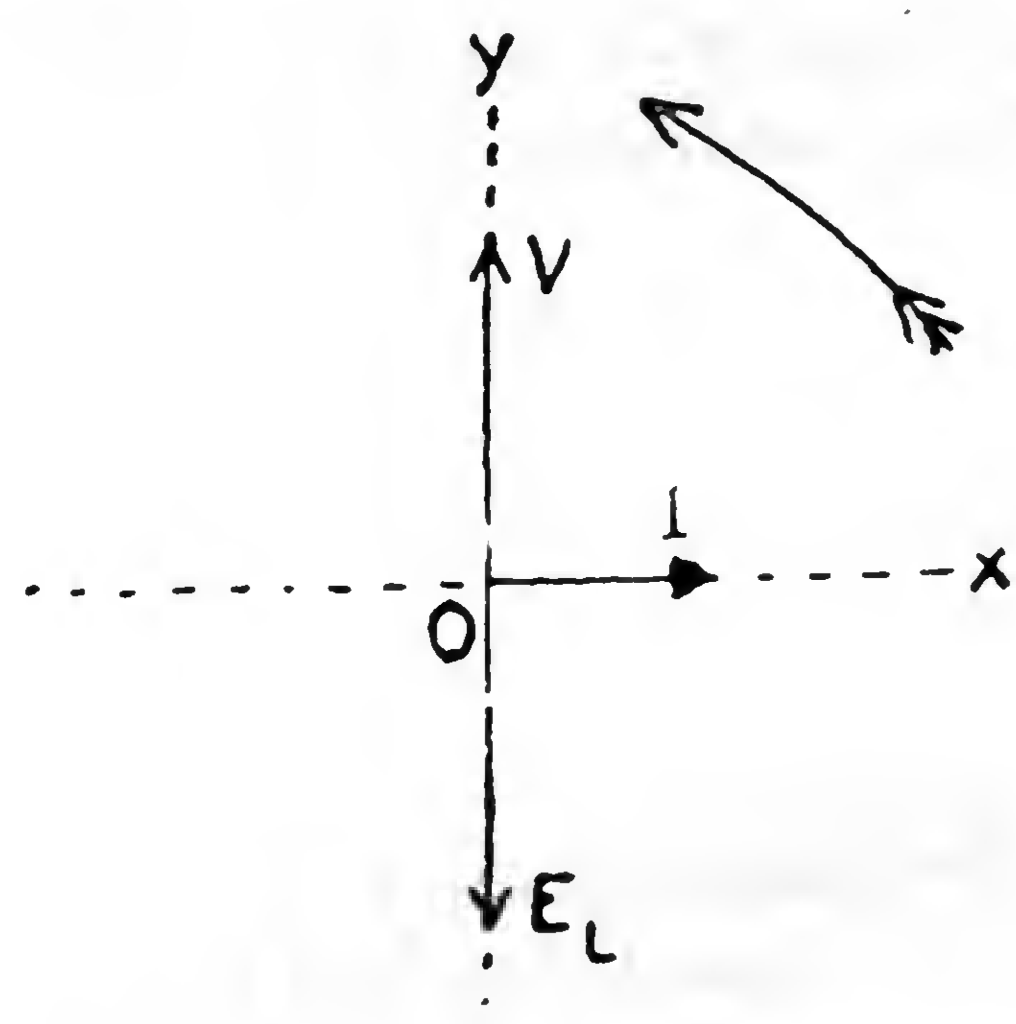
Thus in a purely inductive circuit the voltage leads the current by 90°. But it is always better to say that *the current lags the applied voltage by 90° in a purely inductive circuit.*

The maximum value of v is $\omega L I_{max}$ and the r.m.s. value is ωLI

$$\therefore \quad \frac{V}{I} = \omega L = 2\pi f L \quad \ldots \quad \ldots \quad \ldots \quad \text{(ii)}$$

ωL is called *inductive reactance* and is constant if the frequency of applied voltage is constant. Its symbol is X_L and its unit is the ohm in practical system of units. Fig. 9 (*b*) shows the voltage e_L along with the current and applied voltage waves, and Fig. 9 (c) shows the vector diagram of the three quantities. The rotation of vectors is always anti-clockwise.





(b) Fig. 9 (c)

C. *Circuit Containing Capacitance Only* : Let a voltage V be applied across a capacitor of C farads. If the instantaneous value of the applied voltage at any instance is v, then the quantity of electricity q at that instant is

$$q = C\,v \text{ coulombs.}$$

Now quantity q is the product of current and time. If the current is varying

$$q = \int i \times dt$$

$$\text{or} \qquad dq = i \cdot dt$$

$$\therefore \qquad i = \frac{dq}{dt} = C\left(\frac{dv}{dt}\right) \quad \ldots \quad \ldots \quad \ldots \quad \text{(i)}$$

If the equation of v is

$$v = V_{max} \sin \omega t$$

$$\therefore \quad i = C\, V_{max}\, \omega \cos \omega t \quad \ldots \quad \ldots \quad \ldots \quad (\text{ii})$$

The maximum value of i is $\omega C\, V_{max}$
and its r. m. s. value is $I = \omega C\, V$, where V is the r. m. s. value of v.

$$\therefore \quad \frac{V}{I} = \frac{1}{\omega C} = \frac{1}{2\pi f C}.$$

If the frequency is constant $\frac{1}{\omega C}$ is constant and is called *capacitive reactance*, symbol X_C and its unit is the ohm in practical system of units.

$$\therefore \quad X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \quad \ldots \quad \ldots \quad \ldots \quad (\text{iii})$$

Thus from (ii), *the current leads the applied voltage by 90° in a purely capacitive circuit.*

Fig. 10 shows a purely capacitive circuit. The current here

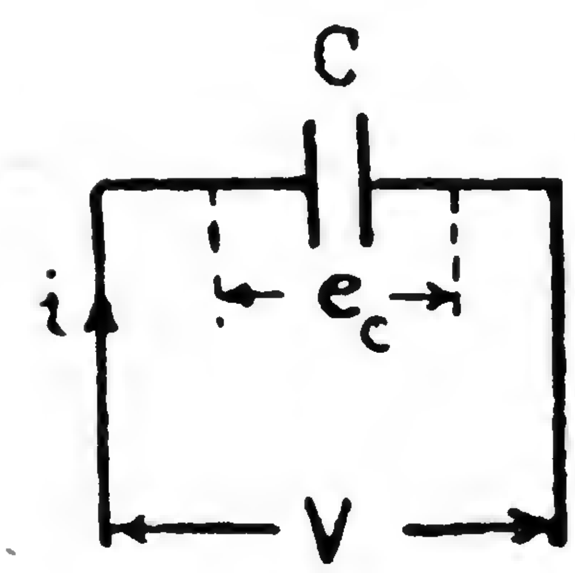


Fig. 10

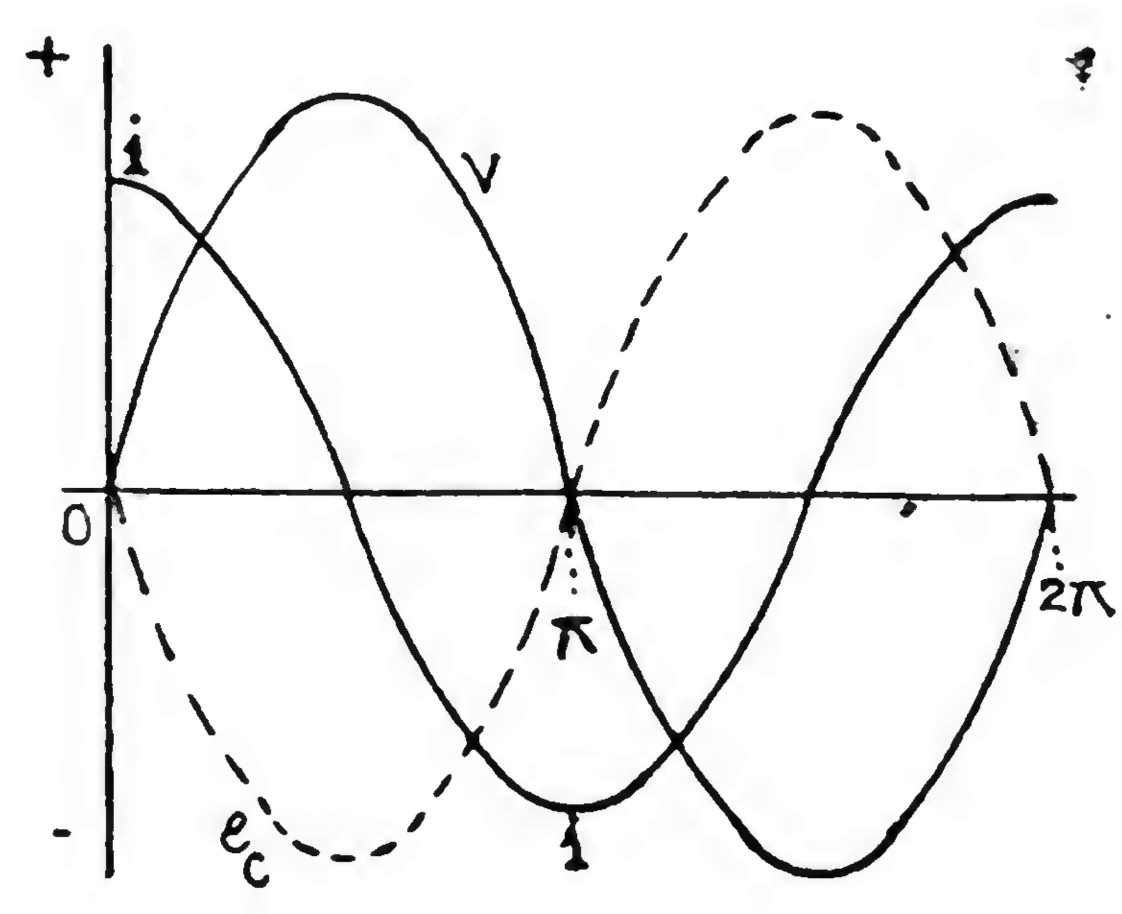


Fig. 11

may be looked upon as the *charging current* of the capacitor for the 1st 1/4 period of the voltage wave when the current decreases from + ve maximum to zero, while the voltage e_c across the capacitor increases from zero to − ve maximum. [e_c is always equal to and opposing the applied voltage from instant to instant]. During the next 1/4 period of voltage wave, the condenser is *discharged*, the current increases from zero to − ve maximum, and the voltage across the capacitor decreases from − ve maximum to zero, and so on. Thus

the capacitor is charged and discharged during alternate quarter cycles and these charge and discharge currents give rise to an alternating current in the circuit when an alternating voltage is applied across the capacitor. In d. c. circuits there is an initial charging current, but this rapidly reaches to zero when the voltage e_c becomes equal to the applied voltage and there is no further flow of current. Fig. 11 shows the graphs of current, applied voltage and the voltage e_c across the capacitor.

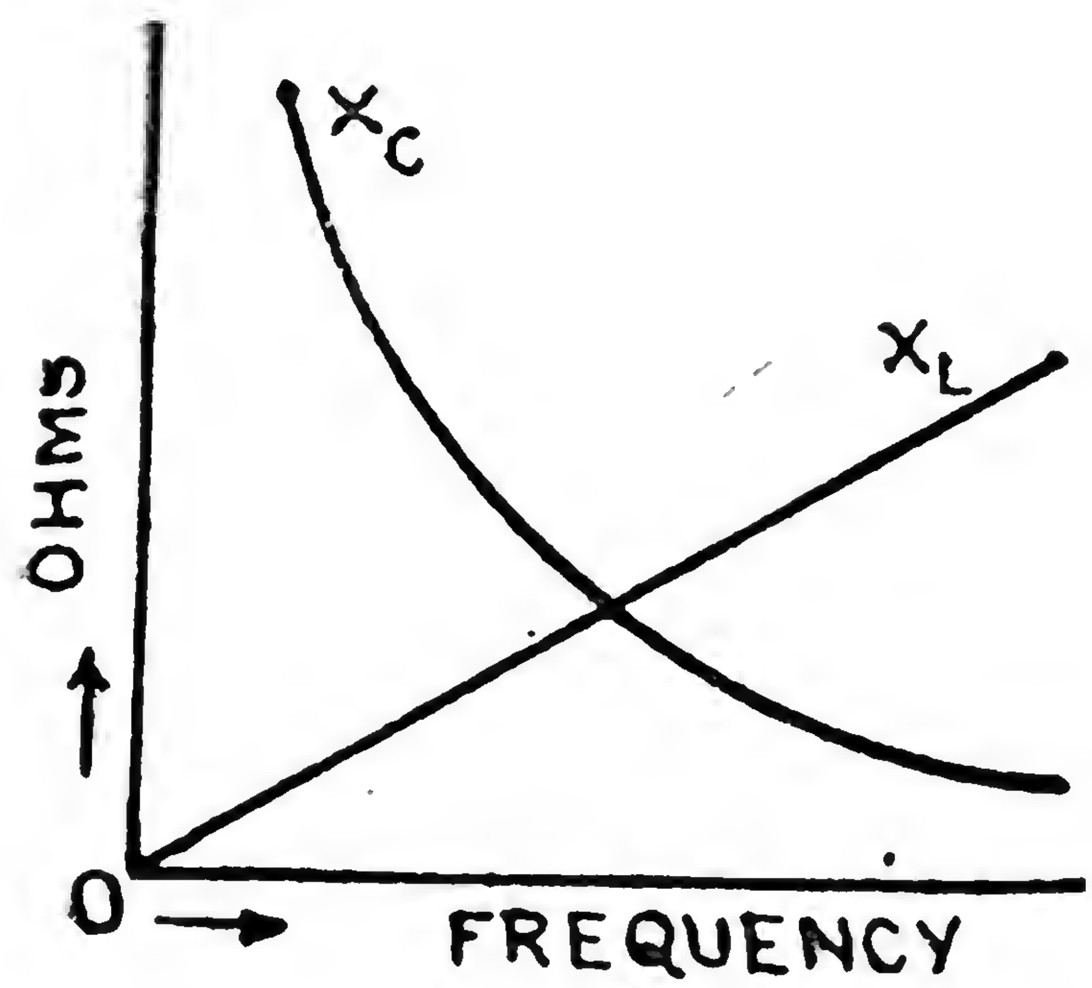


Fig. 12

The effect of varying the frequency of applied voltage on inductive reactance X_L and on capacitive reactance X_C is shown by Fig. 12. If frequency = 0

$X_L = 0$ and $X_C = \propto$ (infinity)

If frequency $= \propto$

$X_L = \propto$ and $X_C = 0$ (almost).

8. Simple Series and Parallel Circuits : Case I—

Resistance and Inductance in Series. Fig. 13 shows the circuit where R and L are in series across a voltage V. The current I is common to R and L, and V may be looked upon as made up of two components $V_R\ (= IR)$ and $V_L\ (= IX_L)$. V_R is in phase with I and it denotes the voltage drop across R (See Fig. 14). V_L is the component equal and opposite to the induced e. m. f. e_L and V_L is ahead of the current by 90° (see Fig. 9 (b)). Hence V is the vector sum of V_R and V_L as shown in Fig. 14. The angle between V and I is the phase difference angle ϕ. The combined effect of resistance R and reactance X_L in the circuit is called *impedance*, symbol Z, and its unit is the ohm in practical system of units.

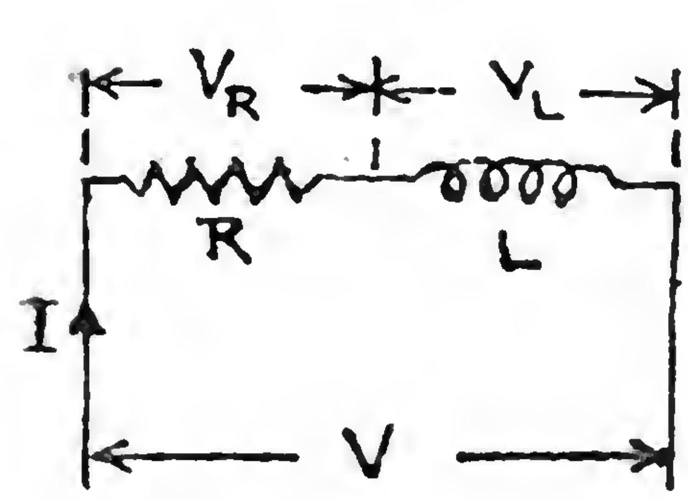


Fig. 13

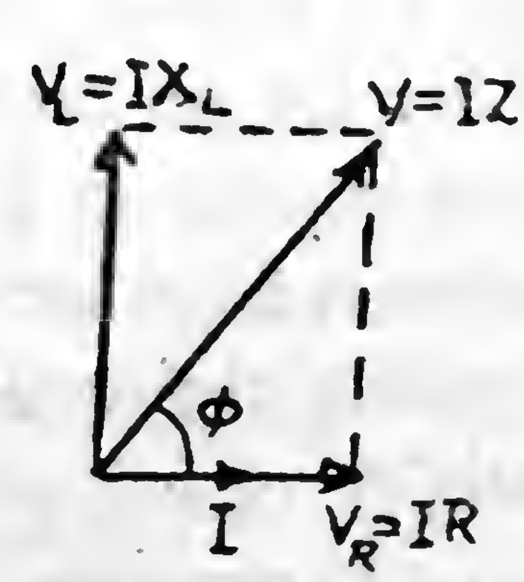


Fig. 14

From the geometry of Fig. 14

$$(IZ)^2 = (IR)^2 + (IX_L)^2$$

Cancelling I,

$$Z^2 = R^2 + X_L^2$$

$$\therefore \quad Z = \sqrt{(R^2 + X_L^2)} \quad \dots \quad \dots \quad \dots \quad (4)$$

i. e. *impedance* = $\sqrt{}$ (*resistance*^{2} + *reactance*^{2})

$$\text{and} \quad \tan\phi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R} \quad \dots \quad \dots \quad \dots \quad (5)$$

Though resistance, reactance and impedance are not vector quantities, it follows from Fig. 14 that their relationship may be shown by Fig. 15, which is called the impedance triangle and that the impedance can be expressed in vectorial form as

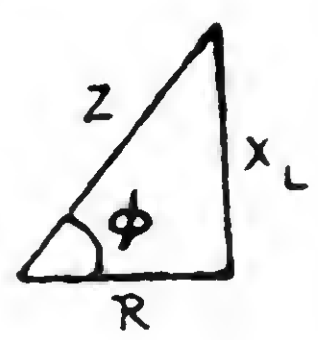


Fig. 15
Impedance Triangle

$$Z = R + j\,X_L$$

and $\tan\phi = \dfrac{X_L}{R}$ as before.

When more than two impedances are connected in series the total impedance is given by

$$Z = \sqrt{[(r_1 + r_2 + r_3 + \cdots)^2 + (x_1 + x_2 + x_3 + \cdots)^2]} \qquad (6)$$

where $Z_1 = (r_1 + jx_1)$; $Z_2 = (r_2 + jx_2)$; $Z_3 = (r_3 + jx_3)$ etc., and Z_1, Z_2, Z_3 are the individual impedances.

In vector notation the total impedance is given by

$$Z = (r_1 + r_2 + r_3 + \cdots) + j\,(x_1 + x_2 + x_3 + \cdots) \quad \dots \quad (7)$$

Example: Three coils A, B and C are connected in series across 200 volt 50 cycle supply mains. R_A = 10 ohms, L_A = 0·05 henry; R_B = 12 ohms, L_B = 0·01 henry; R_C = 5 ohms, L_C = 0·08 henry. Calculate (a) the voltage across each coil and (b) the current and its phase angle to the supply voltage.

Solution: $Z_A = \sqrt{[10^2 + (2\pi \times 50 \times 0{\cdot}05)^2} = \sqrt{(10^2 + 15{\cdot}7^2)}$
$= \sqrt{346{\cdot}49} = 18{\cdot}6$ ohms.

$Z_B = \sqrt{[12^2 + (2\pi 50 \times 0{\cdot}01)^2]} = \sqrt{[12^2 + 3{\cdot}14^2]}$
$= 12{\cdot}4$ ohms.

$Z_C = \sqrt{[5^2 + (2\pi 50 \times 0{\cdot}08)^2]} = \sqrt{[5^2 + 25{\cdot}12^2]}$
$= 25{\cdot}75$ ohms

$Z_T = \sqrt{[(10 + 12 + 5)^2 + (15{\cdot}7 + 3{\cdot}14 + 25{\cdot}12)^2]}$
$= \sqrt{(27)^2 + (43{\cdot}96)^2}$
$= 51{\cdot}6$ ohms.

The current in the circuit

$$I = \frac{V}{Z_T} = \frac{200}{51{\cdot}6} = 3{\cdot}88 \text{ amperes}$$

voltage across coil A, $V_A = IZ_A = 3{\cdot}88 \times 18{\cdot}6 = 72{\cdot}1$ volts

" " coil B, $V_B = IZ_B = 3{\cdot}88 \times 12{\cdot}4 = 48{\cdot}1$ volts

" " coil C, $V_C = IZ_C = 3{\cdot}88 \times 25{\cdot}75 = 99{\cdot}8$ "

Phase angle between applied voltage and current

$$\phi_T = \tan^{-1}\left(\frac{43{\cdot}96}{27}\right) = 1{\cdot}629 = 58{\cdot}5^\circ$$

Phase angle between V_A and current

$$\phi_A = \tan^{-1}\frac{15{\cdot}7}{10} = \tan^{-1} 1{\cdot}57 = 57.5^\circ$$

Phase angle between V_B and current

$$\phi_B = \tan^{-1}\frac{3{\cdot}14}{12} = \tan^{-1} 0{\cdot}262 = 14{\cdot}8^\circ$$

Phase angle between V_C and current

$$\phi_C = \tan^{-1}\frac{25{\cdot}12}{5} = \tan^{-1} 5{\cdot}024 = 78{\cdot}8^\circ$$

Fig. 16 shows the vector diagram of voltages and current.

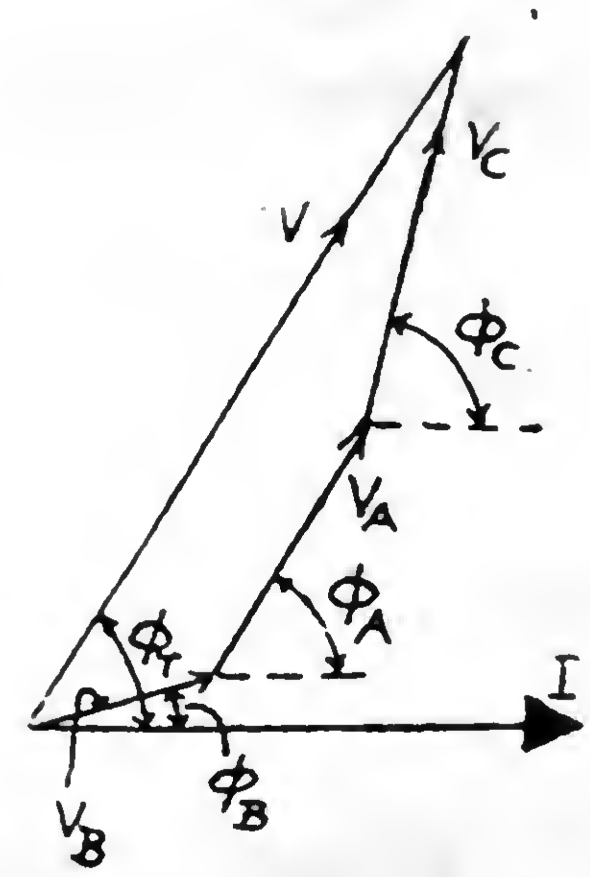


Fig. 16

Case II—*Resistance and Capacitance in Series.* Fig. 17

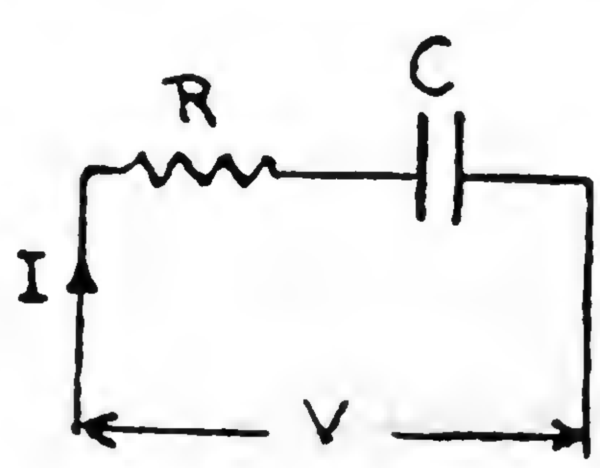


Fig. 17

Fig. 18

shows a circuit where R and C are in series across a voltage V. The current I is common, and V may be split into two components

$$V_R (= IR) \quad \text{and} \quad V_C (= IX_C)$$

V_R is in phase with I and V_C lags the current by 90°. Fig. 18 shows their vector addition which gives V. The angle between V and I is the phase difference angle ϕ. The combined effect of R and C in the

circuit is called impedance Z. Hence

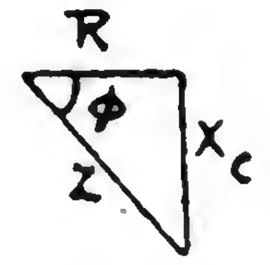


Fig. 19

$$Z^2 = R^2 + \left(\frac{1}{2\pi fc}\right)^2$$
$$= R^2 + X_C{}^2$$
$$\therefore \quad Z = \sqrt{R^2 + X_C{}^2}$$
$$\text{and} \quad \tan\phi = \frac{V_C}{V_R} = \frac{IX_C}{IR} = \frac{X_C}{R} \quad \ldots \quad \ldots \quad (8)$$

The impedance triangle in this case is as shown in Fig. 19.

In vector notation $Z = R - jX_C$

since Z is in the 4th quadrant and X_C is on the negative side of the y–axis.

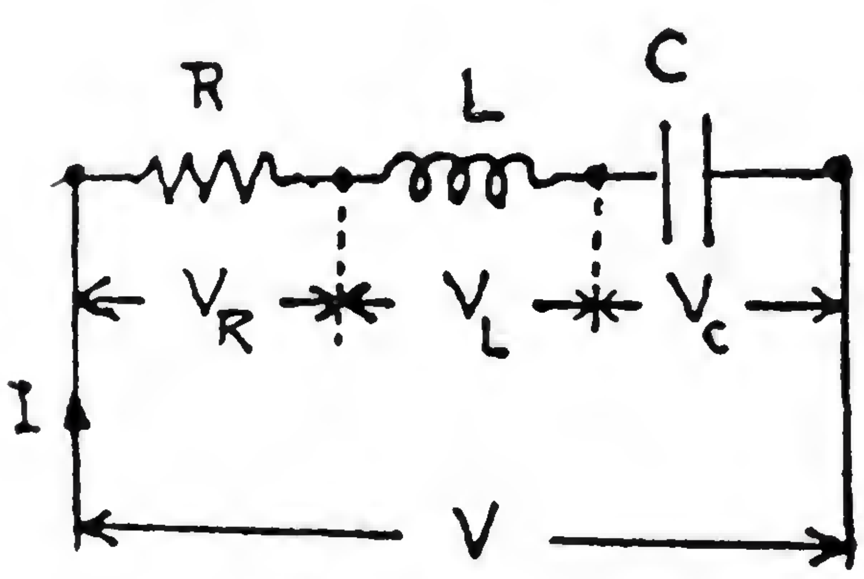


Fig. 20

Case III—*R, L and C in series* : Fig. 20 shows the circuit in which R, L and C are connected in series and the whole circuit is across a voltage V. Let the voltage drops across R, L and C be V_R, V_L and V_C respectively. Then

$$V_R = IR$$
$$V_L = IX_L = I\,(2\pi f L)$$
$$V_C = I\,X_C = I \times \frac{1}{2\pi f C}$$

I is the phase with V_R, 90° lagging to V_L and 90° leading to V_C.

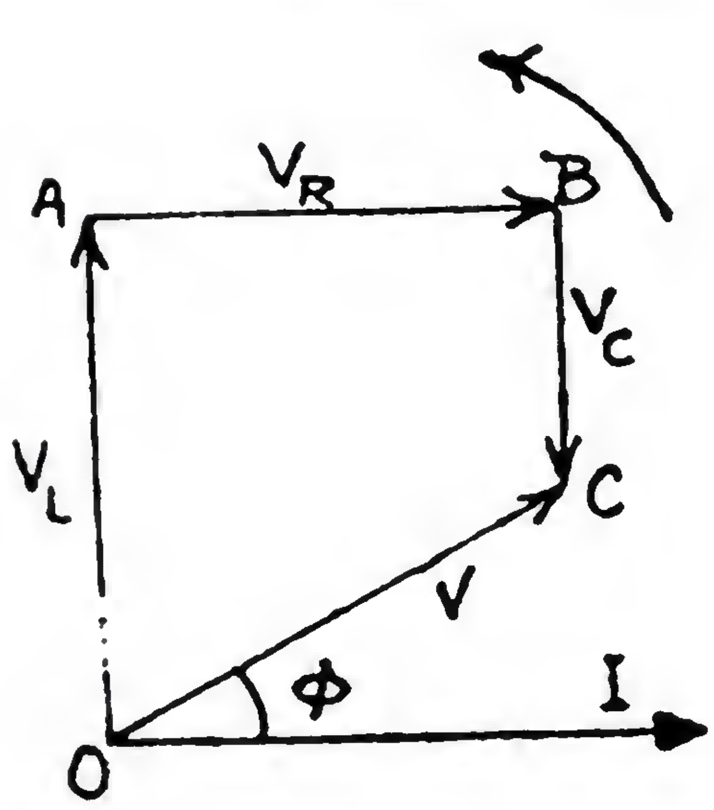


Fig. 21

Hence the vector diagram is shown by Fig. 21 in which I is along the reference axis because it is common to the three units. Starting with V_L from O, OA = V_L and 90° out of phase with I. AB = V_R in phase with I and BC = V_C 90° out of phase with I, and lagging. The closing side OC = V the applied voltage. The phase angle ϕ between the current and the applied voltage is given by ∠ IOC.

Note that V_C and V_L have a phase difference of 180°, i. e. C tries to neutralise the effect of L in a circuit. If $V_L = V_C$ the phase angle $\phi = 0°$. In any circuit containing L and C there is only one value of frequency at which $V_L = V_C$. This frequency is called

resonant frequency (f_r). In a *series circuit* the expression for f_r is derived as shown below :—

$$V_L = I X_L, \text{ and } V_C = I X_C$$

For *series* or *voltage resonance*,

$$I X_L = I X_C$$

$$2\pi f_r L = \frac{1}{2\pi f_r C}$$

$$\therefore \quad f_r = \frac{1}{2\pi} \sqrt{\left(\frac{1}{LC}\right)} \quad \dots \quad \dots \quad \dots \quad (9)$$

At resonance in a series circuit $I = \dfrac{V}{R}$.

In vector notation

$$Z = R + j(X_L - X_C)$$

Since at resonance $X_L = X_C$

$$Z = R + j0$$

$$\therefore \quad I = \frac{V}{R} \quad Q.\ E.\ D.$$

Case IV—*Parallel Circuit* : Fig. 22 shows three branches in a parallel combination, one branch has L, another branch has C, while R is present in all the three branches. The numerical values are given in the diagram. It is required to calculate the total current taken from the supply and the phase angle between the total current and the applied voltage.

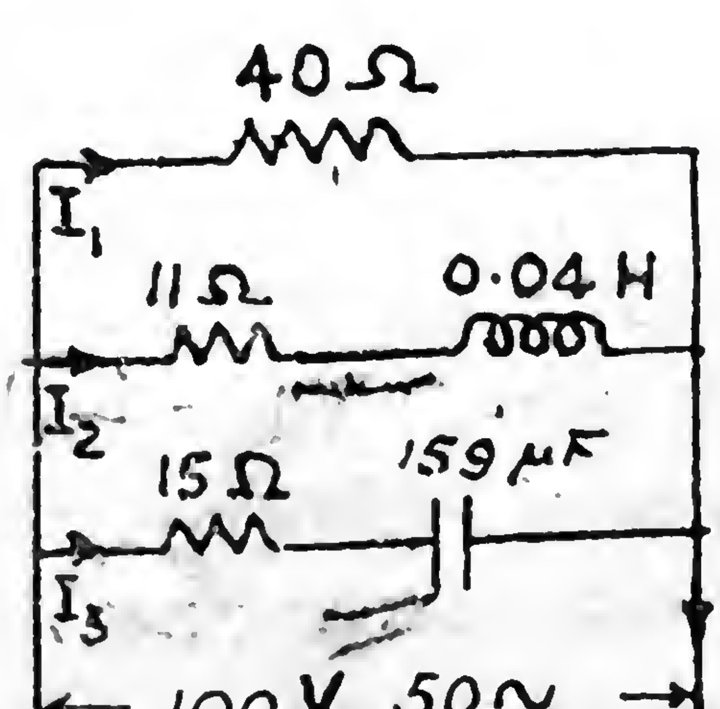


Fig. 22

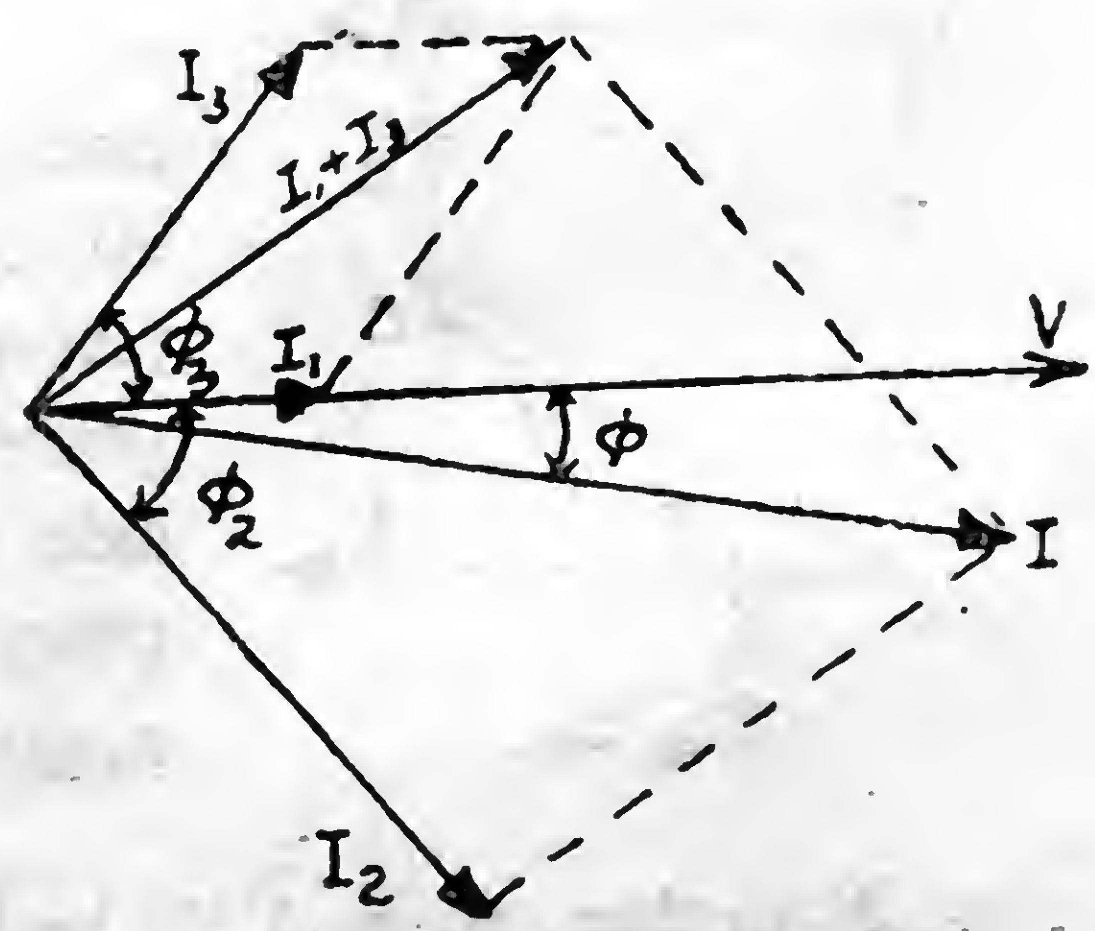


Fig. 23

A. It is best to calculate the current in each branch and the total current is then the vector sum of the three currents.

Branch (i) $I_1 = \dfrac{100}{40} = 2{\cdot}5$ A; phase angle 0°.

Branch (ii) $X_L = 2\pi \times 50 \times 0{\cdot}04 = 12{\cdot}56\ \Omega$

$$Z_2 = \sqrt{11^2 + 12{\cdot}56^2} = 16{\cdot}7\ \Omega$$

$$I_2 = \frac{100}{16{\cdot}7} = 5{\cdot}99 \text{ A}$$

$$\cos\phi_2 = \frac{11}{16{\cdot}7} = 0{\cdot}66 \quad \therefore \quad \phi_2 = 48^\circ{\cdot}40' \text{ (lagging)}$$

$$\therefore \quad \sin\phi_2 = 0{\cdot}75.$$

Branch (iii) $X_C = \dfrac{10^6}{2\,\pi\, 50 \times 159} = 20\ \Omega$

$$Z_3 = \sqrt{15^2 + 20^2} = 25\,\Omega$$

$$I_3 = \frac{100}{25} = 4 \text{ A.}$$

$$\cos\phi_3 = \frac{15}{25} = 0{\cdot}6 \qquad \therefore \qquad \phi_3 = 53^\circ - 10' \text{ (leading)}$$

$$\sin\phi_3 = 0{\cdot}8.$$

By graphical method the addition of these three vectors is shown in Fig. 23. By measurement the total current is 9 amperes. By taking the x– and y–components the addition of the three vectors is as follows :—

	x	y
$I_1 \rightarrow$	2·5	0
$I_2 \rightarrow$	5·99 × 0·66	− 5·99 × 0·75
	(3·96)	− (4·5)
$I_3 \rightarrow$	4 × 0·6	+ 4 × 0·8
	(2·4)	(3·2)

Total of x–components $= 2{\cdot}5 + 3{\cdot}96 + 2{\cdot}4 = 8{\cdot}86$

Total of y–components $= 0 - 4{\cdot}5 + 3{\cdot}2 = -\,1{\cdot}3$

$\therefore$ *total current* $I = \sqrt{8{\cdot}86^2 + 1{\cdot}3^2} = 8{\cdot}95$ A.

Phase angle between I and V is

$$\phi = \tan^{-1}\frac{1{\cdot}3}{8{\cdot}86} = \tan^{-1} 0{\cdot}146 = 8^\circ\text{–}20'.$$

Note : The component of current which is in phase with the voltage vector is called the *active component.* The other component

which is at 90° with the voltage vector is called the *reactive* or *idle* or *wattless* component.

B. *By the use of Admittance, Conductance and Susceptance*.

Let Z_1, Z_2 and Z_3 denote three impedances. When these are connected in series the total impedance Z_T is

$$Z_T = Z_1 + Z_2 + Z_8 \text{ (vector sum)} \quad \ldots \quad \ldots \quad \text{(i)}$$

When connected in parallel

$$\frac{1}{Z_T} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} \text{ (vector sum)} \quad \ldots \quad \ldots \quad \text{(ii)}$$

The reciprocal of impedance is called *admittance*, symbol Y and its unit is the *mho* in practical system of units, i. e. $Y = \frac{1}{Z}$.

So that (ii) reduces to

$$Y_T = Y_1 + Y_2 + Y_3 \text{ (vector sum)} \quad \ldots \quad \ldots \quad \text{(iii)}$$

Just as Z has two components R and X so also Y has two components G and B. G is called *conductance* and B is called *susceptance*. The unit is the mho for both G and B. The total conductance of the three branches in parallel is

$$G_T = G_1 + G_2 + G_3 \text{ (arithmetic sum)} \quad \ldots \quad \ldots \quad \text{(iv)}$$

and total susceptance

$$B_T = B_1 + B_2 + B_3 \text{ (algebraic sum)} \quad \ldots \quad \ldots \quad \text{(v)}$$

and the total admittance

$$Y_T = \sqrt{(G_T^2 + B_T^2)} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \text{(vi)}$$

Since there are inductive reactance (X_L) and capacitive reactance (X_C), so also there are two kinds of susceptances B_L and B_C.

In the impedance triangles, Figs. 15 and 19, X_L has a + ve sign while X_C has a − ve sign. In parallel circuits B_L has a − ve sign and B_C has + sign. The impedance and admittance triangles shown in Fig. 24 are for the same quantities. Hence

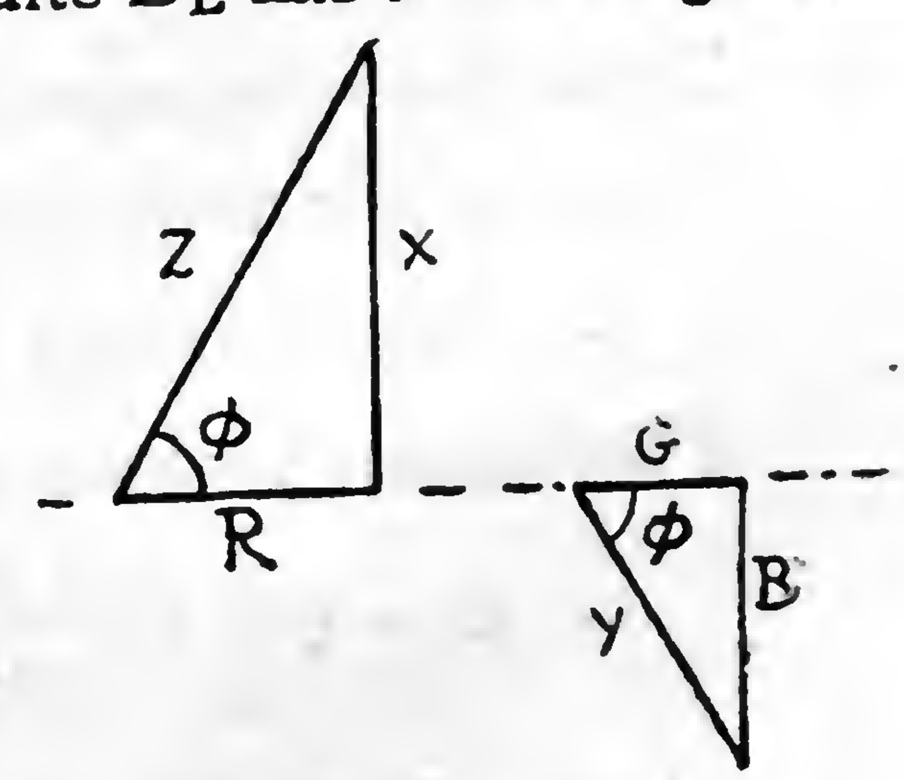


Fig. 24

$$\cos\phi = \frac{R}{Z} = \frac{G}{Y}$$

$$\therefore \quad G = \frac{RY}{Z} = \frac{R}{Z^2} = \frac{R}{R^2+X^2}$$

Similarly, $B = \frac{X}{R^2+X^2}$;

$$R = \frac{G}{G^2+B^2} \quad \text{and} \quad X = \frac{B}{G^2+B^2}.$$

Solving the last problem,

$$G_1 = \frac{R_1}{R_1^2+X_1^2} = \frac{40}{40^2+0^2} = \frac{1}{40} = 0{\cdot}025 \text{ mho}$$

$$G_2 = \frac{R_2}{R_2^2+X_2^2} = \frac{11}{11^2+12{\cdot}56^2} = \frac{11}{278{\cdot}7} = 0{\cdot}04 \text{ mho}$$

$$G_3 = \frac{15}{15^2+20^2} = 0{\cdot}024 \text{ mho}$$

$$B_1 = 0$$

$$B_2 = \frac{12{\cdot}56}{11^2+12{\cdot}56^2} = 0{\cdot}045 \text{ mho } (-\text{ve})$$

$$B_3 = \frac{20}{15^2+20^2} = 0{\cdot}032 \quad (+\text{ve})$$

Total $G = 0{\cdot}025 + 0{\cdot}04 + 0{\cdot}024 = 0{\cdot}089$ mho

total $B = 0 - 0{\cdot}045 + 0{\cdot}032 = -\ 0{\cdot}013$ mho

Total $Y = \sqrt{(G^2+B^2)} = \sqrt{[(0{\cdot}089)^2 + (-0{\cdot}013)^2]} = 0{\cdot}09$

$\therefore \quad I = VY = 100 \times 0{\cdot}09 = 9$ amperes.

$$\tan\phi = \frac{0{\cdot}013}{0{\cdot}089} = 0{\cdot}146.$$

From Tables $\phi = 8^\circ - 20'$ (check)

Note that when R alone is present and X is absent

$$G = \frac{R}{R^2+X^2} = \frac{R}{R^2} = \frac{1}{R}$$

Similarly, when X is present and $R = 0$

$$B = \frac{X}{R^2+X^2} = \frac{X}{X^2} = \frac{1}{X}.$$

In symbolic or vector notation

$$Y = G \mp jB.$$

The *minus* sign is used when the circuit is inductive and the *plus* sign when the circuit is capacitive.

Resonance in parallel circuit occurs when the algebraic sum of all the reactive components of current in the parallel branches is zero, the voltage vector being along the x-axis.

When L is in parallel with C, and R is absent from both the branches, resonance occurs when the currents in the two branches are equal, i. e.

$$\frac{V}{2\pi f L} = V \times 2\pi f C.$$

Hence from the above relation, the resonant frequency for this particular circuit is derived as follows :—

$$\frac{1}{2\pi f_r L} = 2\pi f_r C$$

$$\therefore \qquad f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

which is the same as Eq. (9). This is known as *current resonance*.

9. Power in A. C. Circuits : In alternating current circuits at any particular instant, the power is given by

$$power = v \times i$$

where v and i are the voltage and current at that instant. And *the average power over one complete cycle* is measured by a wattmeter.

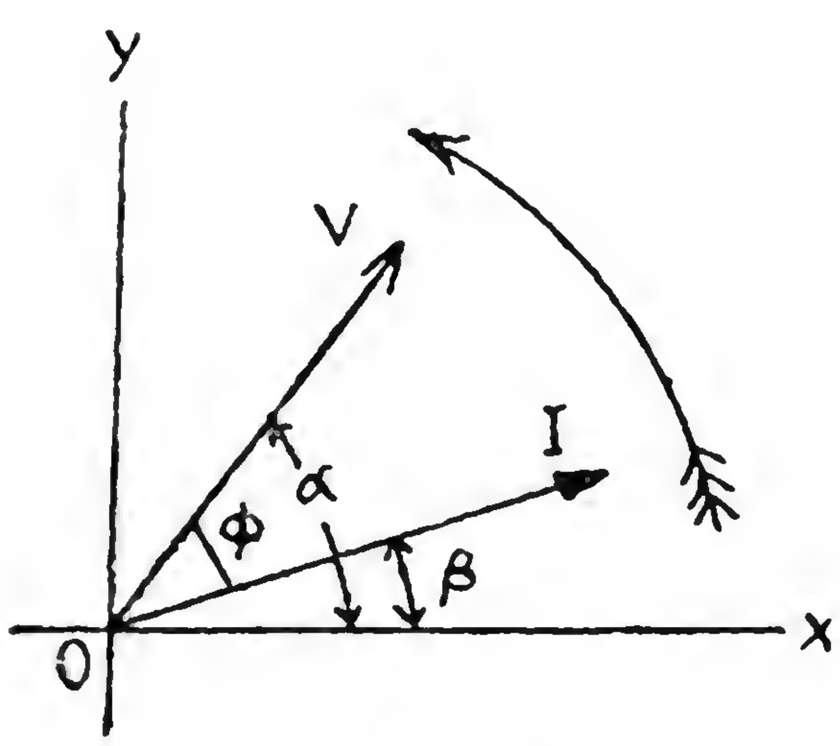


Fig. 25

If the instantaneous value v of the voltage is given by

$$v = V_{max} \sin \alpha$$

and that of the current by

$$i = I_{max} \sin \beta$$

then the power at any instant is

$$power\ p = v \times i.$$

Substituting the values of v and i

$$p = V_{max} \times I_{max} (\sin \alpha \sin \beta)$$
$$= V_{max} \times I_{max} \left[\tfrac{1}{2}\cos(\alpha - \beta) - \tfrac{1}{2}\cos(\alpha + \beta)\right]$$

From Fig. 25 $(\alpha - \beta) = \phi$, and $(\alpha + \beta) = (2\alpha - \phi)$

$$\therefore \quad p = \frac{V_{max} \times I_{max}}{2} \cos\phi - \frac{V_{max} \times I_{max}}{2} \cos(2\alpha - \phi)$$

The average value of $\left[-\frac{V_{max} \times I_{max}}{2} \cos(2\alpha - \phi)\right]$ over one complete cycle is zero and the curve is of double the supply frequency. Therefore, *average power* $= \frac{V_{max} \times I_{max}}{2} \cos\phi$, and is constant.

since I_{max}, V_{max} and ϕ are constants for that circuit. Now

$$\frac{V_{max} \times I_{max}}{2} = \frac{V_{max}}{\sqrt{2}} \times \frac{I_{max}}{\sqrt{2}} = V \times I$$

where V and I are the r. m. s. values. Hence the average power in an a. c. circuit is given by the expression

$$\textbf{power} = VI \cos \phi \qquad \ldots \qquad \ldots \qquad \ldots \qquad (10)$$

Case I. When the circuit contains only resistance, the current is in phase with the voltage and therefore $\cos \phi = 1$. Hence,

$$\textbf{power} = VI$$

as in the case of d. c. circuits.

Case II. When the circuit contains either inductance or capacitance or both and resistance is absent, the current either lags or leads the voltage by 90°. Therefore $\cos \phi = 0$. Hence,

$$\textbf{power} = V \times I \cos \phi = 0.$$

Case III. When the circuit contains resistance and inductance or capacitance or both, the current will either lead or lag depending upon whether $X_L < X_C$ or $X_C < X_L$. Hence **power** $= VI \cos \phi$.

Fig. 26 (a) shows the power curve for case I; Fig. 26 (b) and (c) shows the power curves for case II and Fig. 26 (d) that for case III. The graph of power is obtained by multiplying the instantaneous values of V and I over one complete cycle. In Fig. 26 (a)

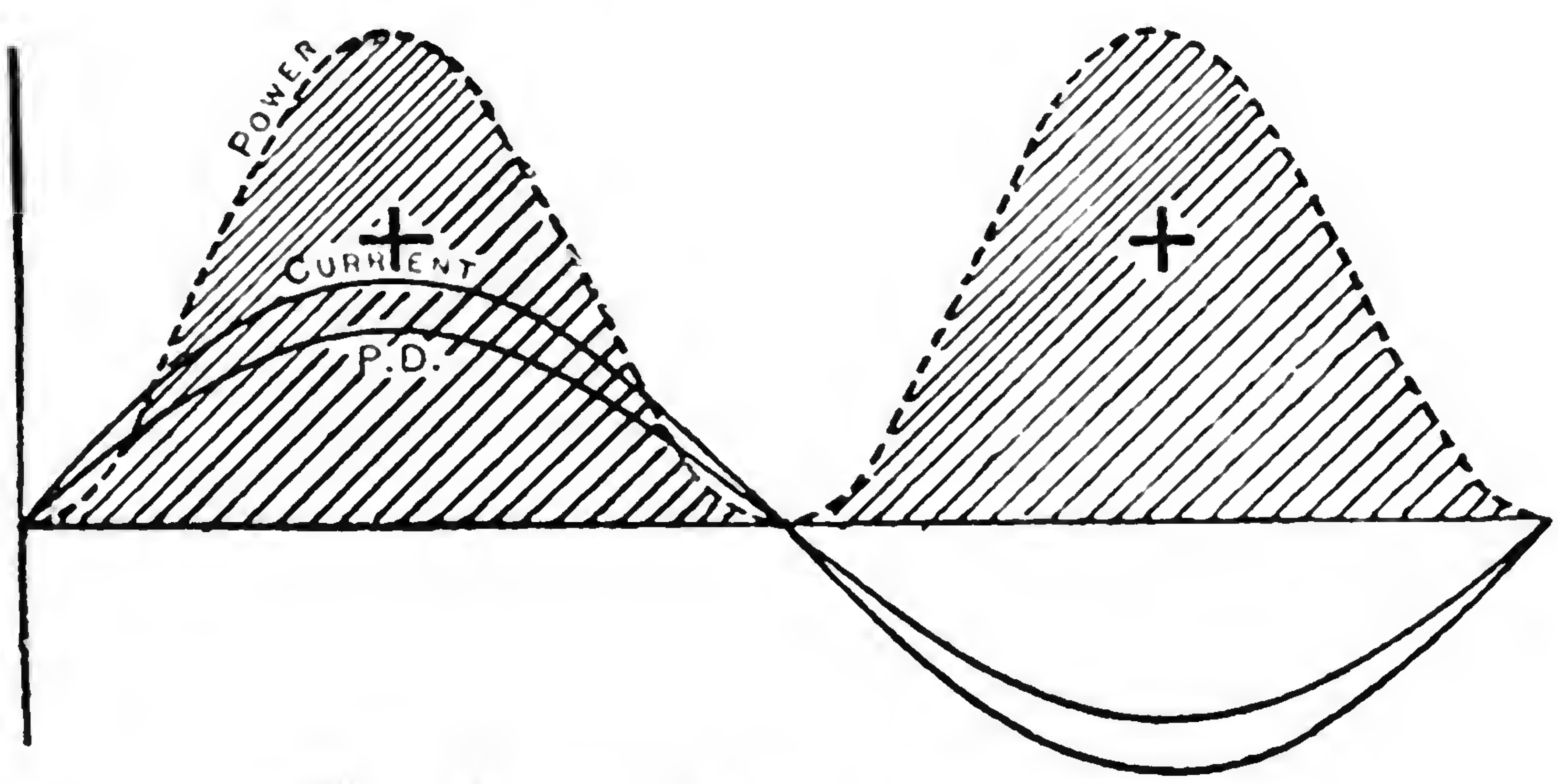


Fig. 26 (a). Unity Power Factor

the shaded portion is the total power, all of it lies above the x-axis and the whole power therefore is ***positive.*** The power is absorbed by the resistance of the circuit. In Fig. 26 (b) the shaded portions indicate the varying values of power from instant to instant. The

curve is for a purely inductive circuit and as much shaded portion is above the x-axis as it is below. Hence the net power is zero. The meaning of this is that during one quarter cycle the power is *absorbed and stored* in the circuit and in the next quarter of a cycle the stored energy is returned to the supply lines.

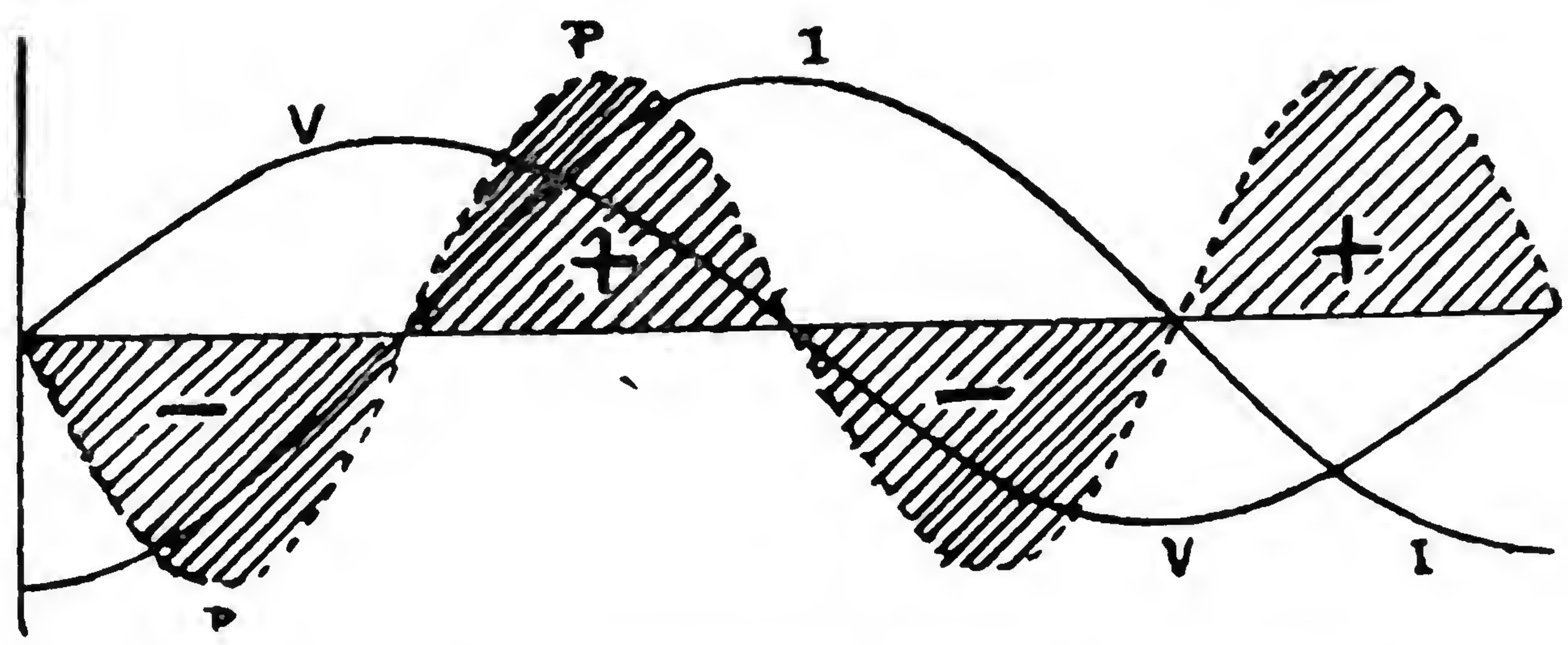


Fig. 26 (b). Zero Power Factor (Lagging).

Fig. 26 (d) shows the power curve for circuits having resistance, and $X_C > X_L$. Only a small portion of the power is stored and

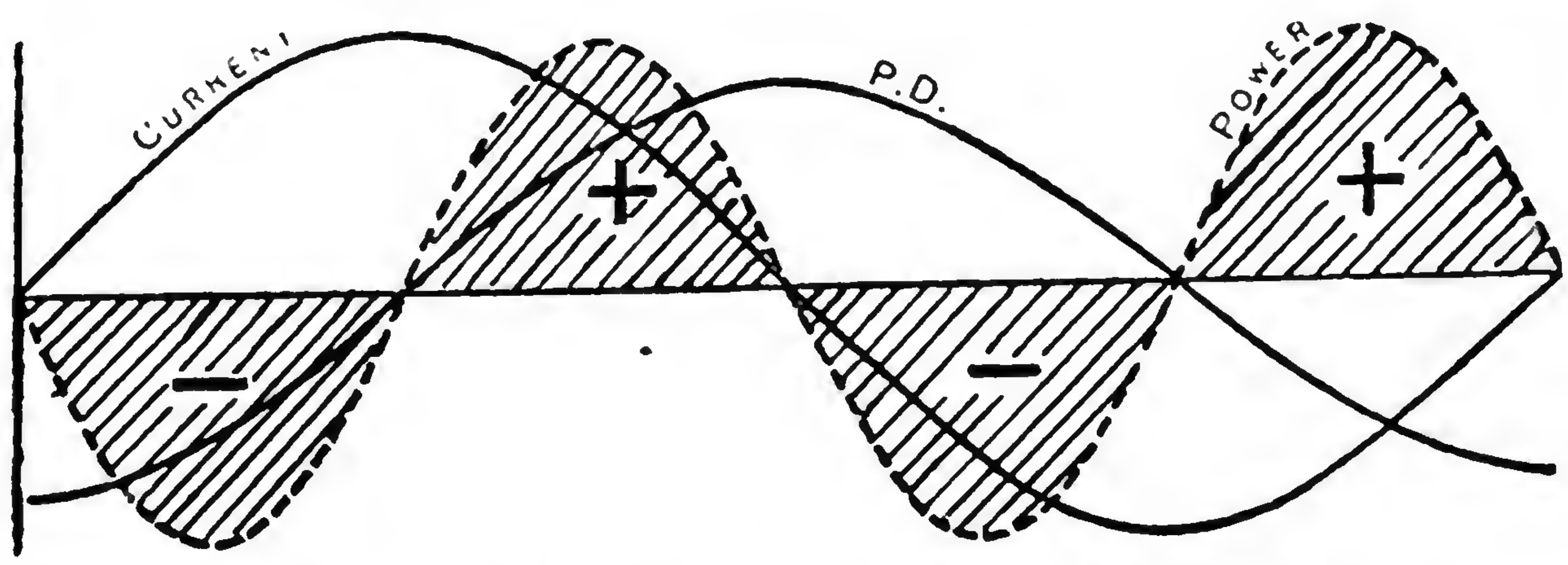


Fig. 26 (c). Zero Power Factor (Leading).

returned to the supply lines. So the net power is the difference between the shaded areas on the two sides of the axis, and for all circuits containing resistance the net power is of course positive.

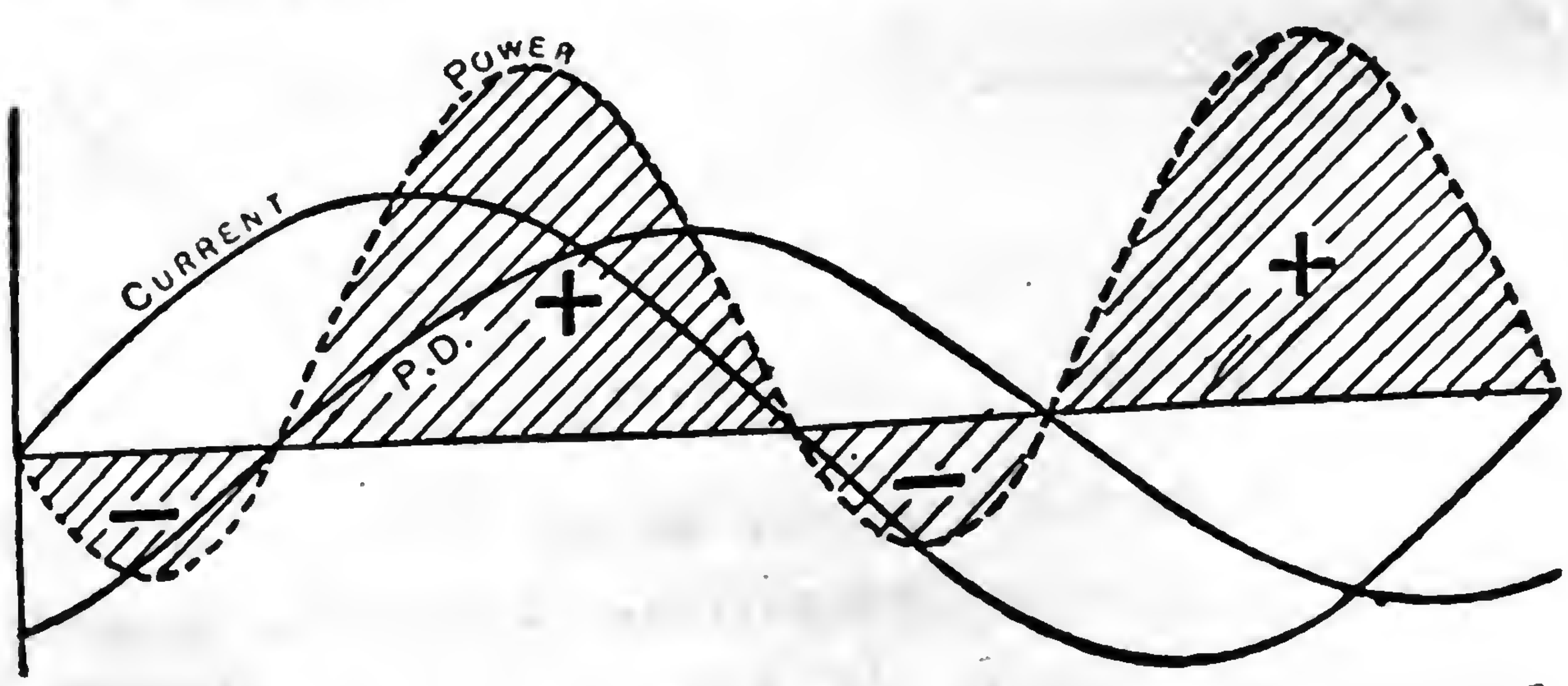


Fig. 26 (d). Power Factor Leading.

Since *power* = $VI \cos \phi$ for a. c. circuits and since $\cos \phi$ is always less than unity, the expression $\cos \phi$ is called *the power factor of the circuit* i. e. a factor by which the apparent power (volts × amperes) must be multiplied to obtain true power.

The value of $\cos \phi$ depends upon the relative values of R and X.

In fact $$\cos \phi = \frac{R}{Z} = \frac{R}{\sqrt{(R^2 + X^2)}} \qquad \ldots \quad \ldots \quad \ldots \quad (11)$$

An ideal circuit is that in which X is absent and $\cos \phi = 1$. But in most circuits inductance creeps is due to various causes and therefore there is a departure from the ideal. The current in such circuits always lags behind the voltage. The effects of power factor of consumers' circuits on Power Supply Undertakings are (a) they have to use a larger sized conductor when the power factor is less than unity; (b) the losses due to I^2R in the supply lines are greater, because the current is greater than when the power factor is unity. The power factor of a circuit can be improved or increased by the introduction of capacitance in the circuit as will be shown later on.

10. Polyphase Systems : Figs. 1 and 2 of this Chapter show an elementary alternator having one coil only and two slip-rings. The alternating wave of this coil when rotated at constant speed is shown is Fig. 3 (a). This is a single-phase system.

A system which is associated with two or more voltages of equal value is called a *polyphase system*. In a true m–phase system,

(i) the voltage vectors have a *time phase* displacement of ($2\pi/m$) radians or ($360°/m$); where m is the number of phases;

(ii) the r. m. s. values of all voltage vectors are the same, and

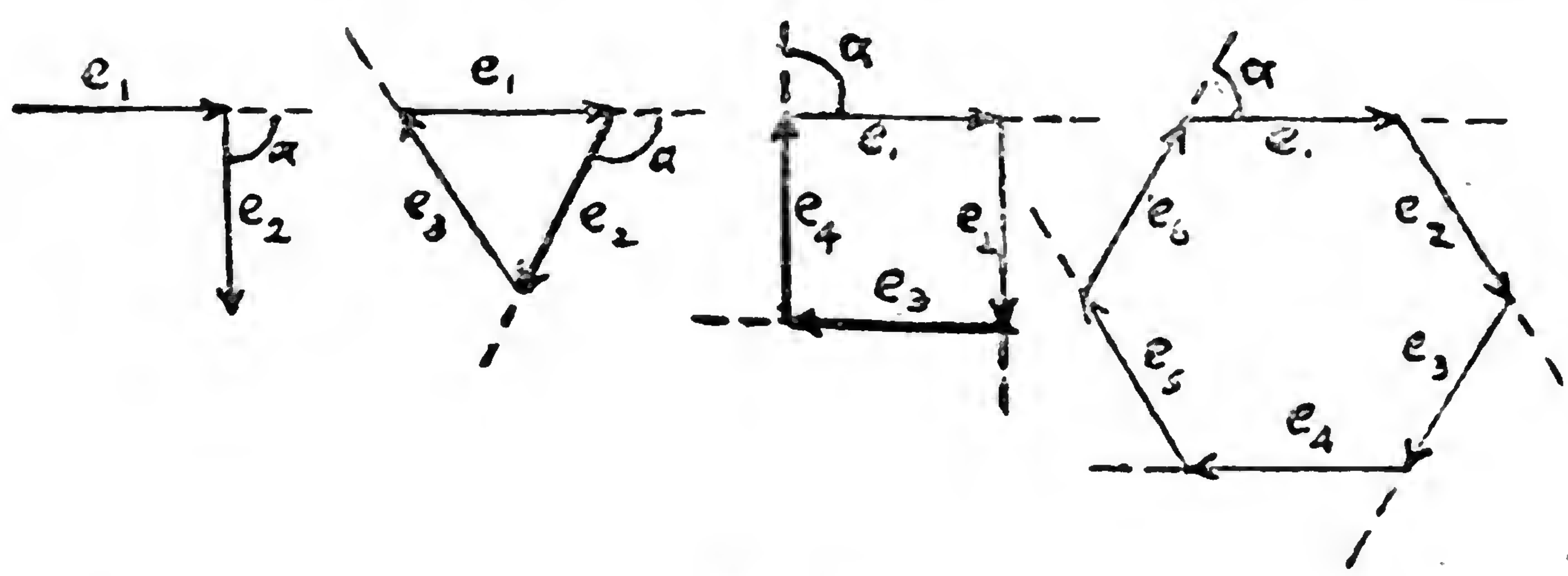


Fig. 27

(iii) the vector sum of all the voltages *at any instant* is zero, i. e.

$$e_1 + e_2 + e_3 + \cdots + e_m = 0$$

where e_1, e_2 etc. are the values of phase voltages at any one instant.

If all the three conditions are fulfilled the system is said to be *symmetrical*, otherwise it is an unsymmetrical system. An exception is the 2-phase system. Fig. 27 shows the voltage vectors of 2, 3, 4, and 6 phase systems. The angle α in the figure is the phase angle between successive voltages of the system. Except for 2-phase system, $\alpha = \frac{2\pi}{m}$, where m is the number of phases of the system.

A. *The 2-Phase System* is obtained by adding a second coil on the armature of the elementary alternator of Fig. 2 at right angles to the first coil. This second coil must have its own two slip-rings. If the two coils have the same number of turns the two e. m. fs. e_1 and e_2 will have the same maximum and r. m. s. values. And since the coils rotate at the same speed and in the same magnetic field the voltages will have the same frequency. The e. m. f. equations of the two voltages may be written as

$$\left.\begin{aligned} e_1 &= E_{max} \sin \omega t \\ e_2 &= E_{max} \sin (\omega t + 90^\circ) \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (12)$$

Fig. 28 shows the two coils of a 2-pole elementary alternator and the two e. m. f. waves. (*Note*: the pole windings are not shown in the figure. These windings require direct current excitation.)

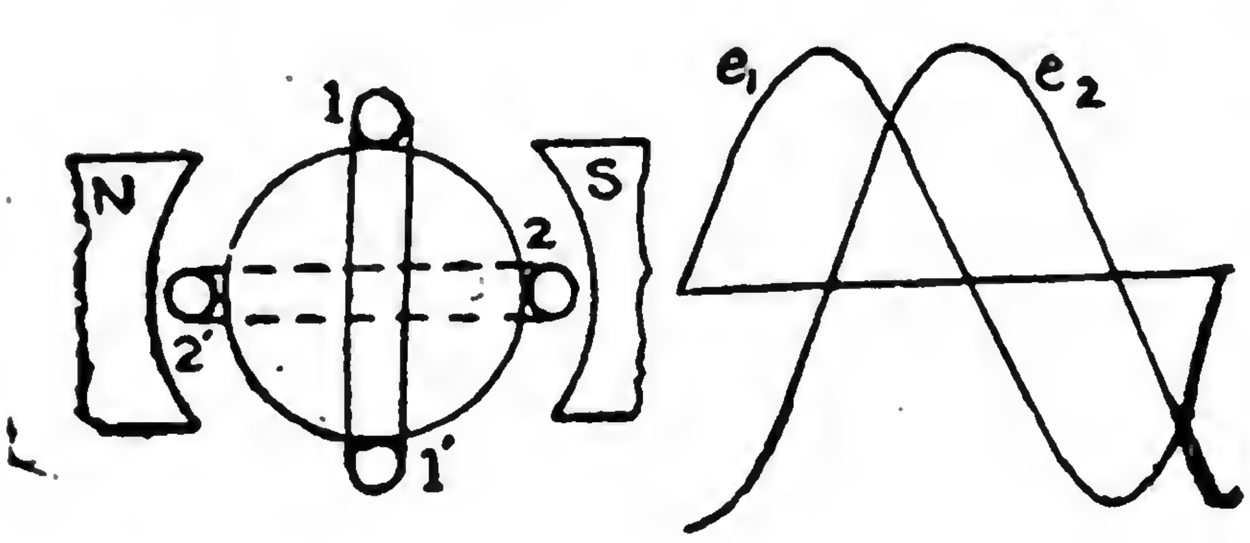


Fig. 28

A 2-phase alternator can supply power to (*a*) two single-phase circuit loads or (*b*) 2-phase circuit loads, or both. In case (*b*) the power consuming device or apparatus must have its windings arranged in a similar manner. If the load impedances on the two phases are identical, the load (on the system) is said to be *balanced*. When the load is balanced the currents in the two phases have the same anlge of phase difference between their respective voltage vectors. Fig. 29 shows a 2-phase alternator supplying loads having impedances of Z_1 and

Z_2 ohms. The power is fed by 4 line conductors. But if the ends ***b*** and ***d*** of the two coils (or the two phase windings) are connected together only 3 line conductors are needed to supply the load. The *common wire* will carry a current which is the vector sum of the two phase winding currents I_1 and I_2. If the loads are balanced, i. e. $Z_1 = Z_2$ in every respect $\left(\frac{R_1}{X_1} = \frac{R_2}{X_2}\right)$, the current in the common wire is $\sqrt{2}$ *times* the current in any one of the other two wires, and the power factor angles of each phase are equal ($\phi_1 = \phi_2$). The vector diagram is also shown in the figure.

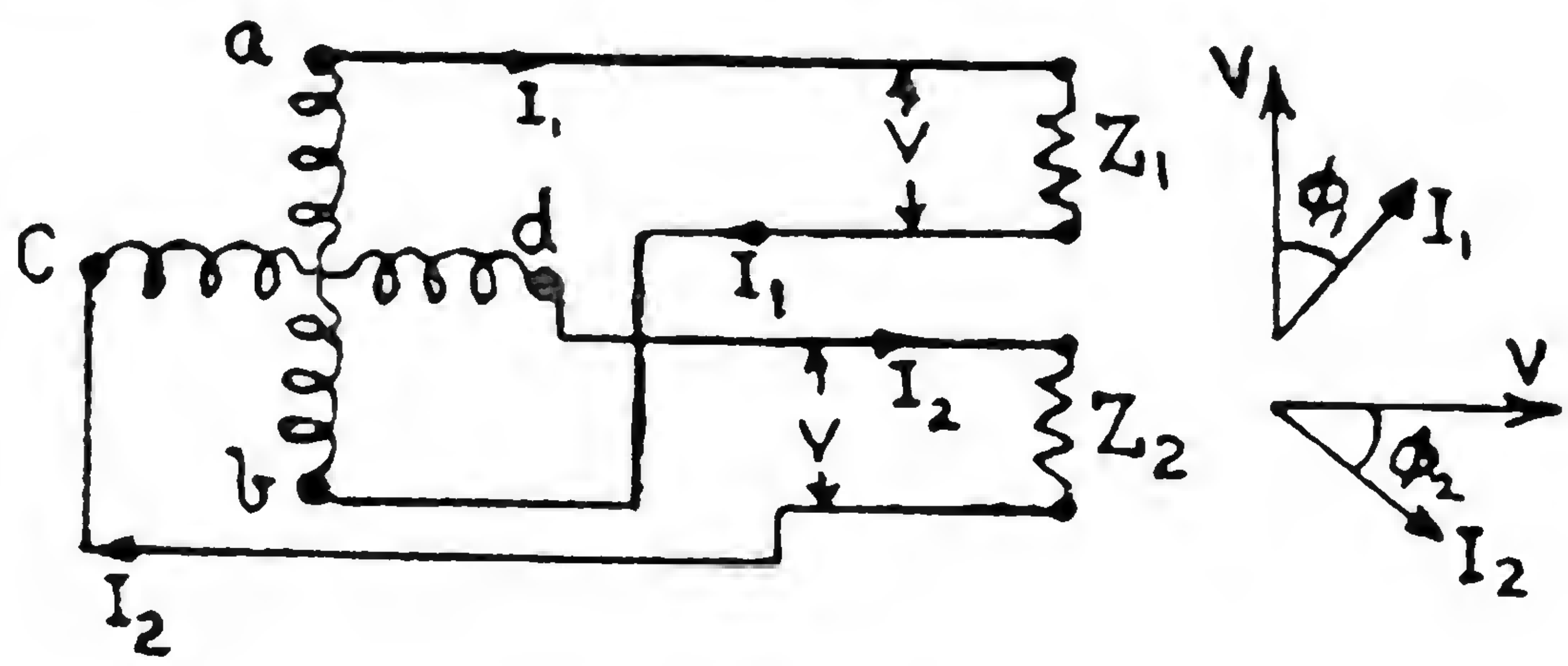


Fig. 29

Example : A 2-phase alternator supplies two single-phase loads by three line conductors. The impedances of the load are equal, $R = 16$ ohms and $X_L = 12$ ohms. Calculate the power factor of each load and the current in each line conductor (a) at full load and (b) when one load is reduced by 50 %. Phase voltage is 100 volts.

Solution : $Z_1 = Z_2 = \sqrt{(16^2 + 12^2)} = 20$ ohms.

$$\cos\phi_1 = \cos\phi_2 = \frac{R}{Z} = \frac{16}{20} = 0{\cdot}8 \text{ (lag.)}$$

$$\therefore \quad I_1 = I_2 = \frac{100}{20} = 5\,A.$$

since $Z_1 = Z_2$ in every respect, current in common conductor is

$$I = \sqrt{2} \times 5 = 7{\cdot}07\,A.$$

$\therefore$ the currents are 5 A, and $7{\cdot}07\,A$.

By reducing the load the power factor of that load does not change. And since the power factors are the same, the load currents are at 90° to each other. $I_1 = 5\,A$ and $I_2 = 2{\cdot}5\,A$. Therefore the current in the common wire is

$$I = \sqrt{5^2 + 2{\cdot}5^2} = 5{\cdot}59\,A.$$

The currents in the other two wires are $5\,A$ and $2{\cdot}5\,A$.

B. *The Three-Phase System* is obtained by having three coils at 120° from each other on the armature of a 2-pole machine. Fig. 30

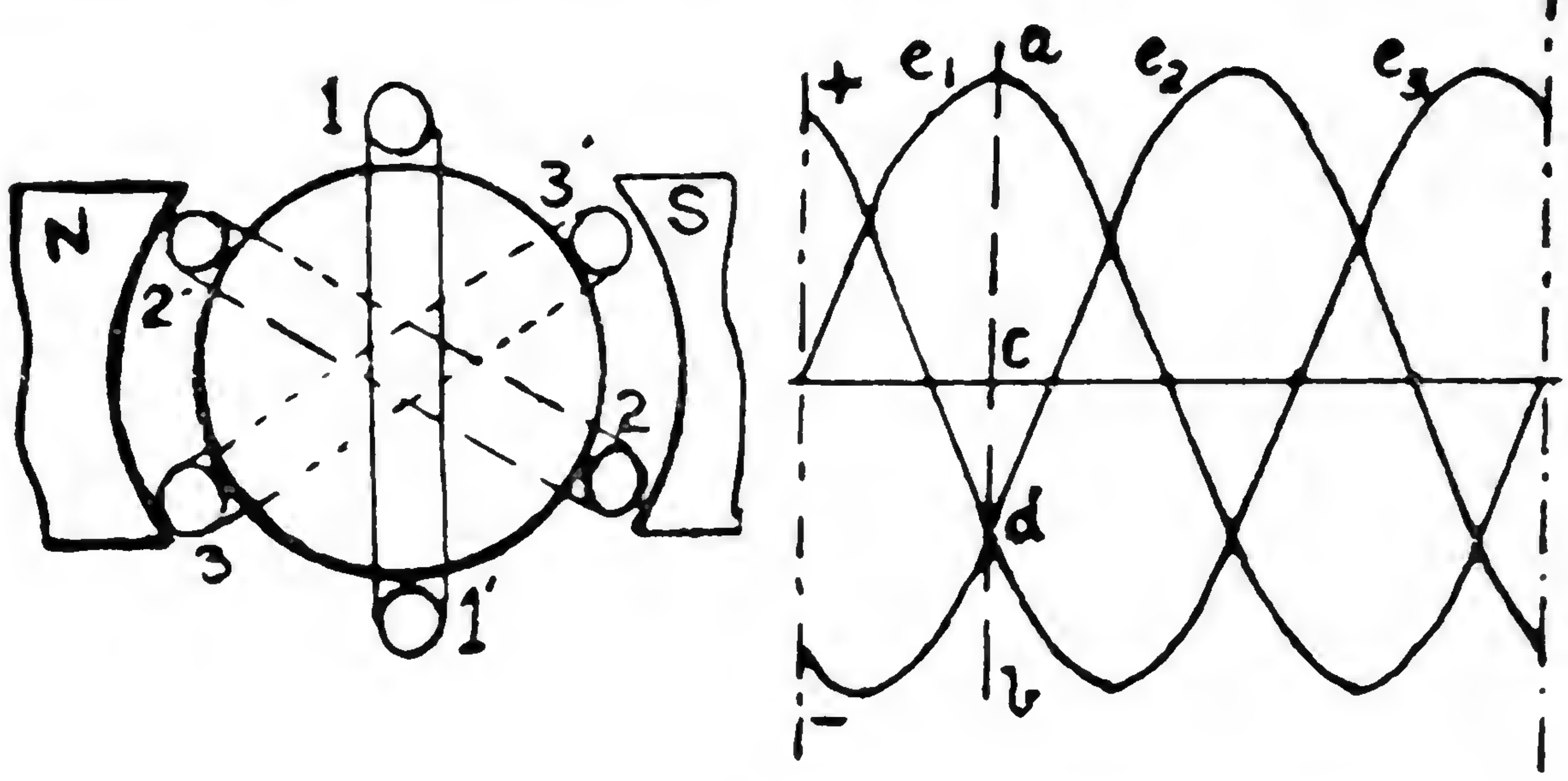


Fig. 30

shows an elementary 3-phase alternator, each coil should have two slip-rings of its own. If this armature is rotated at constant speed in a uniform field the three e. m. f. waves will be as shown in the figure. These waves have the same frequency, and the maximum and r. m. s. values, if the number of turns of the coils are the same. The maximum and zero values of one wave occur after $\frac{1}{3}$rd periodic time from that of the preceding wave. The voltage vectors are shown in Fig. 31. The algebraic sum of three voltages at any instant is zero. The **phase sequence** of Fig. 31 is ***RYB***, i. e. if the voltage of ***R*** phase reaches its positive maximum value at instant *t*, the voltage of ***Y*** phase attains its positive maximum value after $\frac{1}{3}$rd periodic time, and after another $\frac{1}{3}$rd periodic time the voltage of ***B*** phase reaches its positive maximum value.

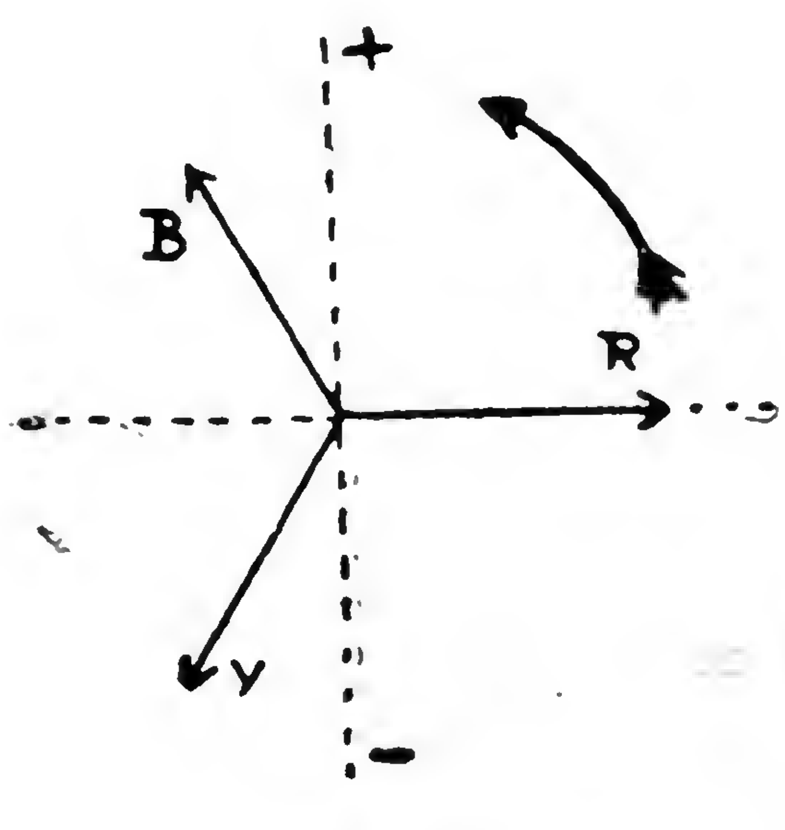


Fig. 31

The e. m. f. equations of the three voltage waves may be written as

$$\left.\begin{aligned} E_R &= E_{max} \sin \omega t \\ E_Y &= E_{max} \sin (\omega t + 240^\circ) \\ \text{or } E_Y &= E_{max} \sin \left(\omega t + \frac{4\pi}{3} \right) \\ E_B &= E_{max} \sin (\omega t + 120^\circ) \\ \text{or } E_B &= E_{max} \sin \left(\omega t + \frac{2\pi}{3} \right) \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (13)$$

Instead of having 6 slip-rings and therefore 6 line conductors from a 3-phase alternator, the standard practice is to have only 3 slip-rings. Thus 3 line conductors supply power to a 3-phase load. This is made possible by having the three windings of an alternator connected either in star or delta.

In order to connect the three windings of an alternator either in star or delta, some convention must be adopted in giving the *positive direction* of induced e. m. f. in a phase winding. The two ends of a phase winding are marked as S (start) and F (finish).

Fig. 32 (a) shows, for simplicity, 3 single-turn coils of a 2-pole elementary alternator. These coils are spaced 120° apart from each

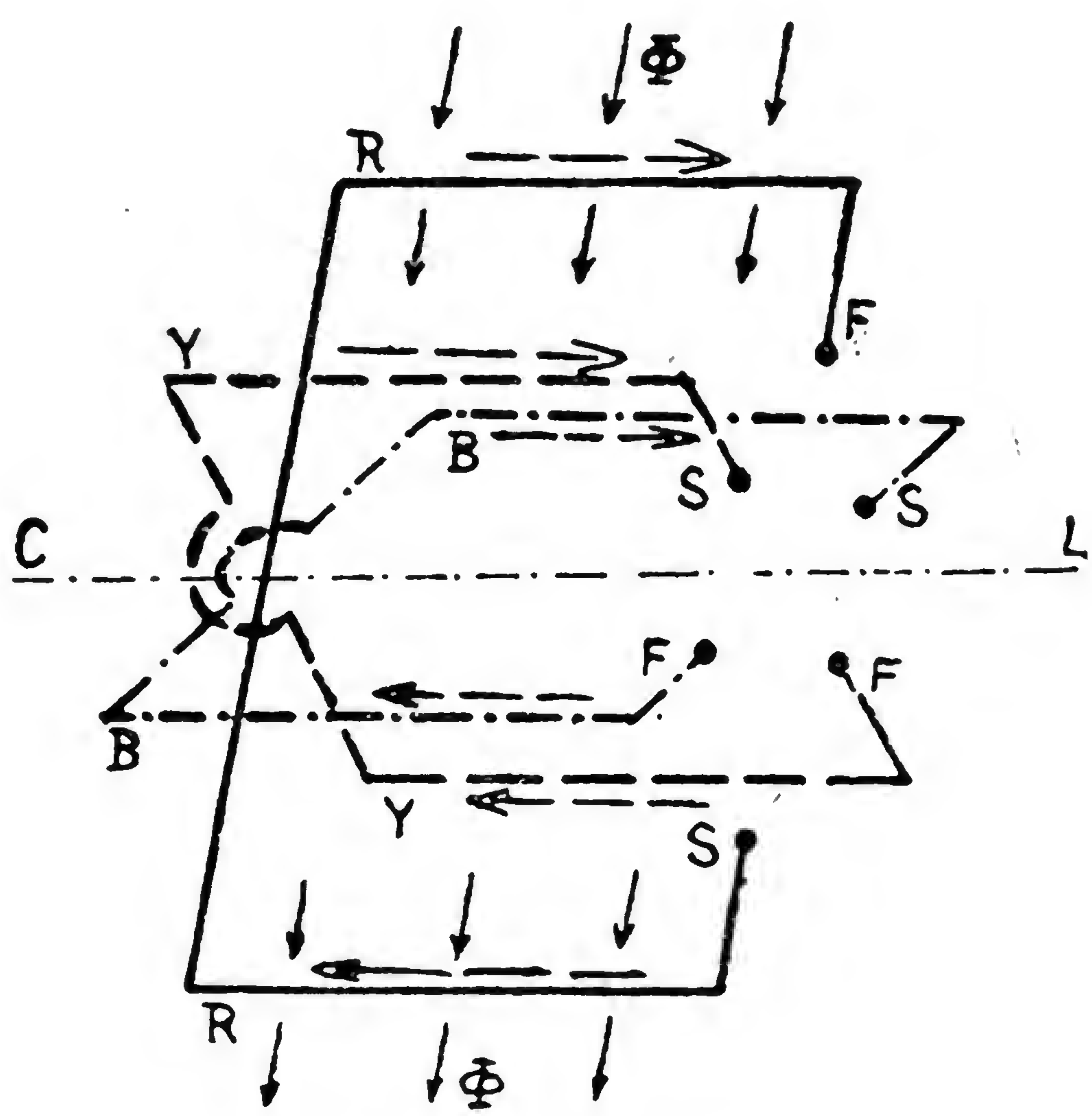


Fig. 32 (a)

other and are shown to cccupy a certain position with respect to the magnetic field Φ at a particular instant. By Fleming's Right Hand Rule the direction of induced e. m. f. in the R phase and in the other phases is shown by dotted arrows. If end F of R phase is considered as positive, its end S is negative. After 1/3 periodic time the Y phase occupies the position in which the R phase is shown in the figure. Therefore the direction of induced e. m. f. in this phase will also be

the same. Hence its ends are marked accordingly F and S. Similarly the ends of B phase are marked.

When all the three S ends are joined together they form a common point called the **star point** of the alternator. What applies to these single-turn coils applies equally well to full phase windings which usually consist of many coils per phase, and each coil sometimes with many turns. Fig. 32 (b) is the schematic representation of a star-connected armature of an alternator.

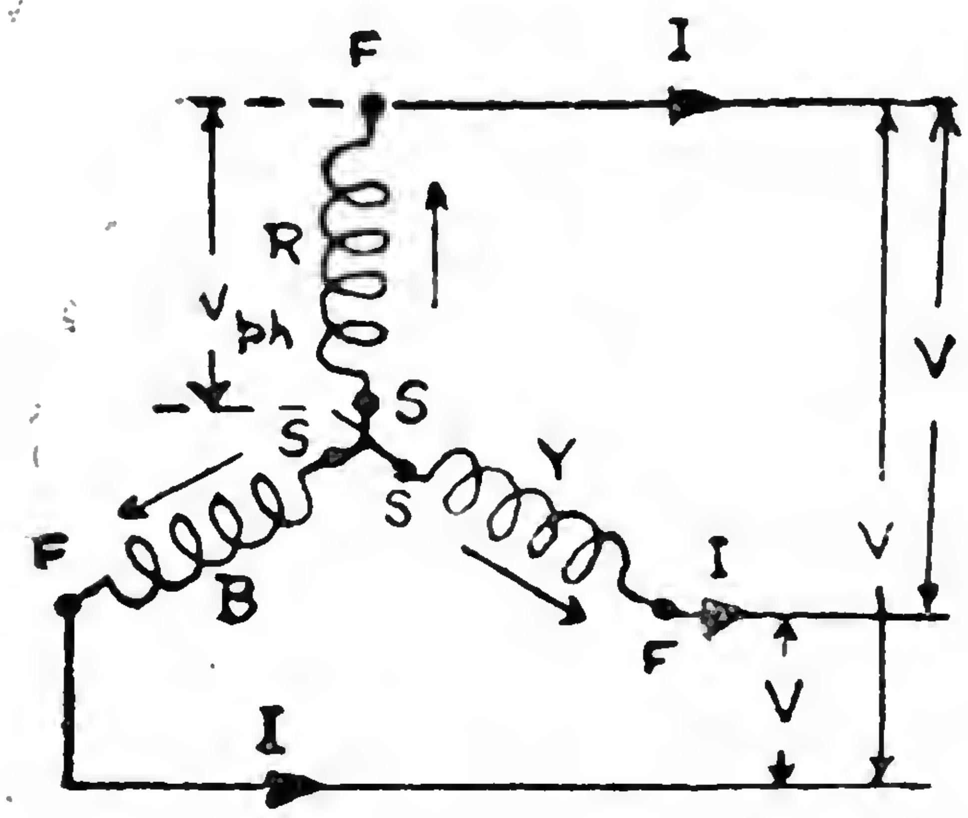


Fig. 32 (b)

The potential of the star point is zero with respect to the potential of any one of the F ends, but not necessarily zero with respect to the Earth potential if the star point is not earthed. The usual practice is to earth the star point. In a star-connected alternator, assuming a balanced load, all phase currents are equal and the line currents are equal to phase currents. But the line voltage V is $\sqrt{3}$ times the phase voltage V_{ph}, i. e.

$$V = \sqrt{3}\ V_{ph} \quad \dots \quad \dots \quad \dots \quad (14)$$

The **delta** or **mesh** connection is made by connecting the S and F points in cyclic order, i. e. (i) the end F of R phase is connected to the S end of Y phase; (ii) the end F of Y phase to S end of B phase and (iii) the F end of B phase to the S end of R phase. This is shown in Fig. 33.

The vector sum of the three phase voltages round the mesh is zero, hence no circulating current flows in the windings of a delta-connected alternator. Further the line voltage V is equal to phase voltage V_{ph}. And assuming a 3-phase balanced load, the line current I is equal to $\sqrt{3}$ times the phase current I_{ph}, i. e.

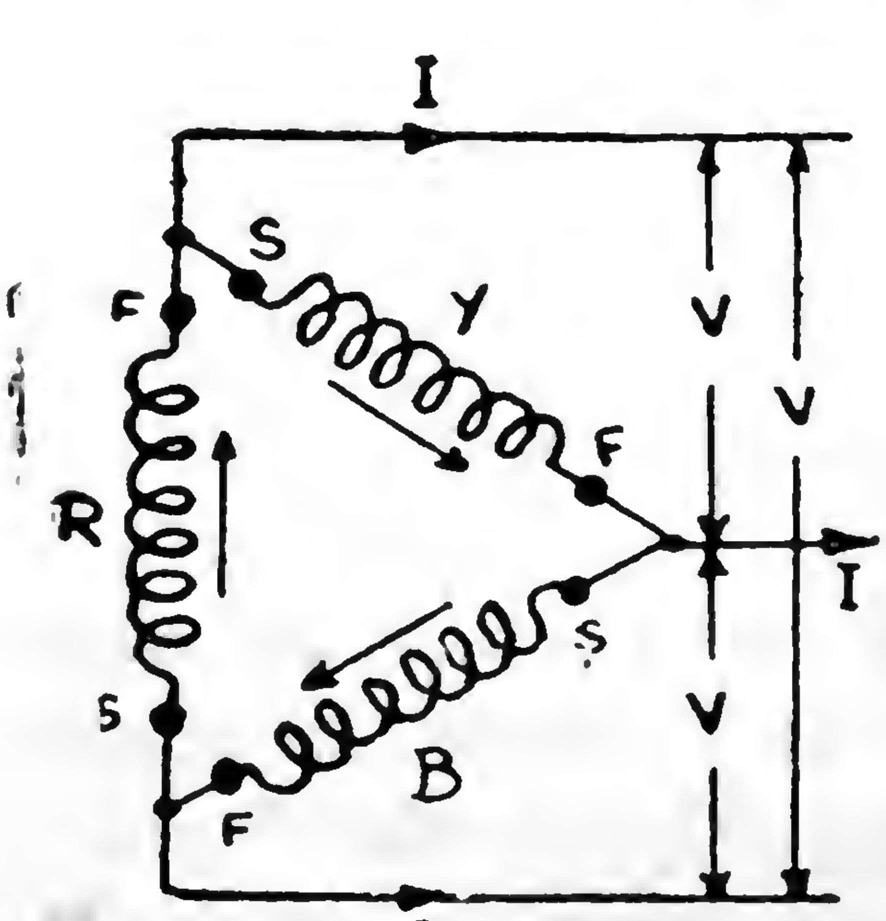


Fig. 33

$$I = \sqrt{3}\ I_{ph} \quad \dots \quad \dots \quad (15)$$

A 3-phase load may have its three impedances Z_1, Z_2 and Z_3 connected either in star or delta. Provided the line voltage is suitable,

(*a*) a star-connected alternator can supply power to either a star-connected or delta-connected load;

(*b*) a delta-connected alternator can similarly supply power to either a star-connected or delta-connected load.

Example: A 100-volt, 3-phase, star-connected alternator is connected to a delta-connected load, the impedances of which are $Z_1 = Z_2 = Z_3 = (16+j12)$ ohms, (i. e. resistance = 16 ohms and inductive reactance is 12 ohms). Calculate (a) current in each impedance; (b) the line currents; (c) phase voltage of alternator, (d) the power factor and (e) the power consumed by the load.

Solution: $Z_1 = \sqrt{(16^2 + 12^2)} = 20$ ohms $= Z_2 = Z_3$

since the line voltage is 100 volts, the current in each impedance is

(*a*) $I_{ph} = \dfrac{100}{20} = 5A.$

(*b*) The line current $I = \sqrt{3} \times 5 = 8{\cdot}66\,A.$

(*c*) Alternator phase voltage $= \dfrac{100}{\sqrt{3}} = 57{\cdot}7$ *volts.*

(*d*) Power factor of the load $\cos\phi = \dfrac{R}{Z} = \dfrac{16}{20}$

$= 0{\cdot}8$ *lagging*

(*e*) Power per phase is either I^2R or $VI\cos\phi$

load phase $I = 5A$; $R = 16$ ohms; $V = 100$ volts, $\cos\phi = 0{\cdot}8$

total power consumed $= 3 \times 5^2 \times 16 = 1200$ *watts*

or „ „ „ $= 3 \times 100 \times 5 \times 0{\cdot}8$

$= 1200$ *watts.*

Example: If the three impedances of the last example are connected in star and is then connected to the same alternator calculate (a) the voltage across each impedance, (b) the current in each impedance, (c) the line currents and (d) the power consumed by the load.

Solution: Because the load is star-connected, the voltages across each impedance is $= \dfrac{\text{line voltage}}{\sqrt{3}} = \dfrac{100}{\sqrt{3}} = 57{\cdot}7$ *volts*

Therefore the current in each impedance $= \dfrac{57{\cdot}7}{20} = 2{\cdot}885$ A.

Line current = phase current = *2·885 A.*

$$\text{Load power factor} = 0{\cdot}8$$

$$\text{Power} = 3 \times 2{\cdot}885^2 \times 16 = 400 \textit{ watts}$$

$$\text{or} \qquad = 3 \times 57{\cdot}7 \times 2{\cdot}885 \times 0{\cdot}8$$

$$= 400 \textit{ watts.}$$

Note.—The power is 1/3 in the star-connected load as compared to when the load is delta connected.

Three-Phase, 4-Wire System is always used for a. c. distribution network. The phases of the alternator (or transformer) are connected in star. The star-point is usually earthed and the voltage of supply is 400/230 volts—400 volts between any two line wires and 230 volts between any line wire and the *neutral wire* which is connected to the star point. Thus if the phase voltage is V, the line voltage is $\sqrt{3}$ times V. 1-phase *Residential* (lighting) loads are connected between the neutral and line wires at 230 volts and 3-phase *Power* loads are connected across the line wires at 400 volts.

The single-phase loads are so distributed over the network that there is a fair amount of balance of load between the three phases. However it is extremely difficult to exactly balance the system When there is no balance of load the line wires carry unequal currents and the out-of-balance current is carried by the neutral wire. This out-of-balance current is the vector sum of three line currents.

11. Measurement of Power: The instrument which measures power is called a **wattmeter.** This instrument has two coils, one a current coil which is stationary, and the other the pressure coil which is movable. When currents pass in these coils, magnetic fluxes are created and the interaction of these two fluxes, deflect the pressure coil. The pointer of the instrument is at one end of the spindle on which the pressure coil turns. Fig. 34 shows a wattmeter measuring power in a single-phase load. Note that this instrument has 2 large terminals and 2 smaller terminals. The larger ones belong to the current coil and the smaller ones to the pressure coil. One terminal of each coil is marked "±". These should be connected together as shown

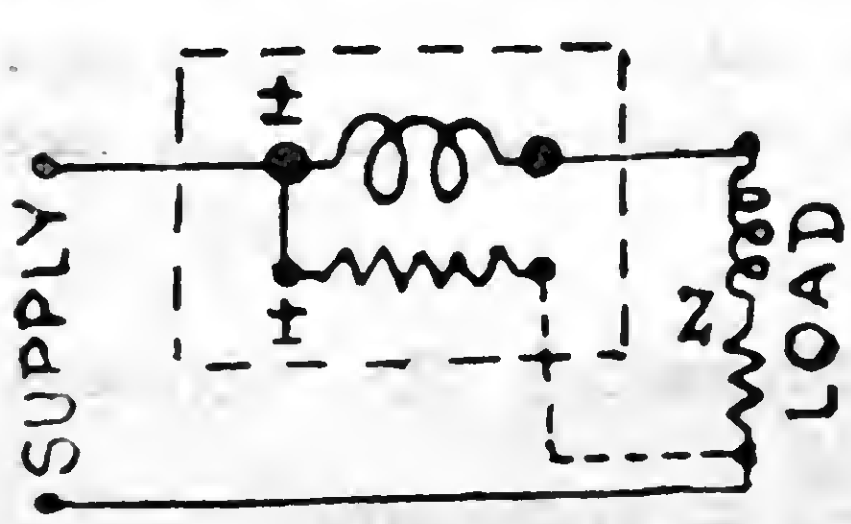


Fig. 34

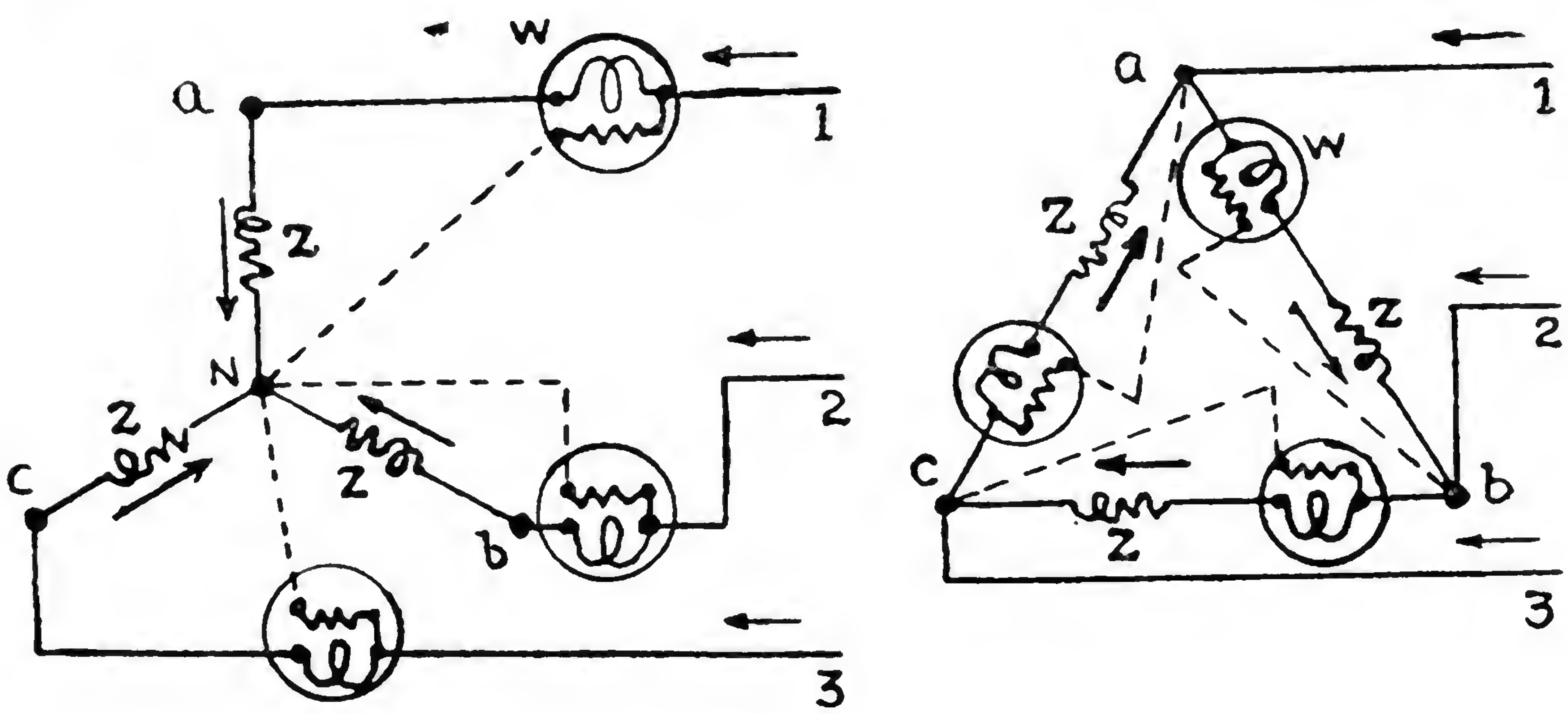
in the figure. These instruments are marked to read in watts or kilowatts. In high voltage systems it is the common practice to use *instrument transformers* in conjunction with wattmeters.

In the case of a 2-phase circuit, power must be measured by employing two wattmeters, one for each phase. The total power then is

$$P = W_1 + W_2$$

where W_1 and W_2 are the readings of the two instruments. If the loads are balanced, i. e. the impedances of the two loads are equal in every respect, then one wattmeter will suffice and the total power is twice that indicated by the instrument.

In the case of 3–phase circuits, power can be measured by 3 wattmeters, one connected in each phase as shown in Fig. 35 (a) and (*b*), provided that the star point is available if star-connected, or if mesh connected the connections can be broken to insert the



(a) Fig. 35 (b)

instruments. In both the figures the total power is the sum of the readings of the three wattmeters.

If the load is balanced, the total power, P, is

$$P = 3\ V_{ph}\ I_{ph} \cos\phi \quad \ldots \quad \ldots \quad (16)$$

where V_{ph} and I_{ph} are the phase values of voltages and currents and $\cos\phi$ is the *phase angle between the two*. In a star-connected load, phase current = line current, and phase voltage = line voltage $\div \sqrt{3}$. Therefore power in terms of line voltage and line current, is

$$P = 3\left(\frac{V}{\sqrt{3}}\right) I \cos\phi = \sqrt{3}\ VI \cos\phi \quad \ldots \quad \ldots \quad (17)$$

where V and I are the line values.

Similarly, for the delta connected load,

$$P = 3\,V\,\frac{I}{\sqrt{3}}\cos\phi$$

$$= \sqrt{3}\,VI\cos\phi, \text{ as before.}$$

However, for the commercial loads where instruments cannot be connected as shown in Fig. 35, another method is adopted and it is described in the next Section.

12. The 2–Wattmeter Method: It will be proved here that the total power in a 3-phase circuit can be measured by two wattmeters, whether the load is balanced or not.

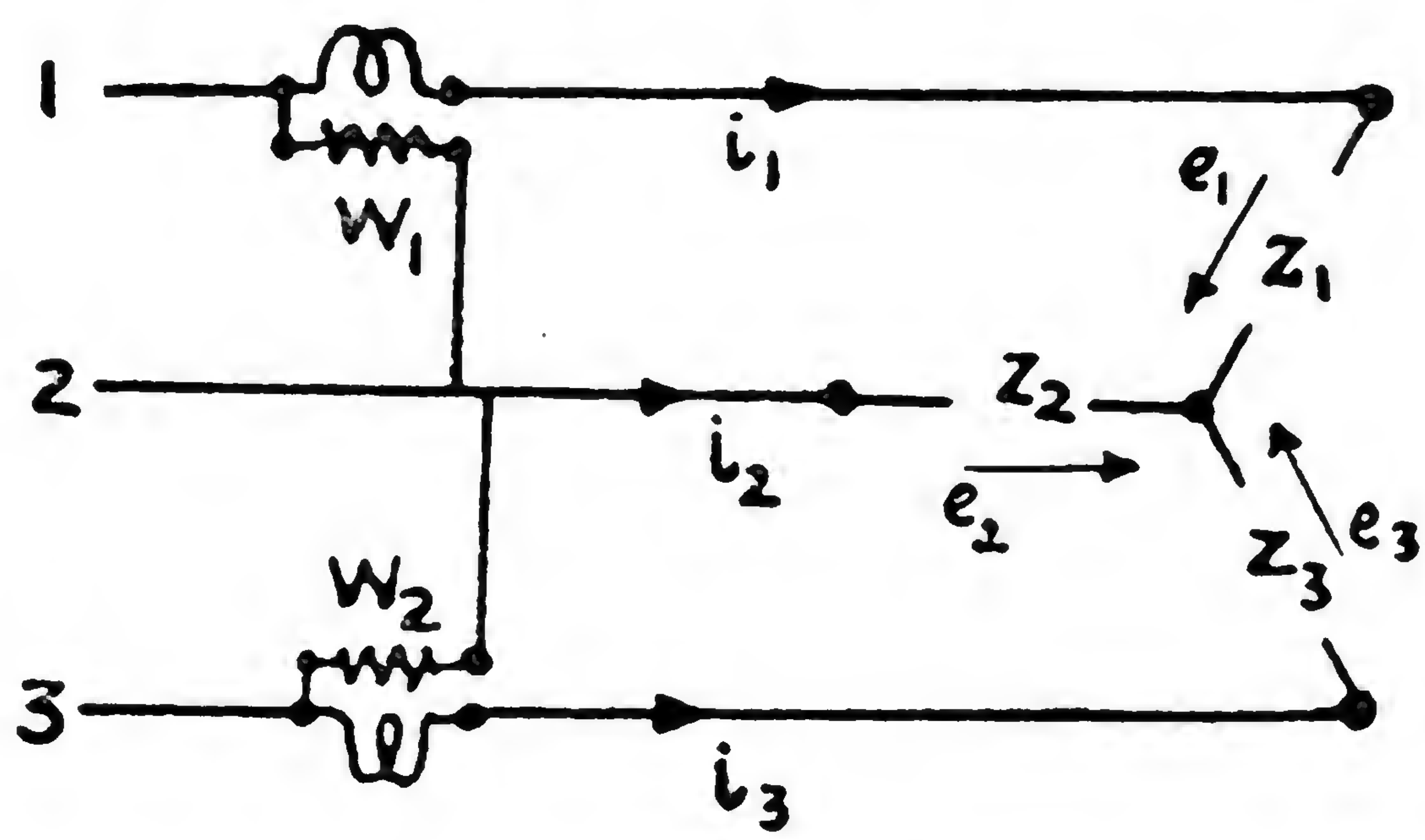


Fig. 36

Fig. 36 shows star-conneced load impedances Z_1, Z_2 and Z_3 being supplied with power from a 3-phase, 3-wire system. The current coil of wattmeter $\mathbf{W}_1$ is in line 1 and its pressure coil across lines 1 and 2. The current coil of wattmeter $\mathbf{W}_2$ is in line 3 and its pressure coil across lines 3 and 2.

Let e_1, e_2 and e_3 be the instantaneous values of voltages across the load impedances and let the instantaneous values of current in them be i_1, i_2 and i_3. The total instantaneous power is given by

$$w = e_1 i_1 + e_2 i_2 + e_3 i_3 \quad \ldots \quad \ldots \quad \ldots \quad (18)$$

Now $\mathbf{W}_1$ measure i_1 and $(e_1 - e_2)$ while $\mathbf{W}_2$ measures i_3 and $(e_3 - e_2)$. Therefore the instantaneous power measured by the two wattmeters is

$$w = i_1(e_1 - e_2) + i_3(e_3 - e_2)$$

Expanding and arranging,

$$w = e_1 i_1 + e_3 i_3 + e_2(-i_1 - i_3)$$

But since $i_1 + i_2 + i_3 = 0$, $(-i_1 - i_3) = i_2$.

Substituting the value of $(-i_1 - i_3)$

$$w = e_1 i_1 + e_3 i_3 + e_2 i_2$$

which is the total instantaneous power. Hence the sum of the readings of the two wattmaters indicates the total average power.

In the above discussion no assumption is made whether the load is balanced or not. The same result is obtained if the load is mesh connected.

It is very interesting and instructive to study the vector diagram of a *balanced load*, and to find out which of the vectors are concerned with the *two wattmeter method* of measuring power.

Fig. 36 is the connection diagram and Fig. 37 is the corresponding vector diagram. In Fig. 37 i_1 is the line current and $(e_1 - e_2)$ is the line voltage measured by wattmeter $\mathbf{W}_1$, while i_3 and $(e_3 - e_2)$ are measured by the wattmeter $\mathbf{W}_2$. Since the line currents are I and the line voltages are V, these values can be substituted for the instantaneous values. Hence the wattmeter $\mathbf{W}_1$ reads

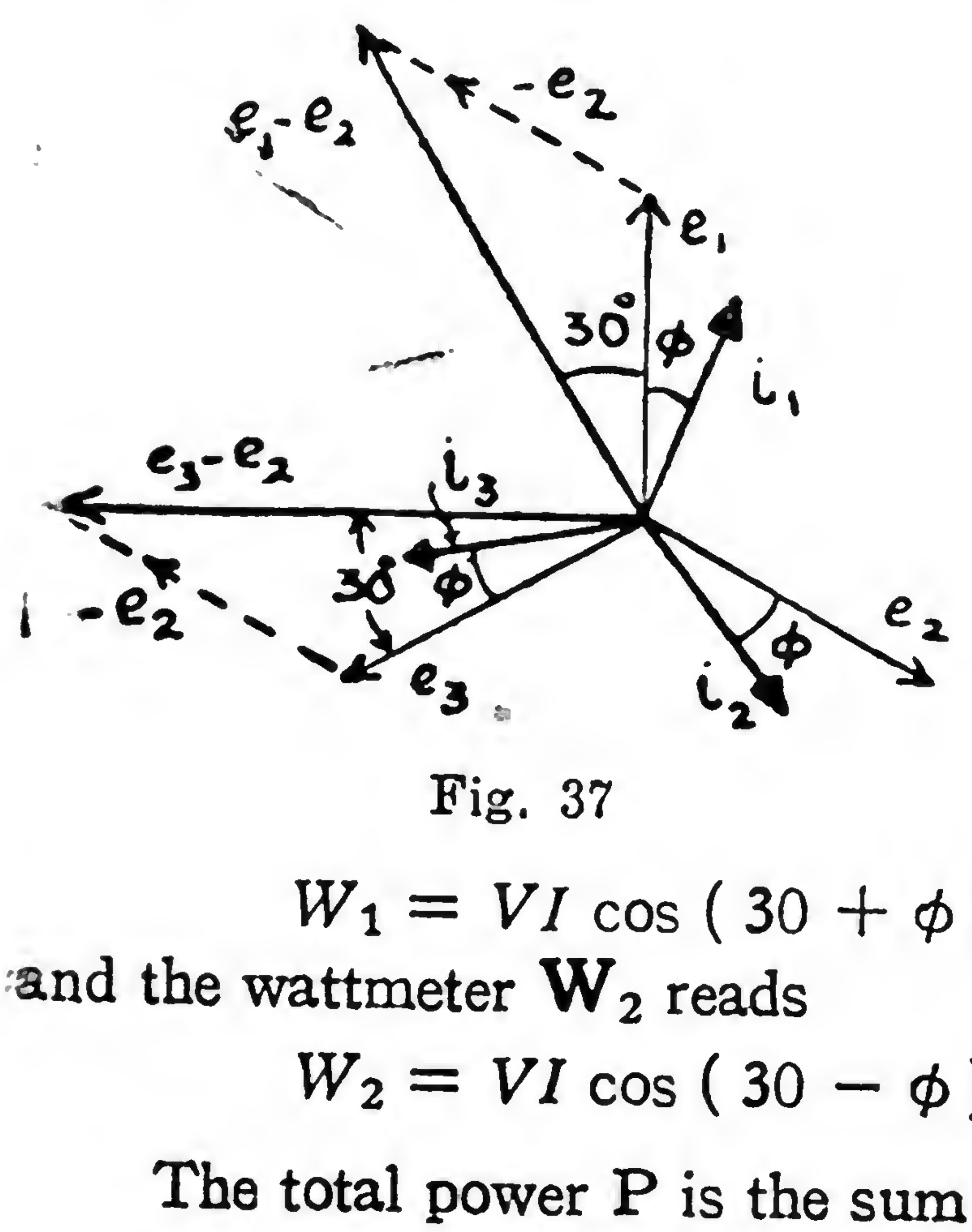


Fig. 37

$$W_1 = VI \cos(30 + \phi) \text{ watts} \quad \dots \quad \dots \quad (19)$$

and the wattmeter $\mathbf{W}_2$ reads

$$W_2 = VI \cos(30 - \phi) \text{ watts} \quad \dots \quad \dots \quad (20)$$

The total power P is the sum of the two wattmeter readings, i. e.

$$P = W_1 + W_2 = VI \; [\cos(30 + \phi) + \cos(30 - \phi)]$$
$$= VI \; [\cos 30 \cos \phi - \sin 30 \sin \phi + \cos 30 \cos \phi + \sin 30 \sin \phi]$$
$$= VI \; [2 \cos 30 \cos \phi]$$
$$P = W_1 + W_2 = \sqrt{3}\, VI \cos \phi$$

Hence power in a 3-phase circuit is given by the expression

$$P = \sqrt{3}\, VI \cos \phi \quad \dots \quad \dots \quad \dots \quad \dots \quad (21)$$

where V and I are the *line values* and $\cos \phi$ is the power factor of phases. This proves that the two wattmeters read the total power in a 3-phase load whether star or mesh connected.

Further, (a) if the angle of lag ϕ increases upto 60°, the reading of wattmeter $\mathbf{W_1}$ will go on diminishing and at p. f. = 0·5 or when $\phi = 60°$, it will read zero. For

$$W_1 = VI \cos(30° + 60°) = VI \cos 90° = 0$$

Under these circumstances, $\mathbf{W_2}$ will alone read the total power.

(b) When the power factor is unity, both the wattmeters will indicate equal power.

(c) If the angle of lag is greater than 60°, i. e. the p. f. is less than 0·5, the reading of $\mathbf{W_1}$ will be negative, while the reading of $\mathbf{W_2}$ remains positive as before. Under this condition, the connection of the pressure coil (or the current coil) must be reversed. So that the total power is

$$P = W_2 - W_1$$

This clearly shows that unless the wattmeter connections are done systematically, it is impossible to know whether the two readings are positive or otherwise.

Now $W_1 + W_2 = \sqrt{3}\ VI \cos\phi \quad \dots \quad \dots \quad \dots \quad (22)$

similarly,

$$\begin{aligned} W_2 - W_1 &= VI \cos(30° - \phi) - VI \cos(30 + \phi) \\ &= VI\ [\cos 30° \cos\phi + \sin 30° \sin\phi - \\ &\qquad \cos 30° \cos\phi + \sin 30° \sin\phi\ ] \\ &= VI\ 2 \sin 30° \sin\phi \end{aligned}$$

$$\therefore \quad W_2 - W_1 = VI \sin\phi \quad \dots \quad \dots \quad \dots \quad (23)$$

Dividing (23) by (22),

$$\frac{W_2 - W_1}{W_2 + W_1} = \frac{VI \sin\phi}{\sqrt{3}\ VI \cos\phi}$$

$$\therefore \quad \frac{W_2 - W_1}{W_2 + W_1} = \frac{\tan\phi}{\sqrt{3}} \quad \dots \quad \dots \quad \dots \quad (24)$$

It follows, therefore, that in the case of a *balanced load*, its phase angle ϕ can be determined without measuring the magnitudes of the voltage and current.

Equation (24) can be slightly altered in form and written as

$$\tan\phi = \sqrt{3}\,\frac{W_2 - W_1}{W_2 + W_1} \quad \dots \quad \dots \quad \dots \quad (25)$$

Again, let the ratio $\frac{W_2}{W_1} = K$, then from Eq. (25)

$$\tan \phi = \sqrt{3}\,\frac{1-\frac{1}{K}}{1+\frac{1}{K}} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (26)$$

or $$\tan \phi = \sqrt{3}\,\frac{K-1}{K+1} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (27)$$

Equation (26) or (27) affords a method of finding the power factor of a *balanced 3-phase load* if the ratio of the two wattmeter readings is known.

Example: The input to a 400-volt, 3-phase a. c. motor is measured by two wattmeters. W_1 reads 6·4 kW and W_2 reads 19·6 kW. Calculate (a) the total power supplied to the motor, (b) the power factor at which the motor works, and (c) the line currents.

Solution. All motor loads should be considered as balanced load.

(a) total power supplied $= 6{\cdot}4 + 19{\cdot}6 = 26\ kW$.

(b) $W_2 - W_1 = 13{\cdot}2$ kW and $W_2 + W_1 = 26$ kW.

$$\therefore \quad \tan \phi = \sqrt{3}\,\frac{W_2 - W_1}{W_2 + W_1} = \sqrt{3}\,\frac{13{\cdot}2}{26} = 0{\cdot}878.$$

$\therefore$ From Tables $\cos \phi = 0{\cdot}75$ (lag).

(c) Since power in a 3-phase balanced load is

power $P = \sqrt{3}\cdot VI\cos\phi$ (V and I are line values)

$$I = \frac{P}{\sqrt{3}\,V\cos\phi} = \frac{26 \times 1000}{\sqrt{3} \times 400 \times 0{\cdot}75} = 50\ A.$$

21. Measurement of Power without the use of a Wattmeter:

I. *The Three-Voltmeter Method.* Consider the circuit of Fig. 38 (a). X is highly inductive and is connected in series with a non-inductive resistance R. It is required to measure the power consumed by X and its power factor. An ammeter is usually connected

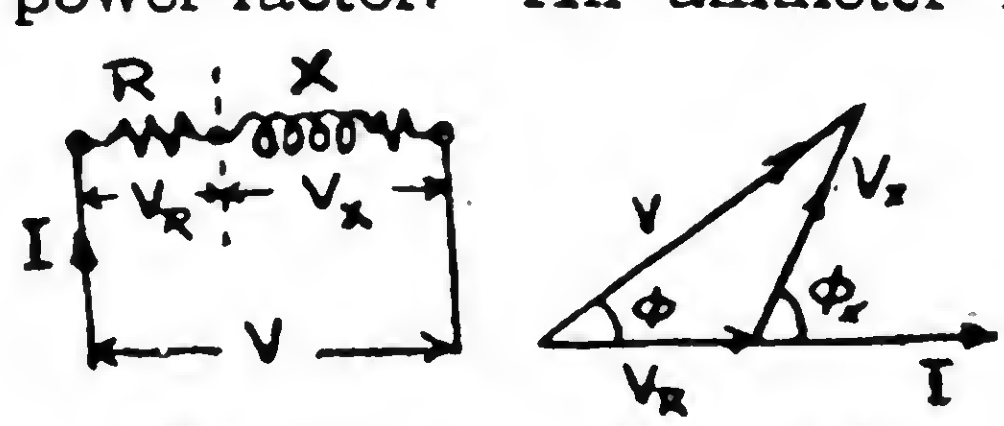


Fig. 38 (a) Fig. 38 (b)

The Three Voltmeter Method.

in the circuit and the three voltages V and V_R and V_X are measured. The vector diagram of the circuit is shown in Fig. 38 (b) where ϕx is the power factor angle of X and $\cos \phi$ is the power factor of the whole circuit.

From the geometry of Fig. 38 (b),

$$V^2 = V_R^2 + V_X^2 + 2V_R V_X \cos \phi_X.$$

Now $V_R = IR$, $\therefore$ $V^2 = V_R^2 + V_X^2 + 2R\,(V_X I \cos \phi_X)$
but $V_X I \cos \phi_X$ is the power consumed by X. Hence,

$$V_X I \cos \phi_X = \frac{V^2 - V_R^2 - V_X^2}{2R}$$

Substituting the value of $R = \dfrac{V_R}{I}$

power consumed by X, $P_X = I\left(\dfrac{V^2 - V_R^2 - V_X^2}{2\,V_R}\right)$

and $$\cos \phi_X = \frac{V^2 - V_R^2 - V_X^2}{2V_R V_X}.$$

From the above relation it can be shown that the total power consumed by the circuit is

$$\textit{total power } P = I\left(\frac{V^2 + V_R^2 - V_X^2}{2\,V_R}\right)$$

and that the total power factor, $\cos \phi = \dfrac{V^2 + V_R^2 - V_X^2}{2V\,V_R}.$

II. *The Three Ammeter Method.* Fig. 39 shows the circuit X is connected in parallel with a non-inductive resistance R. The

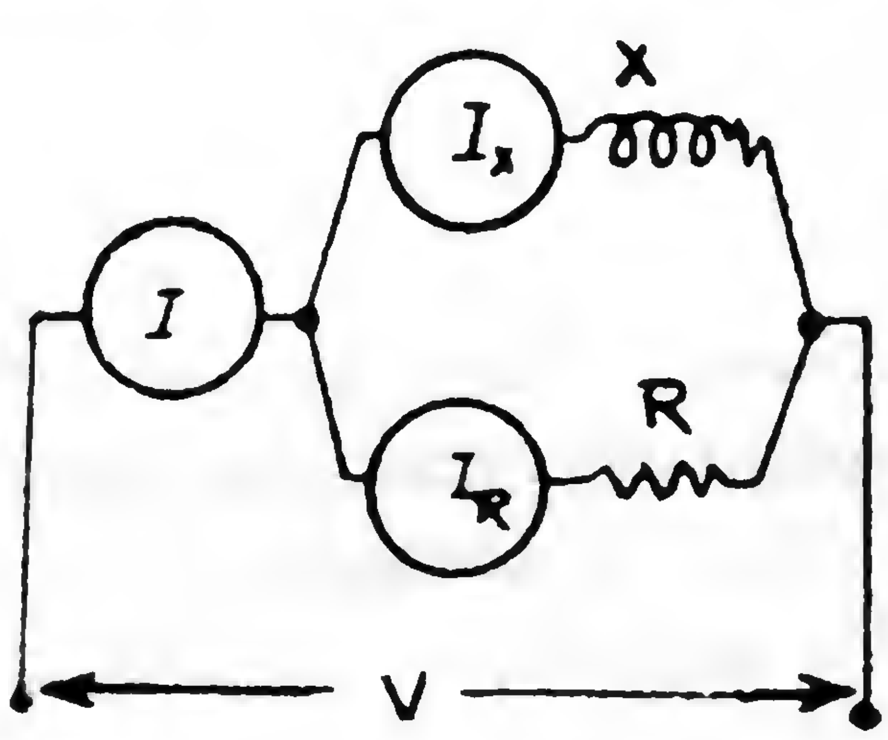


Fig. 39. The Tree Ammeter Method.

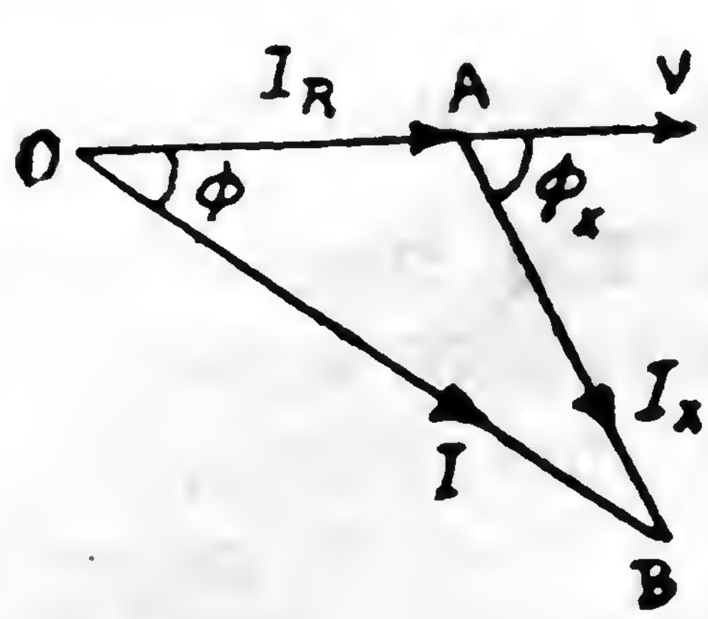


Fig. 40

vector diagram is shown in Fig. 40. By measuring the three currents I, I_R and I_X and the applied voltage V, we have from the geometry of Fig. 40

$$I^2 = I_R^2 + I_X^2 + 2\,I_R\,I_X \cos \phi_X$$

Now $I_R = \frac{V}{R}$, $\therefore \quad I^2 = I_R^2 + I_X^2 + \frac{2}{R}(VI_X \cos\phi_X)$

But $VI_X \cos\phi_X$ is the power consumed by X. Hence

$$P_X = VI_X \cos\phi = \frac{R}{2}(I^2 - I_R^2 - I_X^2)$$

$$\text{Now} \quad R = \frac{V}{I_R}$$

$$\therefore \quad P_X = V\frac{(I^2 - I_R^2 - I_X^2)}{2\,I_R}$$

$$\text{and} \quad \cos\phi_X = \left(\frac{I^2 - I_R^2 - I_X^2}{2\,I_R\,I_X}\right)$$

Example: A choke coil is connected in series with a non-inductive resistance of 5 ohm. The combination is connected across a 100 volts 50 cycle supply. If the voltage across the resistance is 40 volts and across the choke coil 80 volts, calculate (a) the current in the circuit, (b) power consumed by the choke coil, and (c) the total power factor of the circuit.

Solution: (a) current in the circuit $I = \frac{V_R}{R} = \frac{40}{5} = 8\,A.$

(b) $P_X = I\frac{V^2 - V_R - V_X^2}{2\,V_R} = 8\frac{100^2 - 40^2 - 80^2}{2 \times 40} = 200$ watts

(c) Power consumed by resistance $= I^2R = 64 \times 5 = 320$ watts
Total power consumed $= 200 + 320 = 520$ watts
Now total power $= V \cdot I \cos\phi$

$$\therefore \quad \cos\phi = \frac{\textit{total power}}{VI} = \frac{520}{100 \times 8} = 0{\cdot}65 \textit{ lag.}$$

CHAPTER IX

THE TRANSFORMER

1. The Single-Phase Transformer : A transformer converts electrical energy from one voltage into another voltage. And since it has no rotating parts it is called *static transformer* and the conversion of energy is at a high efficiency.

A single-phase transformer consists essentially of two coils wound on a closed magnetic circuit of low reluctance. The magnetic path is made up of *sheet steel laminations*. These coils therefore, have high mutual inductance. The coil which is connected to a source of supply voltage is called the *primary* and the other, which supplies electrical energy to an external load, is called the *secondary*.

These windings are usually former wound in sections and are so arranged on the two limbs of a core-type frame that half the number of sections of each winding is on one limb and the remaining sections on the other limb. The low voltage (*l. v.*) winding is next to the core and the high voltage (*h. v.*) winding surrounds the *l. v.* winding. There is however, a small gap between the two windings.

Fig. 1 shows, for oonvenience, the primary of a 1-phase transformer on one limb and the secondary on the other limb. With the secondary on open circuit, the primary, which has a high inductance, takes a small current I_0 lagging behind the applied voltage by a fairly large angle ϕ_0. I_0 consists of two components.

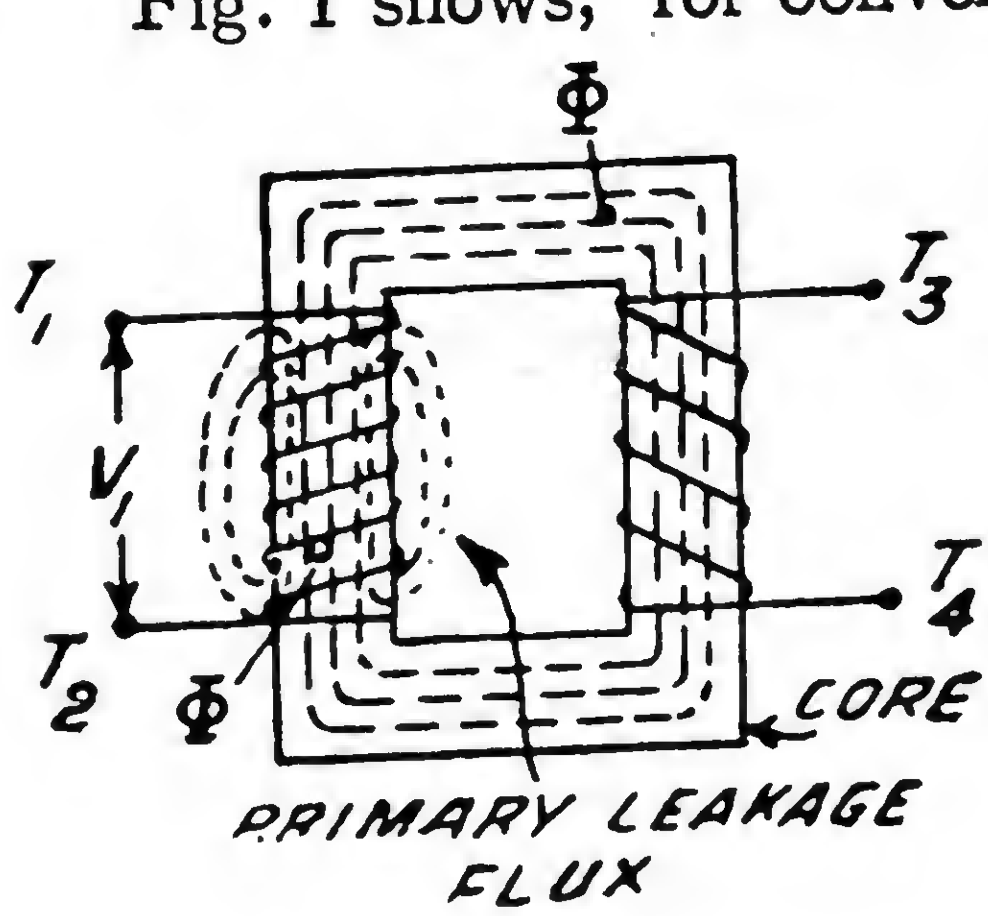


Fig. 1

(1) $I_0 \sin \phi_0$ the wattless magnetising component which produces the magnetic flux Φ. This flux induces an e. m. f. of self-induction E_1 in the primary. This component is the true magnetising current I_m.

(2) $I_0 \cos \phi_0$ is in phase with the applied voltage. This component is the iron loss current (I_i). There are hysteresis and eddy current losses.

The phase relationship between the above quantities is indicated in Fig. 2. I_0 is a small fraction of the full load current of the primary and I_i is small compared to I_0 or I_m. E_1 is very slightly less than V_1 the applied voltage and is also in phase opposition to V_1.

If the terminals of the secondary are connected to a load having a power factor of $\cos \phi_2$, a current I_2 flows through it due to the induced e. m. f. E_2 in it. E_2 is induced by the same flux. The effect of I_2, according to Lenz's Law, is to reduce the mutual flux. But the reduction in Φ causes reduction of E_1, so that, momentarily, V_1 sends more current through the primary until this increased current restores the original value of Φ. E_1 then increases and is again very slightly less than V_1. All this happens instantaneously and automatically. This action is known as the *balancing of ampere-turns* of both sides. For instance, if the primary and the secondary turns are 500 and 200 respectively and if the secondary supplies a current of 10 amperes, the extra current that the primary will draw from the supply is

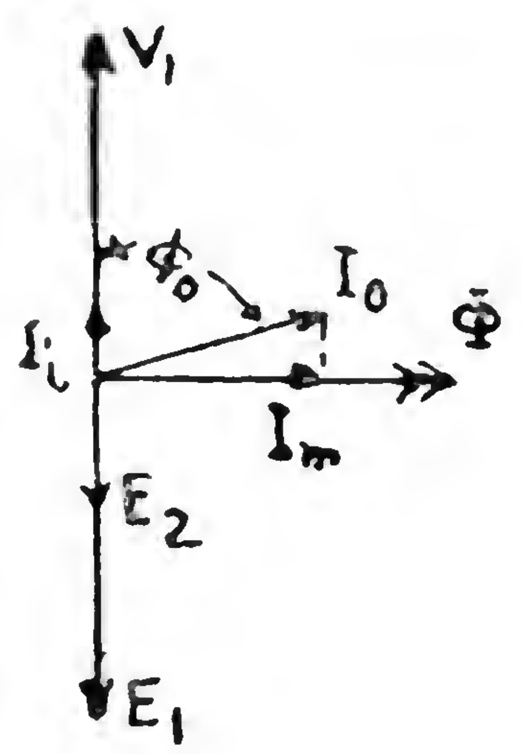


Fig. 2

$$I_1' = 500 \times \frac{200}{10} = 4 \text{ amperes.}$$

So that the current in the primary is the vector sum of I_0 and I'_1.

2. The E. M. F. Equation of a Transformer : Let

f = frequency of the supply voltage in cycles per second,

T_1 = number of primary turns

T_2 = ,, ,, secondary ,,

Φ_{max} = maximum number of lines in the magnetic flux.

$= B \times a$

B = maximum flux density

a = cross-sectional area in sq. cm. of the core.

In $\frac{1}{4}$ cycle the flux change from 0 to Φ_{max} or from Φ_{max} to zero i. e. in time $\frac{1}{4f}$ second. Therefore the average rate of change of flux is $\Phi_{max} \Big/ \frac{1}{4f} = 4\Phi_{max} f$ lines per second. Hence

average e. m. f. induced per turn $= \dfrac{d\,\Phi_{max}}{dt} \times 10^{-8}$ volts.

$= 4\,\Phi_{max} f \times 10^{-8}$ volts.

If there are T_1 turns

average e. m. f. induced in primary $= 4\,\Phi_{max} f \times T_1 \times 10^{-8}$ volts

Since $\dfrac{r.\,m.\,s.}{average}$ for a since wave is 1·11

r. m. s. value of induced e. m. f. in primary

$= 1{\cdot}11 \times 4\,\Phi_{max} \times f \times T_1 \times 10^{-8}$ volts.

$\therefore\quad E_1 = 4{\cdot}44\,\Phi_{max} f\, T_1\, 10^{-8}$ volts ... ... (1)

similarly, $E_2 = 4{\cdot}44\,\Phi_{max} f\, T_2\, 10^{-8}$ volts ... ... (2)

from Eqs. (1) and (2),

$$\frac{E_1}{T_1} = \frac{E_2}{T_2} = \textit{volts induced per turn.}$$

Thus we arrive at the following relationships :—

$$\frac{E_1}{E_2} = \frac{T_1}{T_2} = \frac{V_1}{V_2} = n, \text{ the } \textit{ratio of transformation.}$$

The usual values of B_{max} for transformer cores lie between 9000 and 14000 lines per sq. cm.

Example : A 1-phase, 220 volt, 50 cycle transformer has 20 turns on its primary and 275 on its secondary. Its core sectional area is 440 cm². Find (i) voltage induced in the secondary and (ii) the maximum flux density in the core.

Solution : (i) Using $\dfrac{E_1}{T_1} = \dfrac{E_2}{T_2}$,

$$E_2 = E_1 \frac{T_2}{T_1} = 220 \times \frac{275}{20} = 3025 \textit{ volts}$$

(ii) $\Phi_{max} = B_{max} \times area$

$$\Phi_{max} = \frac{E_1 \times 10^8}{4{\cdot}44 \times T_1 \times f} = \frac{220 \times 10^8}{4{\cdot}44 \times 20 \times 50}$$

$$\therefore\quad B_{max} = \frac{\Phi_{max}}{area} = \frac{220 \times 10^8}{4{\cdot}44 \times 20 \times 50} \times \frac{1}{440} = 11260 \textit{ lines/cm.}^2$$

3. Leakage Reactance : It has been already mentioned that the current in either winding produces flux in the core. But there is only

one resultant useful flux when the transformer is on load. This flux is due to magnetising current I_m of the primary. However, there is a primary leakage flux surrounding the primary winding as shown in Fig. 1 and when the secondary is loaded there exists a secondary leakage flux surrounding the secondary winding. These leakage fluxes produce in their respective windings an e. m. f. of self-induction, which is proportional to the current. This is therefore equivalent to an inductance placed in series with each winding, the reactance of which is called *leakage reactance.*

The effect of leakage reactance is to cause a volt drop at the secondary terminals. In practice, leakage reactance is reduced considerably by

(a) placing the windings on both limbs of the core and

(b) sub-dividing and interleaving the sections of the windings.

4. Transformer on Load: Each winding of a transformer possesses resistance and reactance. So that V_1 is really greater than E_1, and V_2, the secondary terminal voltage, is less than E_2, the induced e. m. f. in the secondary. Fig. 3 shows the complete vector diagram of a transformer on load. In order to make the figure easy to understand, the relative values of voltages and currents are drawn in such a manner that voltage vectors represent "volts per turn" and the current vectors represent "ampere-turns". Thus the figure can be used for any transformation ratio, or what amounts to the same thing that the transformation ratio is 1 : 1.

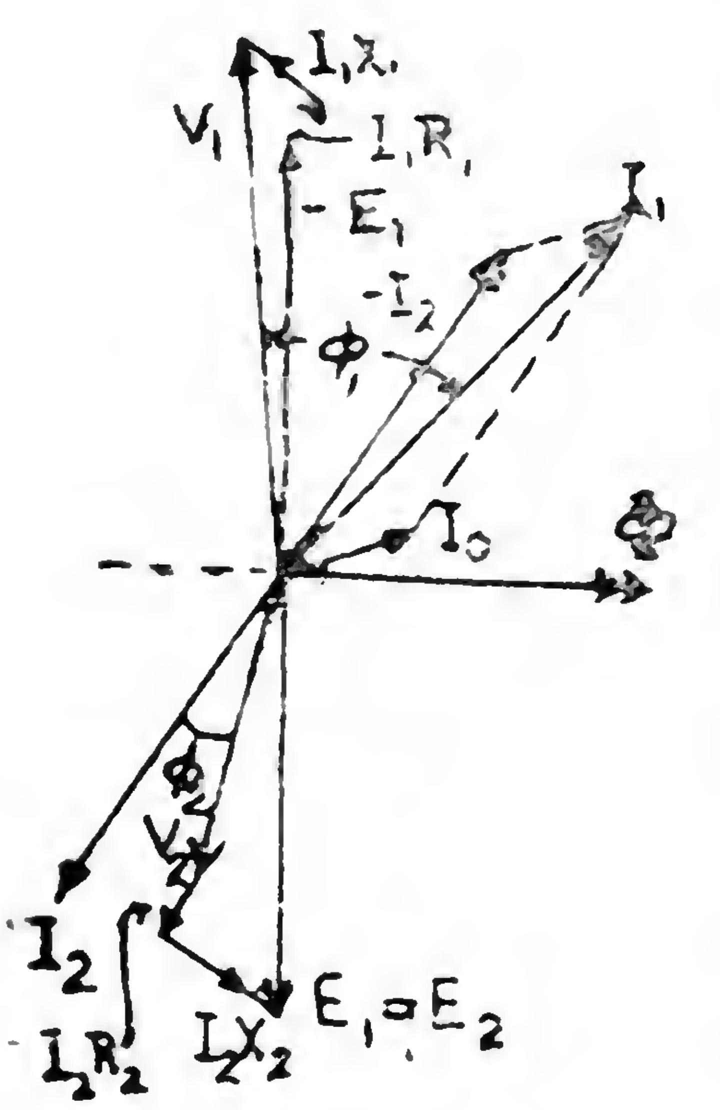


Fig. 3

Φ is taken along the x–axis, E_2 (induced e. m. f. in secondary) is at 90° lagging to Φ. $E_1 = E_2$ (assumed by the scales used). $-E_1$ as shown is balanced or opposed by a part of applied voltages V_1. V_1 is the vector sum $-E_1$, $I_1 R_1$ and $I_1 X_1$ where $I_1 R_1$ is the resistance drop of primary and $I_1 X_1$ reactance drop of primary. Or

$$V_1 - (I_1 R_1 + I_1 X_1) = - E_1 \quad \ldots \quad \ldots \quad \text{(vectorially)}$$

I_2 is the secondary load current,

$-I_2$ is the primary balancing current, $= I'_1$

$I_1 = (-I_2 + I_0)$ ... (vectorially), is the primary current

$V_2 = E_2 - (I_2 R_2 + I_2 X_2)$ ... (vectorially).

The power factor of the load is $\cos \phi_2$, and the primary side power factor is $\cos \phi_1$.

Note that $I_1 R_1$ is in phase with I_1 and $I_2 R_2$ in phase with I_2. And $I_1 X_1$ is at 90° leading $I_1 R_1$ or I_1, $I_2 X_2$ is at 90° leading $I_2 R_2$ or I_2. V_2 is the terminal voltage on the load side.

5. Losses and Efficiency: The losses in a transformer consist of (*a*) the iron losses and (*b*) the copper losses. There are no mechanical losses such as friction, windage and bearing losses. Hence the efficiency of a transformer is higher than that of a rotating machine of equal rating.

The **iron losses** consist of (i) hysteresis and (ii) eddy current losses. Since the frequency is constant and the flux in the core is also constant and independent of load, the iron losses are assumed to be constant at all loads. By using high grade silicon steel stampings the iron losses are kept at a low value. The iron losses of a transformer can either be calculated from the design data or by the open-circuit test.

The Open-Circuit Test: Fig. 4 shows the diagram of connections for this test. One winding, preferably the low voltage winding, is connected to the supply of rated voltage and frequency. The other winding is on open circuit. The current taken is I_0 and is very small. Under these conditions normal magnetic flux is set up in the core and therefore the wattmeter records normal iron losses. The copper loss is negligibly small since the current is very small.

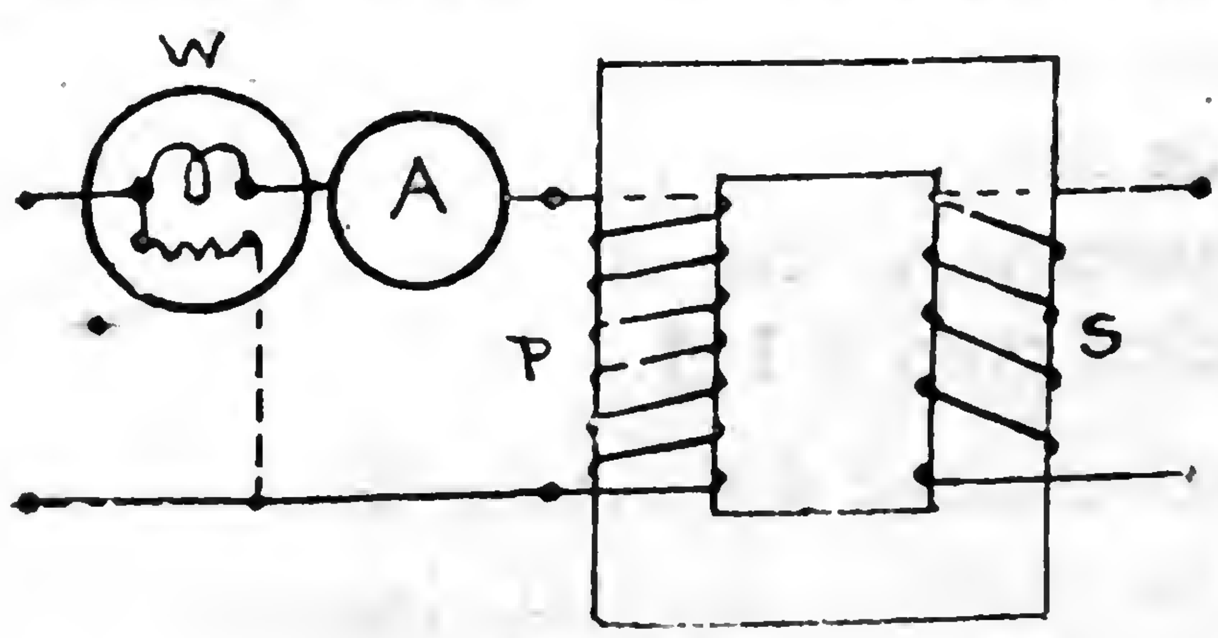


Fig. 4

Hysteresis loss varies as $B^{1 \cdot 6} f$, where B is the flux density and f is the frequency of supply voltage. The eddy current loss varies as $B^2 f^2 t^2$, where t is the thickness of the laminations.

The **copper losses** are due to the resistance of the two windings and they vary as the square of the current. These losses can be accurately obtained either from the design data or by means of the short-circuit test.

The Short-Circuit Test. Fig. 5 shows the diagram of connections for the test. The low tension winding is usually short-circuited

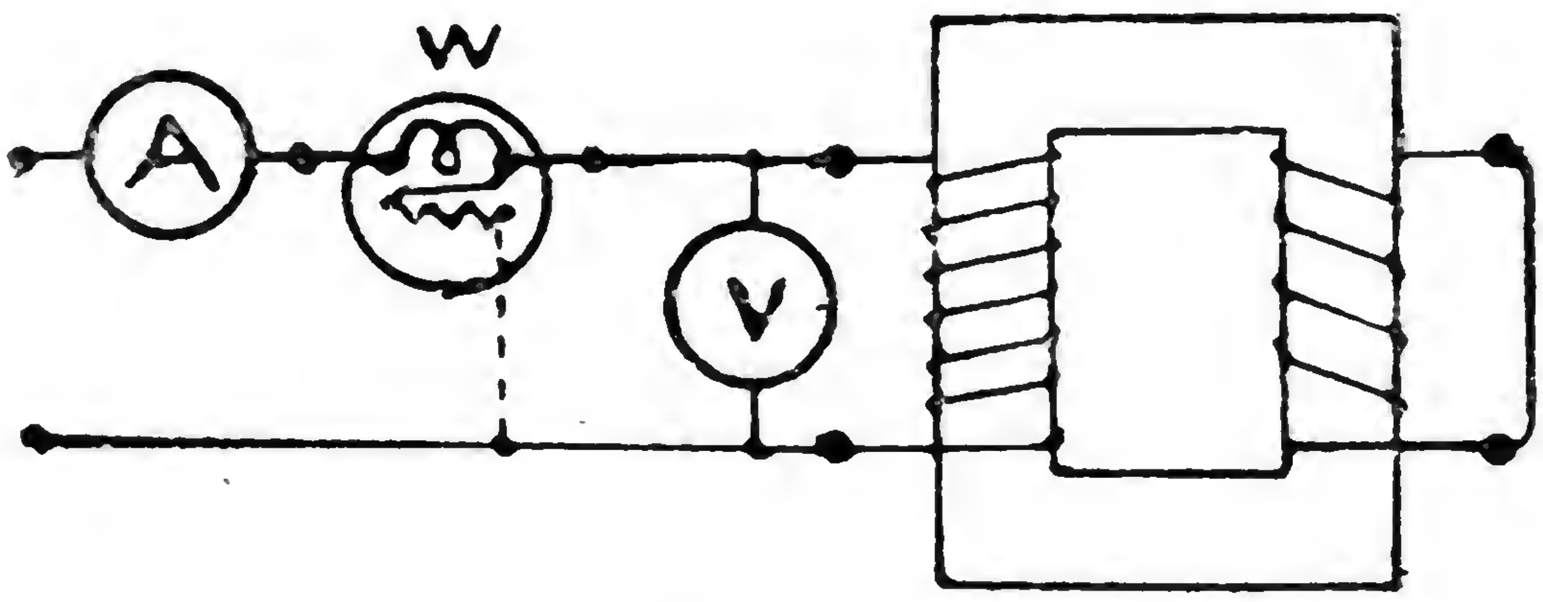


Fig. 5

by connecting its terminals with a wire of large cross-sectional area so that the resistance of this wire is negligible compared to that of the winding.

A very low voltage is applied to the other winding at the start, say 1% of the normal voltage. The voltage is gradually increased until the ammeter shows the full load current of the winding. The wattmeter will then record the copper losses of both the windings. The iron losses are negligible at this low induction.

The **efficiency** of a transformer then can be determined from the readings of the above mentioned tests. Thus

$$efficiency = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + \text{cu. loss} + \text{fe. loss}} = \left(\frac{output}{output + losses}\right)$$

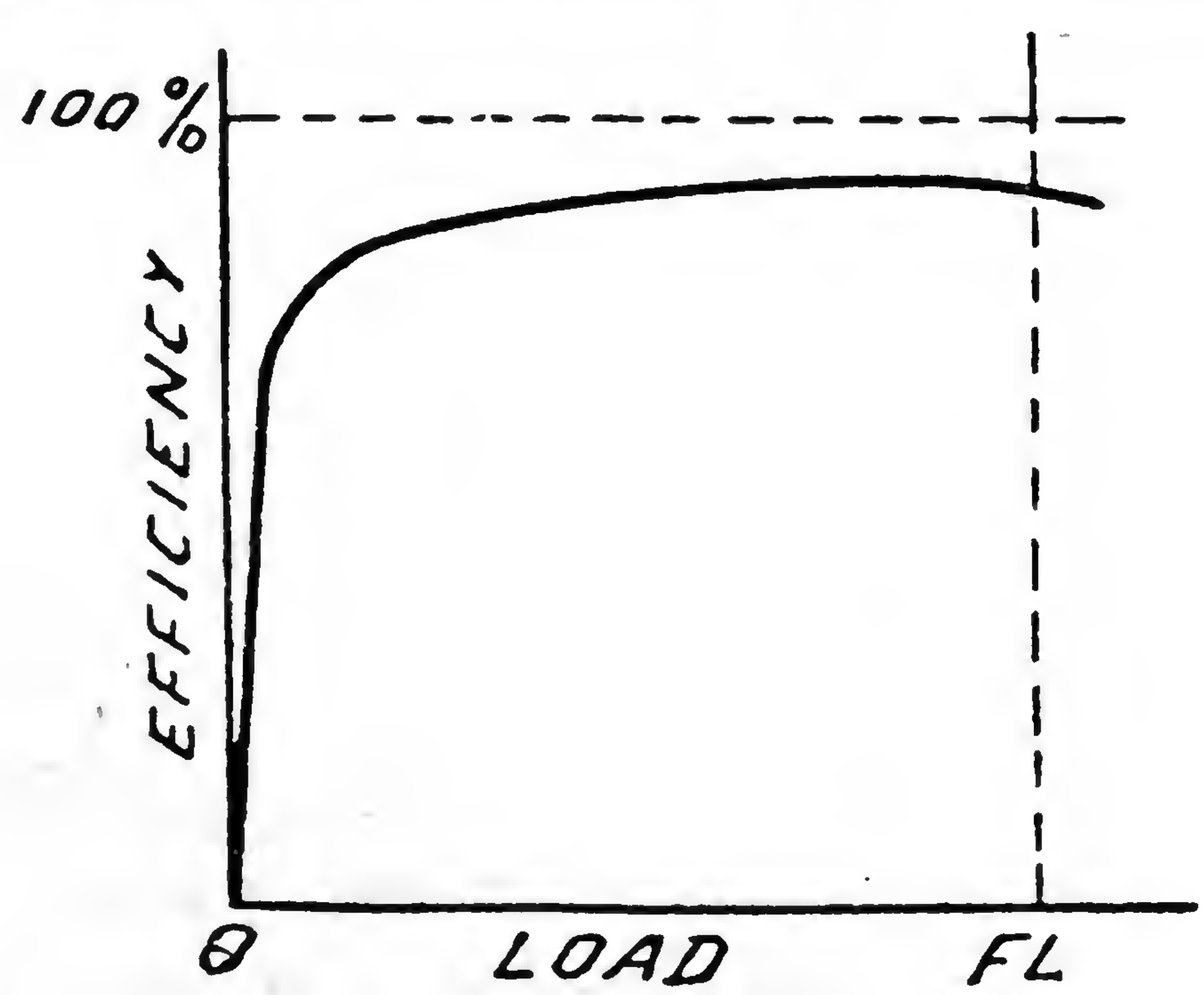


Fig. 6

The copper losses vary as the square of the current. Thus if W_c is the loss at full load, the copper losses at half load are ($\frac{1}{4} W_c$). At $\frac{1}{4}$ load copper losses are ($\frac{1}{16} W_c$) and so on. Fig. 6 shows the efficiency curve of a medium sized transformer. The rating of transformers is always in kVA rather than in kW. The efficiency suffers if the power factor at which the transformer works is low.

Maximum efficiency occurs when the constant losses are equal to variable losses. Putting the equation for efficiency and differentiating with respect to I and equating the result to zero we can obtain the required result [copper loss = $I^2 R$ and W_i = iron loss].

$$\eta = \frac{V I \cos\phi}{V I \cos\phi + I^2 R + W_i}$$

$$\frac{d\eta}{dI} = \frac{V \cos\phi\,(V I \cos\phi + I^2 R + W_i) - V I \cos\phi\,(V \cos\phi + 2IR)}{(V I \cos\phi + I^2 R + W_i)^2} = 0$$

from which $V I \cos\phi + I^2 R + W_i = V I \cos\phi + 2I^2 R$

$$\therefore \quad W_i = I^2 R \quad Q.\,E.\,D$$

Example: From the test readings on a 5 kVA, 500/250 volt, 50 cycle, 1-phase transformer, calculate its efficiency for full load at (i) unity power factor and (ii) 0·7 p. f. lagging.

Also calculate at what load at unity power factor the maximum efficiency occurs and the value of this efficiency.

O. C. Test: 500 V: 1 A; 75 watts. (*h. v. side*)

S. C. Test: 15 V; 8 A; 80 watts. (*h. v. side*).

Solution: Full load current (*h. v. side*) $= \frac{5000}{500} = 10$ A.

$\therefore$ full load copper losses $= 80 \times \frac{10^2}{8^2} = 125$ watts

iron losses = 75 watts (constant at all loads)

$\therefore$ total losses on full load = 75 + 125 = 200 watts

(i) % efficiency at u. p. f. $= \frac{5000}{5000 + 200} = 96{\cdot}15\ \%$

(ii) % efficiency at 0·7 p. f. $= \frac{5000 \times 0{\cdot}7}{5000 \times 0{\cdot}7 + 200} = 94{\cdot}6\ \%$

Load for maximum efficiency is calculated as follows:—Let I be the primary load current at which the maximum efficiency occurs. At this load the copper losses are 75 watts (i. e. equal to the iron

losses). Since copper losses vary as the square of the current we have

$$R = \frac{80}{8^2} = \frac{80}{64}\ \Omega$$

$$\therefore\ I^2 \times \frac{80}{64} = 75 \quad \therefore\ I = \sqrt{\frac{75 \times 64}{80}} = 7{\cdot}75A.$$

The **all-day efficiency** of a distributing transformer is given by

$$\%\ efficiency = \frac{\text{kilo-watt-hours output}}{\text{kilo-watt-hours input}} \times 100.$$

Example: A 50 kVA distribution transformer has no load losses equal to 900 watts and full load losses equal to 2500 watts. Calculate the all-day efficiency of this transformer if it has to supply during 24 hours of the day the following loads:—

(i) full load at 0·8 p. f. for 2 hours
(ii) 3/4th full load at 0·8 p. f. for 8 hours
(iii) 1/2 full load at 0·75 p. f. for 10 hours
(iv) no load ... ... for 4 hours.

Solution: Energy loss due to iron losses in 24 hrs.

$$= \frac{24 \times 900}{1000} = 21{\cdot}6 \text{ kw-hr.}$$

Full load copper losses = 2500 − 900 = 1600 watts

Output kw-hr in 24 hrs.

$$= 50 \times 0{\cdot}8 \times 2 + \tfrac{3}{4} \times 50 \times 0{\cdot}8 \times 8 + \tfrac{1}{2} \times 50 \times 0{\cdot}75 \times 10$$
$$= 507{\cdot}5 \text{ kw-hr.}$$

Energy loss due to copper losses in 24 hrs.

$$= 2 \times 1{\cdot}6 + 8 \times 1{\cdot}6 \times (\tfrac{3}{4})^2 + 10 \times 1{\cdot}6 \times (\tfrac{1}{2})^2$$
$$= 3{\cdot}2 + 7{\cdot}2 + 4 = 14{\cdot}4 \text{ kw-hr.}$$

Total energy losses = 21·6 + 14·4 = 36 kw-hr.

∴ total input in kw-hr

= output + losses = 507·5 + 36 = 543·5 kw-hr.

$$\therefore\ \%\ all\text{-}day\ efficiency = \frac{507{\cdot}5}{543{\cdot}5} \times 100 = 93{\cdot}2\%.$$

6. Regulation of a Transformer: Regulation of a transformer is usually expressed as a percentage difference between the no load voltage and full load voltage at the secondary terminals.

$$\%\ Regulation = \frac{no\ load\ voltage\ (V_0) - full\ load\ voltage\ (V)}{V_0} \times 100.$$

The fall in voltage is due to resistance and reactance of windings.

Typical values of voltage regulation for a medium sized transformer at 0·9 p. f. lagging is from 1 – 1·5%. In smaller transformers it is about 3%.

The Kapp diagram, Fig. 7, shows the graphical method of determining regulation of a transformer at any power factor, leading or lagging. The only disadvantage this method has, is that to get very accurate results, the diagram has to be drawn to a very large scale owing to the small values of drops as compared to the no load and full load voltages.

In Fig. 7 (a) the full load current I is taken along the x–axis for all power factors. OI = full load current. OA_1 terminal voltage

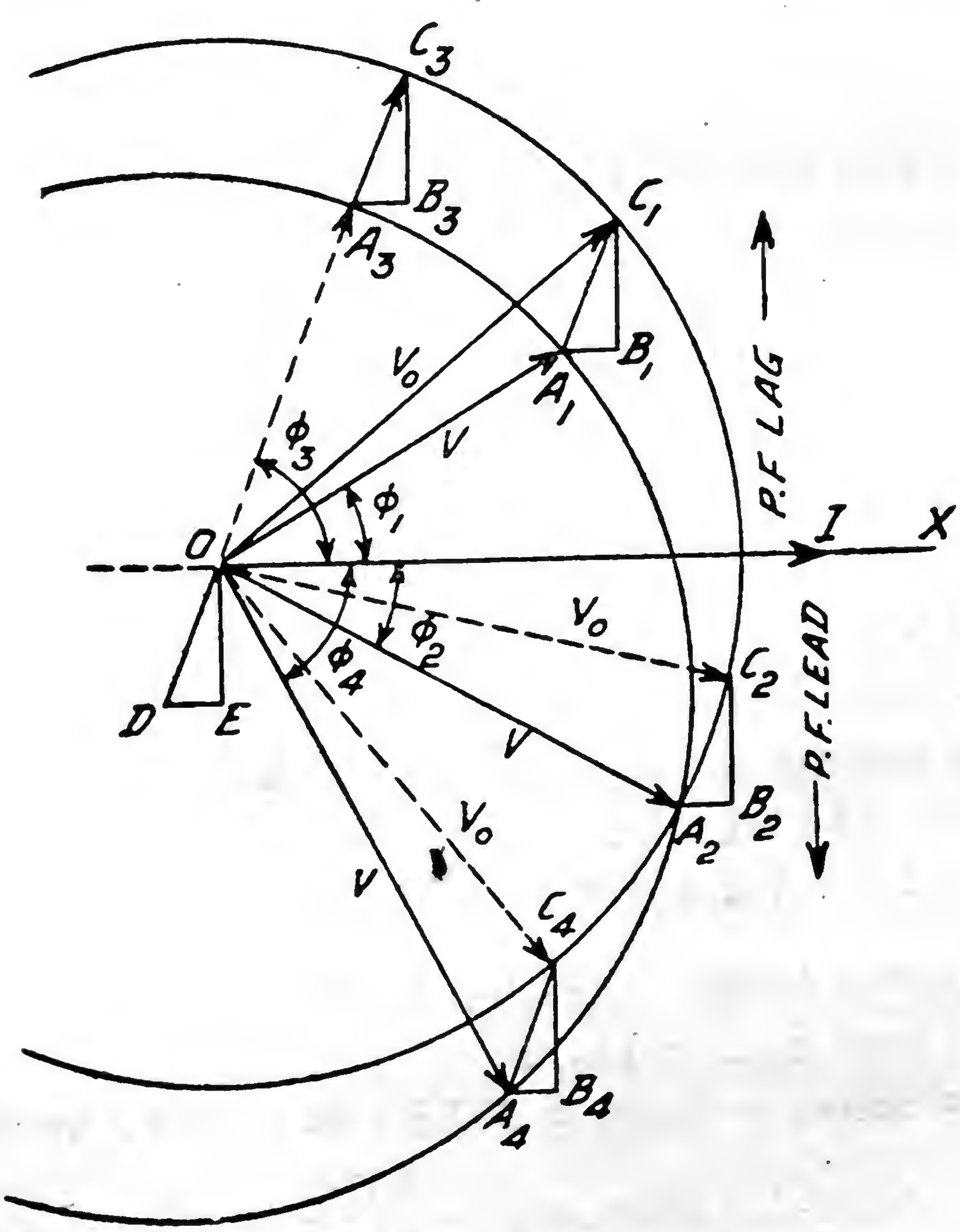


Fig. 7 (a)

on full load at p. f. ϕ_1 lagging. $A_1 B_1 = IR$, the drop in the windings due to resistance; $B_1 C_1 = IX$, the drop in the windings due to reactance and at right angles to $A_1 B_1$. Then OC_1 is the no load voltage. With centre at O and radius OC_1 a circle is drawn. Drawing a triangle DEO at O, equal and parallel to $A_1B_1C_1$, a circle is drawn with the radius = OC_1 and centre at D.

Notice that at cos ϕ_2 (leading) the regulation is zero. At cos ϕ_3 lagging p.f. the regulation is maximum. The regulation value becomes negative if the angle ϕ_2 goes on increasing when V is greater than V_0. This is shown when the power factor is cos ϕ_4.

Fig. 7 (b) shows the vector diagram in which the terminal voltage

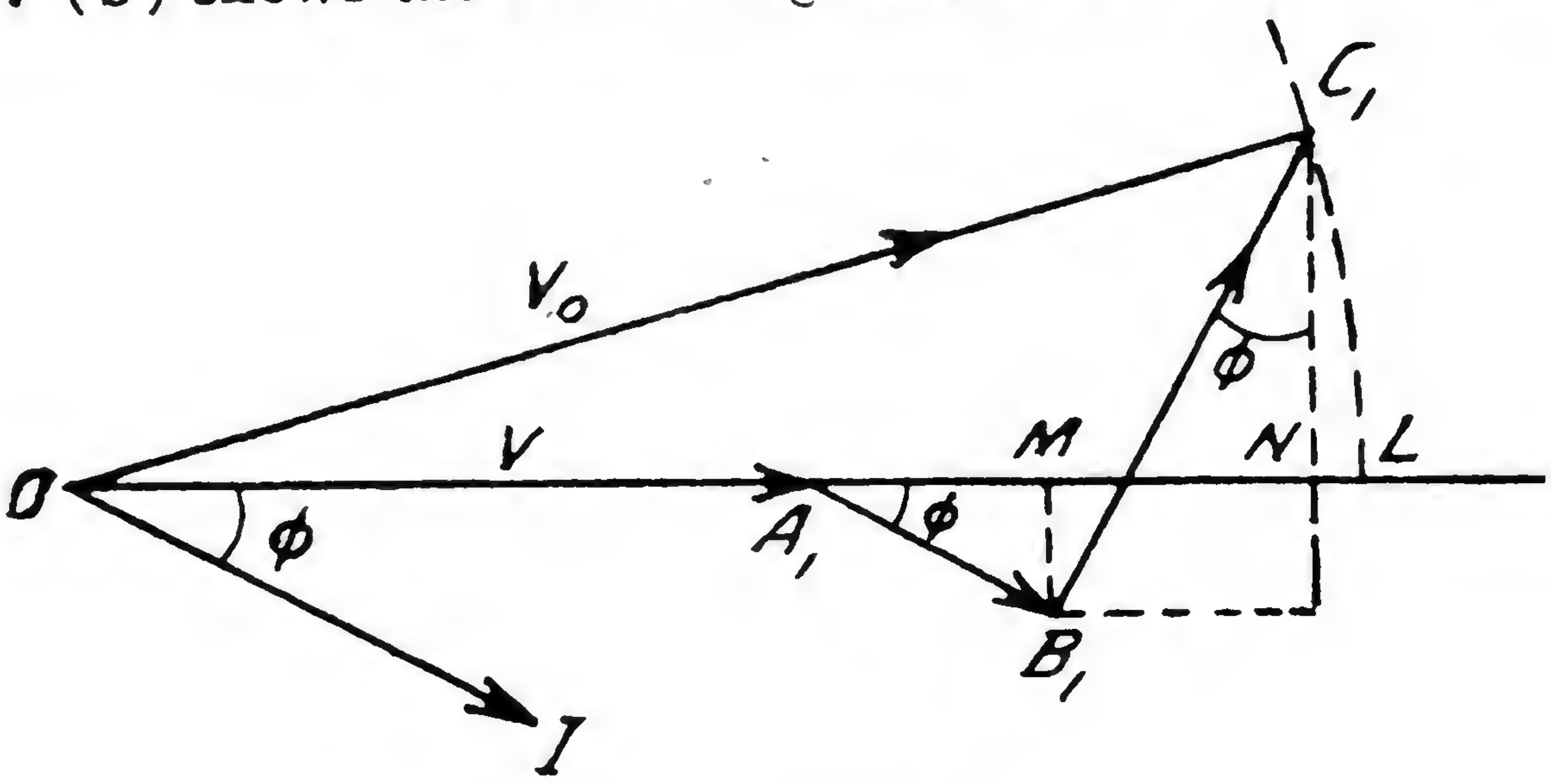


Fig. 7 (b)

on load, V, is taken along the x–axis instead of the current as in Fig. 7 (a).

As before $OA_1 = V$ the voltage on load; OI = the load current lagging V by an angle ϕ; $A_1B_1 = IR$ drop; $B_1C_1 = IX$ drop and $OC_1 = V_0$, the no load voltage. The total voltage drop is

$$\textit{total voltage drop} = OC_1 - OA_1 = OL - OA_1 = A_1L$$

where $\quad OL = OC_1$

Dropping a perpendicular from B_1 at M and from C_1 at N, we have

$$A_1M = A_1B_1 \cos\phi = IR \cos\phi$$

$$MN = B_1C_1 \sin\phi = IX \sin\phi$$

$$\therefore \quad A_1N = IR\cos\phi + IX\sin\phi$$

and A_1N is almost equal to A_1L

Therefore the total voltage drop in a transformer is

$$\textit{total voltage drop} = IR\cos\phi + IX\sin\phi \text{ (approx.)} \quad \ldots \quad (3)$$

7. The Equivalent Circuit of a Transformer: Determination of regulation, or the secondary terminal voltage on load, of a transformer is simplified by the use of the equivalent circuit. For this purpose it is convenient to consider the resistance and the leakage reactance of the two windings as being external to the windings. Further, the no–load current I_0 is assumed to be passing through an

imaginary shunt Z_0, consisting of a resistance R_0 in parallel with a reactance X_0. The two components of I_0 therefore pass through this circuit, I_i through R_0 and I_m through X_0. Thus $I_0 = \sqrt{I_i^2 + I_m^2}$.

By the above reasoning the two windings of the transformer become *ideal,* i. e. there are neither any losses nor any voltage drops in the windings themselves. The windings now only serve the purpose of changing the value of induced e. m. f. and current in accordance with the ratio of turns. Thus Fig. 8 is the *actual circuit diagram* of a transformer modified according to above assumptions.

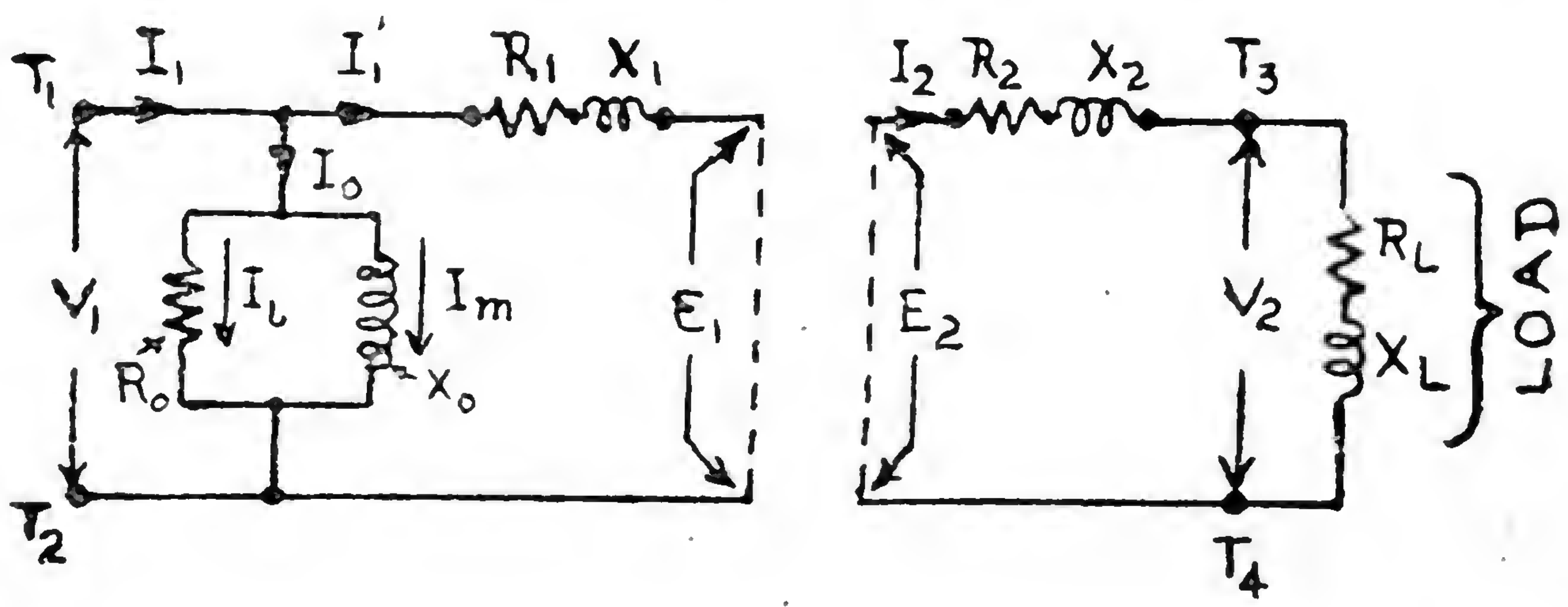


Fig. 8

In the above diagram, the two windings are shown dotted. Placing the shunt impedance at the beginning of the circuit is justifiable, since it does not affect the voltages across the windings, and the difference between I_1 and I'_1 is very small and sometimes ignored, so that, the shunt impedance may be ignored and is omitted from the equivalent circuit of the transformer.

The various quantities of the transformer as shown in Fig. 8 are listed below :—

V_1 applied voltage, (assumed constant),

E_1 induced voltage in the primary,

E_2 induced voltage in the secondary, $= E_1 \left(\frac{T_2}{T_1}\right)$,

V_2 secondary terminal voltage on load,

I_1 current taken from the supply,

I_0 no-load current (constant),

I_m magnetising current (constant) $= \frac{V_1}{X_0}$,

I_i iron loss current (constant) $= \frac{V_1}{R_0}$,

I_2 secondary load current,

I'_1 primary balancing current $= I_2 \left(\frac{T_2}{T_1}\right)$,

R_1 resistance of primary winding,

X_1 leakage reactance of primary winding,

R_2 resistance of secondary winding,

X_2 leakage reactance of secondary winding,

R_0 shunt resistance,

X_0 shunt reactance,

R_L load resistance,

X_L load reactance,

The primary side terminals are marked as T_1 T_2 and secondary side T_3 T_4 to which the load is connected.

The equivalent circuit of a transformer must represent all the quantities shown in Fig. 8, but either in terms of primary side values or secondary side values. Now in order to transfer resistance or reactance from one side to the other, the transferred value shown should be such as to cause the same percentage drop it did on the other side.

For example, let the ratio of turns (n) of the transformer shown in Fig. 8 be 5 (primary) : 1 (secondary). If the voltage and current on the primary side are 1000 volts and 5 amperes respectively, then the voltage and the current on the secondary side should be 200 volts and 25 amperes assuming no losses. If the resistance R_2 of the secondary winding is 0·16 ohm, the voltage drop $I_2 R_2$ is $0{\cdot}16 \times 25 = 4$ volts, i. e. 2%. When R_2 is transferred on the primary side, the voltage drop in it must be also 2% of 1000 volts when 5 amperes are passing in it, i. e. the drop must be 20 volts. Therefore

$\overline{R_2}$ (equivalent resistance of the secondary) $= \frac{20}{5} = 4$ ohms.

4 ohms $= (5 \times 5) \times 0{\cdot}16 = n^2 \times 0{\cdot}16$.

Alternatively :

$$\frac{I_2 R_2}{200} = \frac{I_1 \overline{R_2}}{1000}$$

$$\therefore \quad \overline{R_2} = R_2 \times \frac{I_2}{I_1} \times \frac{1000}{200} = R_2 \times n \times n = R_2 n^2$$

Similarly, $\overline{X_2} = n^2 X_2$; $\overline{R_L} = n^2 R_L$; $\overline{X_L} = n^2 X_L$.

When the primary side resistances and reactances are transferred to the secondary side the results are :—

$$\overline{R_1} = R_1 \left(\frac{1}{n}\right)^2 ; \ \overline{X_1} = X_1 \left(\frac{1}{n}\right)^2 \quad \ldots \quad \ldots \quad (4)$$

Therefore the *total equivalent resistance* referred to the **primary side** becomes :—

$$\overline{R} = R_1 + \overline{R_2} \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

The *total equivalent reactance* referred to the **primary side** becomes :—

$$\overline{X} = X_1 + \overline{X_2} \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

Similarly, the *total equivalent resistance* referred to the **secondary side** becomes :—

$$\overline{R} = R_2 + \overline{R_1} \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

And the *total equivalent reactance* referred to the **secondary** side becomes :—

$$\overline{X} = X_2 + \overline{X_1} \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

Thus Fig. 9 is the equivalent circuit of a transformer referred to the primary side and Fig. 10 is the equivalent circuit referred to the secondary side.

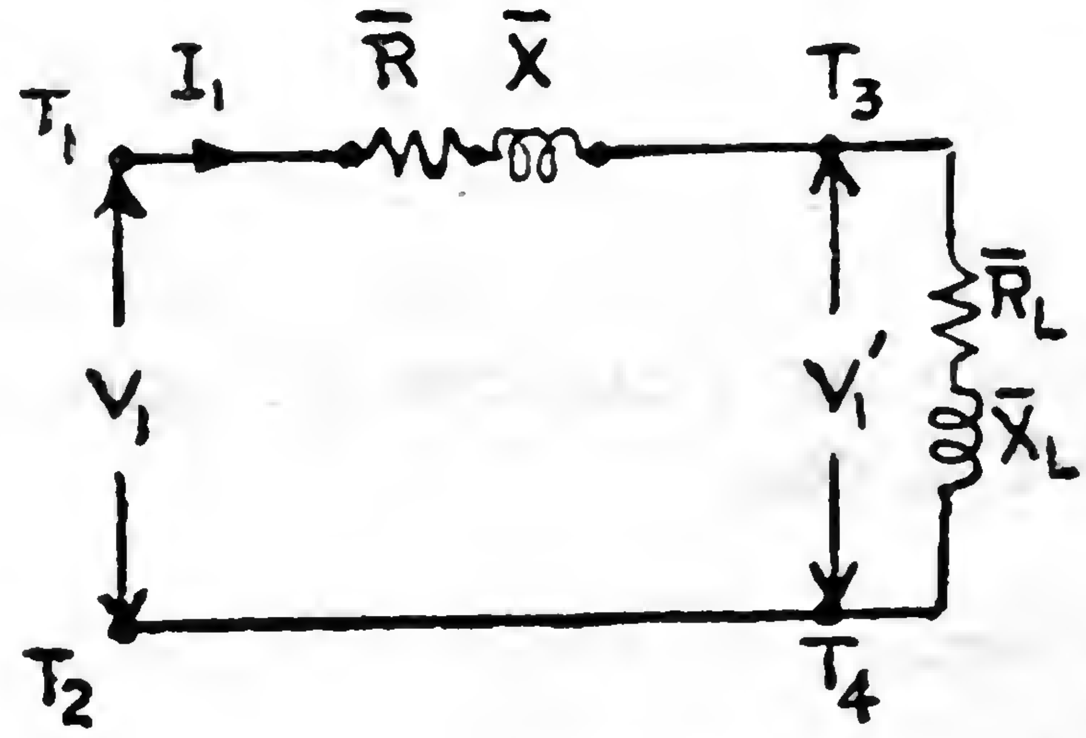


Fig. 9

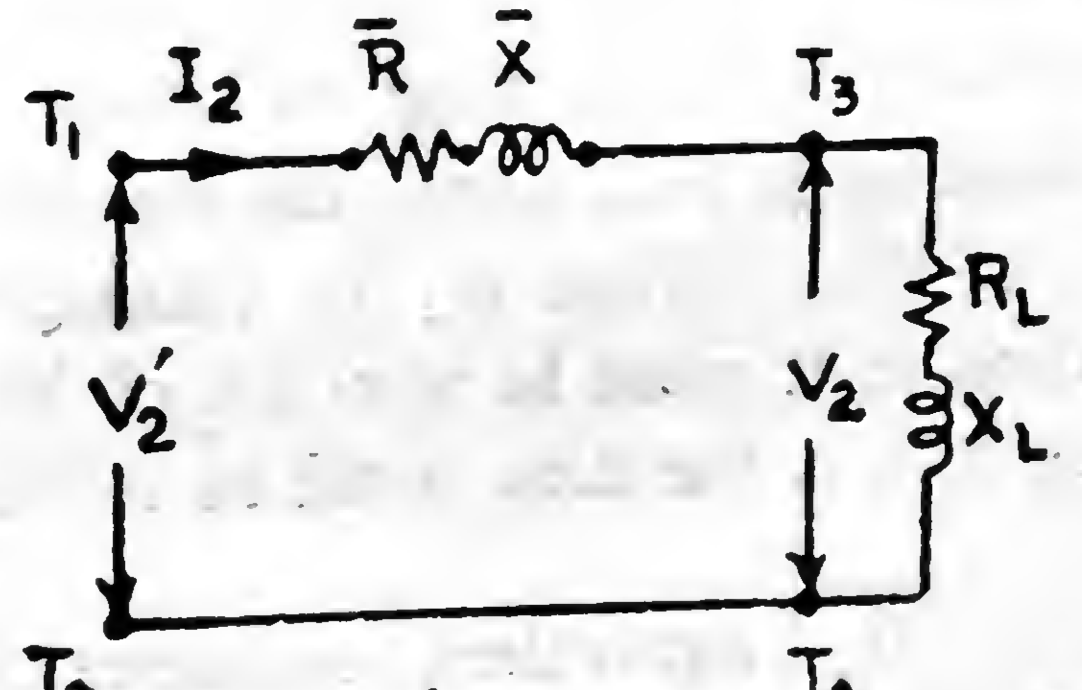


Fig. 10

The secondary load impedance ($R_L + j X_L$), if desired to be transferred on the primary side, becomes

$$\overline{R_L} = n^2 R_L; \text{ and } \overline{X_L} = n^2 X_L.$$

Example : A 20 kVA, 1000/250 V, 50 cycle, 1-ph. transformer on test gave the following results:—

O. C. Test :	1000 V ;	1·75 A,	300 W	(*h. v. side*)
S. C. Test:	5·5 V ;	60 A,	180 W	(*l. v. side*)

Calculate for full load at 80% p. f. lagging (i) the percentage regulation and (ii) the percentage efficiency.

Solution : From O. C. Test iron losses are 300 W (constant). From S. C. Test equivalent impedance $\overline{Z}$ referred to the secondary side

$$\overline{Z} = \frac{5\cdot5}{60} = 0\cdot0916 \text{ ohm}$$

$$\overline{R} = \frac{180}{60 \times 60} = 0\cdot05 \text{ ohm}$$

$$\therefore \quad \overline{X} = \sqrt{\overline{Z}^2 - \overline{R}^2} = \sqrt{0\cdot0916^2 - 0\cdot05^2} = 0\cdot09 \text{ ohm.}$$

$$\text{Full load current (sec.)} = \frac{20000}{250} = 80 \text{ A.}$$

Approximate voltage drop by Eq. (3)

$$= 80\,(0\cdot05 \times 0\cdot8 + 0\cdot09 \times 0\cdot6) = 7\cdot52 \text{ volts.}$$

Thus V_0 the no load voltage is 257·52 volts, and V = 250 volts

$$\%\ Regulation = \frac{V_0 - V}{V_0} \times 100 = \frac{7\cdot52}{257\cdot2} \times 100 = 2\cdot82\,\%$$

$$\text{Full load copper losses} = \frac{80^2}{60^2} \times 180 = 320 \text{ W}$$

$$\text{Total losses on full load} = 300 + 320 = 620 \text{ W}$$

$$\therefore \quad \%\ efficiency = \frac{20000 \times 0\cdot8}{20000 \times 0\cdot8 + 620} \times 100 = 96\cdot3\,\%$$

It is better to calculate the regulation by constructing the vector diagram, taking I along the reference axis as shown in Fig. 11. This method gives correct results. In the figure, OI is the current and OA = V the voltage on load. Cos ϕ is the load power factor. AB = IR drop and BC = IX drop. Hence OC = V_0, the no load voltage. OM = V cos ϕ, and

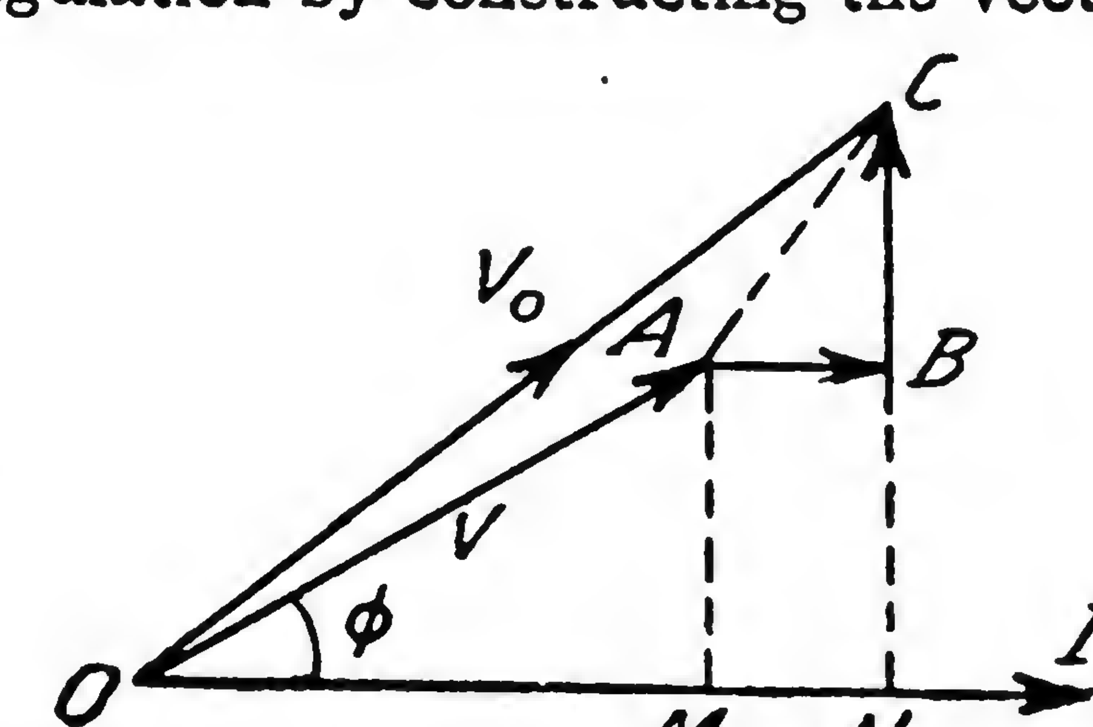


Fig. 11

BN $= V \sin \phi$. Therefore

$$V_0^2 = (OM + MN)^2 + (BN + BC)^2$$
$$= (V \cos \phi + IR)^2 + (V \sin \phi + IX)^2$$

$$\therefore \quad V_0 = \sqrt{[(V \cos \phi + IR)^2 + (V \sin \phi + IX)^2]} \quad \dots (9)$$

Thus calculating V_0 for the last problem,

$$V_0 = \sqrt{[(250 \times 0{\cdot}8 + 80 \times 0{\cdot}05)^2 + (250 \times 0{\cdot}6 + 80 \times 0{\cdot}09)^2]}$$

from which $V_0 = 257{\cdot}54$ volts.

8. Per Cent Resistance and Reactance Drops: It is sometimes convenient to express resistance and reactance drops as a percentage of the no load secondary voltage. Thus

$$\textit{percentage resistance drop} = \frac{IR}{V_0} \times 100$$

$$\textit{percentage reactance drop} = \frac{IX}{V_0} \times 100$$

Thus in the last problem,

$$\%\ \textit{resistance drop} = \frac{80 \times 0{\cdot}05}{257{\cdot}54} \times 100 = 1{\cdot}55\%$$

$$\%\ \textit{reactance drop} = \frac{80 \times 0{\cdot}09}{257{\cdot}54} \times 100 = 2{\cdot}80\%.$$

9. The Auto-Transformer: The auto-transformer is used in cases where the ratio of transformation differs little from 1. It has only one winding as shown in Fig. 12. This winding has a tapping point. Considering a step-down auto-transformer,

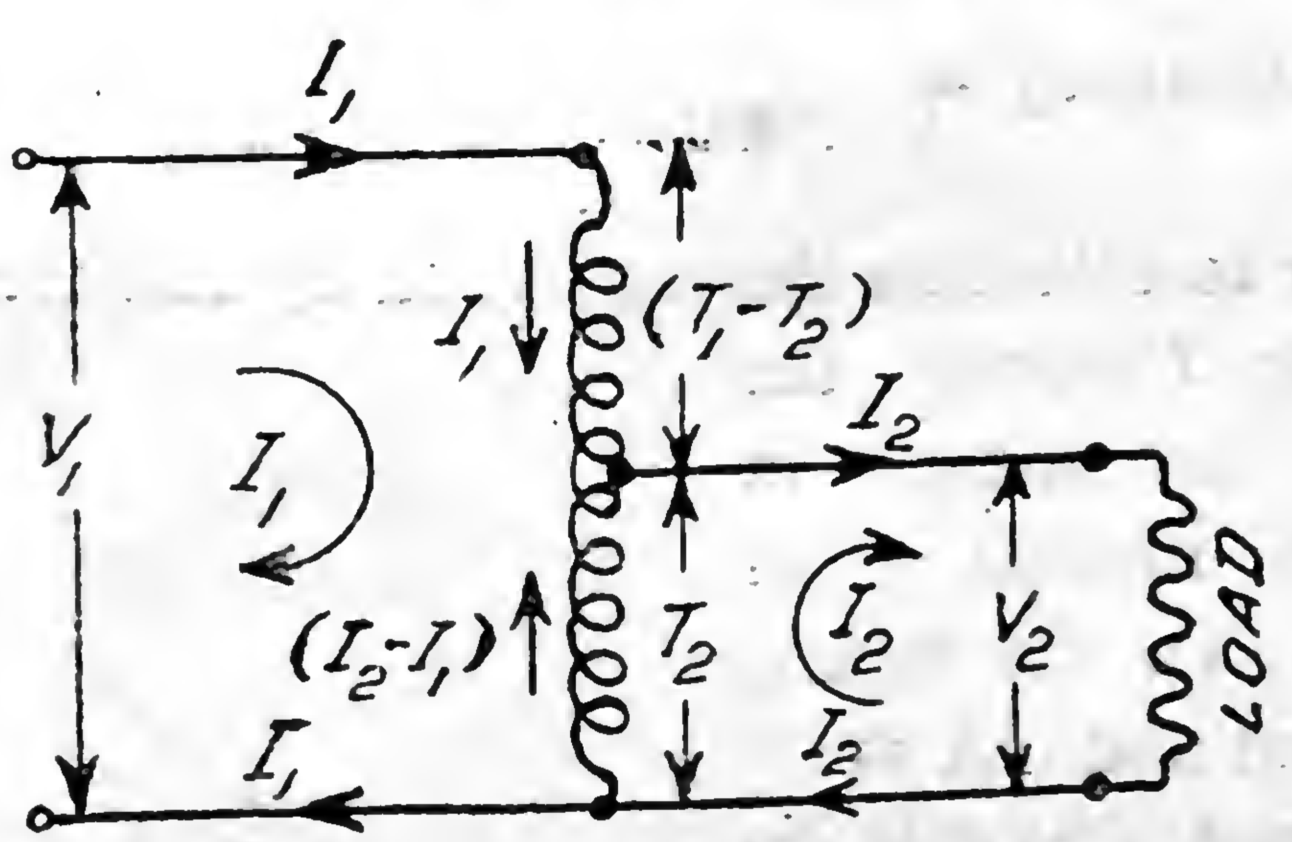


Fig. 12

let V_1 = supply voltage across the whole winding of T_1 turns

I_1 = primary current

I_2 = secondary current

V_2 = secondary voltage across T_2 turns which form the secondary turns.

Assuming no losses

$$V_1 I_1 = V_2 I_2$$

and $\dfrac{V_1}{V_2} = \dfrac{I_2}{I_1} = \dfrac{T_1}{T_2}$ = ratio of transformation = n

since $V_1 > V_2$, $I_2 > I_1$

Therefore current flowing in $(T_1 - T_2)$ portion of turns is I_1 and current in portion T_2 turns is $I_2 - I_1$. Hence the cross-sectional area of these T_2 turns is smaller than the remaining turns. Thus the auto-transformer requires less copper than a two winding transformer of the same output. This can be proved as follows :—

area $\propto$ current; and length $\propto$ number of turns

$\therefore$ area $\times$ length = volume of copper $\propto$ current $\times$ turns

$$\frac{\textit{weight of copper in auto-transformer}}{\textit{weight of copper in a 2-winding transformer}}$$

$$= \frac{I_1(T_1 - T_2) + (I_2 - I_1)T_2}{I_1 T_1 + I_2 T_2} = \frac{I_1 T_1 + I_2 T_2 - 2 I_1 T_2}{I_1 T_1 + I_2 T_2}$$

$$= 1 - \frac{2 I_1 T_2}{I_1 T_1 + I_2 T_2} = 1 - \frac{2\dfrac{T_2}{T_1}}{1 + \dfrac{T_2}{T_1} \times \dfrac{I_2}{I_1}}$$

But $\dfrac{T_2}{T_1} = \dfrac{1}{n}$; $\dfrac{I_2}{I} = n$. Substituting these values

$$= 1 - \frac{\dfrac{2}{n}}{1 + \dfrac{1}{n} \times \dfrac{n}{1}} = 1 - \frac{2}{2n} = 1 - \frac{1}{n}$$

$\therefore$ The ratio of weights $= 1 - \dfrac{1}{n}$ ... ... (10)

There is not much saving if n exceeds 3.

Example: An auto-transformer is connected to 400 V supply mains and its output is 50 A at 230 volts. Draw a sketch and show on it the magnitudes and direction of currents in the transformer windings, the load and the supply lines. Ignore losses.

Solution: Output $= 50 \times 230$ watts

$$\therefore \quad \text{primary line current} = \frac{50 \times 230}{400} = 28{\cdot}75\ A$$

$\therefore$ Supply line current $= 28{\cdot}75\ A$.

Load line current $= 50\ A$.

Current in one section of the winding $= 28{\cdot}75\ A$.

Current in the other section $= 50 - 28{\cdot}75 = 21{\cdot}25\ A$.

10. Wave Form of No-load Current: The shape of the no-load current departs considerably from the assumed sine wave form of alternating currents and voltage. This is due to the hysteresis of the core material, the B–H curve of which assumes a shape shown on the left hand side of Fig. 13. The resultant current wave is plotted from the known values of the hysteresis loop and the flux density wave B. This current wave contains 3rd and 5th harmonics and their amplitudes increase as the flux density B increases. This sets the limit to the maximum flux density employed in the design of transformers.

At light loads the primary current shows slight distortion, which becomes negligible as the load on the transformer increases, say after 30 % of full load value.

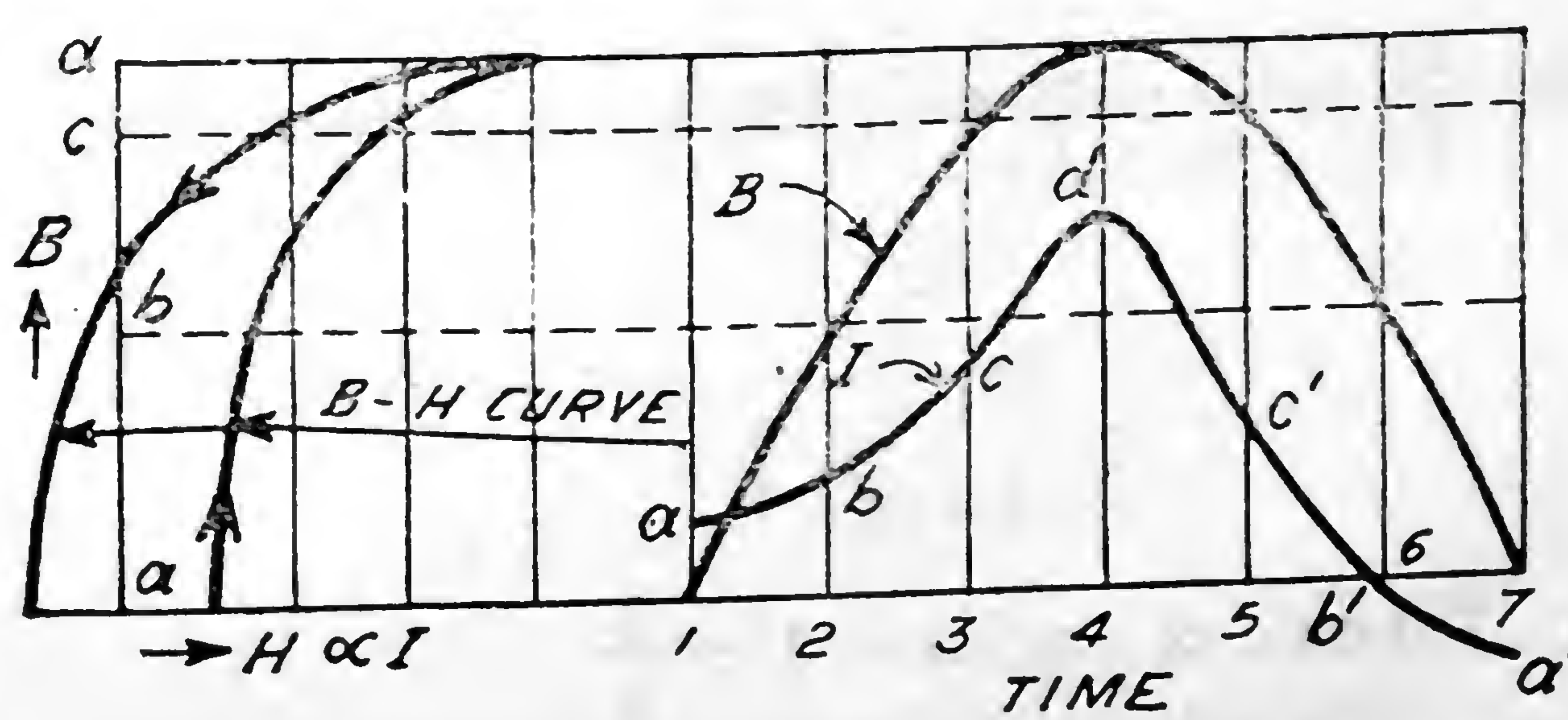


Fig. 13

11. Construction and Types of Transformers: There are two principal types of transformers, viz. (*a*) *core-type* and (*b*) *shell-type*. The transformer of Fig. 1 is a core-type while Fig. 14 shows a shell-type transformer. The advantage of the core-type is that it is possible to inspect and do minor repairs to it on site, while the shell-type affords better bracing to the coils and therefore is more robust. The shell-type is adopted for capacities of less than 50 kVA, and usually for single-phase working. But the shell type is falling in popularity with the manufactures.

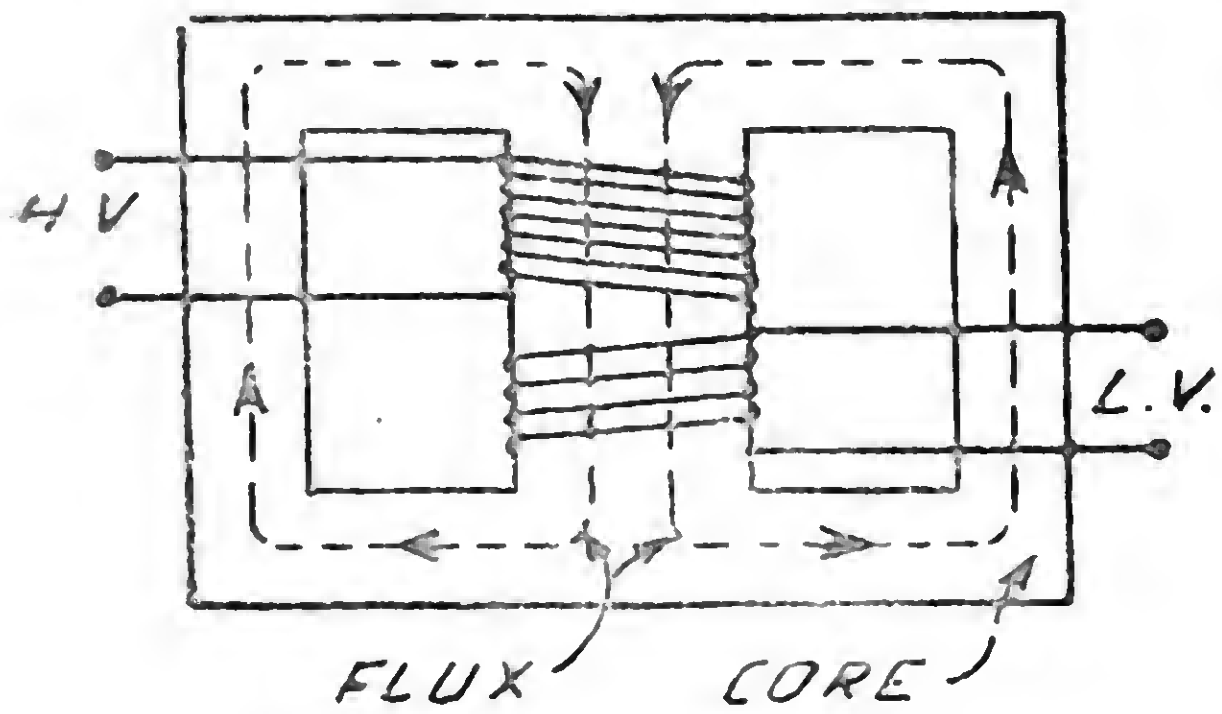
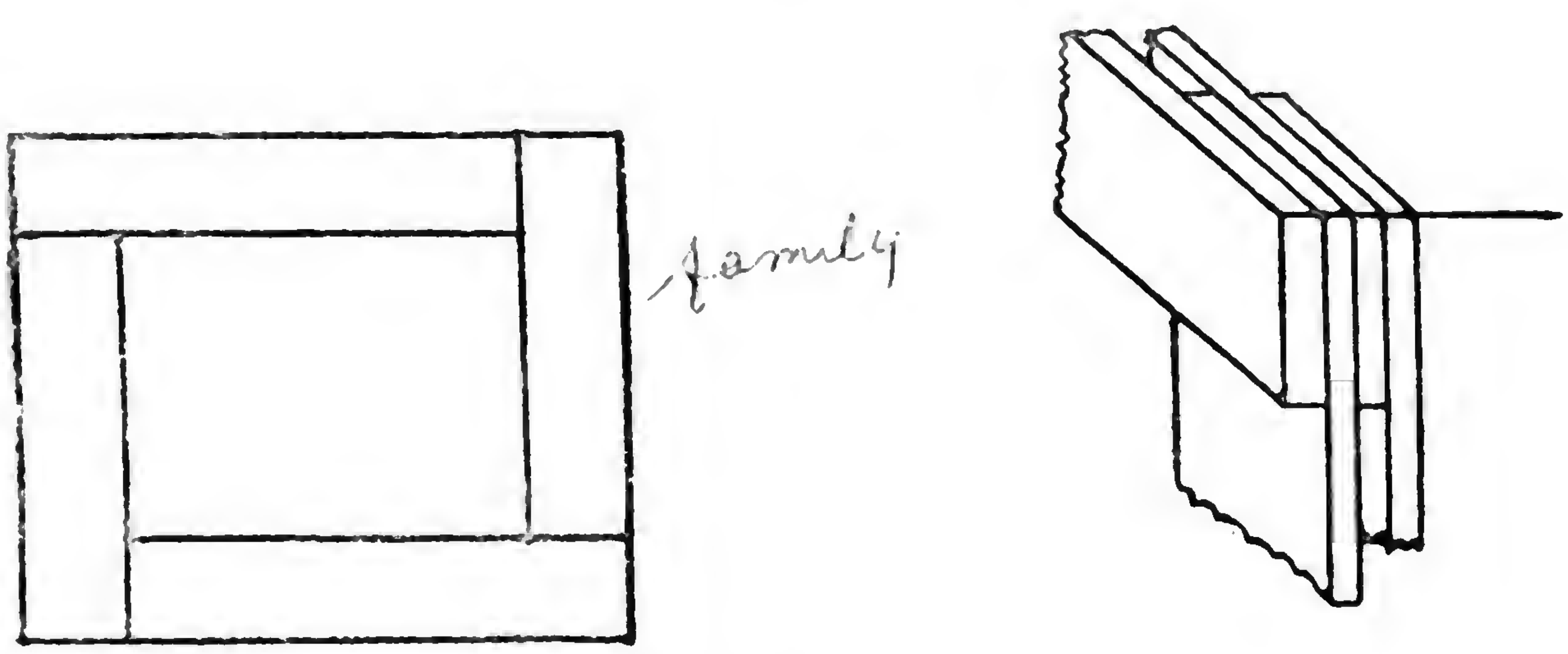


Fig. 14 Shell-Type Transformer, Single-Phase.



(a) Fig. 15 (b)

For three-phase transformers, and the larger single-phase core-type transformers, the core is constructed with vertical limbs of *cruciform section*. For small single-phase shell-type transformers the core is constructed with one *main limb* of rectangular or cruciform section on which the windings are located, and the magnetic circuit is completed through *horizontal yokes* and two outer limbs of rectangular section. In both the types the laminations of the limbs and the yoke are interleaved to avoid increase of reluctance of the magnetic path. See Fig. 15.

The clamping of laminations of small cores is done by binding the limbs with very strong webbing. The yokes are clamped with steel frames, the top and bottom frames being connected by long steel bolts. In larger cores, the limbs are clamped with a single row of

insulated bolts with individual clamping-plates. At the four outside corners of the core, there are rectangular notches into which are

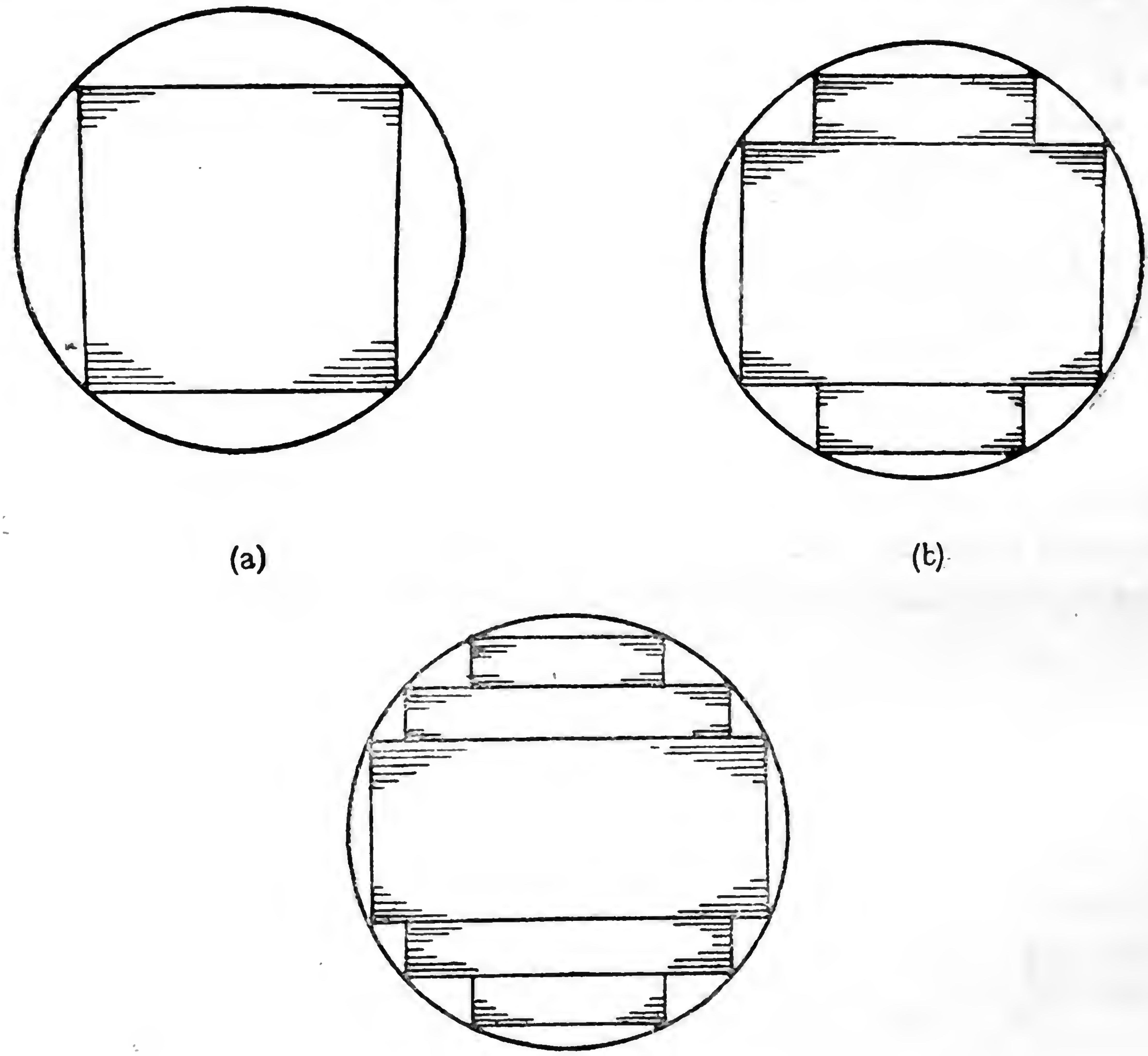


Fig. 16 Cross Section of Trans. Cores.

inserted heavy insulated bolts, which thus key together the yoke and limb laminations, and also serve to clamp the main frames to the yoke. Fig. 16 shows the cross-section of limbs of various sizes of cores.

To ensure low temperature gradient between copper and oil the *h. v.* and *l. v.* windings are arranged concentrically, the *l. v.* winding being next to the core. The insulation between the two windings consists of oil and one or more cylinders of highest quality insulating material. The *l. v.* winding is insulated from the core by a hard and tough insulating material and the *h. v.* winding from the yokes by washers and sectors of good quality insulating materials.

In the very small sizes, the coils are rectangular but in all other types the coils are circular. This shape is economical. The spaces

formed between the circumscribing circle and the iron core are utilised as ventilating ducts through which hot oil can travel from bottom to top.

The laminations are usually 0·014 to 0·018 inch thick. The material of laminations must possess high permeability and low core loss. Each lamina is insulated on one side by either a thin paper or a coating of varnish not thicker than 1 mil. The flux densities in the core lie between 9,000 to 14,000 gauss, the lower value is always preferred to keep down the iron losses and where the transformer is air-cooled.

For small currents the coil conductor used is circular in cross-section and the coil is wound in a number of layers, each layer having several turns. Insulation between layers is provided by a sheet of paper. The complete winding consists of a number of such coils assembled together, with sectors or washers of insulation between them; or, as in the smaller transformers, the coils are all wound directly on to a cylinder of insulation material. The sectors between coils also provide oil ducts radially.

The *h. v.* and the *l. v.* windings are so placed relative to each other that their magnetic centres are as nearly as possible coincident; also their total effective heights are made equal. This reduces the mechanical stress produced on the axial direction under short-circuit condition to absolute minimum.

12. 3–Phase Transformers: Either core-type or shell-type construction can be adopted for these transformers. Fig. 17 shows the usual core-type construction. The primary and the secondary windings belonging to one phase **must** be assembled on one leg only. Both windings are connected in star and their neutral points are shown as **N** and ***n***.

Fig. 18 shows the shell-type construction with the three phase windings. Winding of phase B is in reverse direction of those of phases A and C. If the winding of phase B is in the same direction as those of A and C, the fluxes in ***b*** and ***c*** portions will be the ***vector difference*** of two fluxes 120° in time phase and the resultant will be $\frac{1 \cdot 73}{2}$ times the flux in the central limb, instead of 0·5 times. The

central limb has twice the cross-sectional area of any other portion of the structure, where the flux density is half of that in the central limb.

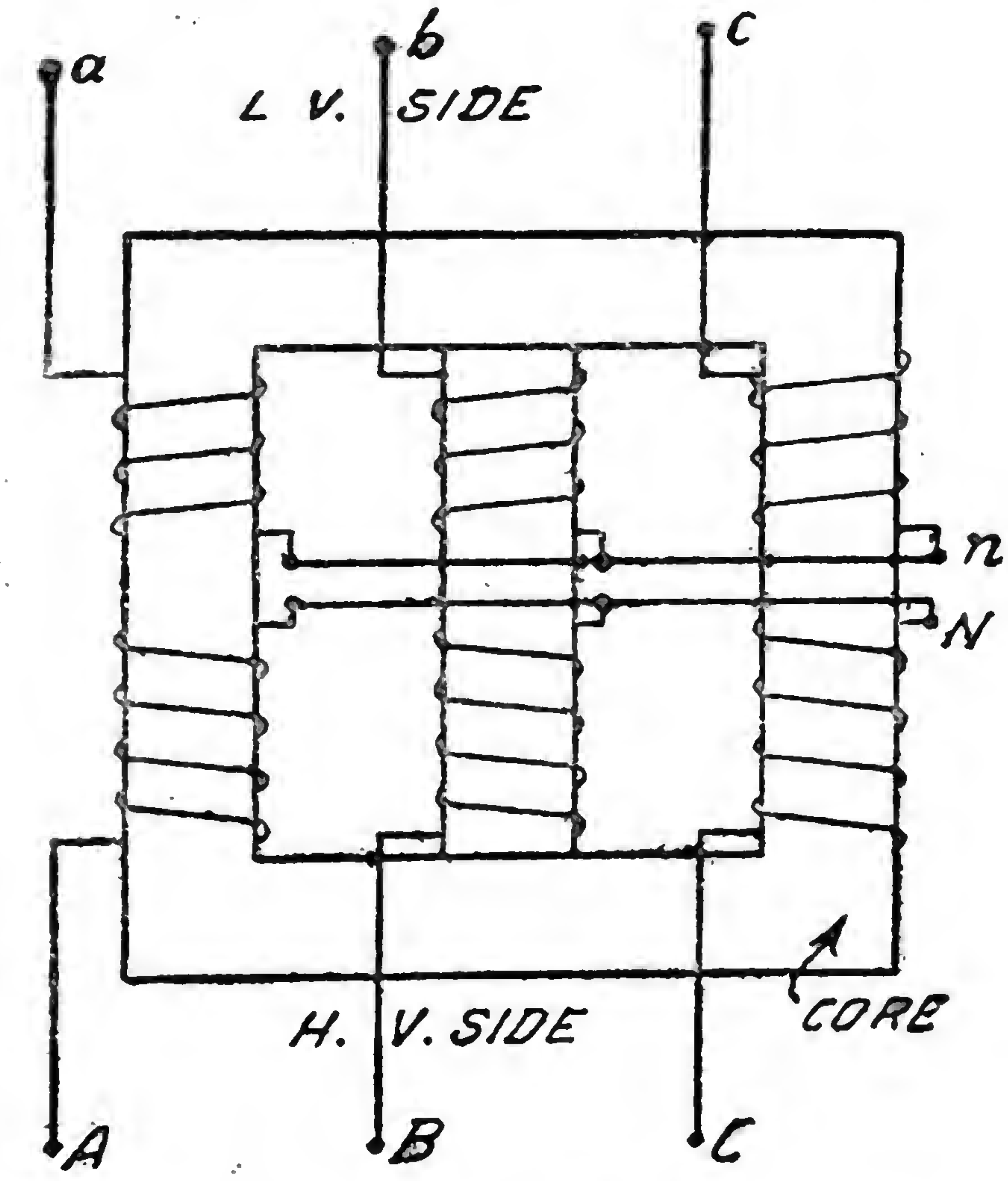


Fig. 17

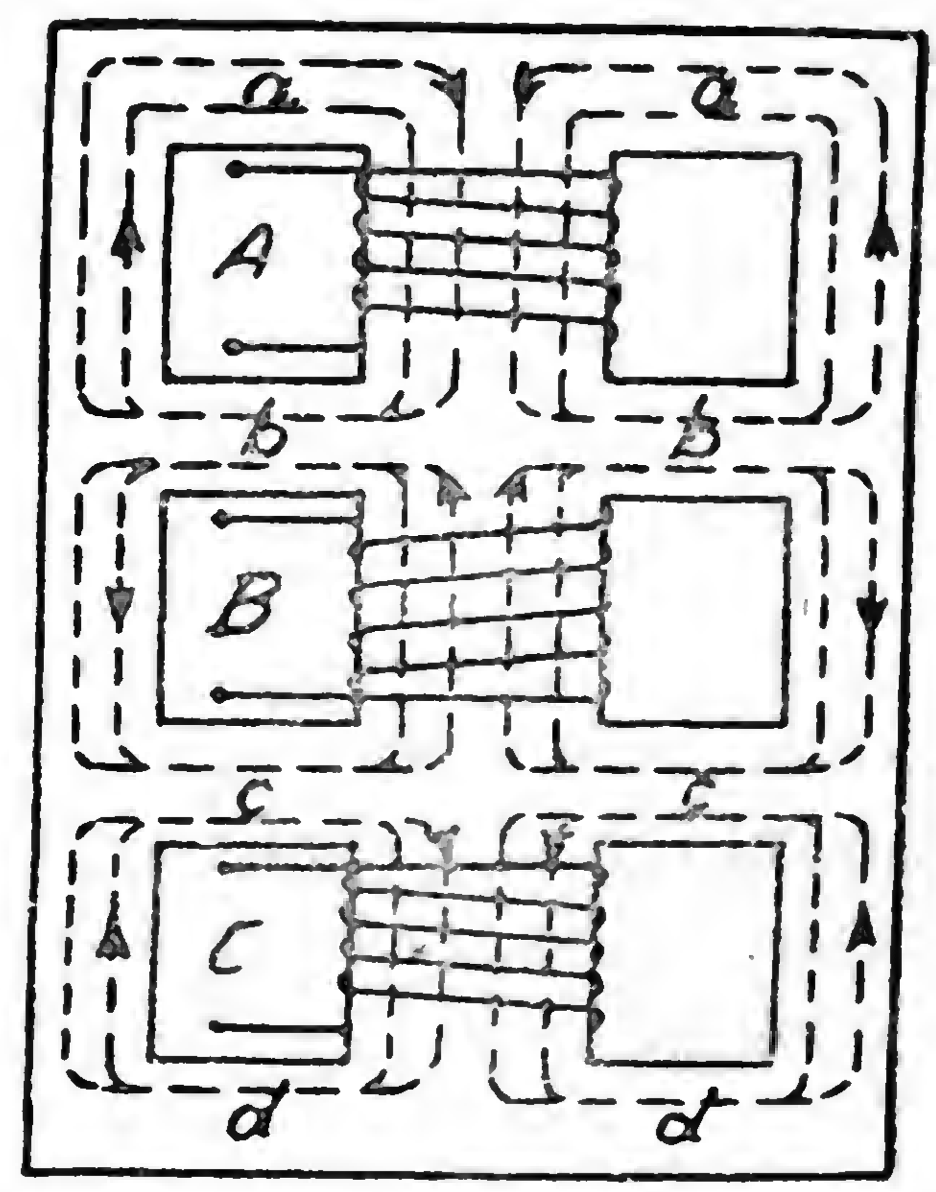


Fig. 18

Since the magnetic circuit of each phase of a 3-phase shell-type transformer is independent of the other two phases, the 3-phase shell-type transformer can be connected in *open delta* if one of its phases is burnt out. But in the core-type transformer the magnetic circuits are not independent, hence a 3-phase core-type transformer cannot be used in *open delta*. (See next Section).

It is the usual practice to earth the neutral point and also the tank frame of 3-phase transformers. The *h. v.* and the *l. v.* terminals are brought out, through proper *bushings*, for making connections to external circuits.

13. Combination of Connections: There are various types of connections possible with 3-phase transformer windings. The most common are the following four types:—

(1) *star – star;* (2) *delta – delta*; (3) *star – delta* and (4) *delta – star.*

The first word of the designation refers to the interconnection of the primary phase windings and the second refers to the interconnection of the secondary phase windings.

If the secondary windings are in two equal sections per phase, the combinations on the secondary side are listed below, the primary phases may be connected either in *star* or *delta*.

(5) *zigzag*; (6) *diametral*; (7) *double-star* and (8) *double-delta.*

The first four and (5) types give 3-phase supply on the secondary side, while (6), (7) and (8) give 6-phase supply on the secondary side. This is how a 3-phase supply is converted into a 6-phase supply.

Besides these, the *open delta* connection is sometimes employed in connection with transformers, but never in connection with alternator phases. This connection is also called the *V*-connection, and is useful in practice for obtaining temporary 3-phase supply from two 1-phase transformers or from the group of three 1-phase transformers connected in delta in the event of one transformer becoming faulty. The faulty transformer is removed for repairs and the two remaining

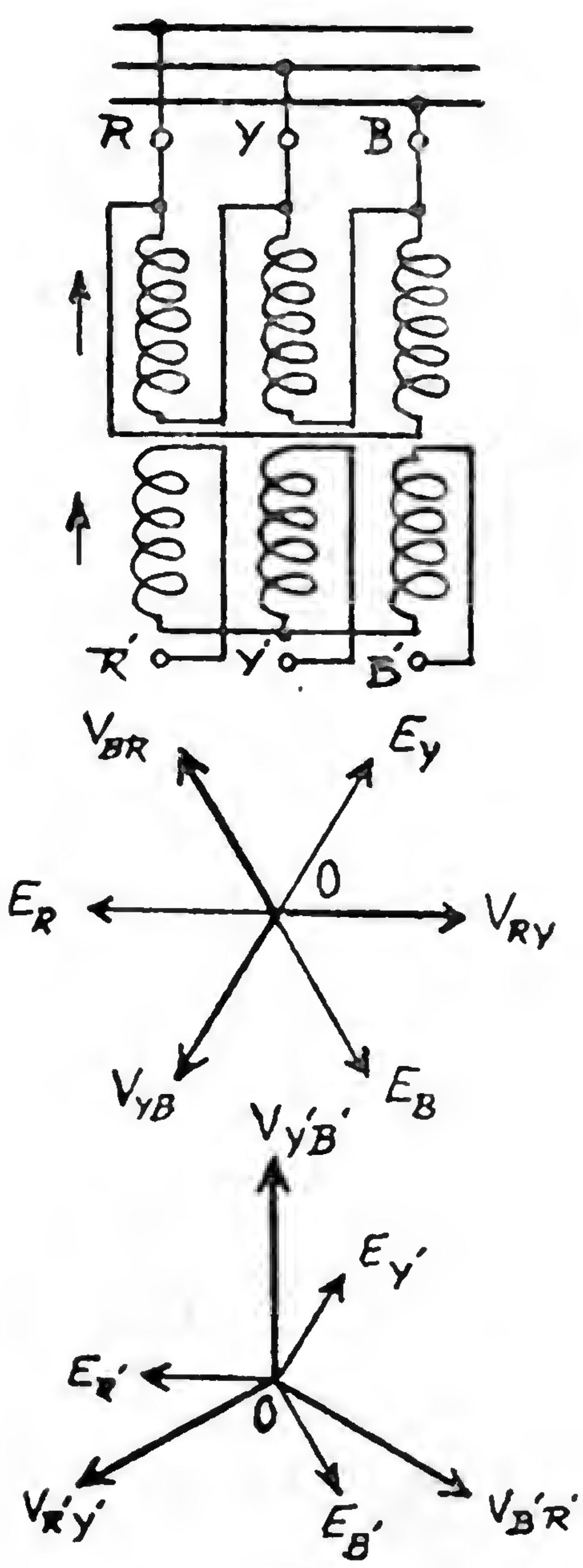


Fig. 19

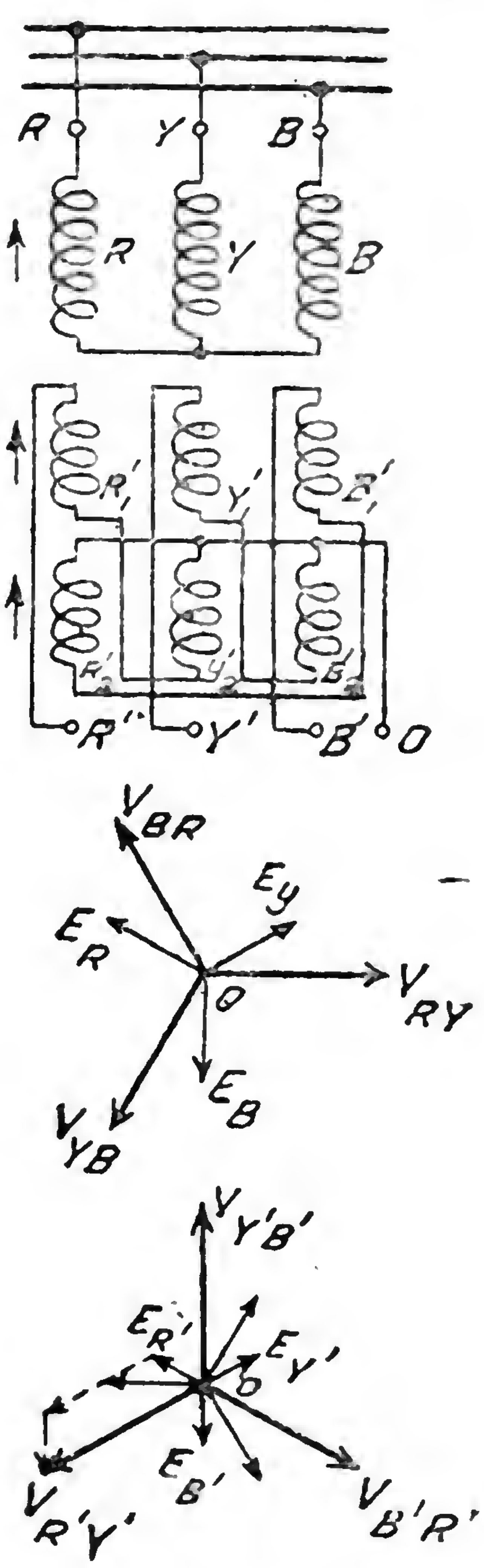


Fig. 20

can operate as before giving a three-phase supply to the load. The rating of $V-V$ connection is $\frac{\sqrt{3}}{2}$ times the rating of *delta-delta* connection. The above are the schematic connections of some of the combinations mentioned above. The combination known as *Scott-connection* is treated in the next Section.

Fig. 19 shows the *delta-star* method of connecting primary and secondary phase windings of a 3-phase transformer, and the vector diagram of voltages. The terminal voltages are indicated by Vs and the induced e. m. f. by Es. The phase sequence is **RYB.** Similarly, Fig. 20 shows the *zigzag* connection and its vector diagrams.

Example: A 3-phase, 50 cycle transformer is connected to 3300 volt supply mains on the primary side and takes 10 A from the line. If the ratio of turns (primary to secondary) per phase is 12·5, calculate the secondary *line* voltage, *line* current for the following winding connections:—

(a) star-star; (b) star-delta; (c) delta-delta and (d) delta-star.

Solution: Note: Always calculate the phase values and then convert them to line values.

(a) *Primary*: voltage per phase $= \dfrac{3300}{\sqrt{3}}$ V

Secondary: voltage per phase $= \dfrac{3300}{\sqrt{3}} \times \dfrac{1}{12{\cdot}5} = 152{\cdot}5$ V

,, line value of sec. voltage $= \sqrt{3} \times 152{\cdot}5$
$= 264$ V

Primary: phase winding current $= 10$ A

Secondary: phase winding current $= 10 \times 12{\cdot}5 = 125$ A

,, line current $= 125$ A.

(b) *Primary*: voltage per phase $= \dfrac{3300}{\sqrt{3}}$ V

Secondary: voltage per phase $= \dfrac{3300}{\sqrt{3}} \times \dfrac{1}{12{\cdot}5}$ V
$= 152{\cdot}5$ V

,, line value $=$ phase value $= 152{\cdot}5$ V

Primary: phase winding current $= 10$ A

Secondary: phase winding current $= 10 \times 12{\cdot}5 = 125$ A

,, line current $= \sqrt{3}$ times phase current

$= \sqrt{3} \times 125 = 216{\cdot}25$ A.

(*c*) *Primary*: voltage per phase = 3300 V

Secondary: voltage per phase $= \dfrac{3300}{12{\cdot}5} = 264$ V

,, line voltage = phase voltage = 264 V

Primary: phase current $= \dfrac{10}{\sqrt{3}}$ A

Secondary: phase current $= \dfrac{10}{\sqrt{3}} \times 12{\cdot}5$ A

,, line current $= \sqrt{3} \times \dfrac{10}{\sqrt{3}} \times 12{\cdot}5 = 125$ A.

(*d*) *Primary*: voltage per phase = 3300 V

Secondary: voltage per phase $= \dfrac{3300}{12{\cdot}5}$ V

,, line voltage $= \sqrt{3} \times \dfrac{3300}{12{\cdot}5} = 459{\cdot}7$ V

Primary: phase current $= \dfrac{10}{\sqrt{3}}$ A

Secondary: phase current $= \dfrac{10}{\sqrt{3}} \times 12{\cdot}5$ A $= 72{\cdot}25$ A

,, line current = phase current $= 72{\cdot}25$ A.

Example : A 3-phase, 50 cycle transformer is connected in delta on the primary side, and takes from the supply mains 10 A at 6600 volts at a p. f. of 0·8 (lag). Each secondary phase winding is split into two equal sections and these sections are connected zigzag. Calculate the secondary line voltage and line currents. Neglect losses and the magnetising current. Primary to secondary turns ratio = 18.

Solution :

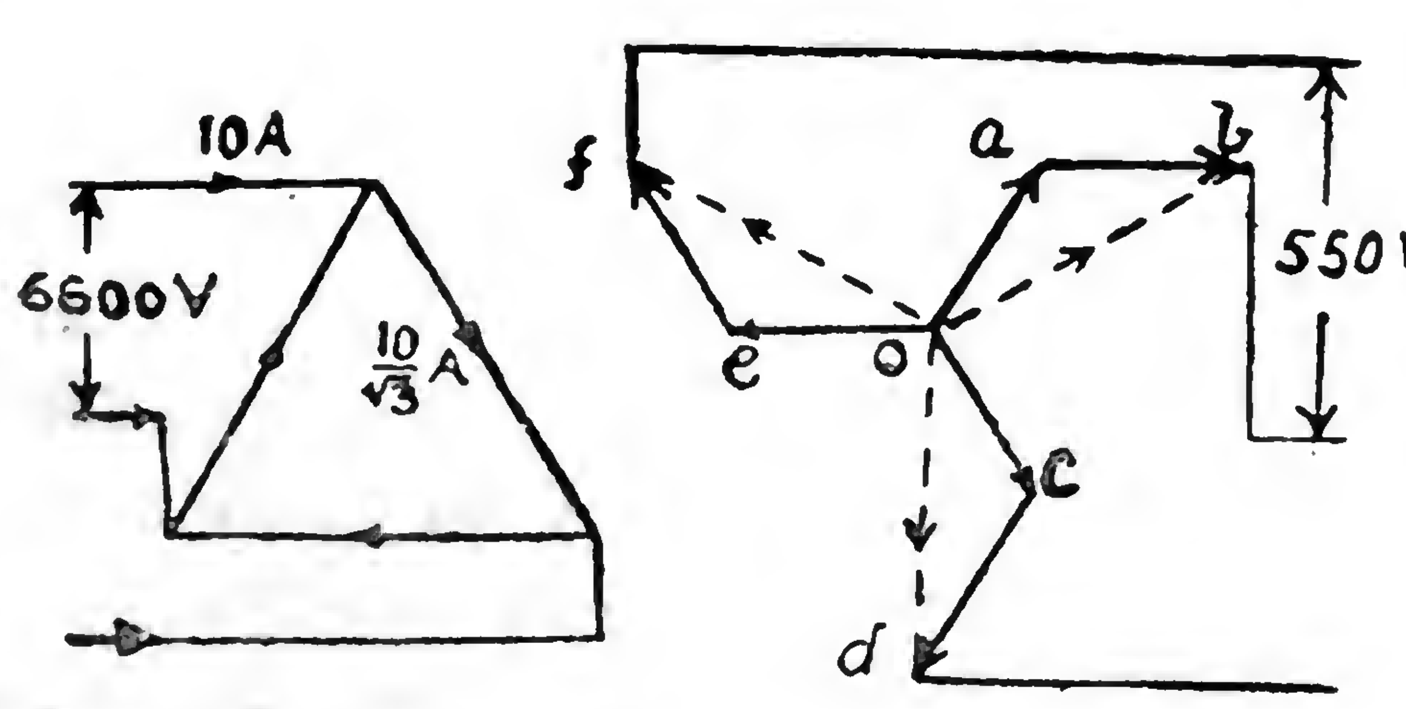


Fig. 21

Primary: phase voltage = 6600 V

Secondary: voltage per section $= \dfrac{6600}{18} \times \dfrac{1}{2}$ (oa)

The secondary phase is composed of two sections at an angle of 120°.

Secondary: phase voltage $= \sqrt{3} \times \dfrac{6600}{18} \times \dfrac{1}{2}$ (ob)

" line voltage $= \sqrt{3}\left(\sqrt{3} \times \dfrac{6600}{18} \times \dfrac{1}{2}\right)$

$= 550\text{ V}$ (fb)

Let the current in the secondary phase section $= I_2$

$$I_2 = I_1 \times \frac{6600}{550} = \frac{6600}{550} \times \frac{10}{\sqrt{3}} = \frac{120}{\sqrt{3}}\text{ A}$$

In the zigzag connection these currents are 120° for one phase. Hence

$$\text{secondary line current} = \sqrt{3} \times \frac{120}{\sqrt{3}} = 120\text{ A.}$$

We can check the answer by considering the power on the primary side being equal to the power on the secondary side, assuming that there are no losses in the transformer and balanced load

power on the primary side $= \sqrt{3} \times 6600 \times 10 \times \cos\phi_1$

power on the secondary side $= \sqrt{3} \times 550 \times 120 \times \cos\phi_2$

when secondary load is balanced $\cos\phi_1 = \cos\phi_2$.

14. The Scott Connection: By connecting two 1-phase transformers as shown in Fig. 22 a balanced 2-phase supply is obtained from a 3-phase balanced system or vice versa. When connected thus one transformer is called the MAIN and the other the TEASER. On the 2-phase side the two transformers have equal number of turns and equal voltages.

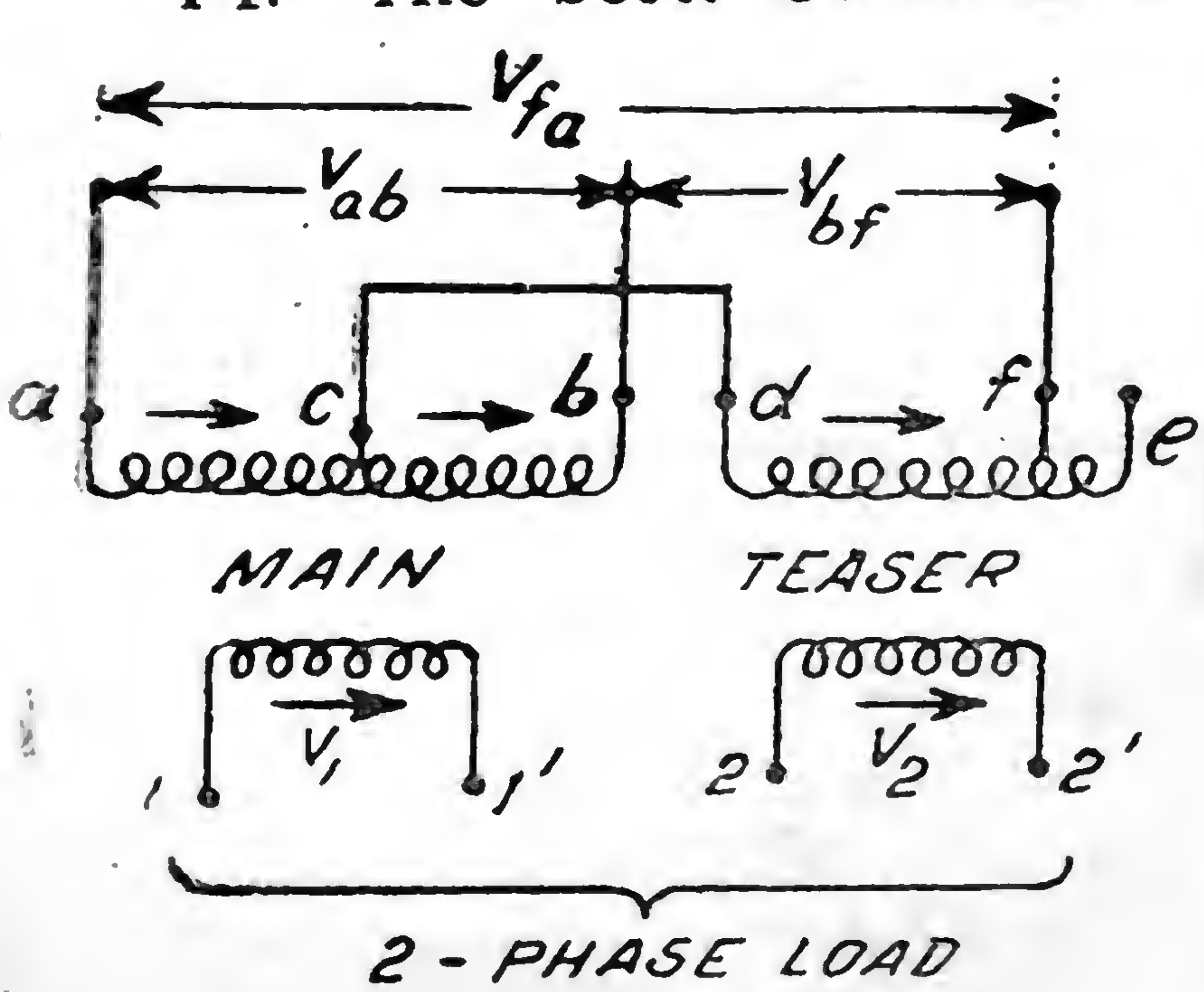


Fig. 22

On the 3-phase side the

Main transformer has a mid-point tapping *c* and two line wires of a 3-phase supply are connected to its ends *a* and *b*. The third line wire is connected to the end *f* of the Teaser and whose other end *d* is connected to the mid-point *c* of the Main. If the number of turns between ***a*** and ***b*** are T_M the number of turns between *d* and *f* must be equal to 0·866 times those on the Main, i. e.

$$T_T = 0{\cdot}866\ T_M$$

where T_T is the number of turns on the 3-phase side of the Teaser.

Usually the two transformer windings are exactly alike, each having a mid-point tapping and a 0·866 tapping. The transformation now-a-days is always from 3-phase to 2-phase, since the 2-phase supply is no more available. The load usually consists of two 1-phase furnaces or two distribution areas as in outlying districts, each getting one 1-phase line.

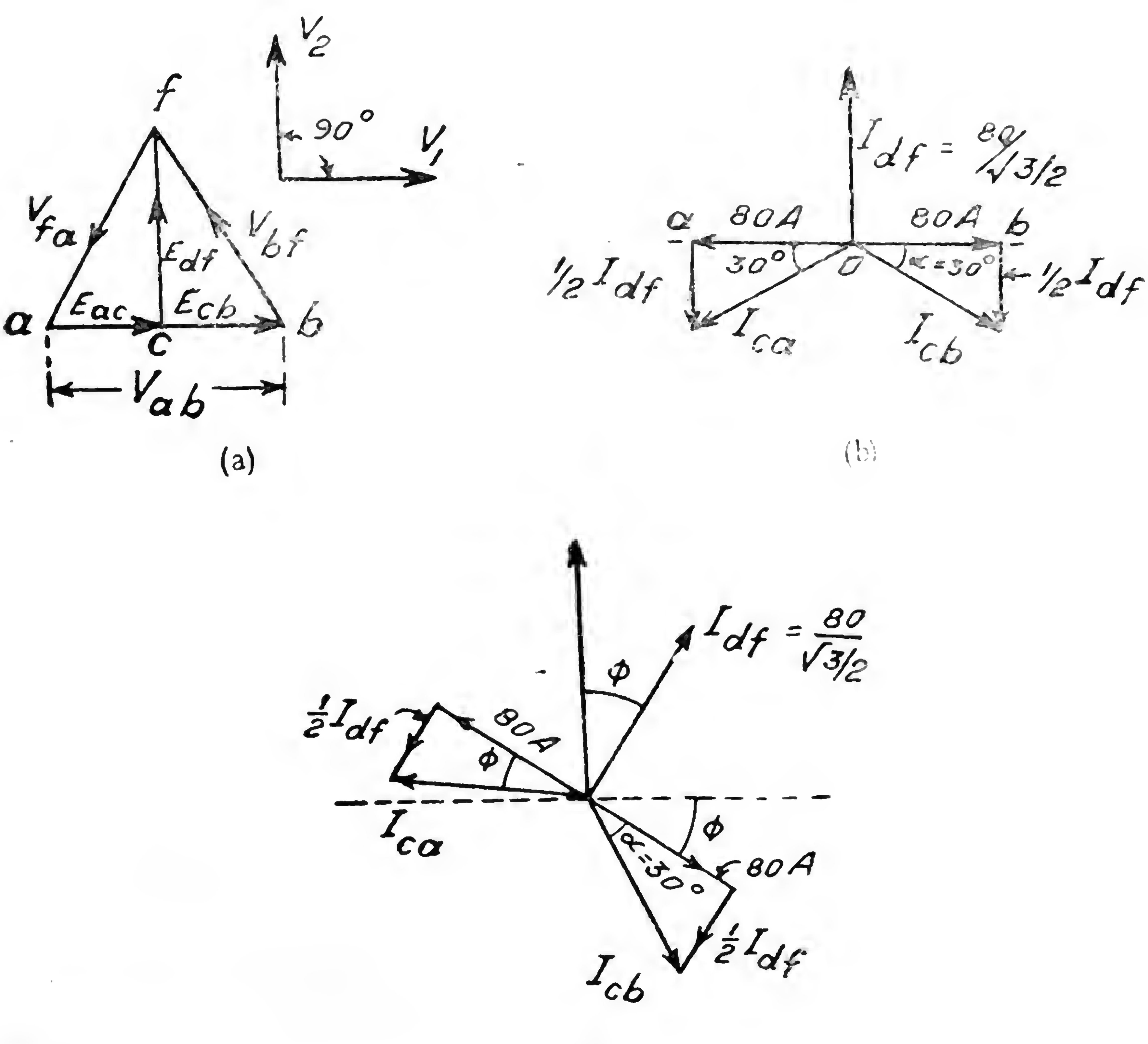


Fig. 23

When the load currents are equal on the 2-phase side and lag (or lead) their respective phase voltages by the same angle, the currents on the 3-phase sides are balanced, i. e. they are equal and have a phase difference of 120° with each other.

Proof : Let the ratio of turns of the Main transformer be 5 : 1 and of the Teaser 5 × 0·866 : 1 (3-phase side to 2-phase side). Let the 2-phase side current be 400 A per phase.

The *balancing* amperes of the Main tr. $= \frac{400}{5} = 80$ A

The " " " " Teaser $= \frac{400}{5 \times 0{\cdot}866} = \frac{80}{0{\cdot}866}$ A.

Since the current in the Teaser splits into two equal parts at the mid-point of the winding of the Main, the resultant current is the vector sum of 80 A and $\left(\frac{1}{2} \times \frac{80}{0{\cdot}866}\right)$ A, these two being at right angles.

$$\therefore \quad I_{ac} = I_{ca} = \sqrt{\left[80^2 + \left(\frac{1}{2} \times \frac{80}{0{\cdot}866}\right)^2\right]}$$

$$= \sqrt{\left[80^2 + \left(\frac{80}{\sqrt{3}}\right)^2\right]}$$

$$= \sqrt{\left[6400 + \frac{6400}{3}\right]} = \sqrt{\left(\frac{25600}{3}\right)}$$

$$= \frac{160}{\sqrt{3}} = \frac{80}{0{\cdot}866} = I_{df}. \quad \text{See Fig. 23 (b).}$$

$$\therefore \quad I_{ac} = I_{ca} = I_{df}$$

Further, $\tan \alpha = \left(\frac{80}{0{\cdot}866} \times \frac{1}{2}\right) \times \frac{1}{80} = \frac{1}{\sqrt{3}} = 0{\cdot}577$

From Tables, $\tan 30° = 0{\cdot}577 \quad \therefore \quad \alpha = 30°$ *Q. E. D.*

Thus the angle between I_{ca} and $I_{df} = 90° + 30° = 120°$

and " " " I_{cb} and $I_{df} = 90° + 30° = 120°$

and " " " I_{ca} and $I_{cb} = 180° - 30° - 30° = 120°$.

Fig. 23 (c) shows the current vectors when the load power factor is cos ϕ. For this case the proof is the same as above.

Example: Two 1-phase Scott-connected transformers supply two furnaces *A* and *B* of equal loading at unity power factor. The 3-phase side is connected

to a 3-phase 6600 volt system and the 2-phase side supplies at 80 volts 480 kW to each furnace. Calculate the line currents on the 3-phase side and prove that the answer is correct.

Solution: Since the 2-phase load is balanced, the 3-phase currents are also balanced, i. e. all the currents are equal and have a phase angle between them of 120°. Hence neglecting the primary (3-phase side) magnetising current, the primary currents = Teaser balancing amperes.

$$\text{Teaser load current} = \frac{480 \times 1000}{80} = 6000 \text{ A.}$$

$$\text{and Teaser } \textit{balancing} \text{ current} = 6000 \times \frac{80}{6600} \times \frac{1}{0{\cdot}866} = \frac{800}{11 \times 0{\cdot}866} \text{ A.}$$

$$\therefore \quad \text{Line currents are} = \frac{800}{11 \times 0{\cdot}866} = \mathit{83{\cdot}9\ A.}$$

To check the answer, the power input must be equal to the power output of $2 \times 480 = 960$ kW, assuming no losses.

$$\therefore \quad \sqrt{3} \times 6600 \times \frac{800 \times 2}{11 \times \sqrt{3}} \times \frac{1}{1000} = 960 \text{ kW } (\textit{check.})$$

15. Parallel Operation of Transformers: The demand of power on a Sub-station varies during the day. So that at light loads only one transformer is in circuit. As the load increases another transformer is put in parallel with the first. The condition for satisfactory operation of two transformers in parallel are:—

(1) Terminals having the same polarity, or markings, are connected to the same bus-bar.

(2) The voltage ratio must be the same for each transformer.

(3) At full load currents, the impedance voltages must have the same value for both transformers, and the ratio *resistance/reactance* must be the same for both.

Failure to fulfil condition (1) causes short circuit. If transformers have slight differences in their voltage ratios, circulating currents flow in the windings and may cause overheating at full load. If at rated kVA the size and shape of the impedance triangles are not exactly the same, the power factors at which the transformers operate will differ from the load power factor, hence the sharing of loads will not be proportional to their *kVA* ratings.

Fig. 24 shows the standard terminal markings of two transformers connected in parallel on *h. v.* and *l. v.* bus-bars. High voltage terminals are marked in capital letters. If there are no markings on the terminals Polarity Test helps to determine the correct polarity. The test consists

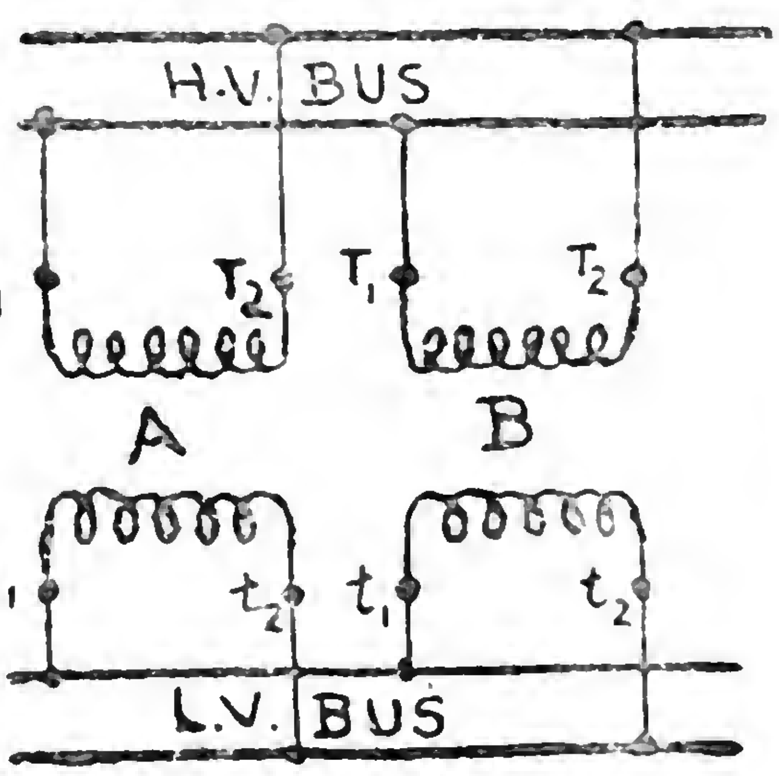


Fig. 24

(*a*) in making permanent connections between the supply bus-bars and the primary terminals;

(*b*) on the secondary side one of the terminals of **A** is connected temporarily to a terminal of **B**; and

(*c*) the remaining two terminals (one of **A** and the other of **B**) are connected through a voltmeter capable of reading twice the value of the secondary normal voltage.

If, when the supply is connected to the primary, the voltmeter reads zero. it indicates that the terminals which are temporarily connected together are of the same polarity. If they are not the voltmeter reads twice the normal secondary voltage.

Load sharing by two transformers, having equal voltage ratios and equal percentage voltage drops, is shown by the vector diagram of Fig. 25 wherein

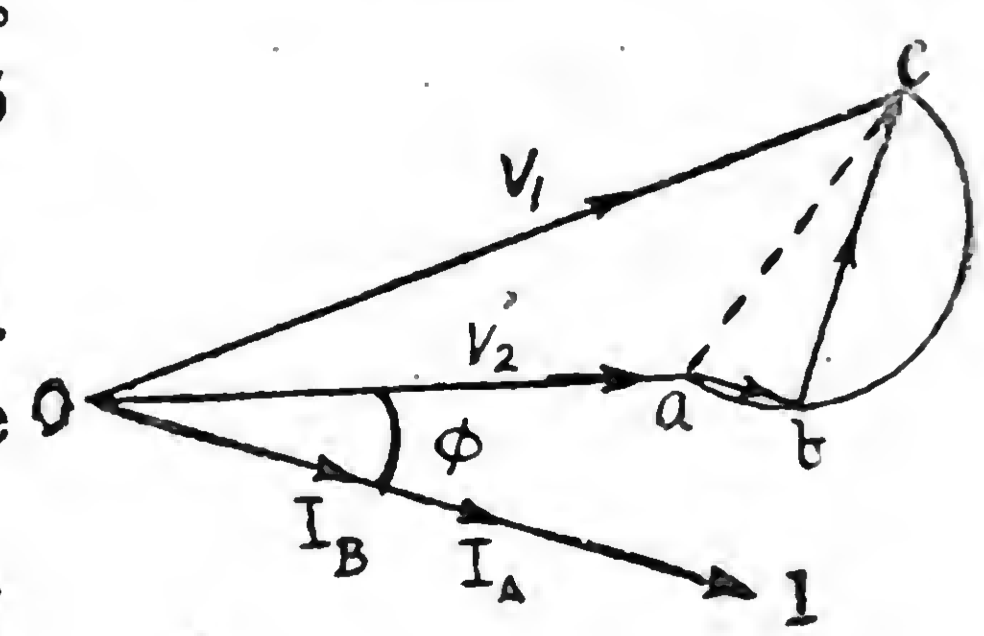


Fig. 25

- *Oa* is the secondary terminal p. d.
- *Oc* is the primary terminal voltage (reversed),
- *abc* is the impedance triangle of voltage drops,
- *ab* is the % resistance drop,
- *bc* is the % reactance drop,
- *OI* total load current,
- OI_A is the current in **A**,
- OI_B is the current in **B**.

Because the % drops are equal for both transformers, the impedance triangles of voltage drops overlap in the figure, and I, I_A and I_B have the same phase angle ϕ with the secondary terminal voltage.

Thus in this case the transformers supply the total load in proportion to their ratings.

If Z_A and Z_B are the impedances of two transformers A and B respectively, then when the two are in parallel

$$I_A Z_A = I_B Z_B; \text{ and } I = I_A + I_B \text{ (vectorially).}$$

From the above two relations

$$I_A = I\left(\frac{1}{1+\frac{Z_A}{Z_B}}\right) = I\left(\frac{Z_B}{Z_A+Z_B}\right) \quad \ldots \quad \ldots \quad (11)$$

$$I_B = I\left(\frac{1}{1+\frac{Z_B}{Z_A}}\right) = I\left(\frac{Z_A}{Z_A+Z_B}\right) \quad \ldots \quad \ldots \quad (12)$$

Usually the resistances and reactances are given in % voltage drops corresponding to full load current and normal secondary terminal voltage. Since in the above equations only the ratio of impedances are involved, the calculations are carried out directly with percentage values, i. e.

$$\frac{Z_A}{Z_B} = \frac{\%\ impedance\ drop\ of\ \mathbf{A}}{\%\ impedance\ drop\ of\ \mathbf{B}} \quad \ldots \quad \ldots \quad \ldots \quad (13)$$

If the transformers are of different ratings

$$\frac{Z_A}{Z_B} = \frac{\%\ impedance\ drop\ of\ \mathbf{A}}{\%\ impedance\ drop\ of\ \mathbf{B}} \times \frac{kVA\ of\ \mathbf{B}}{kVA\ of\ \mathbf{A}} \quad \ldots \quad (14)$$

For instance, if for transformer A the resistance drop is 1 % and % reactance drop 4 % and for B the resistance drop is 2 % and the % reactance drop is 5 %, then with equal ratings

$$\frac{Z_A}{Z_B} = \frac{1+j\,4}{2+j\,5}$$

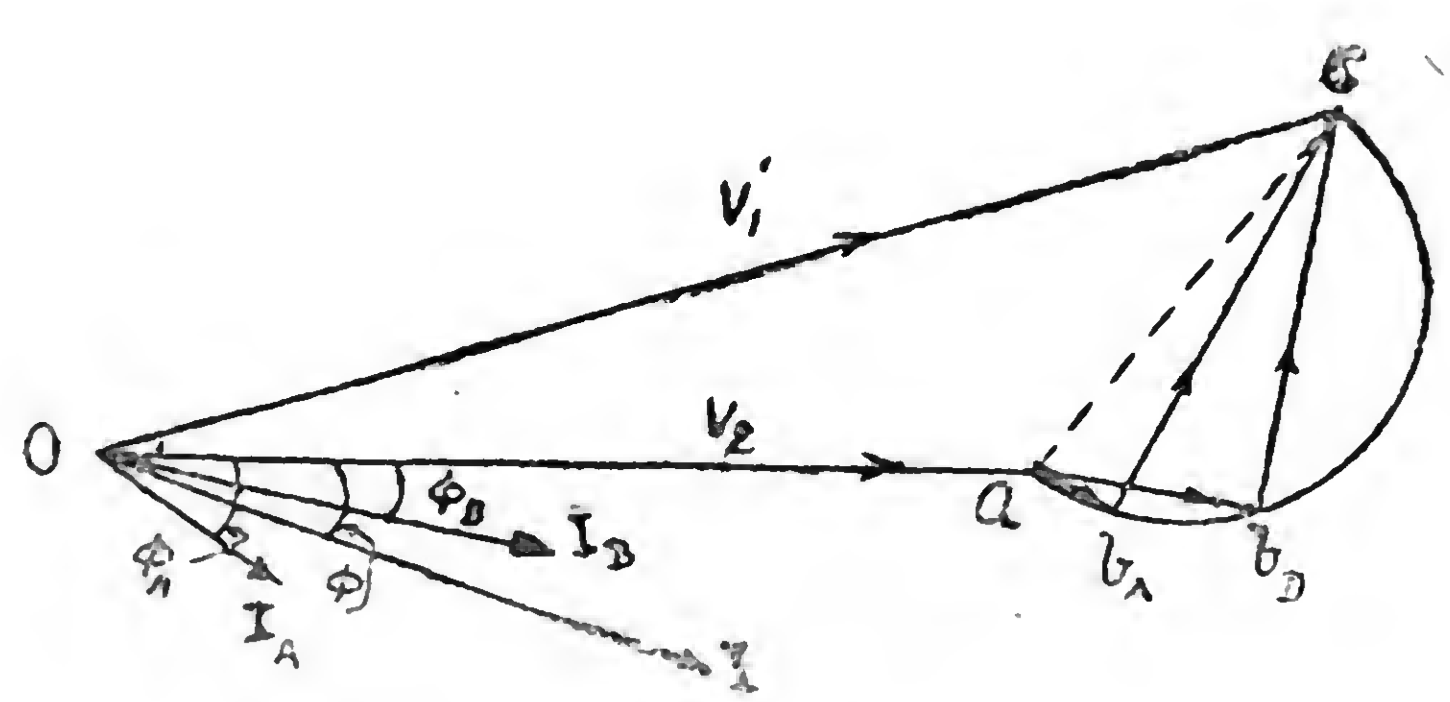


Fig. 26

If the rating of **A** is 500 kVA and that of **B** is 300 kVA, then

$$\frac{Z_A}{Z_B} = \frac{1 + j\,4}{2 + j\,5} \times \frac{300}{500}\,.$$

When the ratio $\dfrac{R}{X}$ is different for each transformer the vector diagram is as shown in Fig. 26.

Example: Two 100 kVA transformers are operating in parallel. Calculate the load and power factor of each transformer, when the load supplied is 16 kW at 0·8 p. f. lag.

A's % resistance drop = 1·5%
% reactance drop = 6 %
B's % resistance drop = 1·2%
% reactance drop = 4%

Solution: (*By symbolic method.*)

$$\frac{Z_A}{Z_B} = \frac{1{\cdot}5 + j\,6}{1{\cdot}2 + j\,4} \times \left(\frac{1{\cdot}2 - j\,4}{1{\cdot}2 - j\,4}\right) = 1{\cdot}478 + j0{\cdot}0688$$

$$\frac{Z_B}{Z_A} = \frac{1{\cdot}2 + j\,4}{1{\cdot}5 + j\,6} \times \left(\frac{1{\cdot}5 - j\,6}{1{\cdot}5 - j\,6}\right) = 0{\cdot}674 - j0{\cdot}0314.$$

$$\text{The load } kVA = \frac{160}{0{\cdot}8} = 200$$

$$\text{in symbolic notation load } kVA = 200\,(0{\cdot}8 - j\,0{\cdot}6)$$
$$= (160 - j\,120)$$

Since in the problem no mention is made of the voltage at which the load is supplied, kVA can be substituted in place of I in the equations. Thus

$$A\text{'s } kVA = \textit{load } kVA \times \frac{1}{1 + \dfrac{Z_A}{Z_B}}$$

$$= (160 - j\,120)\left(\frac{1}{1 + 1{\cdot}4{,}8 - j\,0{\cdot}0688}\right)$$

Rationalising

$$= (160 - j\,120)(0{\cdot}403 - j\,0{\cdot}0112)$$
$$= 63{\cdot}14 - j\,50{\cdot}15$$
$$= 80{\cdot}5\ kVA.$$

$$\text{A's power factor} = \frac{63{\cdot}14}{80{\cdot}5} = 0{\cdot}782 \text{ lag.}$$

$$\text{A's kW} = 63{\cdot}14$$

$$\text{B's kVA} = (160 - j\,120)\left(\frac{1}{1 + 0{\cdot}674 - j\,0{\cdot}0314}\right)$$

$$= (160 - j\,120)\,(0{\cdot}596 + j\,0{\cdot}0112)$$

$$= 96{\cdot}7 - j\,69{\cdot}73$$

$$= 119{\cdot}2\ kVA$$

$$\text{B's power factor} = \frac{96{\cdot}7}{119{\cdot}2} = 0{\cdot}812 \text{ lag.}$$

$$\text{B's kW} = 96{\cdot}7$$

[Note that **B** is overloaded.]

16. Instrument Transformers: Instrument transformers are special types of transformers used with measuring instruments in high-voltage a. c. circuits. These are of two types :—

(*a*) Ammeters and current coils of wattmeters are operated through **current transformers**, in which the primary is a thick bar round which the secondary is wound. The secondary current is a constant fraction of the primary current. Full load secondary current is usually 5 amperes.

(*b*) Voltmeters and potantial coils of wattmetars are connected to the secondary windings of **potential transformers**, in which the secondary p. d. is a constant ratio of the primary p. d. The normal secondary voltage is usually 110 volts. The construction of this type of transformer is similar to that of power transformers.

The advantages of using instrument transformers are :—

Measuring instruments can be placed at a convenient place away from the h. v. system, and can be handled safely, since one side of the secondary of the instrument transformers is earthed.

The current transformer should never be left open-circuited. To remove an ammeter from the circuit, the procedure is first to short circuit the secondary terminals by a thick wire and then to remove the instrument. If the transformer is left open-circuited the primary ampere-turns are alone present causing a very high flux density and iron loss in the core. This usually damages the transformer.

CHAPTER X

THE ALTERNATOR

1. **Introductory:** An alternator is an alternating voltage generator and consists essentially of two parts : the field magnet system and the armature.

Since no commutator is required in the case of an alternator, the armature is the stationary member called the **stator** and the field magnet system is the rotating member called the **rotor.** The advantages of having a stationary armature are :—

(1) it is simpler to brace and insulate the armature coils. The usual voltages of generation are 6600, 11000 volts and 33000 volts;

(2) high potentials between slip-rings are avoided;

(3) the number of slip-rings is only two, and they carry current at either 250 or 125 volts d. c.

The Stator is built up of stampings as in a d. c. machine and is clampped on to a fabricated steel frame. This frame does not carry any magnetic flux.

The Rotor is of two types : (1) the **Salient pole** type and (2) the **smooth cylindrical** type. The former is suitable for slow speeds upto 600 r. p. m. The armatures of slow speed alternators have a large diameter and short axial length. The number of poles is sometimes large (upto 40 poles) and these poles are either bolted or dovetailed to a large diameter spider wheel. The poles are built up of steel stampings and the overhang of the pole-shoes gives support to the field winding coils. These coils are, usually, made up of copper strips when the field current required is very large.

The smooth cylindrical type is used for high speed alternators which are driven by steam turbines (max. speed = 3000 r. p. m.). In this case the rotor diameter has to be small to prevent stresses due to centrifugal force. But the axial length is long. The field winding is embedded in the grooves of the rotor. The pole-faces are formed at the ungrooved portions of the rotor. The number of poles is usually 2 and in some cases 4.

The frequency (f), the number of poles (p) and the speed (n) of an alternator are related to each other by the expression

$$f = \frac{pn}{120}$$

where f = cycles per second, p = number of poles and n = speed in revolutions per minute.

The standard frequency in this country is 50 c. p. s. Hence an alternator must run at only one speed and no other speed. This speed is called the **synchronous speed.**

Alternators are not self-excited, the current to the field windings must be supplied by either an **exciter** installed in the Power House to supply all the alternators, or a small exciter fixed on each alternator shaft. The exciters are in fact d. c. generators.

Forced ventilation is often adopted, especially in turbo-alternators to carry away the heat. In large capacity alternators, axial ducts are provided by punching holes in the armature stampings. These ducts are in addition to the radial ducts provided in the assembly of armature stampings.

The usual ranges of voltage of alternators are :—

(i) 250, 420, 1100, 3300 volts (small capacity alternators)

(ii) 3300, 6600, 11000 volts (medium capacity alternators)

(iii) 6600, 11000, 33000 volts (large capacity alternators).

2. Shape of E. M. F. Waves: The induced e. m. f. in a conductor moving in a magnetic field is proportional to the rate of cutting magnetic lines of force at that instant. Hence the wave shape, or wave-form, is very similar to that of the flux in the air-gap. If the flux wave-form is sinusoidal the e. m. f. wave-form will also be sinusoidal. But in practice the flux wave-form is far from the ideal.

To obtain an e. m. f. wave which resembles very closely a sine wave-form various methods are employed. These are :—

(1) The pole-shoes are **skewed.** This method is simple and interesting. The induced e. m. f. equation is

$$e = B\,l\,v \times 10^{-8} \text{ volts}$$

since for an alternator v, the peripheral velocity, is constant, either B, the flux density, or l, the active length of conductor, must vary

sinusoidally. In this case l is made to vary according to a sine law as seen in Fig. 1 (a) which shows a pair of skewed poles. Fig. 1 (b) shows the wave-form.

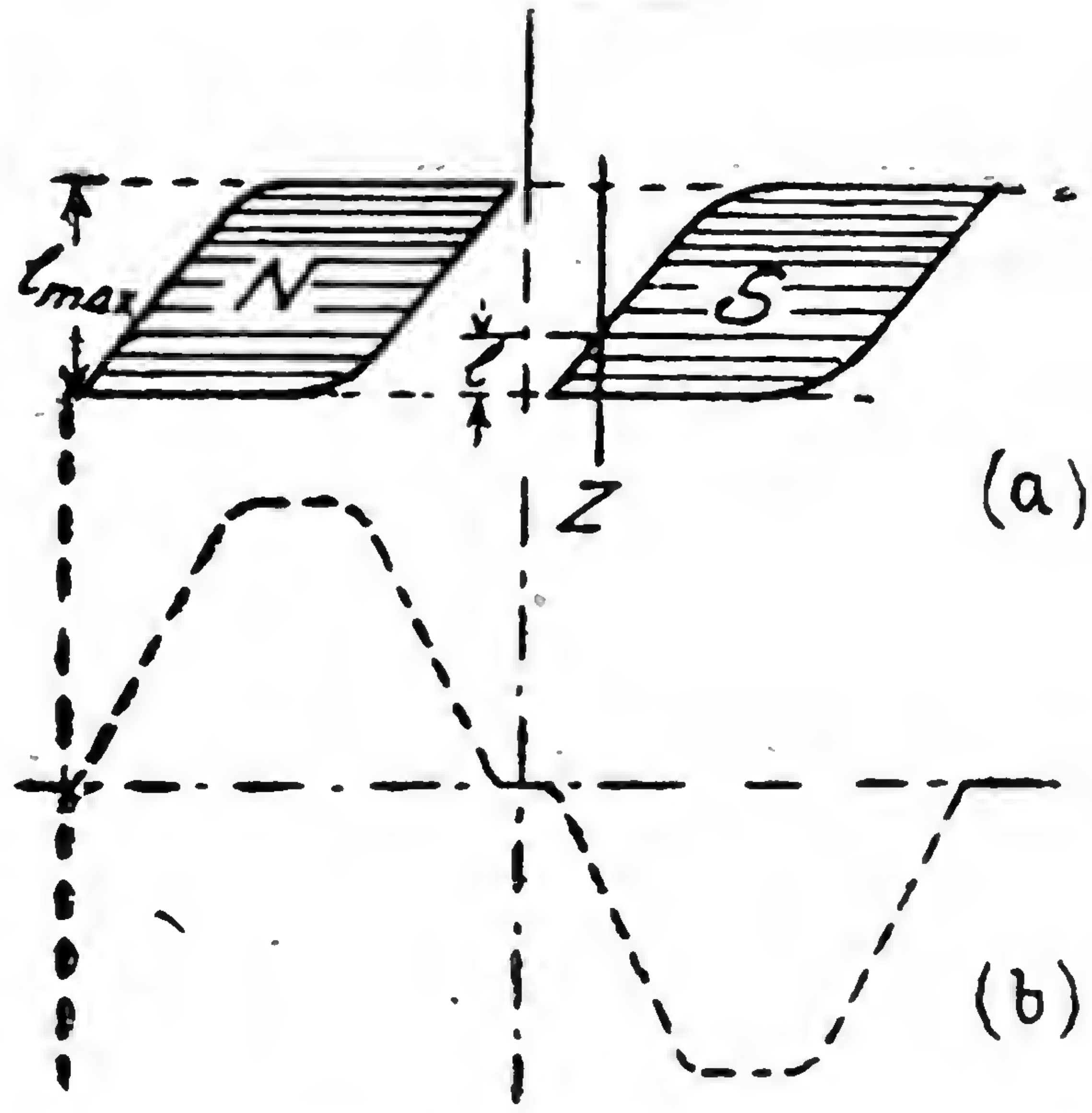


Fig. 1 (a) & (b)

(2) The flux density B in the air-gap can be made to vary according to a sine-wave law if the air-gap length varies from the centre of a pole to the tips of the pole-shoes. The increase in the gap length is made nearly twice, see Fig. 1 (c) and the ratio *pole arc / pole pitch* is kept round about 0·67

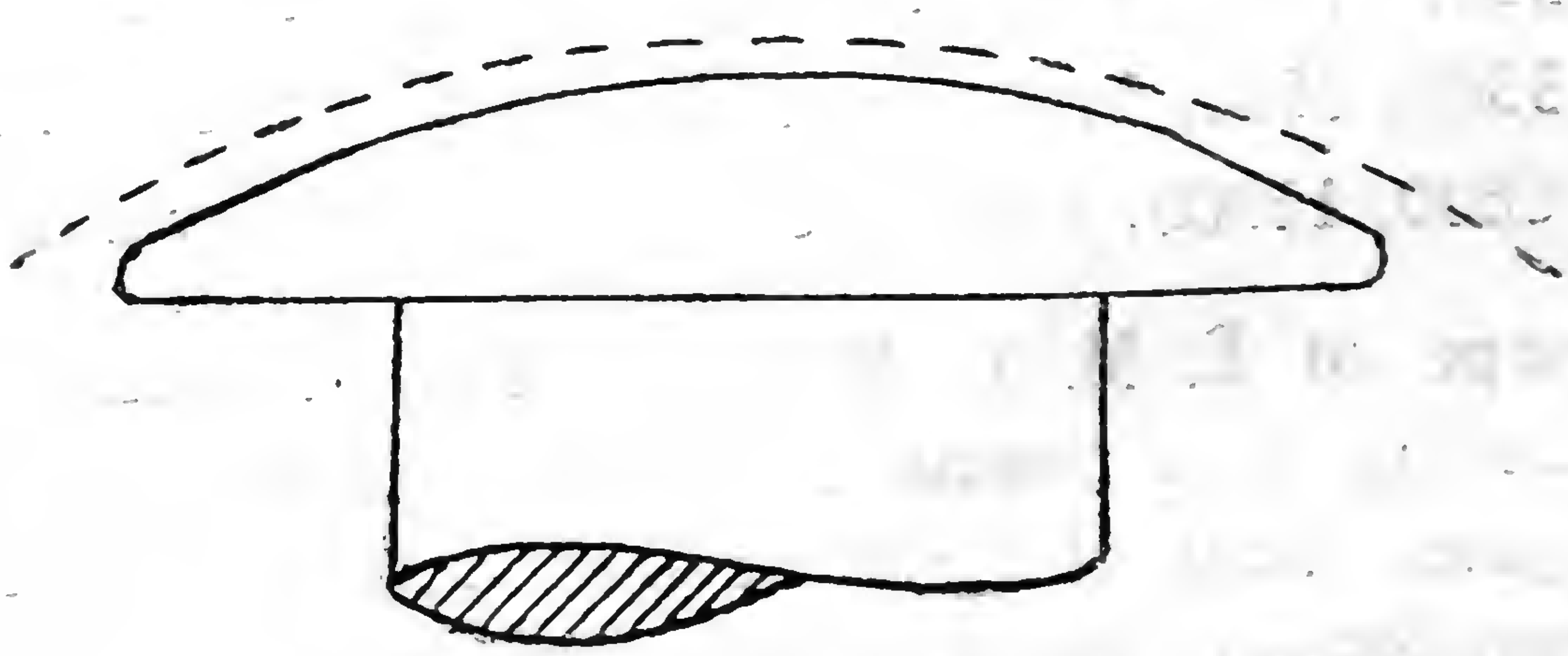


Fig. 1 (c)

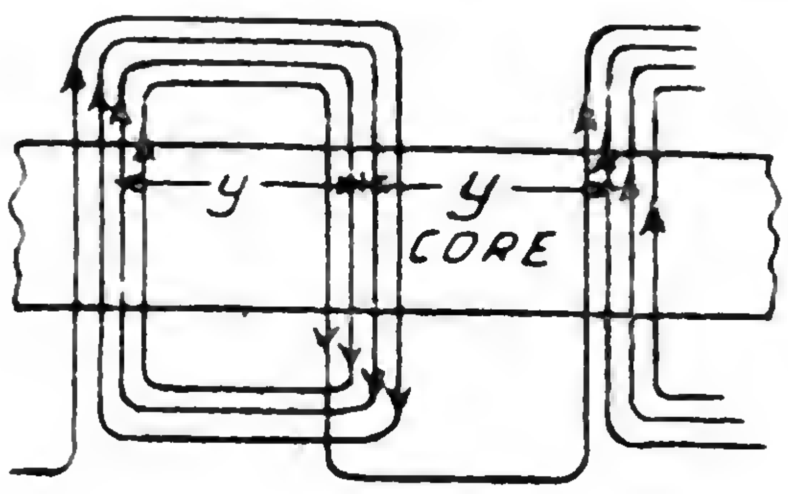
(3) In turbo-alternators, the unslotted portion of the rotor is 0·3 times the pole pitch, and the field coils are accommodated in several pairs of slots, the windings of each pair contribute towards the total m. m. f. The resultant air-gap density variation approaches very near to a sine wave law.

3. Armature Windings: The armature coils of an alternator are wound in the slots of the armature assembly, as in d. c. armatures. But where the d. c. winding is "closed" the alternator windings must

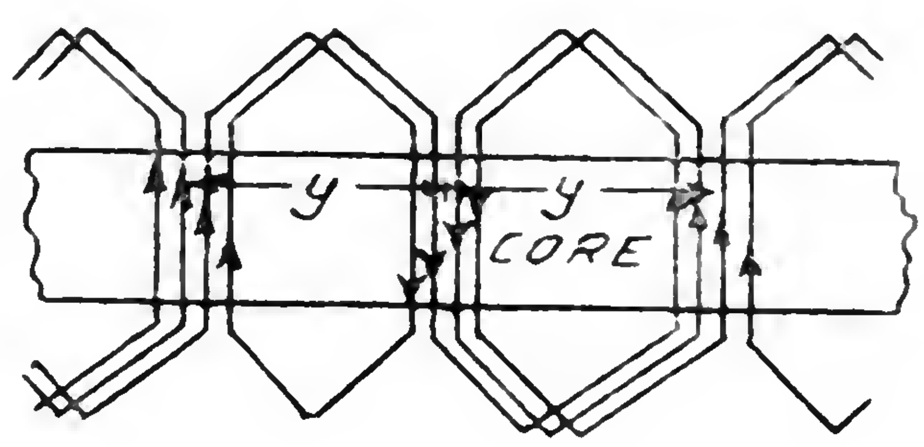
be "open". Both ends of the windings are brought out for making suitable connections. In order to improve the e. m. f. wave form, the two sides of a coil do not span a pole-pitch. The number of slots per pole range from 6 to 12.

Most of the a. c. stator windings are done by arranging the coils in the slots in one or two layers, called single-layer and two-layer windings. In the former case there is only one coil-side per slot, so that one coil completely fills up two slots.

Single-phase winding is done in several ways as shown in Fig. 2.



(a)



(b)

Fig. 2

The winding of Fig. 2 (*a*) is called *concentric* and that of Fig. 2 (*b*) is called *split-lattice*. Both are single-layer windings.

A 2-phase winding is obtained by placin two independent windings spaced 90° (electrical) apart. For instance, if there are 8 slot per pole, then one winding is placed in the first four consecutive slots and the other winding in the remaining slots of the pole. Fig. 3 shows a 2-phase concentric winding.

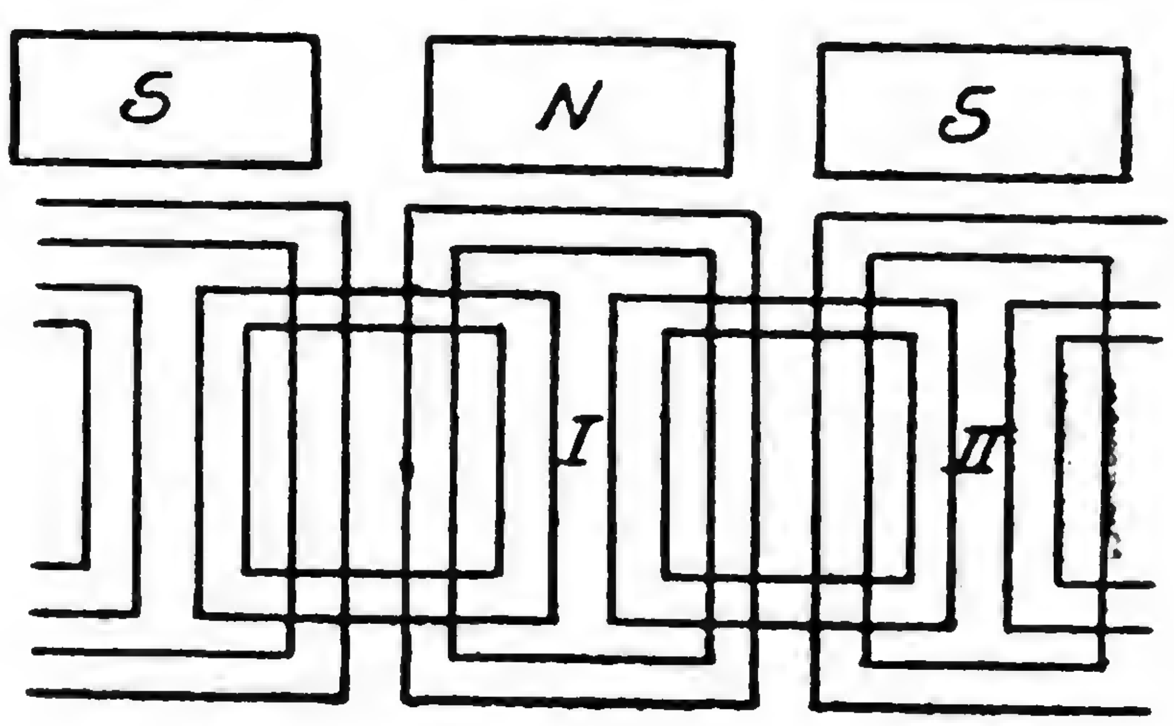


Fig. 3

In 3-phase windings, the winding of each phase occupies 1/3 the total number of slots per pole. Fig. 4 shows a 3-phase concentric winding for a 4-pole machine. Notice that the coils have two different shapes. Fig. 5 shows a 3-phase winding for an odd number of pairs of poles, (in this case number of poles = 6). Notice that

one coil side length (of only one coil) is longer than the other. This coil is called *mush.*

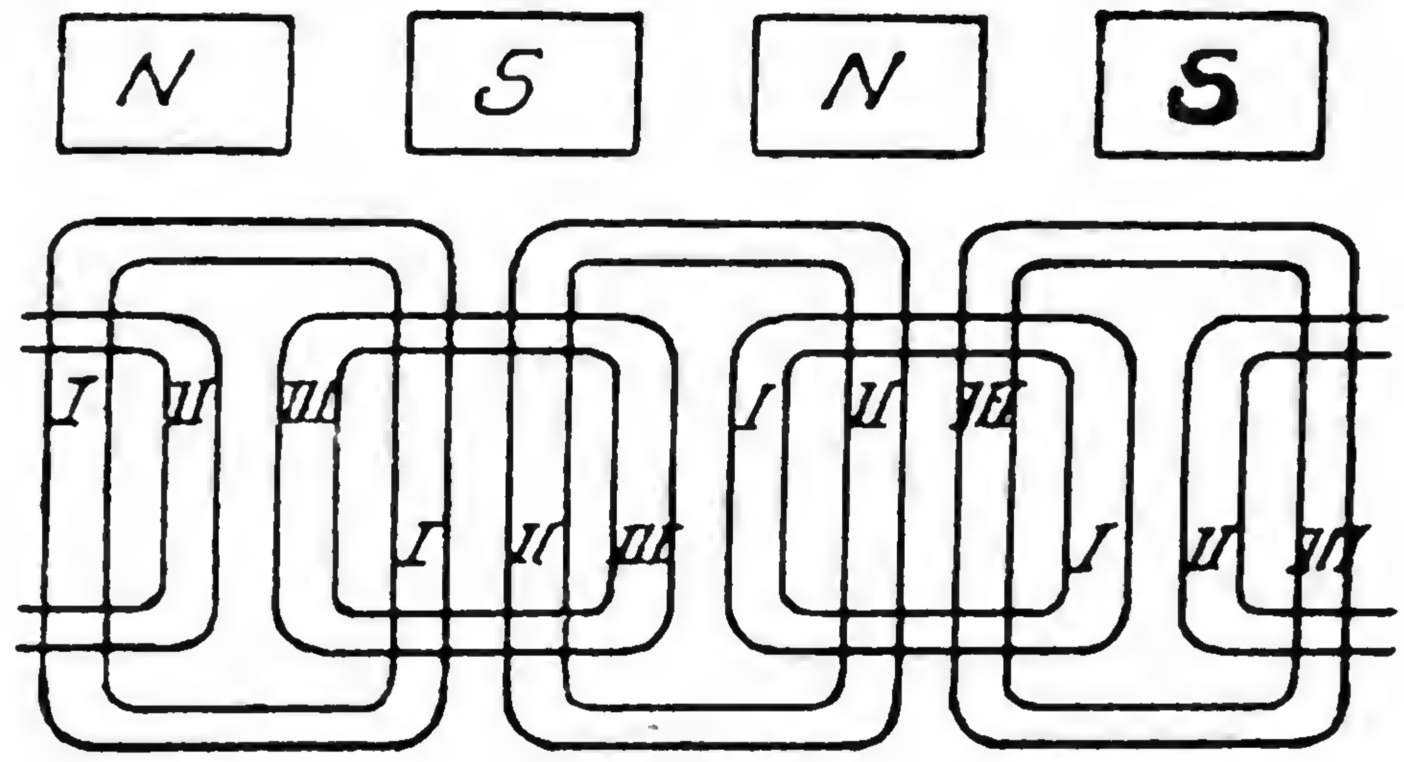


Fig. 4

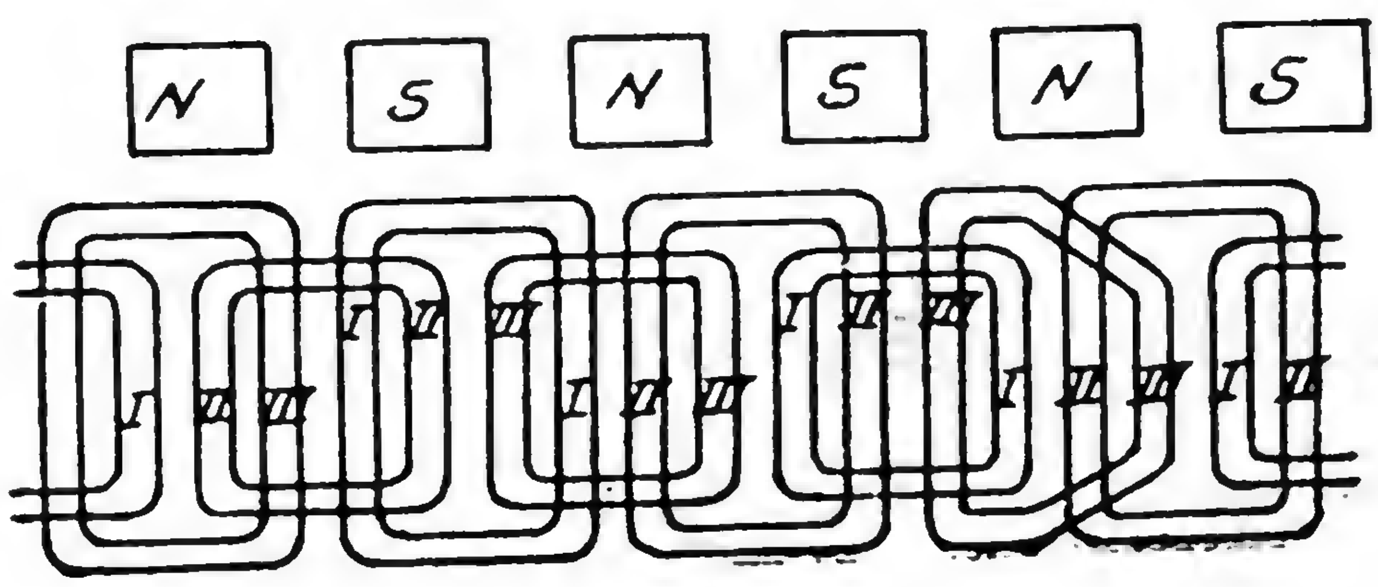


Fig. 5

If the number of slots per pole per phase = N, then the

$$\textit{phase spread } \sigma = \frac{\pi}{N} \text{ (electrical radians)}$$

and $$\textit{angular phase pitch } \beta = \frac{2\pi}{\textit{no. of phases}} \text{ (elec. radians)}$$

Thus for a 3-phase machine $\beta = 120°$ (electrical).

4. **Winding Factors:** (*a*) *the Breadth Factor* (k_b):—Consider a 3-phase winding where the number of slots per pole is 9. Hence the number of slots per pole per phase = 3 = N. Thus the winding of one phase occupies 3 consecutive slots under a pole and the coils of this phase occupy 3 slots one pole-pitch away, assuming full-pitch coils. If the e. m. f. generated in one coil-side is $e/2$, then the total voltage per coil is e volts. The three coils under a pole pair are joined in series. Therefore the total voltage of the three coils is the vector sum of the individual coil voltages, *if the coils are in separate slots.*

The angular displacement between each consecutive coil voltage is

$$= \frac{\pi}{N} \times \frac{1}{\textit{number of phases}} = \frac{\pi}{3} \times \frac{1}{3} = 20° \text{ (elec.)}$$

Thus Fig. 6 shows the 3 voltage vectors having a phase displacement of 20°. The total voltage is given be ad in the figure.

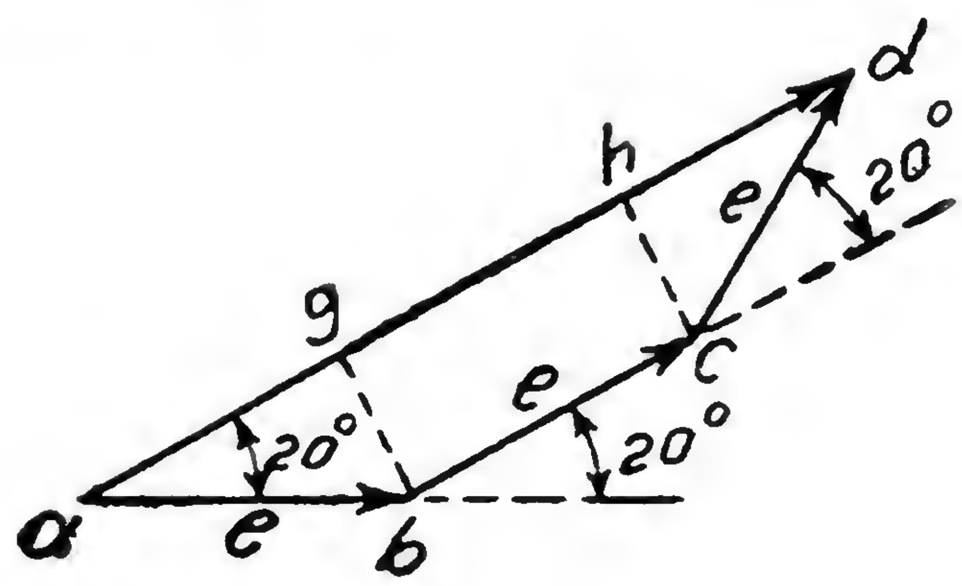


Fig. 6

$$ad = 2e \cos 20° + e \qquad [\cos 20° = 0{\cdot}94]$$
$$= e(2 \times 0{\cdot}94 + 1)$$
$$= 2{\cdot}88\, e.$$

$$\text{The breadth factor} = \frac{2{\cdot}88\, e}{3\, e} = 0{\cdot}96 \text{ for this case}$$

i. e. the breadth factor is a ratio $\dfrac{\textit{vectorial sum}}{\textit{arithmetic sum}}$ of individual coil voltages.

Consider a general case as shown in Fig. 7. Let N = number of slots per pole per phase,

α = angle of displacement between the two consecutive coil voltages, or, angle between two consecutive slots in elec. degrees.

If $ab = e = bc = cd = dg$ etc. the resultant voltage of these N coils is given by ag.

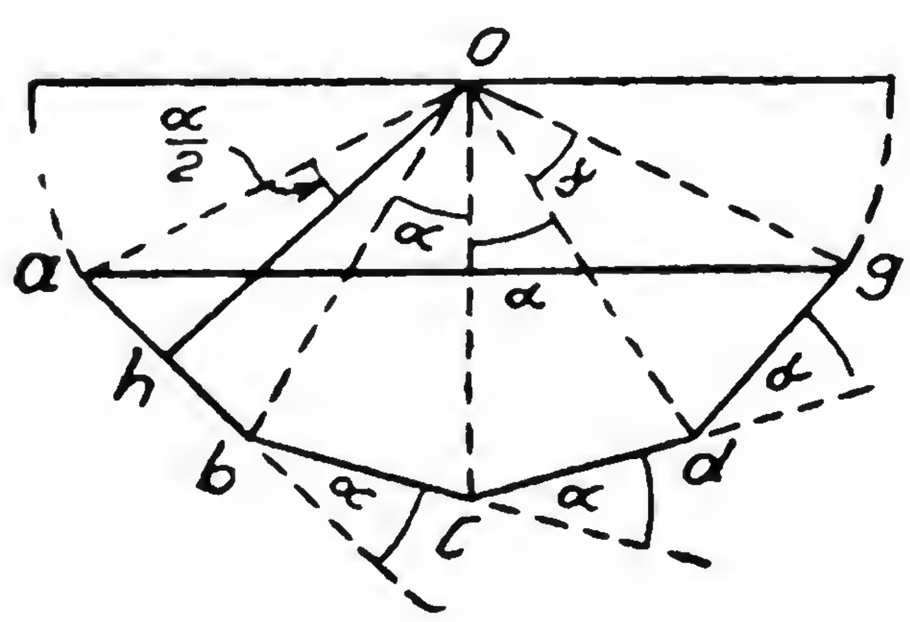


Fig. 7

From the figure

$$e = ab = 2ah = 2\,(oa) \sin \frac{\alpha}{2}$$

Similarly, $ag = 2\,(oa) \sin \dfrac{N\alpha}{2}$.

$$\text{The breadth factor} = \frac{ag}{3ab} = \frac{2\,(oa) \sin \frac{N\alpha}{2}}{2\,(oa)\, N \sin \frac{\alpha}{2}} = \frac{\sin \frac{N\alpha}{2}}{N \sin \frac{\alpha}{2}} \quad \dots (1)$$

(*b*) *Coil-span Factor* (k_c): If two coil-sides of a coil do not span exactly a distance of one pole-pitch, but a less distance, the coil is said to be **short-corded.** Thus the induced e. m. f.s of the two coil-sides has a phase displacement. See Fig. 8 where the angle of displacement is ψ in electrical degrees, or rather $(180° - \psi)$. The

sum of the two voltages is given by the line oq, each voltage is represented by op and pq. $op = pq = e$

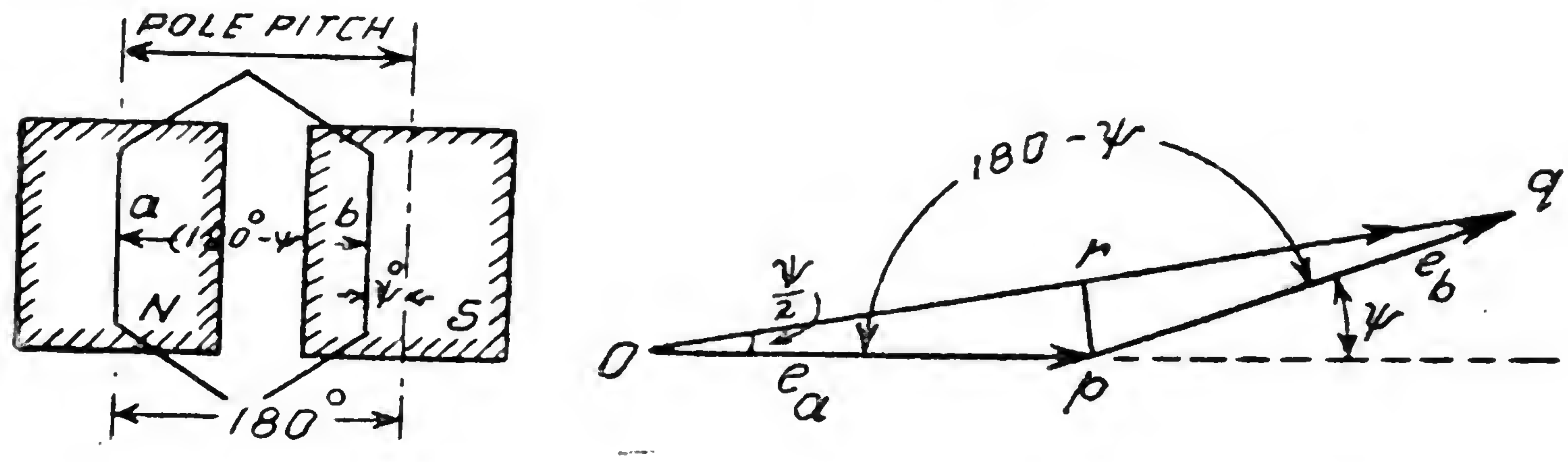


Fig. 8

$$oq = or + rq \quad \text{and} \quad or = rq \quad \therefore \quad oq = 2or$$

$$or = e\cos\frac{\psi}{2} \quad \therefore \quad oq = 2e\cos\frac{\psi}{2}.$$

Hence the coil-span factor $= \dfrac{2e\cos\frac{\psi}{2}}{2e} = \cos\dfrac{\psi}{2}$ ... (2)

5. The E. M. F. Equation :

Let f = frequency in cycles per second $= \dfrac{p \cdot n}{120}$

p = number of poles,

Φ = flux per pole (in lines),

Z_{ph} = number of conductors in series per phase,

$T_{ph} = \left(\dfrac{Z_{ph}}{2}\right)$ number of turns per phase,

n = revolutions per minute.

Hence

no. of lines cut per second by a conductor $= p\,\Phi\,\dfrac{n}{60}$

$\therefore$ ave. e. m. f. induced $= p\,\Phi\,\dfrac{n}{60} \times 10^{-8}$ volts. (per conductor)

Ave. e. m. f. induced per phase $= k_b\, k_c\, \Phi\,(2f)$

$\times\,(2T_{ph}) \times 10^{-8}$ volts

$= 4\,k_b\, k_c\, \Phi\, f\, T_{ph} \times 10^{-8}$ volts.

$\therefore$ r. m. s. value of voltage induced per phase

$= 4 \times 1{\cdot}11\, k_b\, k_c\, \Phi f\, T_{ph} \times 10^{-8}$ volts

$= 4{\cdot}44\, k_b\, k_c\, \Phi f\, T_{ph} \times 10^{-8}$ volts ... ... (3)

where k_b = breadth factor and k_c = coil-span factor, and 1·11 is the form factor for sine waves.

Example: A 50 cycle, 3-phase star-connected alternator has a synchronous speed of 375 r. p. m. and its armature has 144 slots and 6 conductors per slot. The flux per pole is 2·5 megalines distributed sinusoidally in the air-gap. The coils are short corded by 20° (elec.). Calculate the number of poles and the line voltage of this alternator on no load,

Solution: Since $f = \dfrac{p \cdot n}{120}$

$$p = \frac{120 f}{n} = \frac{120 \times 50}{375} = 16 \textit{ poles.}$$

$$N = \text{slots per pole per phase} = \frac{144}{16 \times 3} = 3;$$

$$\text{and} \qquad T_{ph} = \frac{144}{3} \times \frac{6}{2} = 144 \text{ turns/phase}$$

$\propto$ = elec. angle between slots

$$\propto = \frac{\pi}{N \times \textit{no. of phases}} = \frac{\pi}{3 \times 3} = 20^\circ \text{ (elec.)}$$

$$\text{breadth factor } k_b = \frac{\sin \dfrac{N\propto}{2}}{N \sin \dfrac{\propto}{2}}$$

$$\frac{N\propto}{2} = \frac{3 \times 20}{2} = 30^\circ,\ \sin 30^\circ = 0{\cdot}5$$

$$\frac{\propto}{2} = 10^\circ,\ \sin \frac{\propto}{2} = \sin 10^\circ = 0{\cdot}17365$$

$$\therefore \qquad k_b = \frac{0{\cdot}5}{3 \times 0{\cdot}17365} = 0{\cdot}96.$$

$$\text{The coil span factor } k_c = \cos \frac{\beta}{2}, \text{ where } \beta = 20^\circ$$

$$= \cos 10^\circ = 0{\cdot}985$$

The winding factor $= k_b \times k_c = 0{\cdot}96 \times 0{\cdot}985 = 0{\cdot}9456.$

Hence induced voltage per phase

$$V_{ph} = 4{\cdot}44 \times 0{\cdot}96 \times 0{\cdot}985 \times 2{\cdot}5 \times 10^6 \times 50$$

$$\times \left(\frac{144}{3} \times 6 \times \frac{1}{2} \right) \times 10^{-8} \text{ volts}$$

$$= 755 \text{ volts.}$$

$\therefore$ line voltage $= \sqrt{3} \times 755 =$ *1307 volts.*

6. **Armature Reaction**: Armature reaction means the effects of magnetic field produced by armature currents upon the main flux of the field windings. These effects may be (i) **cross-magnetising**, (ii) **de-magnetising** or (iii) **magnetising**.

Fig. 9 shows the direction of flux due to armature currents at unity power factor. The coils belonging to different phases are marked with Roman numerals, and the usual convention is adopted to indicate the direction of currents. The rotation of the machine is anti-clockwise, as indicated by the arrow. Therefore the relative motion of armature conductors with respect to the flux is *clockwise*, and by using Fleming's Right Hand Rule, the direction of currents in the armature coils is determined. Phase II has no voltage induced in it since its conductors, at the instant, cut no lines of force. Hence there is no current in the coils of phase II. The positions of the other two phases are such that equal voltages are induced in them, therefore the currents are in the directions shown. The direction of the flux produced by the coil (or armature) currents is marked, and it is evident that the effect is purely **cross-magnetising**.

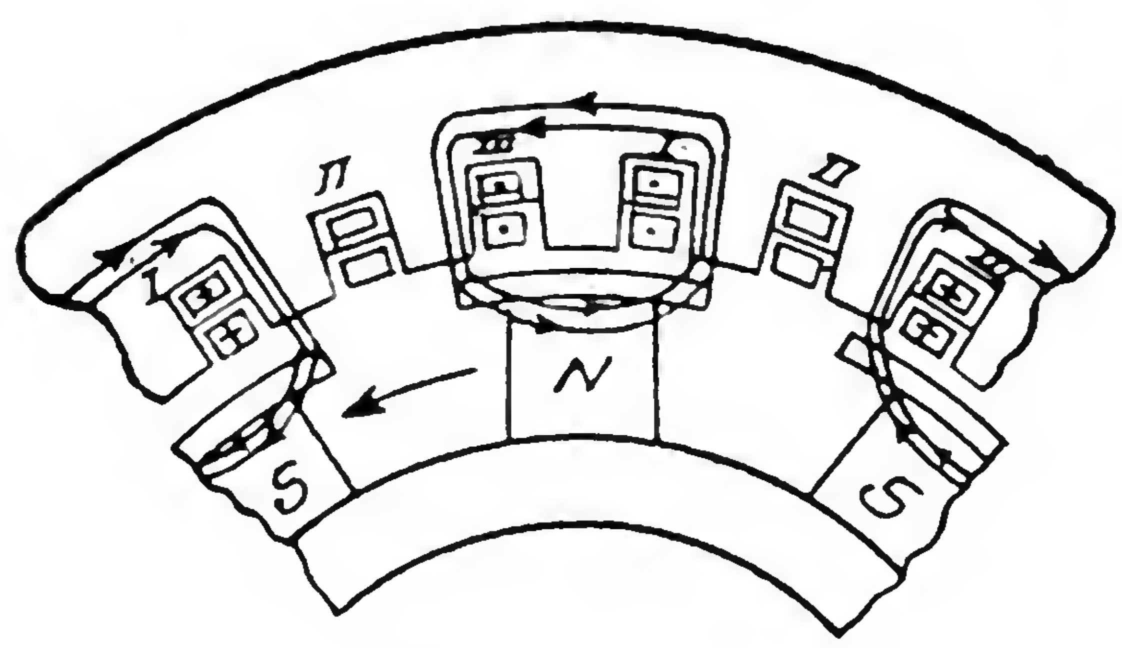


Fig. 9

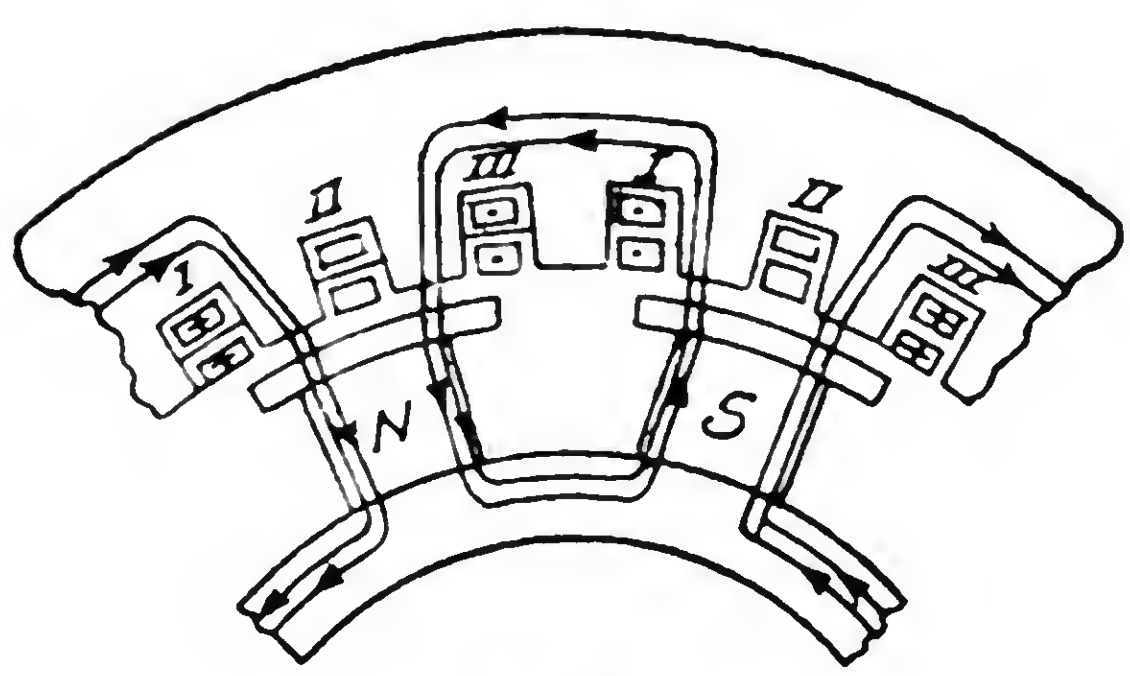


Fig. 10

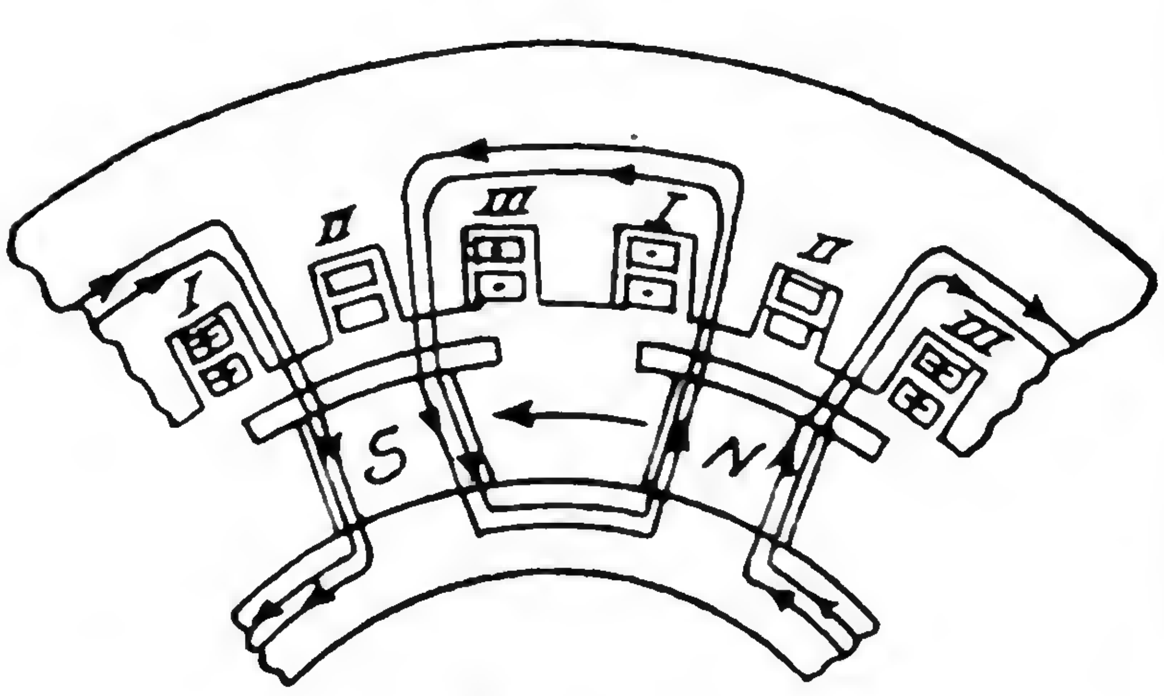


Fig. 11
Effects of Armature Reaction.

Fig. 10, which is for the condition of zero power factor (lagging), shows phase II under the centres of poles. The induced e. m. f. in its conductors is maximum, but the current is zero since it lags behind the voltage by 90° (electrical). The induced e. m. fs. in the conductors, or coils, of phases I and III, for the same instant, are equal and so also the currents, the directions of which are marked.

The direction of flux produced by these currents is seen to be altogether **de-magnetising.** The armature flux direction is opposite to the direction of flux of the main poles. The main flux is not shown but it is very easy to visualise its direction from the figures.

With similar reasoning, the effect of armature flux when the power factor is zero (leading), is seen to be **magnetising** as shown in Fig. 11. The interaction of armature flux with the main flux always produces a resultant flux.

Summarising, the conclusions are that

(*a*) at unity power factor, the effects of armature flux is **cross-magnetising**:

(*b*) at lagging power factors the effect is **de-magnetising**; and

(*c*) at leading power factors the effect is **magnetising.**

In other words, when the power factor is lagging more field ampere-turns are required, i. e. the main field system will require a larger field current to maintain a certain terminal voltage on load as the angle of lag increases. But when the power factor is leading the field current has to be reduced as the angle of lead increases.

Fig. 12 shows three vector diagrams:—(*a*) is for unity power factor, (*b*) for a large lagging power factor and (*c*) for a large leading power factor. IR is the ohmic resistance drop in phase with the current and IX_s is the reactance drop leading the current by 90°.

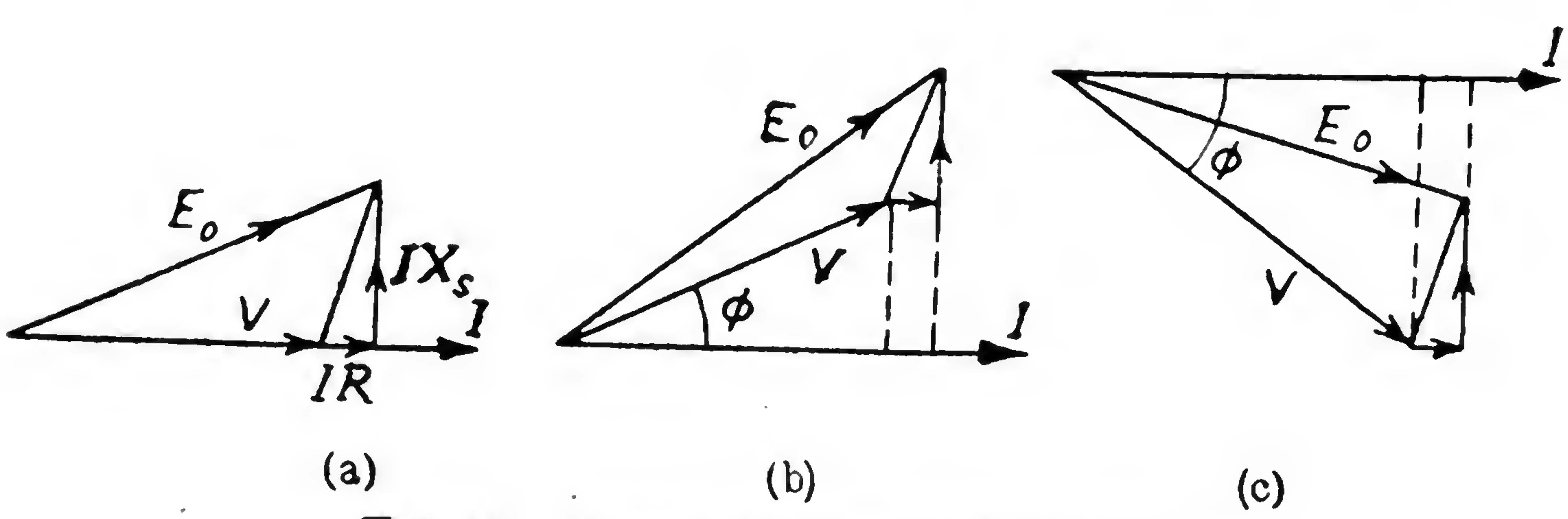


Fig. 12. Vector Diagrams: Regulation

Since I, R and X_s are assumed constant, the magnitudes of IR and IX_s drops in the three figures are the same. The vector sum of V, IR and IX_s is E_0. In the case of 3-phase alternators the above quantities must be considered to be phase values.

7. Synchronous Reactance:—The interaction of the main flux and the armature flux produces a resultant flux. If we assume that the permeability of the magnetic circuit is constant under all conditions, then the armature circuit may be considered to possess not only leakage reactance X but also an added reactance X_a, which is physically non-existent, but which, nevertheless, takes care of the effects of armature reaction in a simplified manner. Hence

$$X + X_a = X_s$$

where, X is the leakage reactance of the armature:

X_a is the ficticous quantity of reactance, assumed to take care of the effects of armature reaction, and

X_s is then the total armature reactance and is given the name of *synchronous reactance.*

With this in view, the vector diagram of Fig. 12 (b) has been altered as shown in Fig. 13, where IX and IX_a drops are separated. Fig. 14 gives the equivalent circuit of an alternator. The winding CC is *ideal,* the quantities X_a, X and R are placed outside. E_0 is the voltage induced by the field current alone. The equivalent drop due to armature reaction takes place in X_a, while the IR and IX drops take place in R and X respectively. The available voltage is then V, or rather, the terminal voltage on load is V. Thus the total impedance of the armature can be written as

$$Z_s = \sqrt{[R^2 + (X + X_a)^2]} = \sqrt{(R^2 + X_s^2)}$$

where Z_s is called the *synchronous impedance.*

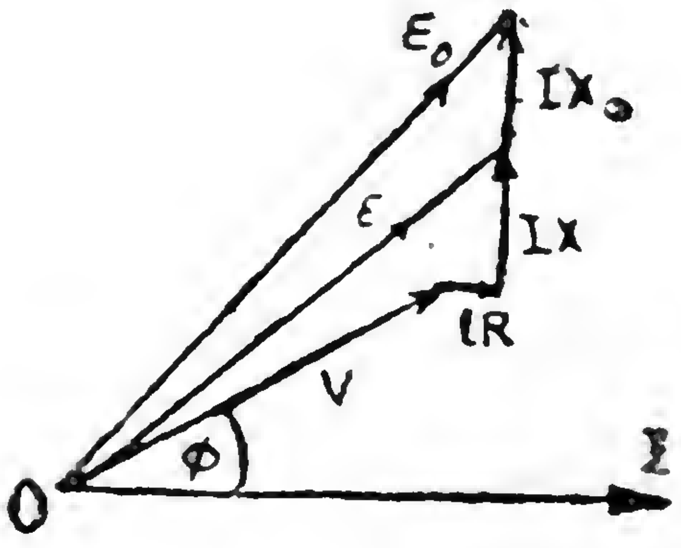


Fig. 13

Vector Diagram; Regulation

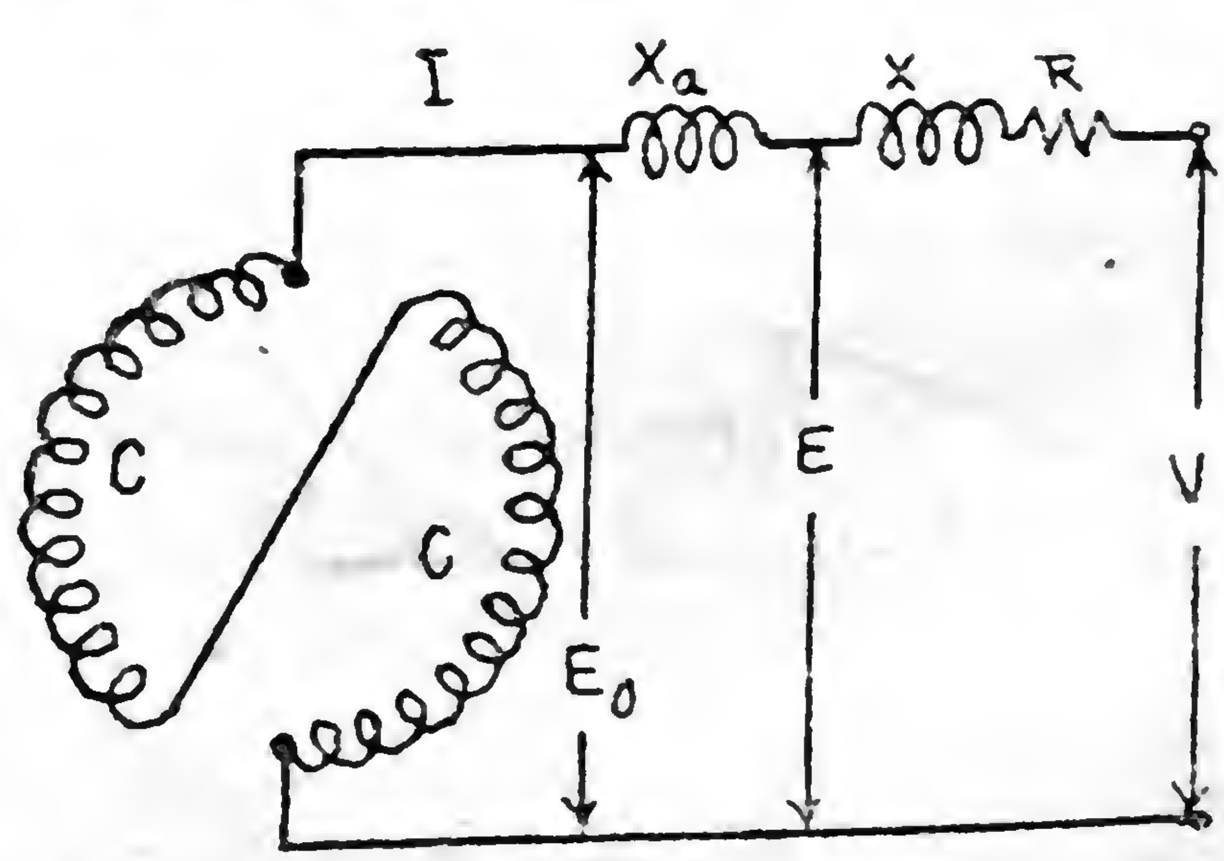


Fig. 14

Equivalent Circuit of Alternator.

The experimental determination of synchronous impedance is done by performing two tests on an alternator. One is the open-

circuit test and the other the short-circuit test. These tests are described in the next Section.

8. Open-Circuit and Short-Circuit Tests:

A. *The Open-Circuit Test*: The open circuit test is important, since it gives data for the calculation of alternator regulation The alternator is driven at constant normal speed and the machine is unloaded. The field excitation is varied over a very wide range and the terminal voltage is recorded for each change in the field current. The graph so obtained is the open-circuit characteristic or the magnetisation curve of the alternator. Its shape very nearly resembles the B–H curve. The circuit diagram is shown in Fig. 15.

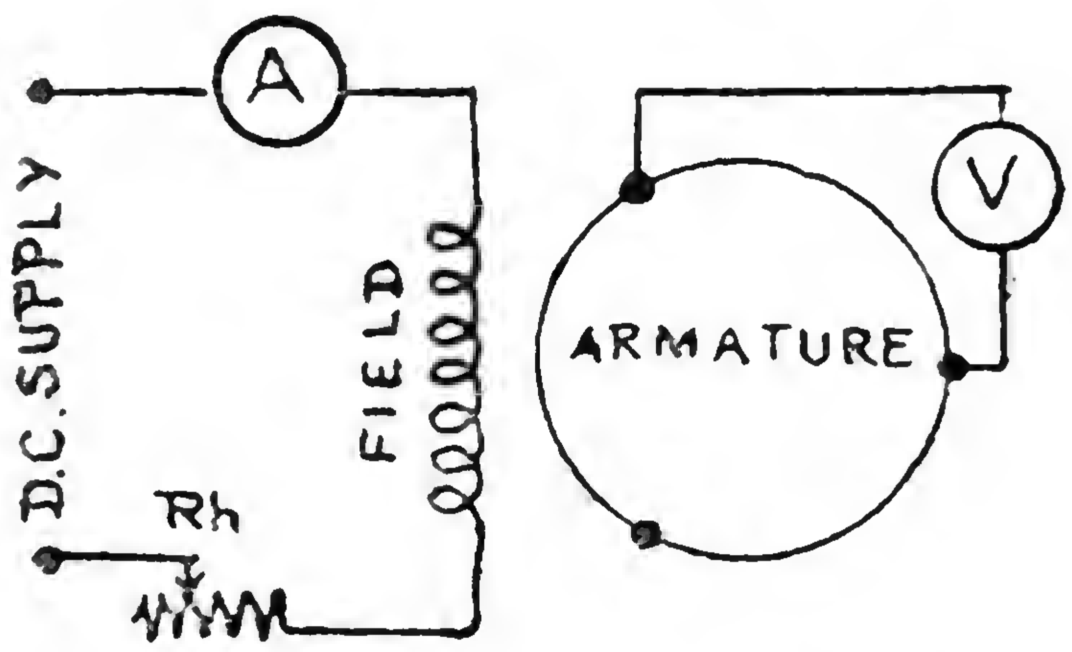


Fig. 15 Open-Circuit Test

B. *The Short-Circuit Test*: The short-circuit test is performed by running the alternator at constant normal speed, with the terminals of machine short-circuited through ammeters. The field excitation current is zero, to begin with. Readings are taken as the

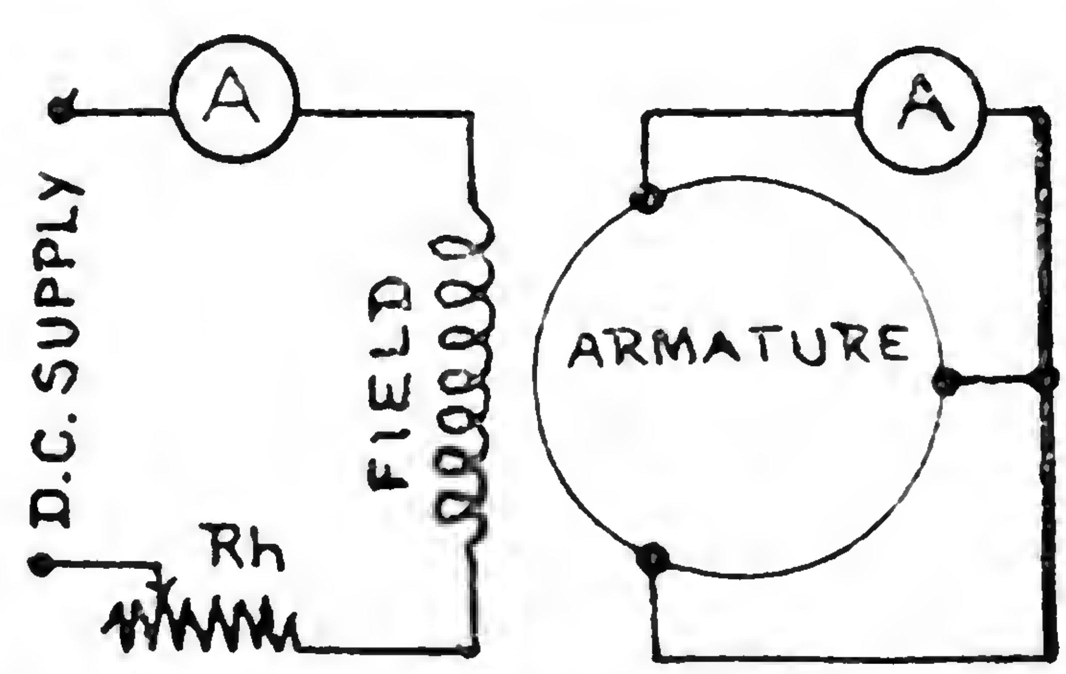


Fig. 16 Short-Circuit Test

field current is gradually increased from zero, until alternator armature current is twice its full load current. The graph of field current against-armature current is called the short-circuit characteristic and it is almost a straight line as shown by the line *OB* in Fig. 17.

When the armature is shorted, the terminal voltage is assumed to have dropped to zero. The whole of the induced e. m. f. is consumed by the synchronous impedance Z_s. The line OB is

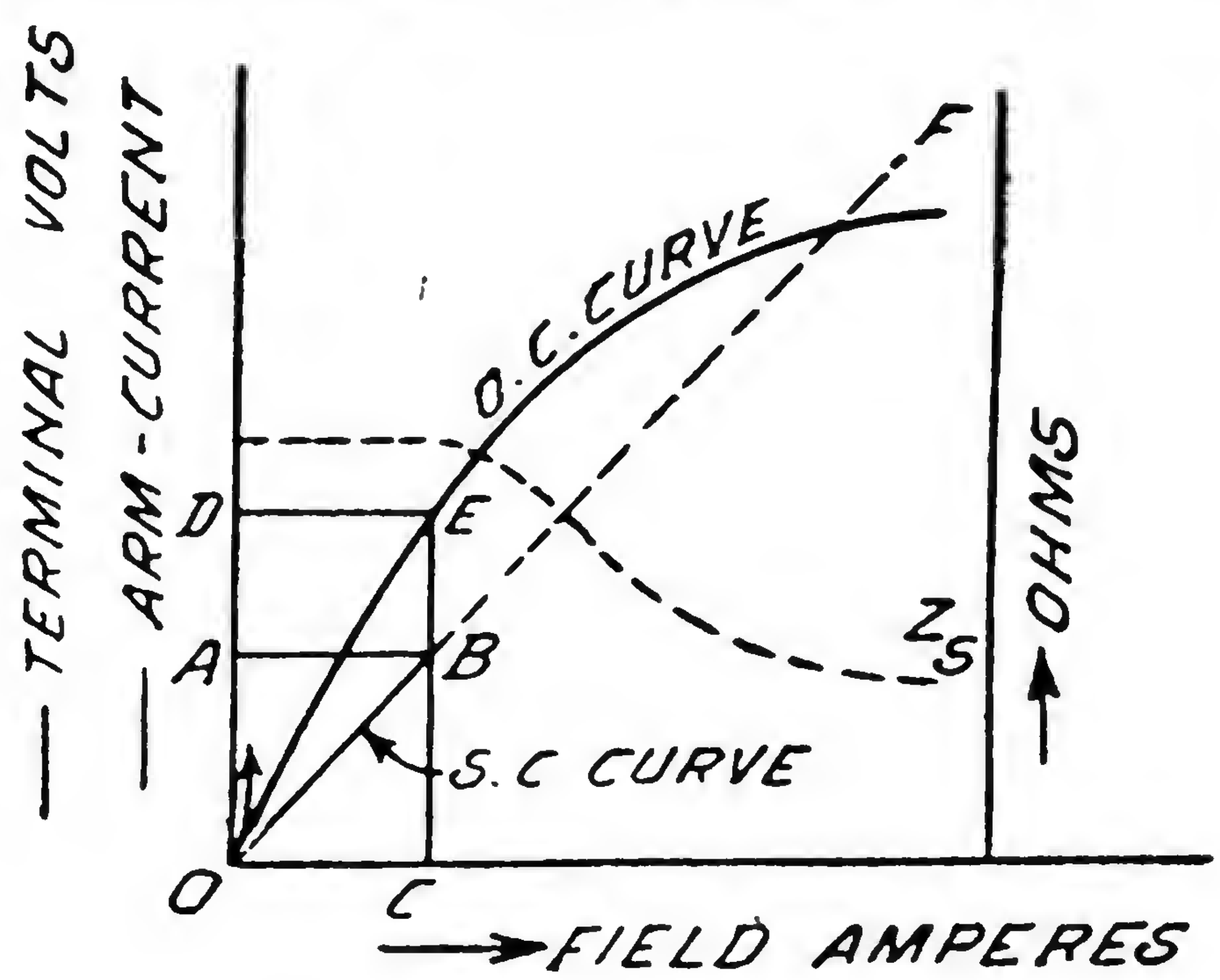


Fig. 17 O. C. and S. C. Curve.

produced to cut the open-circuit characteristic curve. Hence, if two values, one on the $O.$ $C.$ curve and the other on the $S.$ $C.$ curve for the same field excitation are taken we get the value of the synchronous impedance Z_s by the ratio

$$\frac{EC\ (\,volts\,)}{BC\ (\,amperes\,)} = Z_s.$$

The ratio is not constant, because the $O.$ $C.$ curve bends in the middle. At low magnetic saturation the values of Z_s are high. An approximate graph of Z_s against field current is drawn by a dotted line as shown in Fig. 17. By measuring the ohmic resistance R of the armature winding the synchronous reactance is obtained from the expression

$$X_s^2 = Z_s^2 - R^2, \text{ or } X_s = \sqrt{(Z_s^2 - R^2)} \quad \ldots \quad (4)$$

In the case of 3-phase machines all calculations should be done on only one of the three phases. Usually 3-phase alternators are star-connected. The neutral point may or may not be available. If the neutral point is available, phase values of resistance, current, voltage, etc. are obtained easily, but if the star point is not available, measurements that are taken must be reduced to phase values.

9. Voltage Regulation of Alternators: The voltage regulation of alternators is defined as "the rise in terminal voltage when full load is thrown off, the field excitation and the speed remaining

constant". Regulation is always stated for full load and in percent of terminal voltage on full load. Moreover the power factor also must be stated, since the value of regulation varies with power factor. If E_0 is the no load terminal voltage and V is the full load terminal voltage, then

$$\% \textbf{ regulation} = \frac{E_0 - V}{V} \times 100 \quad \ldots \quad \ldots \quad (5)$$

Thus from Fig. 12 it is seen that the difference between E_0 and V varies as the power factor. Referring to Fig. 12,

(i) for unity power factor,
$$E_0 = \sqrt{[(V + IR)^2 + (IX_s)^2]} \quad \ldots \quad \ldots \quad (6)$$

(ii) for lagging power factor,
$$E_0 = \sqrt{[(V\cos\phi + IR)^2 + (V\sin\phi + IX_s)^2]} \ldots \ (7)$$

(iii) for leading power factor,
$$E_0 = \sqrt{[(V\cos\phi + IR)^2 + (V\sin\phi - IX_s)^2]} \ldots \ (8)$$

This is similar to that adopted for finding the regulation of transformers. But the approximate formula must not be used in the case of alternators because the magnitudes involved are much greater in percentages in the case of alternators.

Since it is very incovenient to determine the regulation of large alternators by direct loading, methods have been devised to determine the regulation without loading them. The two most common methods are :—

(*a*) **the synchronous impedance method,** and

(*b*) **the ampere-turn method.**

The drop of voltage in the armature of an alternator is due to (i) IR_a drop, (ii) IX_a drop and (iii) armature reaction. The armature resistance drop is very small compared to (ii) and (iii) and therefore it is many times ignored for the purpose of determining the regulation.

In the first method (*a*) the effect of armature reaction is taken into account by assuming a ficticious reactance in series with the leakage reactance of the armature winding and these two constitute synchronous reactance X_s. See Section 7.

In the other method (b) armature leakage reactance is assumed to be additional armature reaction, which is just the converse of method (a). Thus assuming the power factor on short circuit to be zero, R_a being neglected, the field ampere-turns required to produce full load current under short-circuit condition must be exactly centralising the demagnetising armature ampere turns.

The data required to determine regulation by the two methods are :

(i) The O. C. characteristic,

(ii) The S. C. characteristic, and sometimes,

(iii) The ohmic resistance of armature winding (per phase value).

A. **The Synchronous Impedance Method:** Procedure :—

(a) Plot the O. C. characteristic from the O. C. test data.

(b) Plot the S. C. characteristic from the S. C. test data.

(c) Measure the ohmic resistance per phase of armature (R_a).

(d) Calculate for full load the synchronous impedance Z_s from two characteristic curves, or from the test data.

(e) Calculate X_s from $X_s = \sqrt{[Z_s^2 - R_a^2]}$.

Example: A 500 volt, 50 kVA alternator on test requires a field excitation of 10 A to give 460 volts on open circuit and to produce 200 A on short circuit. Its armature resistance is 0·22 ohm. Calculate percentage regulation for full load at (a) 0·8 p. f. lag. and (b) 0·8 p. f. leading.

Solution: Full load current $= \dfrac{50 \times 1000}{500} = 100$ A.

$$\text{Syn. impedance } Z_s = \frac{460}{200} = 2{\cdot}3 \text{ ohms.}$$

$$\therefore \quad X_s = \sqrt{[(2{\cdot}3)^2 - (0{\cdot}22)^2} = 2{\cdot}287 \text{ ohms.}$$

(a) $E_0 = \sqrt{[(500 \times 0{\cdot}8 + 100 \times 0{\cdot}22)^2 + [500 \times 0{\cdot}6 + 100 \times 2{\cdot}287)^2]}$
$= 676\ V$

$$\therefore \quad \%\ regulation = \frac{676 - 500}{500} \times 100 = 35{\cdot}2\ \%$$

(b) $E_0 = \sqrt{[(500 \times 0{\cdot}8 + 100 \times 0{\cdot}22)^2 + (500 \times 0{\cdot}6 - 100 \times 2{\cdot}287)^2]}$
$E_0 = 428$ volts

$$\therefore \quad \%\ regulation = \frac{428 - 500}{500} \times 100 = -\ 14{\cdot}4\ \%.$$

B. The Ampere-Turn Method: Procedure :—

(*a*) Construct the O. C. and S. C. characteristics from the test data.

(*b*) Determine the vector sum of V and IR_a [OA in Fig. 18]. Hence OC is the field current (or field ampere-turns).

(*c*) Since OD is the full-load current, OE is the field current necessary to pass full load current on short-circuit.

(*d*) Draw a line from C at an angle $(90+\phi)$ making $CF=OE$.

(*e*) Vector sum of OC and CF is OF $(=OG)$ the necessary field current (or ampere-turns).

(*f*) At field current OG the induced e. m. f. from O. C. C. is $GQ = OH = E_0$.

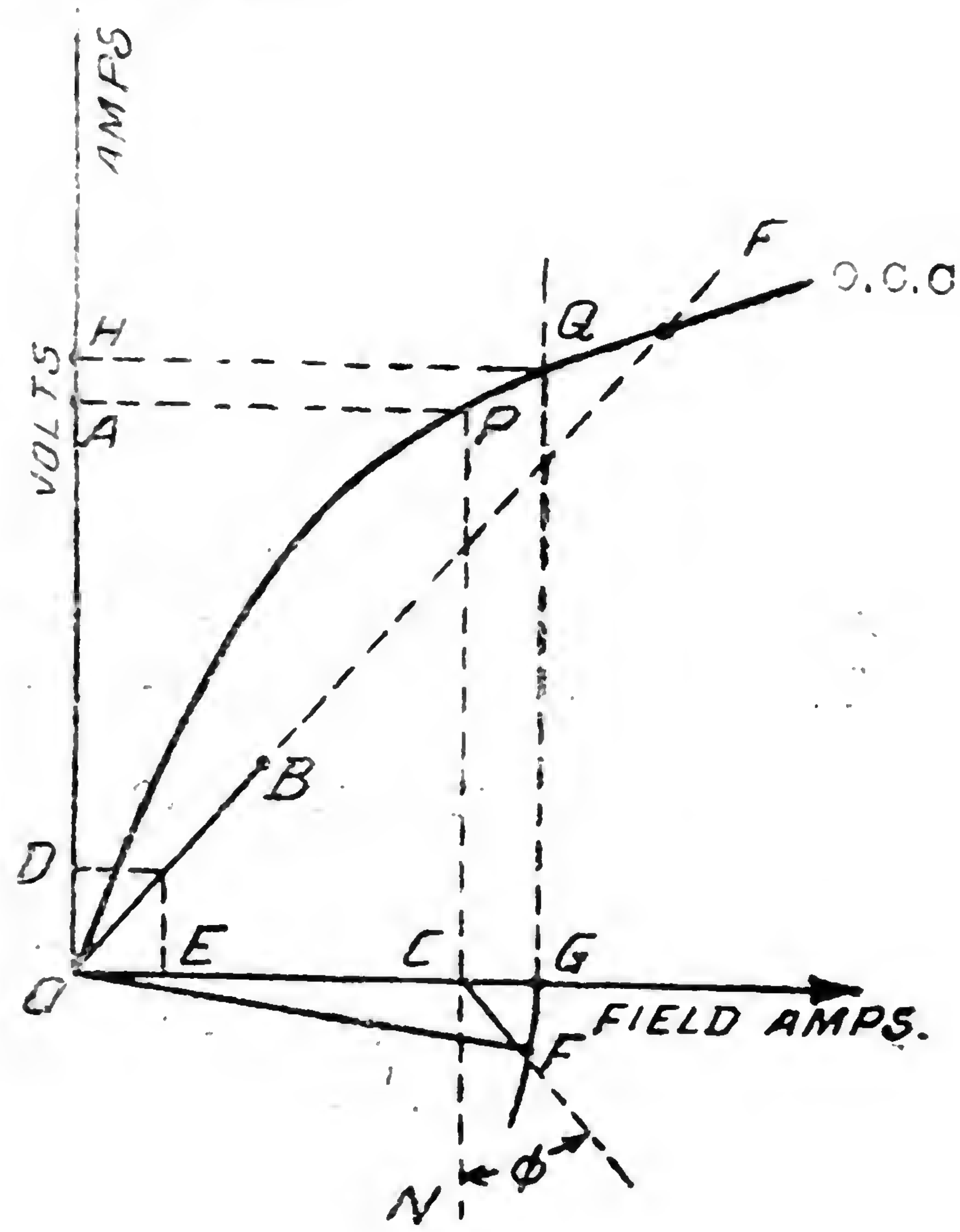


Fig. 18

Thus $\% \; regulation = \dfrac{E_0 - V}{V} \times 100$ as before.

The results by this method are nearer the correct answer than that given by the synchronous impedance method.

The right-hand side of Fig. 19 shows the vector diagram of voltages and the left-hand side that of ampere-turns or excitation currents,

$$V_1 = V + IR_a \text{ (vectorially)}$$

where I is the load current at angle ϕ to V the terminal voltage. In the diagram

I_a is the field current (or amp-turns) necessary to circulate full load current on short circuit.

I'_f is the field current to produce V_1.

I_f is the vector sum of I'_f and I_a

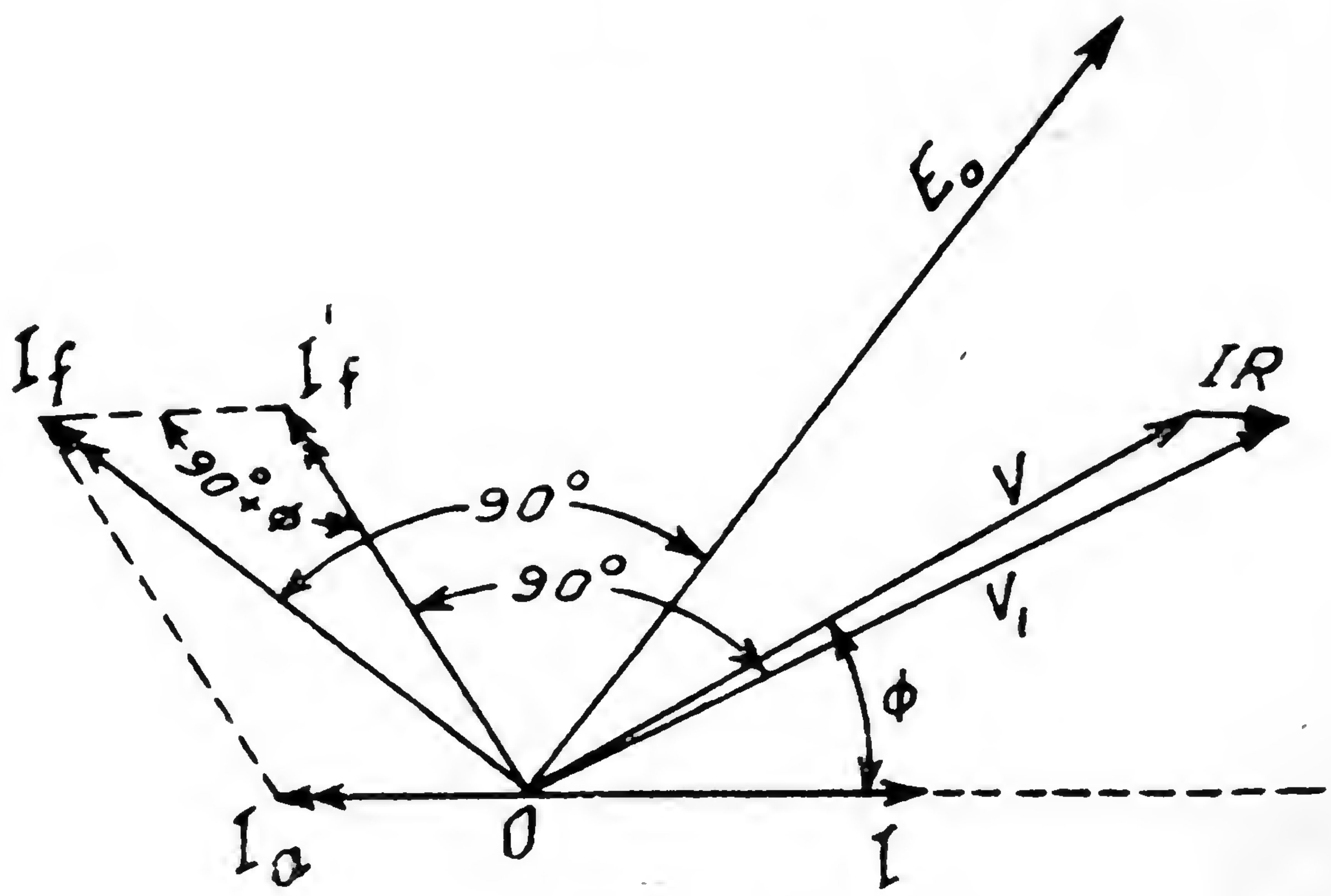


Fig. 19

A line is drawn at an angle of 90 degrees to I_f. E_o lies along this line, and from the O. C. C. the value of E_o is obtained when the excitation is I_f. Thus regulation is calculated by this method.

10. Efficiency: It is not possible to find the efficiency of large capacity alternators by direct loading. Therefore the losses of a machine are determined indirectly. The losses in an alternator are:-

(*a*) **Armature copper loss** ($I^2_{ph} R_{ph}$), where I_{ph} current per phase and R_{ph} resistance per phase. The resistance is measured by "ammeter-voltmeter" method by passing direct current. Due to skin effect this value is increased by 20 p. c.

(*b*) **Iron and friction losses** are measured by coupling an alternator to a d. c. motor whose efficiency is known at all loads. The motor drives the alternator at synchronous speed with the alternator getting its normal excitation. The input to the motor is carefully measured. Hence from the efficiency of the motor for this loading the output is determined. This output is equal to the iron and friction losses of the alternator.

(*c*) **Field winding loss** is easily determined since the circuit is d. c.

11. Alternators in Parallel: For two allernators to run in parallel the following conditions must be fulfilled:—

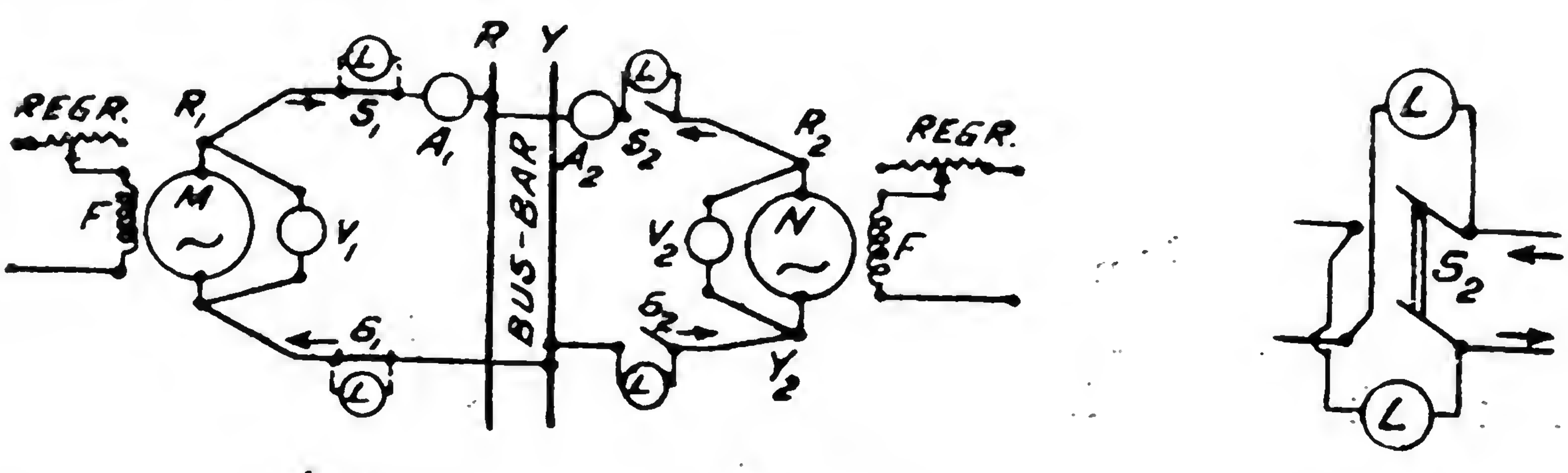
(1) The voltage of the "incoming" machine must be equal to the bus-bar voltage, (or equal to the terminal voltage of the other machine, in the case where only two alternators are to run in parallel).

(2) The frequency of the "incoming" machine must be equal to the bus-bar frequency.

(3) At the instant of paralleling, the two voltages must be in phase.

(4) In the case of 3-phase machines the phase sequence must be the same.

The operation of paralleling an alternator with other alternators is called **synchronising.** This is done with help of a synchroscope or by using incandescent lamps connected across the paralleling switch in a particular manner. When once paralleled, an alternator stays in synchronism. Any tendency to upset synchronism (due to sudden load etc.) is at once counterbalanced by setting up a **synchronising torque** in the machine.



(a) Fig. 20 (b)

In Fig. 20 (a) are shown two single-phase alternators M and N. M is supplying power to a load through the bus-bar RY. The procedure of paralleling M and N is as follows:—

(i) The prime mover of N is started and is brought to its normal speed.

(ii) The alternator field is supplied with direct current, and by adjusting its field regulator the terminal voltage of N is made equal to that of M. This is indicated on the two voltmeters.

(iii) At this stage the lamps across the double-pole switch S_2 will become alternately dark and bright if the speeds of the two alternators are different. The changes in the two lamps cccur less

frequently as the difference in the two speeds is reduced. Hence the speed of N is so adjusted that the interval of time between dark and bright condition of the lamps is longer, say about 5 seconds.

(iv) When the voltmeters show equal readings and the interval between the two conditions of the lamps is long, the switch S_2 is closed *at the instant* when the lamps are dark. Thus N is paralleled with M.

The arrows in the diagram indicate the direction of currents at the instant when the terminals R_1 and R_2 are positive with respect to the terminals Y_1 and Y_2 respectively.

Alternatively, if the lamps are connected across the paralleling switch in the manner shown in Fig. 20 (b), the switch, S_2 must be closed *at the instant* when the lamps are bright.

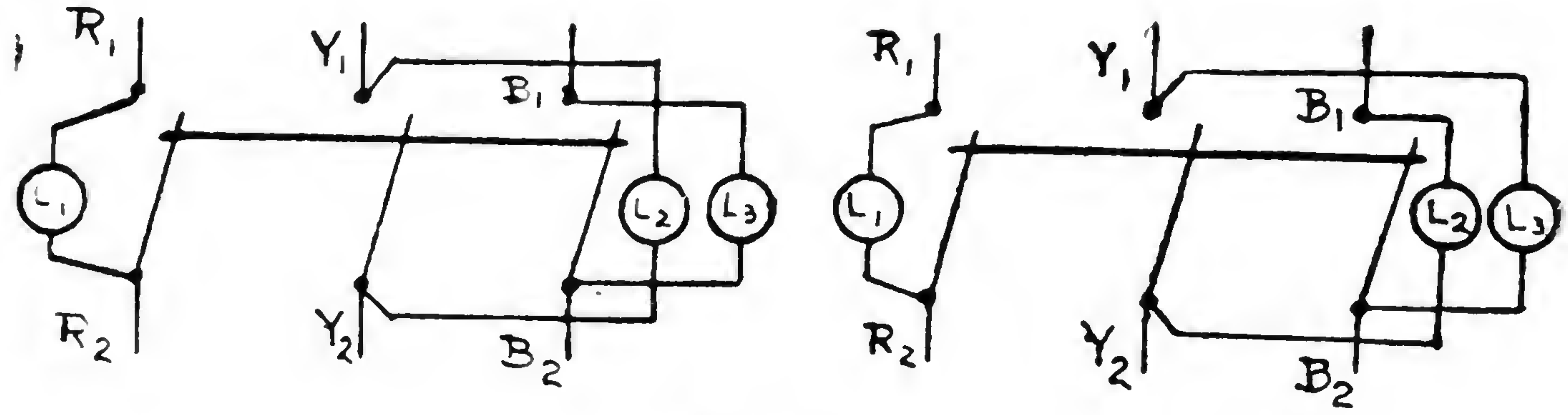
The voltage of machine N is equal and in phase opposition to that of machine M *with respect to the local circuit*, and these two voltages are in phase with each other *with respect to the external circuit.* The local circuit consists of the armature winding resistances of both mechines and the external circuit means the load resistance. Thus when the two machines are to be parallel at a particular instant, then at that instant there is no voltage across the lamps of Fig. 20 (a).

In the case of paralleling 3-phase alternators the phase squence should be determined before connecting the lamps across the paralleling switch which in this case is a triplepole switch.

Various methods are employed to determine the phase sequence of voltages. One of them is to use a phase-sequence tester which has a rotating disc marked with an arrow. In fact the tester is a miniature induction motor having three (and sometimes six) terminals with identification marks. When the tester is connected to the paralleling switch on the bus-bar side, the indicator disc rotates, say, in the clockwise direction. The corresponding phase sequence of the in-coming machine is determined by connecting the terminals of the tester to the terminals of the paralleling switch on the in-coming machine side in such a manner that the rotation of the disc is in clockwise direction.

Lamps are connected in one of the two ways across the paralleling switch. When the lamps are connected as shown in Fig. 21 (a)

the paralleling switch is closed at the instant when all the lamps are dark. But when the connections are as in Fig. 21 (b) the switch is closed at the instant when lamp L_1 is dark and the lamps L_2 and L_3 are equally bright. The connection of Fig. 21 (b) is better and therefore more popular.



(a) Fig. 21 (b)

If the paralleling switch is closed when the vectors are a little out of phase a small circulating current will flow through the armature winding of the in-coming machine and will pull up the machine into exact synchronism. However, if the phase difference is large at the time of synchronising, the circulating current will be very heavy. This may cause a serious disturbance on the whole system.

The modern practice is to use **rotary synchroscopes** instead of lamps. Both the rotor and the stator of the synchroscope motor are wound for 2-phase operation. The stator is connected to the bus-bars and the rotor to the corresponding terminals of the in-coming machine. The splitphasing is done inside the instrument by having an inductive coil X in series with one winding and a non-inductive resistance R in series with the other winding as shown in Fig. 22.

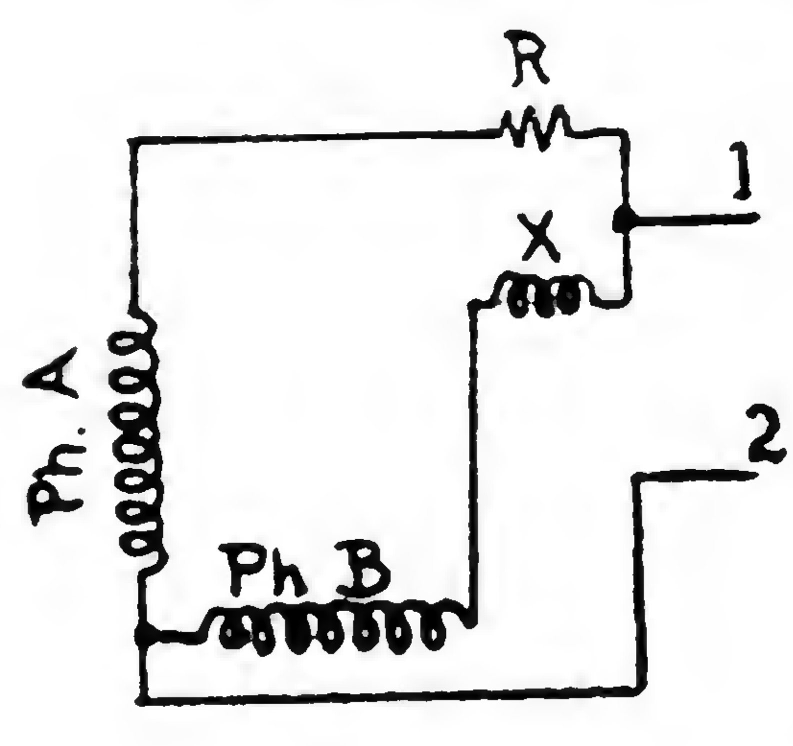


Fig. 22

A pointer is provided on the spindle of the rotor. If the in-coming machine runs slower the pointer rotates in anti-clockwise direction, and in clockwise direction if the in-coming machine runs faster. This action is due to the difference between the frequency of the in-coming machine and that of the bus-bar. When the two frequencies are equal there is no movement of the rotor. The exact moment for synchronising is when the pointer of the instrument moving very slowly is over the top vertical thick line of the synchroscope.

Within the limit of maximum power nothing will upset the parallel running of alternators. Suppose for some cause if the prime mover of M runs a little faster, the phase position of its voltage vector will advance by a small angle δ. Hence there will be a resultant voltage E_s which will cause a circulating current I_s to flow in the local circuit, i. e. in the two armatures. This current lags behind E_s by more than 80°, since the impedance of the local circuit is mostly inductive. This means that I_s is almost in phase with the voltage of M, and I_s is nearly at 180° with the voltage of N. In other words I_s is the "load" current of M and the "motoring" current of N. In Fig. 23 these vectors are shown to illustrate what has been stated so far. Angle θ is more than 80° and

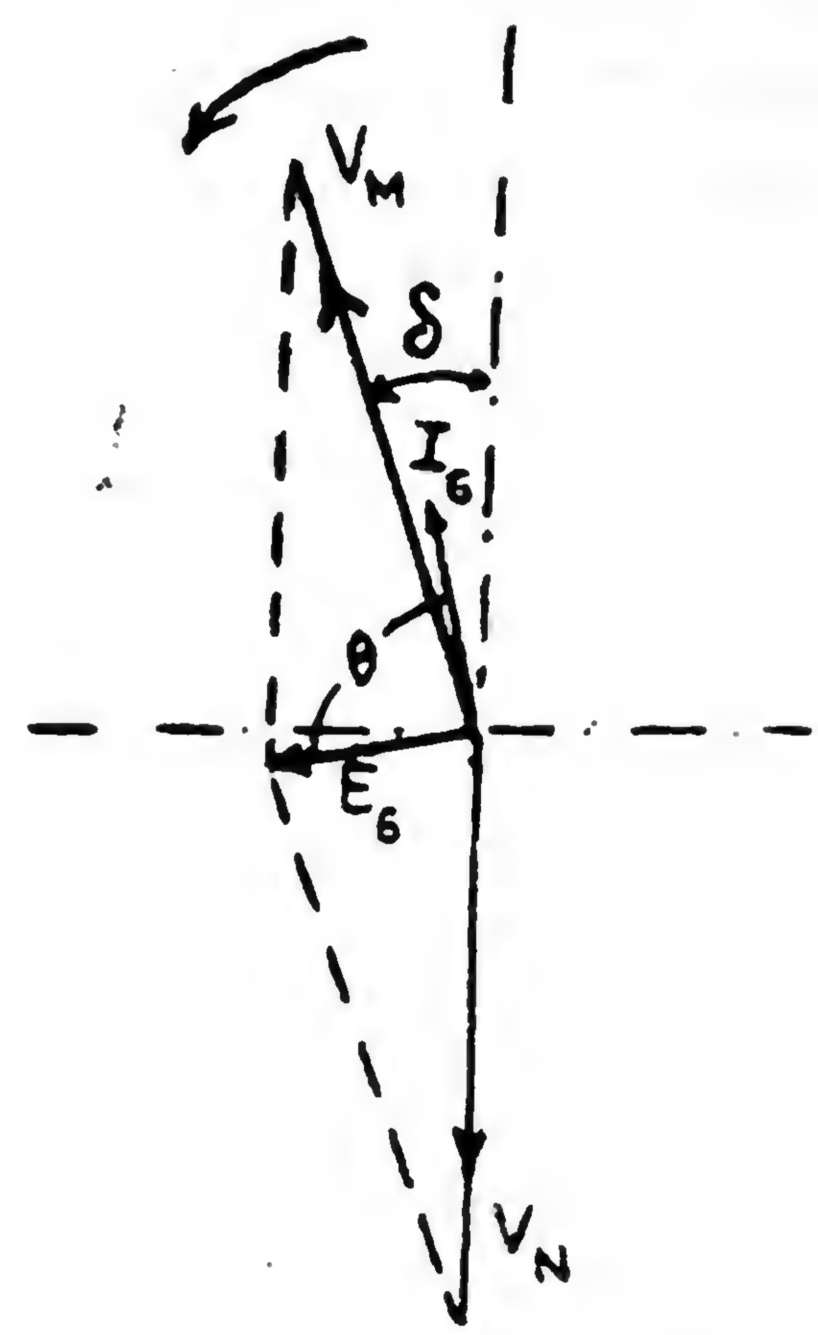


Fig. 23

$$\theta = \tan^{-1}\frac{X}{R}$$

where R and X are the resistance and reactance of the local circuit.

Generally, an increase in load means a reduction in speed and a decrease in load causes an increase in speed. Thus M will fall back towards its original phase position, and N will advance in its phase position. Thus the two machines will again assume their original phase positions.

The extra power of M is

$$P_s = V_M \times I_s \text{ (approx,)}$$

P_s is called the ***synchronising power*** and the current I_s is called the ***synchronising current.*** Thus an alternator, running in synchronism with another is pulled up in step immediately and automatically if it tries to run a little faster than the other.

12. Effect of Changing the Field Excitation of one Machine when in Synchronism with other Machines: When one machine ***alone*** supplies a load, a change in its field excitation results in the change of its voltage, while its power factor does not change. The power factor is determined solely by the load impedance.

But when a machine works in parallel with other machines, the state of affairs is entirely different. In what follows, it is assumed that the capacity of the alternator under discussion is a small percentage of the total capacity of the other generators connected to the same bus-bars. Under this assumption, the voltage and frequency of the bus-bars is constant, and the alternator is then said to be connected to *infinite bus-bars.* As far as Fig. 24 is concerned let

V = bus-bar voltage (assumed constant),
E_0 = induced voltage of the alternator,
Z_s = impedance of the armature of the alternator,
I_s = current in the armature of the alternator. (All are phase values).
E_s = $I_s Z_s$.

(a) If the excitation of the alternator is such as to make E_0 equal to V, the alternator does not contribute power to the bus-bars, and no current will flow through the alternator armature. E_0 will be in phase with V *with respect to the external circuit.*

(b) If the excitation is less so that E_0 is less than V and the alternator is still contributing no power, a current I_s will circulate through the armature such that the vector addition of E_0 and $I_s Z_s$ will be equal to V. In other words, I_s will have to be a magnetising (or leading) current drawn from the bus-bars.

(c) If the excitation is more than normal, so that $E_0 > V$, and that the alternator is on no load, a current I_s will circulate through the armature such that the vector difference of E_0 and $I_s Z_s$ will be

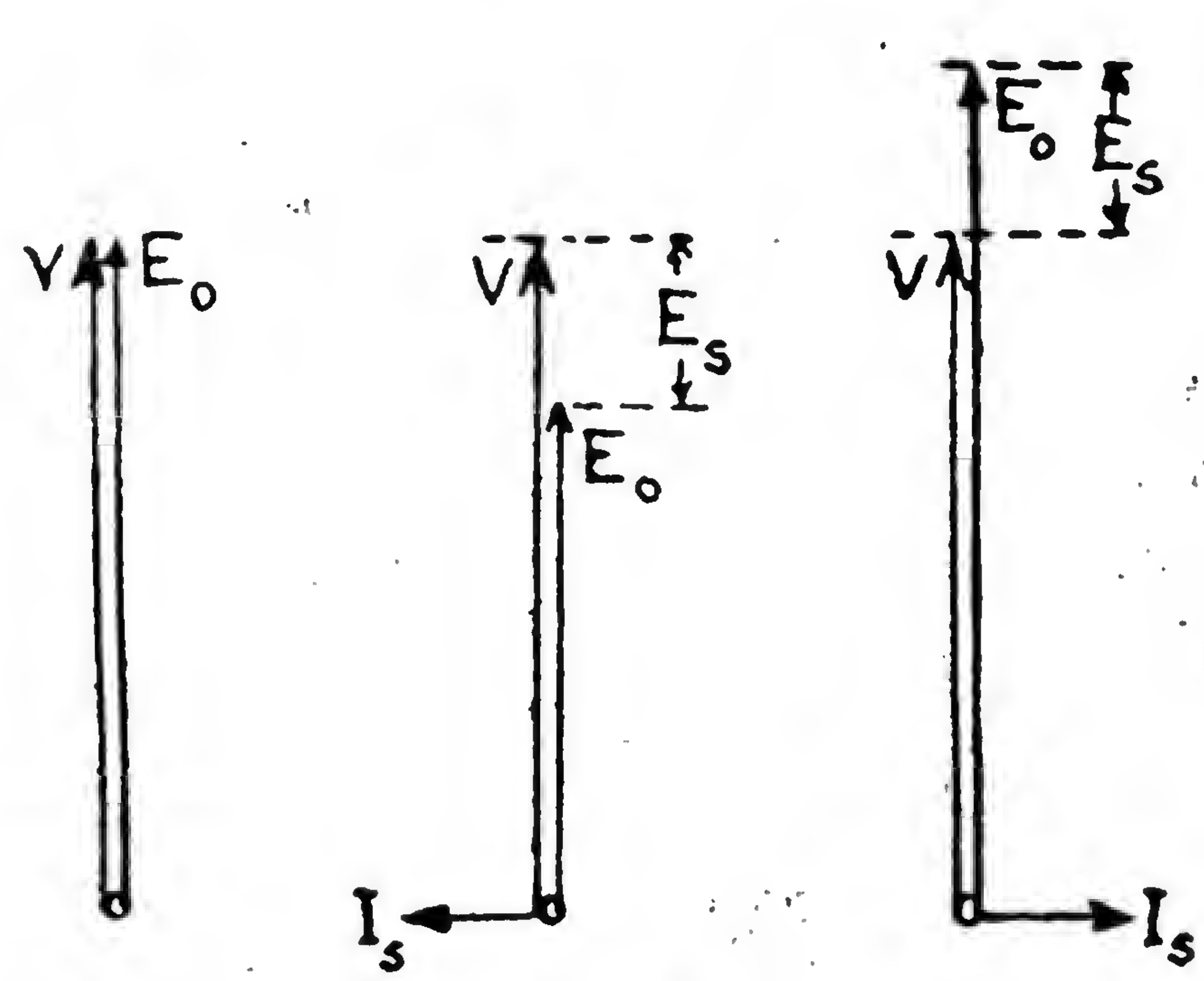


Fig. 24

equal to V. I_s in this case will have to be a demagnetising (or lagging) current drawn from the bus-bars. The resistance drop per phase of the alternator has been ignored in the above cases.

Let us now consider the alternator on load. In order to share the bus-bar load the mechanical input to the alternator must be increased. This will advance E_0 with respect to the bus-bar voltage. In other words E_0 leads V.

Let the excitation at this load be so adjusted that the alternator power factor is unity. Fig. 25 (a) shows the relation between E_0, V and I. The power supplied by the alternator is VI watts per phase,

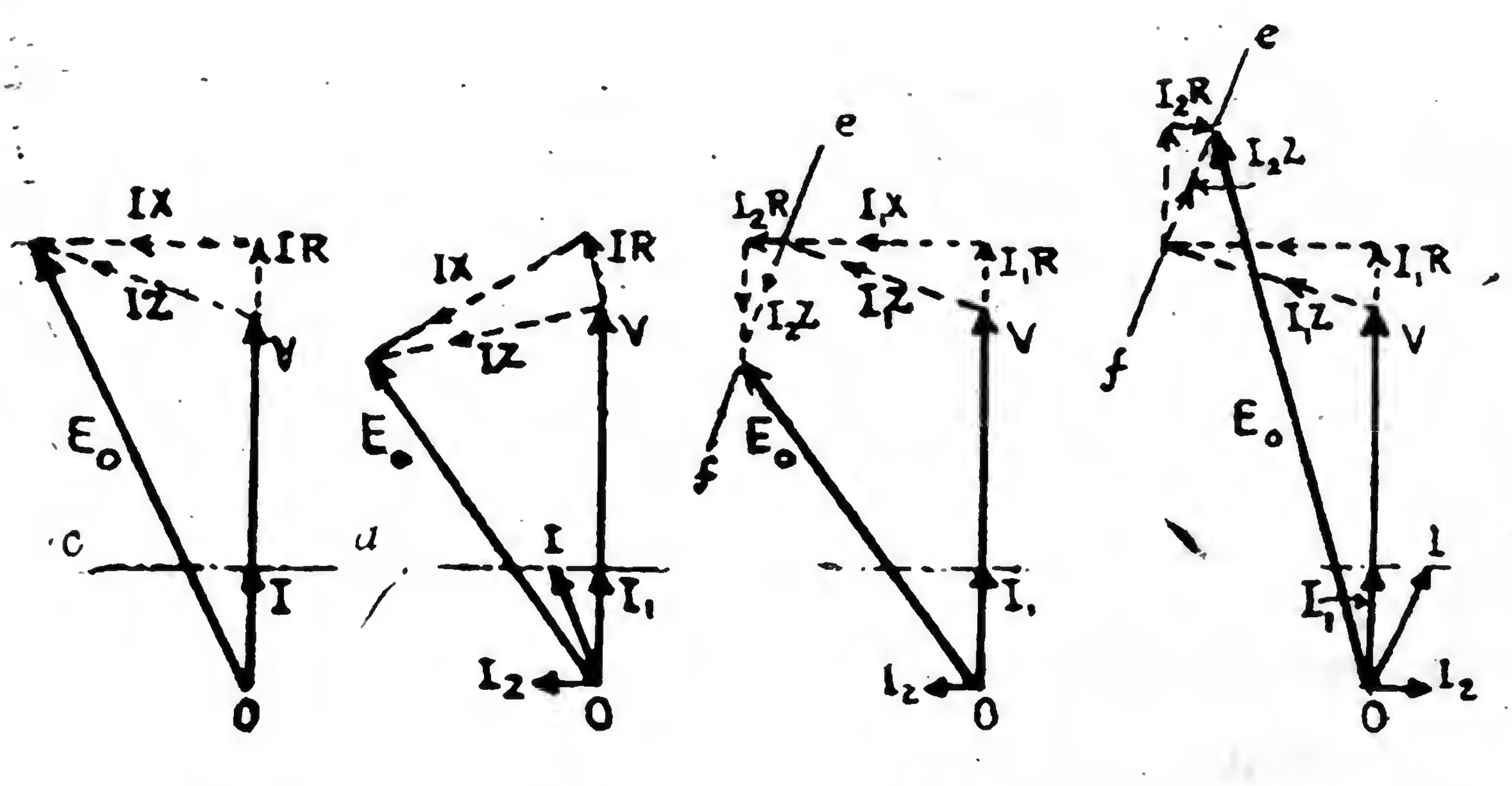


Fig. 25

the power factor being unity. The dotted line cd is a constant *power line* of value $= \sqrt{3}\ VI$ watts, while IR and IX are the resistance and impedance drops respectively (per phase values).

If the excitation is reduced the position of E_0 and IZ_s change with respect to V, which is assumed to remain constant. A wattless component I_2 circulates through its armature and this component must needs be leading to make up for the reduced field ampere-turns. Therefore the total current I in the armature will be the vector sum of I_2 and the current I_1 (the power component). See Fig. 25 (b). Note that the power factor of the machine is not unity but it is *leading*. Fig 25 (c) is constructed by taking into account separately the drops due to the two components I_1 and I_2. For any other

excitation, but supplying the same power, the extremity of E_0 lies on the dotted line ef.

Similarly, Fig. 25 (d) is constructed where the excitation has been increased beyond the normal. The extremity of E_0 lies on the same dotted line ef if the excitation is increased to any other value. The dotted lines of both the figures (c) and (d) are the same,

Summarising, a change in excitation changes the power factor of the alternator connected to infinite bus-bars but does not change it power output. To change the power output the steam supply should be altered if the prime mover is a steam engine.

Example: Two 250 kVA, 3-phase, star-connected alternators run in parallel and share equally a total load of 300 kW at 3,300 volts at 0·8 p. f. lagging. The resistance and reactance per phase of each machine are 2 % and 40 % respectively.

Calculate, (i) the current, (ii) the power factor and the induced e. m. f. of each machine when (a) they are supplied with normal excitation and (b) when the exciation of one machine is so changed that its power factor is 0·9 lagging.

Solution: Full load current of each machine is

$$I_{f.l.} = \frac{250 \times 1000}{\sqrt{3} \times 3300} = 43{\cdot}8 \text{ amperes,}$$

$$\text{Voltage per phase} = \frac{3300}{\sqrt{3}} = 1903 \text{ volts.}$$

$$\text{Resistance per phase} = \frac{1903 \times 2}{100 \times 43{\cdot}8} = 0{\cdot}87 \text{ ohm}$$

$$\text{reactance per phase} = 20 \times 0{\cdot}87 = 17{\cdot}4 \text{ ohms.}$$

$$\text{The total load current} = \frac{300 \times 1000}{\sqrt{3} \times 3300 \times 0{\cdot}8} = 65{\cdot}65 \text{ amperes,}$$

Since the machines are indentical and have the same excitation the load shared by each machine will be equal and at the same power factor, namely 0·8 lagging.

The power and wattless components of the load current are

$$= 65{\cdot}65\,(\cos\phi - j\sin\phi)$$

$$= 65{\cdot}65 \times 0{\cdot}8 - j65{\cdot}65 \times 0{\cdot}6$$
$$= 52{\cdot}52 - j39{\cdot}39 \text{ amperes.}$$

(a) Hence the current supplied by each machine is

$$\frac{52{\cdot}52}{2} - j\,\frac{39{\cdot}39}{2}$$
$$= 26{\cdot}26 - j19{\cdot}695 = 32{\cdot}7 \text{ amperes.}$$

Using Fig. 20 (b), the value of induced e. m. f. per phase can be calculated as

$$E_0 = [(1903 \times 0{\cdot}8 + 32{\cdot}7 \times 0{\cdot}87)^2 + (1903 \times 0{\cdot}6 + 32{\cdot}7 \times 17{\cdot}4)^2]^{1/2}$$
$$= 2313 \textit{ volts.}$$

Then the line value of induced voltage is

$$\sqrt{3} \times 2313 = 4000 \textit{ volts.}$$

(b) Since the steam supply of both machines is the same they supply the same power as before. There is a change, however, in the wattless component of current of both machines.

At 0·9 p. f. lagging the wattless component of current is

$$\frac{26{\cdot}26}{\cos\phi_2} \times \frac{\sin\phi_2}{1} = 26{\cdot}26 \times \frac{0{\cdot}435}{0{\cdot}9} = 12{\cdot}7 \text{ amperes.}$$

The remaining wattless current is supplied by the other machine,

i. e. $\quad 39{\cdot}39 - 12{\cdot}7 = 26{\cdot}69$ amperes.

Hence the wattless component of the other machine is 26·69 and its power component is 26·26 amperes and the total current is

$$I_B = \sqrt{[(26{\cdot}26)^2 + (26{\cdot}69)^2]} = 37{\cdot}5 \textit{ amperes.}$$

$$\text{and its p. factor is } \frac{26{\cdot}26}{37{\cdot}5} = 0{\cdot}703 \textit{ lagging}$$

The total current of the first machine is

$$I_A = \sqrt{[(26{\cdot}26)^2 + (12{\cdot}7)^2]} = 29{\cdot}2 \textit{ amperes.}$$

The two induced e. m. fs. are

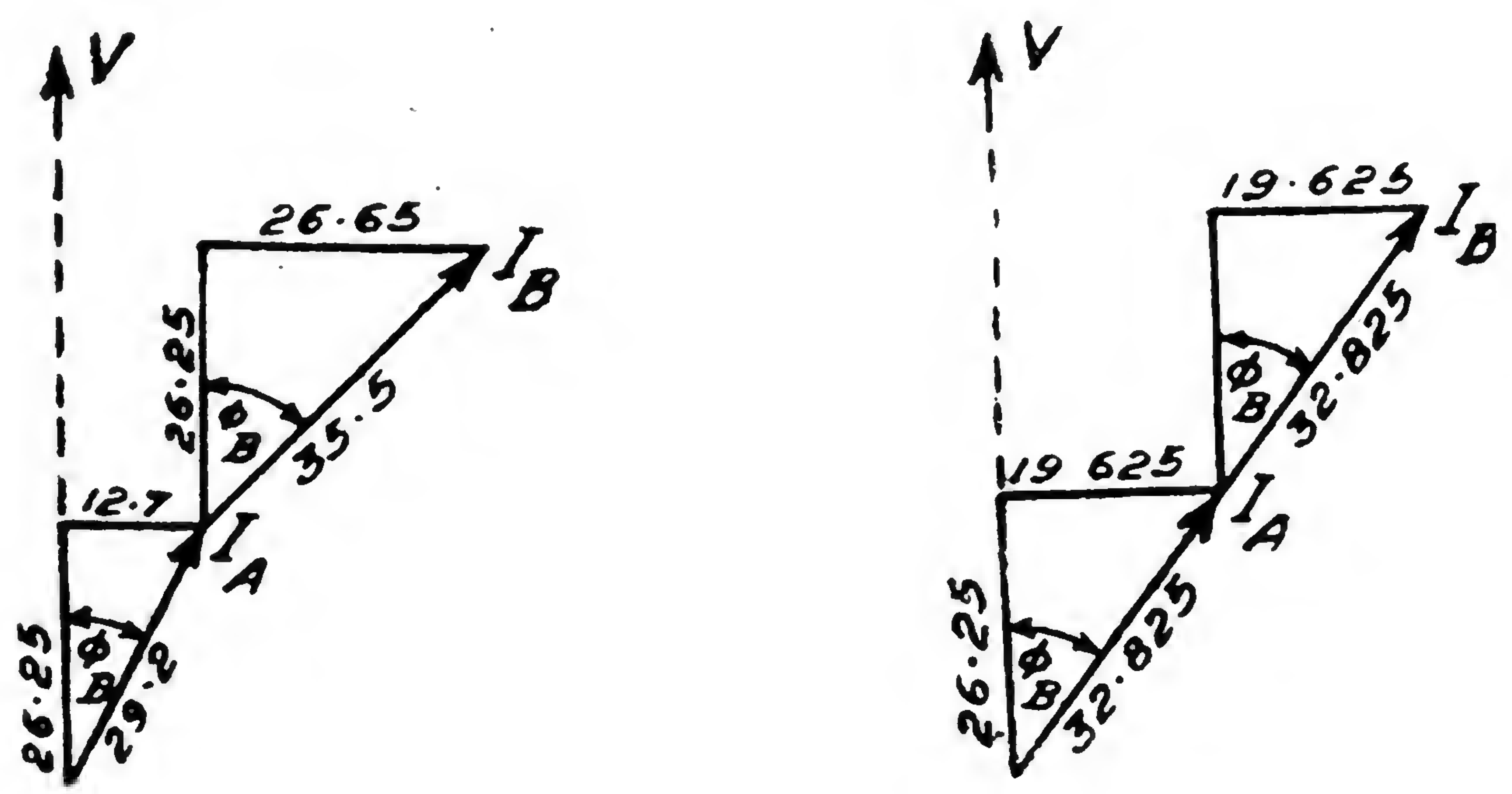
$$E_A = [(1903 \times 0{\cdot}9 + 29{\cdot}2 \times 0{\cdot}87)^2 + (1903 \times 0{\cdot}435 + 29{\cdot}2 \times 17{\cdot}4)]^{1/2} \times \sqrt{3}$$

$= 3790$ *volts.*

$$E_B = [(1903 \times 0{\cdot}703 + 37{\cdot}5 \times 0{\cdot}87)^2 + (1903 \times 0{\cdot}71 + 37{\cdot}5 \times 17{\cdot}4)^2]^{1/2} \times \sqrt{3}$$

$= 4170$ *volts*

Fig. 26 shows the vector diagrams of currents for the two cases.



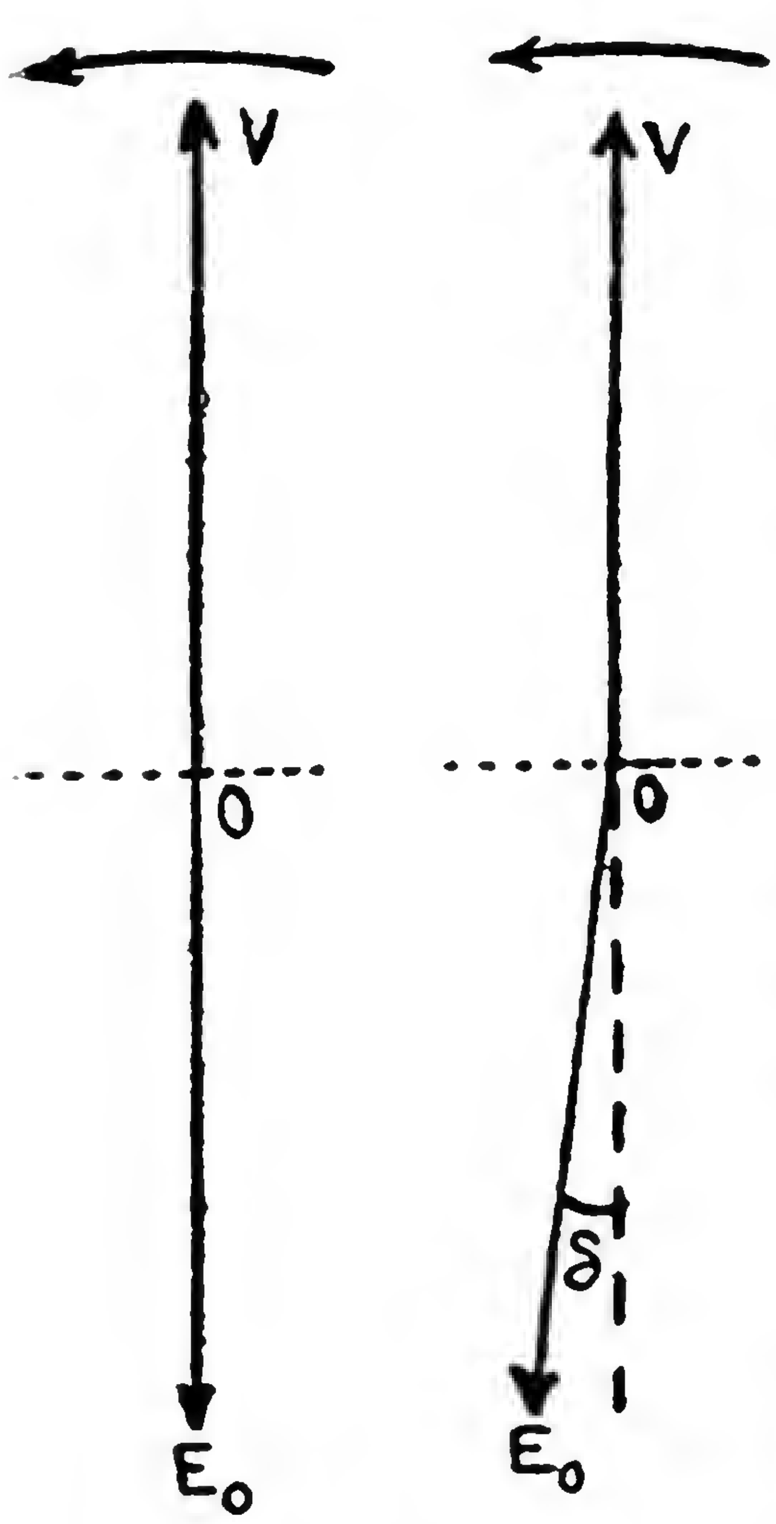
(a) Fig. 26 (b)

CHAPTER XI

THE SYNCHRONOUS MOTOR

1. Introductory: The alternator is a reversible machine i. e., it can be worked as a motor, as does a d. c. generator. Whether working as a generator or as a motor its field windings must be excited by direct current.

Imagine an alternator synchronised to a bus-bar of voltage V, with its excitation so adjusted that its induced e. m. f. E_0 is equal to V. If the alternator does not supply any load, the vector diagram of voltages is as shown in Fig. 1 (a). Under this condition there is no current in the armature since the resultant of E_0 and V is zero.



(a) Fig. 1 (b)

If the alternator is now disconnected from its prime mover it continues to run as a motor at its synchronous speed. *It does not and cannot run at any other speed.* Hence the name "synchronous" motor.

Though there is no load torque at the shaft, the motor takes a small amount of power required to rotate its mass. The power drawn (per phase) $= V \cdot I \cos\phi$; and $I = \dfrac{E_s}{Z}$

where V = voltage per phase across the motor

I = current per phase

ϕ = phase angle between V and I

E_s = resultant voltage of V and E_0

Z_s = impedance per phase of armature.

The vector E_0 is slightly retarded by an angle δ, which is called the *angle of retard.* See Fig. 1 (b), and since the armature winding impedance is mostly inductive the phase angle θ between E_s and I is more than 80°, I lagging E_s. If the load torque on the shaft now comes on (i. e. if the motor is expected to perform mechanical work), E_0 shifts more making δ larger, E_s is also shifted and is larger and

consequently I increases. This is shown by the vector diagram of Fig 2. This is the vector diagram of the synchronous motor. The following points are very important:—

(1) Magnitude of E_0 depends upon the field excitation.

(2) Magnitude of angle δ depends upon the mechanical output of the motor.

(3) Angle θ is constant for a machine $\left(\tan\theta = \dfrac{X_s}{R}\right)$.

(4) E_s is the vector sum of V and E_0.

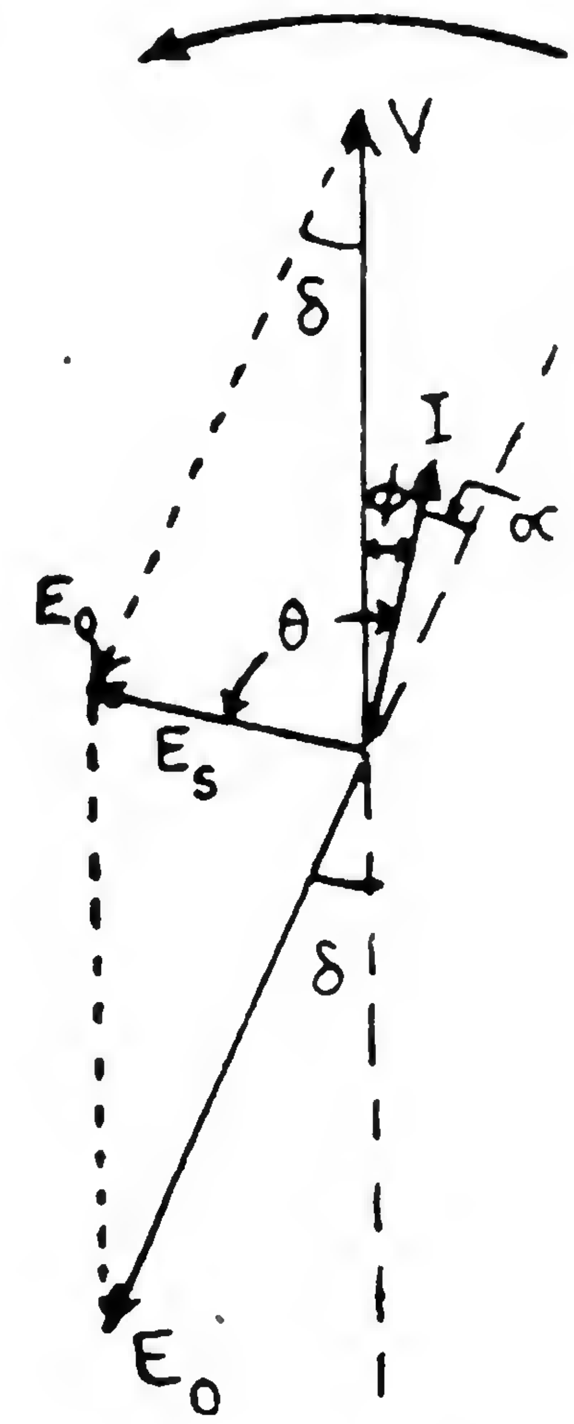


Fig. 2

The following mathematical relations should be noted:—

$$I = \frac{E_s}{Z_s} = \frac{E_s}{\sqrt{(R^2 + X_s^2)}} \quad \cdots \quad (1)$$

where R and X_s are the resistance and synchronous reactance per phase of the armature winding.

$$\textit{input to motor} = \sqrt{3}\cdot VI\cos\phi \quad \ldots \quad (2)$$

$$\textit{mechanical power developed} = \sqrt{3}\,E_0 I\cos\alpha \quad \ldots \quad \ldots \quad (3)$$

where α is the phase angle between I and E_0 reversed.

$$\sqrt{3}\,VI\cos\phi = 3I^2R + \sqrt{3}\,E_0 I\cos\alpha$$

$$\sqrt{3}E_0 I\cos\alpha = (\textit{iron and friction losses}) + (\textit{useful mechanical output})$$

$$\sqrt{3}E_0 I\cos\alpha = \textit{mech. output} + \textit{iron and friction losses.} \quad (3a)$$

For computing the overall efficiency, the field loss must be taken into consideration.

The effects of excitation on the motor are as follows:—

(1) With normal excitation, the current is lagging; $E_0 = V$.

(2) With excitation less than normal, the current lags more for the same mechanical output than when the excitation is normal and E_0 is less than V.

(3) With excitation more than normal, E_0 is greater than V the current may lead or lag, depending upon the output and the amount of excess excitation over the normal.

Fig. 3 (a) shows the vector diagram for less than normal excitation while 3 (b) shows the vector diagram with excitation more than normal.

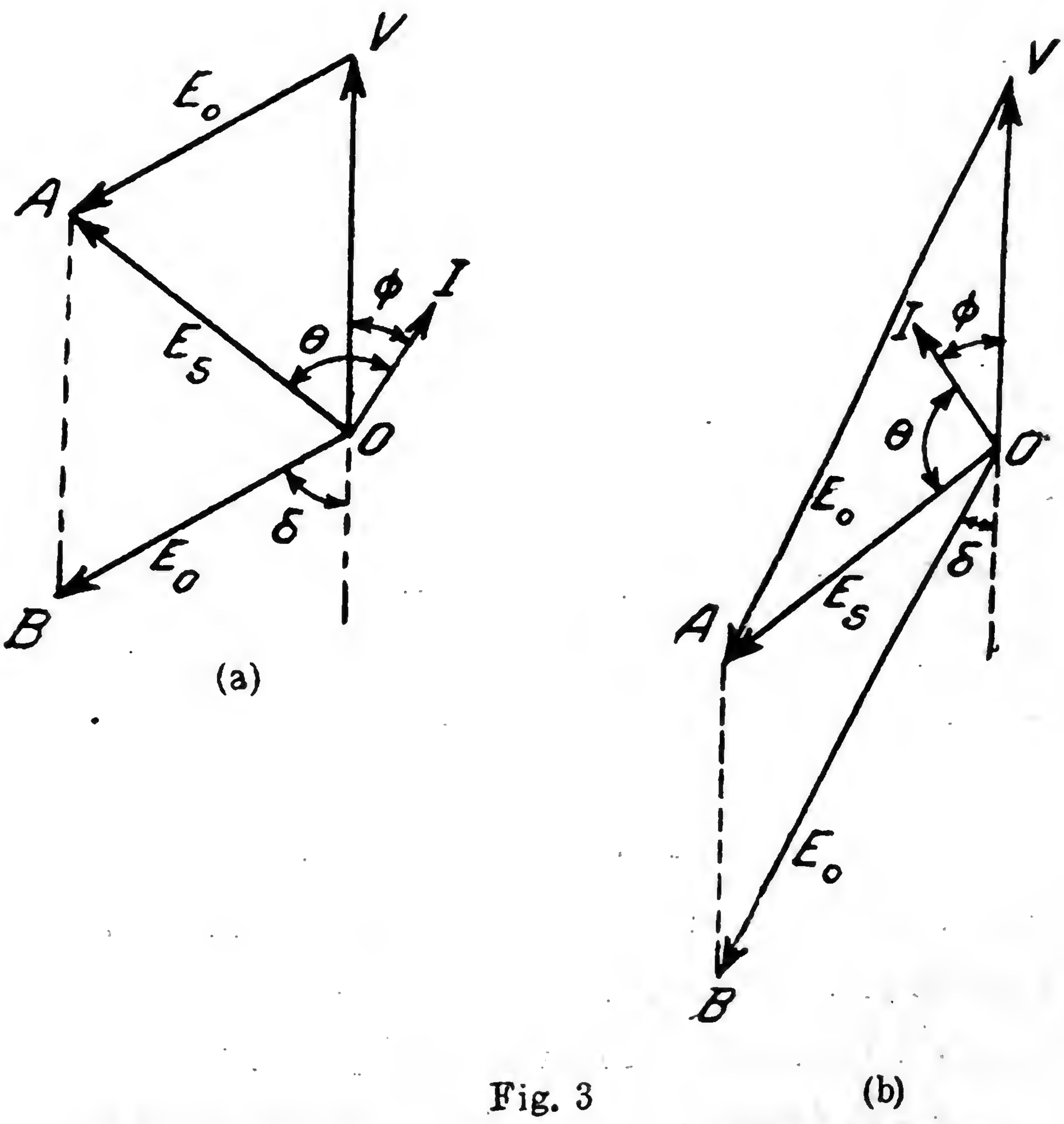


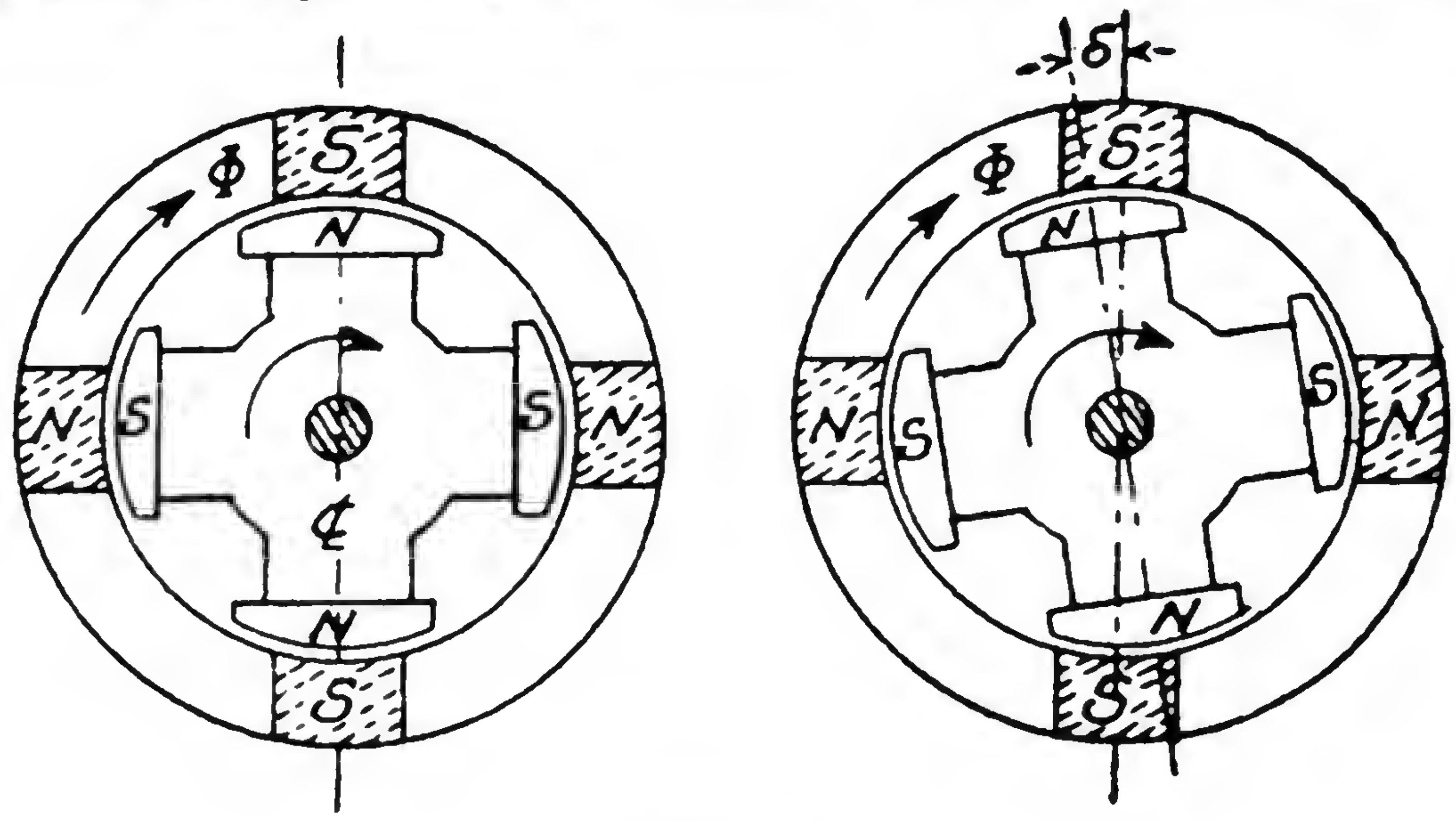
Fig. 3

Current Lagging. Current Leading.

2. Production of Torque: If a polyphase stator winding is given a polyphase a. c. supply, a magnetic field of constant magnitude and rotating at synchronous speed is produced.

Suppose that at a particular instant the stator *S*–pole is at the position shown in Fig. 4 (a), and that the rotor is not rotating. The *S*–pole of the stator and the *N*–pole of the rotor attract each other. But the *N*–pole of the rotor cannot travel from rest with the *S*–pole of the stator due to the inertia of the mass of the motor. When at the next instant the *N*–pole comes opposite the *N*–pole of the rotor, the rotor is repelled in another direction. So that there is no movement of the rotor and it remains stationary. This shows that a synchronous motor is not self-starting.

If an auxiliary motor drives the synchronous motor and brings it to synchronous speed, the two unlike poles will lock themselves and continue to rolate at synchronous speed and at no other speed. And



(a) Fig. 4 (b)

when the synchronous motor is unloaded, the centre lines of the two poles almost coincide. But when the load comes on the shaft of the synchronous motor the rotor poles slightly lag behind the stator poles by an angle δ, **the angle of retard.** The rotor still runs at synchronous speed. If more load comes on the angle δ increases. This is shown in Fig. 4 (b).

Hence summarising,

(*a*) a synchronous motor is not self-starting ;

(*b*) a synchronous motor runs at only one speed, i. e. at synchronous speed, and

(*c*) the torque of a synchronous motor is independent of its speed.

The synchronous speeds for various number of poles and for a fixed frequency of 50 cycles per second are given in the following Table

Number of Poles	2	4	6	8	10	12	16	20	24
Syn. speed (R. P. M.)	3000	1500	1000	750	600	500	375	300	250

To convert the synchronous speed for any other frequency f_2 use the following relationship.

$$\frac{syn.\ speed\ at\ f_2}{syn.\ speed\ at\ 50} = \frac{f_2}{50} \qquad \dots \qquad \dots \qquad \dots \qquad (4)$$

3. The V–Curves : When the excitation (i. e. the field current) of an unloaded synchronous motor is gradually reduced to zero, the motor draws from the supply Mains a large lagging current. Since E_0, the induced e. m. f., is zero, the current makes an angle θ with the supply voltage V.

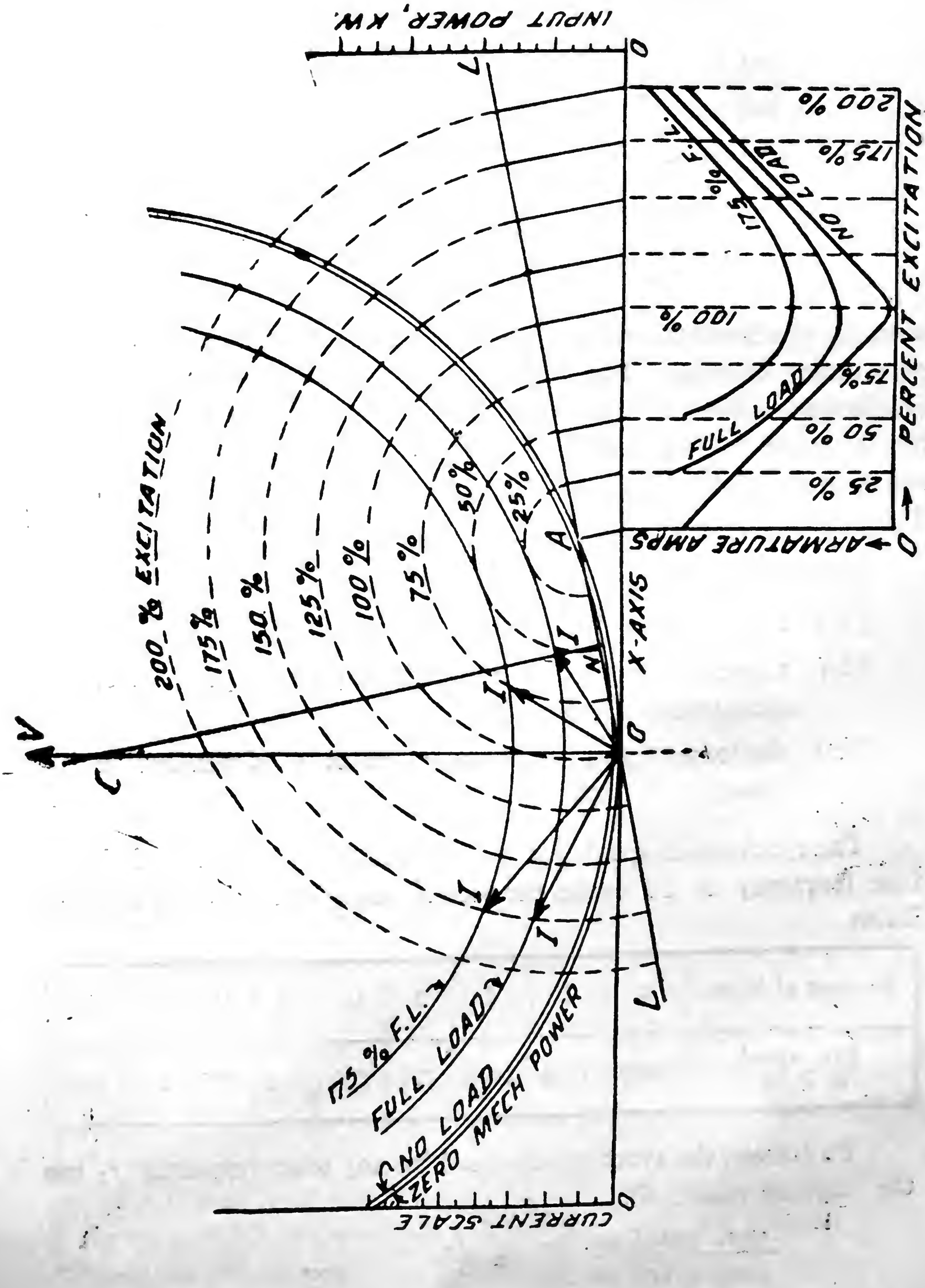


Fig. 5 (a)

If the excitation is gradually increased, the extremity of the current vector travels along the circumference of the no-load circle of Fig. 5 (a) from point A towards point O. OA is the no-load current with zero excitation. At 100 % (i. e. normal) excitation E_0 is almost equal to V and the current is minimum, while the power factor is unity. If the excitation is increased more than 100 % (i. e. above normal), the line current goes on increasing and it leads the voltage. In other words the power factor is leading.

When the motor is loaded, say at full load, the extremity of the current vector travels along the circumference of the full load circle shown in Fig. 5 (a). Here too the current falls gradually as the excitation is increased from a low value of 25% until 100% excitation. Afterwards increasing the excitation beyond 100% the current goes on increasing and is now leading the bus-bar voltage.

Thus if a graph is plotted showing the variation of armature current with excitation, particularly at no load, the shape of the graph resemble the capital letter **V**. Hence these curves are called the *V-curves* of a synchronous motor. Fig. 5 (a) shows three *V-curves* at various loads.

Usually these curves are plotted from experimental data, or they may be derived from the upper portion of Fig. 5 (a) as shown. The circular loci, for constant mechanical power shown by full lines, are called the *O-curves*. The limit of stability can be determined from the positions of intersection of the *O-curves* with the dotted semicircles.

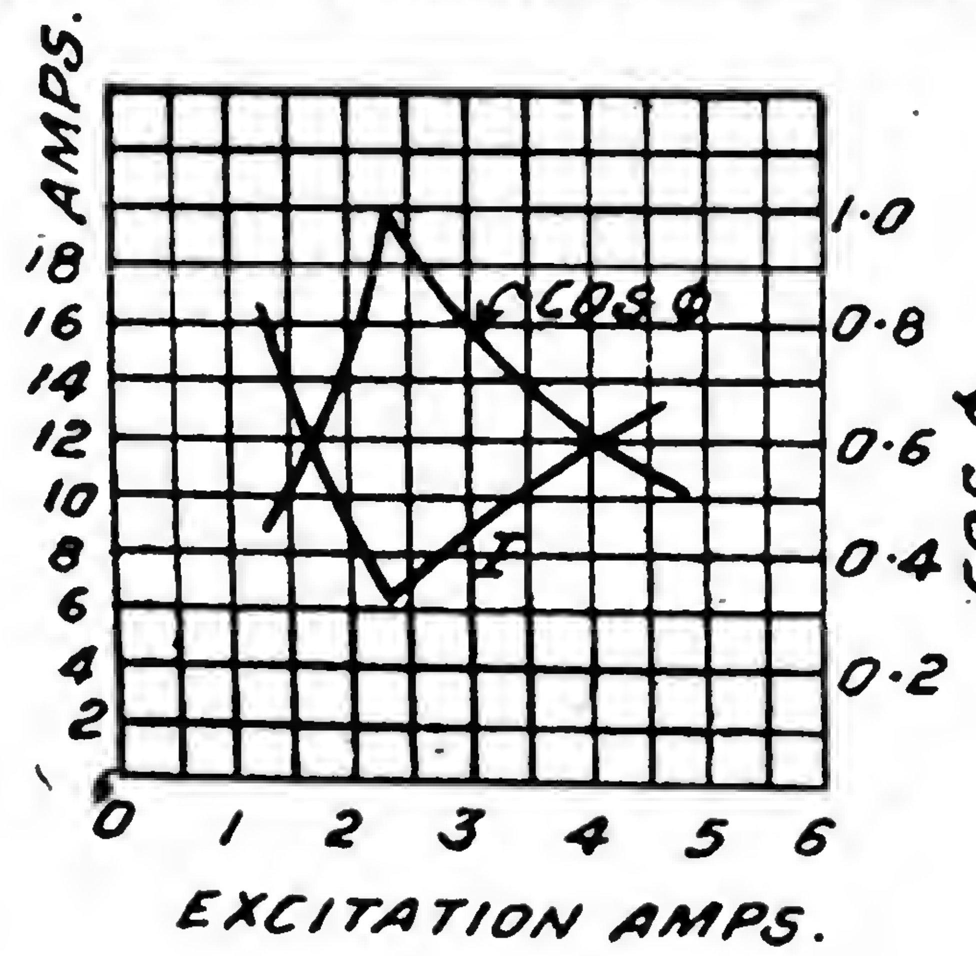


Fig. 5 (b)

One such V-curve is shown in Fig. 5 (b) and the variation of power factor is also shown for the same load when the excitation is increased from a low value to about 150% normal excitation. The current at low values of excitation is high and lags the supply voltage. When the excitation is such that the current is minimum, the power factor is unity. After this the current goes on increasing and leads the supply voltage.

Example: An 11000-V, 3-phase, star-connected synchronous motor takes 66 A from the line. The effective resistance per phase is 1 ohm and the synchronous reactance per phase is 22 ohms. Calculate the motor induced e. m. f. for a power factor of 0·8 (a) lagging and (b) leading. Draw complete vector diagrams for the two cases.

Find also for case (b) the $kVAR$ taken from the line.

Solution:

$$\text{Voltage per phase} = \frac{11000}{\sqrt{3}} = 6350 \text{ V.}$$

$$\text{Impedance per phase} = \sqrt{(1^2+22^2)} = 22{\cdot}02 \text{ ohms} = Z_s$$

$$E_s = IZ_s = 66 \times 22{\cdot}02 = 1453{\cdot}3 \ V$$

$$\theta = \tan^{-1}\tfrac{22}{1} = \tan^{-1} 22 = 87^\circ \cdot 24; \ \cos\phi = 0{\cdot}8$$

$$\therefore \ \phi = 36^\circ \cdot 54'.$$

Case (a): Vector diagram is shown in Fig. 4 (a), where

$$OV = 6350; \ OA = 1453{\cdot}3; \ (\theta-\phi) = 50^\circ \cdot 30' = \angle VOA$$

From the triangle VOA,

$$AV = E_o \text{(per phase)} = [\,OV^2+OA^2-2\,OV\times OA\cos\angle VOA\,]^{1/2}$$

$$= [\,6350^2+1453{\cdot}3^2-2\times6350\times1453{\cdot}3\times0{\cdot}6361\,]^{1/2}$$

$$= 5541 \text{ V.}$$

$$\therefore \quad \text{line value} = \sqrt{3}\times5541 = \mathit{9586\ V.}$$

Case (b): Vector diagram is shown in Fig. 4 (b), where

$$OV = 6350; \ OA = 1453{\cdot}3; \ (\theta+\phi) = 124^\circ \cdot 18';$$

$$\cos 124^\circ \cdot 18' = -0{\cdot}5635$$

$$\therefore AV = E_0 = [6350^2+1453{\cdot}3^2-2\times6350\times1453{\cdot}3\cos(124^\circ{\cdot}18')]^{1/2}$$

$$= [6350^2+1453{\cdot}3^2+2\times6350\times1453{\cdot}3\times0{\cdot}5635]^{1/2}$$

$$= 7268 \text{ V}$$

$$\therefore \quad \text{line value of } E_0 = \sqrt{3}\times7268 = \mathit{12574\ V.}$$

The kVA taken from the line $= \sqrt{3} \times V \times I \times 10^{-3}$

The kW taken from the line $= \sqrt{3} \times V \times I\cos\phi \times 10^{-3}$

The $kVAR$ taken from the line $= \sqrt{3} \times V \times I\sin\phi \times 10^{-3}$

$\therefore$ The $kVAR$ in this case (b)

$$= \sqrt{3}\times11000\times66\times0{\cdot}6 \div 1000 = \mathit{753{\cdot}6\ kVAR.}$$

4. Synchronous Condenser : It has been shown that an over-excited synchronous motor takes a leading current from the supply lines. In fact, the motor draws from the supply lines leading reactive kilo-voltamperes (*kVAR*). This is a very useful characteristic of a synchronous motor and is taken advantage of by Power Supply Undertaking Authorities to improve the power factor of their transmission lines. When used thus the machine is called a **Synchronous Condenser.**

These machines are usualls located at the Receiving End of a long transmission line. They simply float on the line, i. e. these machines do no mechanical work, and the power drawn from the supply lines is almost all reactive.

$$\textit{Reactive power} = \sqrt{3} \times V I_{sy} \sin \phi_{sy}$$

where I_{sy} = current drawn by a motor from the line;
ϕ_{sy} = phase angle between V and I_{sy}.

Ignoring the resistance per phase, since it is negligibly small compared to the synchronous reactance per phase, the vector diagram of voltage and current of a synchronous motor, which is over-excited and is not doing any mechanical work is shown in Fig. 6 (a). In this case $\theta = 90°$ and ϕ_{sy} is also $90°$, if we ignore the iron and friction losses. This reactive power, which an over-excited motor draws from the supply lines, oscillates between the motor and the bus-bars (or supply lines).

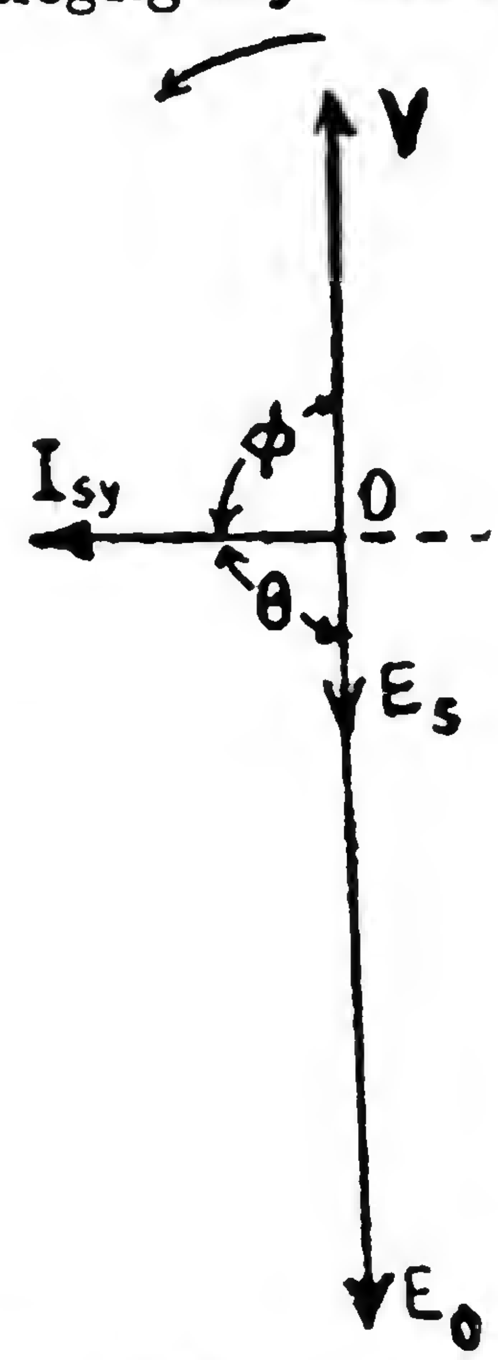


Fig. 6 (a)

Sometimes the power factor of an installation is improved by installing a synchronous motor. This motor does some mechanical work as well and its field current is above the normal value. The following example illustrates the case.

Example : A factory has a total motor load of 500 kW at 0·75 p. f. lagging. It has to install another motor giving a full load output of 175 h. p. Assuming the efficiency of the new motor to be 87 %, suggest the type and kVA of the new motor so that the overall power factor will improve to 0·94 lagging.

Solution: When

$$\cos\phi_1 = 0{\cdot}75;\ \sin\phi_1 = 0{\cdot}6614;\ \tan\phi_1 = 0{\cdot}881$$

$$\cos\phi_2 = 0{\cdot}94;\ \sin\phi_2 = 0{\cdot}341;\quad \tan\phi_2 = 0{\cdot}363$$

$$\text{old load kVA} = \frac{500}{0{\cdot}75} = 666{\cdot}67 \text{ kVA}$$

$$\text{old load kVAR} = 666{\cdot}67 \times 0{\cdot}6614 = 441 \text{ kVAR}$$

$$\text{new load kW} = \frac{175 \times 746}{0{\cdot}87 \times 1000} = 150 \text{ kW}$$

$$\text{Total load in kW} = 150 + 500 = 650 \text{ kW}$$

$$\text{Total}\quad \text{kVA} = \frac{650}{0{\cdot}94} = 691{\cdot}5 \text{ kVA}$$

$$\text{Total}\quad \text{kVAR} = 691{\cdot}5 \times 0{\cdot}341 = 235{\cdot}8 \text{ kVAR.}$$

The type of motor to be used is synchronous motor, since it is required to improve the total power factor.

The kVAR to be neutralised by the synchronous motor

$$= 441 - 235{\cdot}8 = 205{\cdot}2$$

This is the reactive power of the new motor and the kW is 150.

If ϕ_{sy} is the angle of phase difference between the supply voltage and the current of the new motor

$$\tan\phi_{sy} = \frac{205{\cdot}2}{150} = 1{\cdot}367$$

From Tables, $\phi_{sy} = 53^\circ - 48'$

$\therefore \quad \cos\phi_{sy} = 0{\cdot}59$ (leading).

The kVA of the new motor must be

$$= \frac{150}{0{\cdot}59} = 254{\cdot}2 \text{ kVA}$$

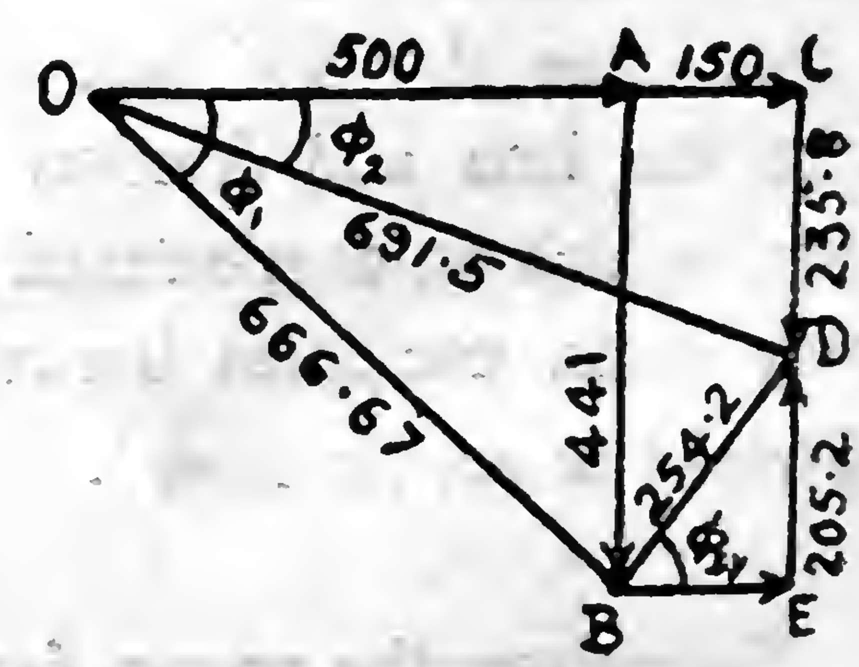


Fig. 6 (b)

In the vector diagram of Fig. 6 (b) DE is the kVAR of the synchronous motor, OA is the old kW, AB is the old kVAR, AC=BE=kW of synchronous motor and CD is the total new kVAR

$$DE = AB - CD$$

$$AB = 500 \tan\phi_1;\ CD = (OA+AC)\tan\phi_2 = 650 \tan\phi_2$$

$$\therefore\quad DE = 500 \times 0{\cdot}881 - 650 \times 0{\cdot}363$$

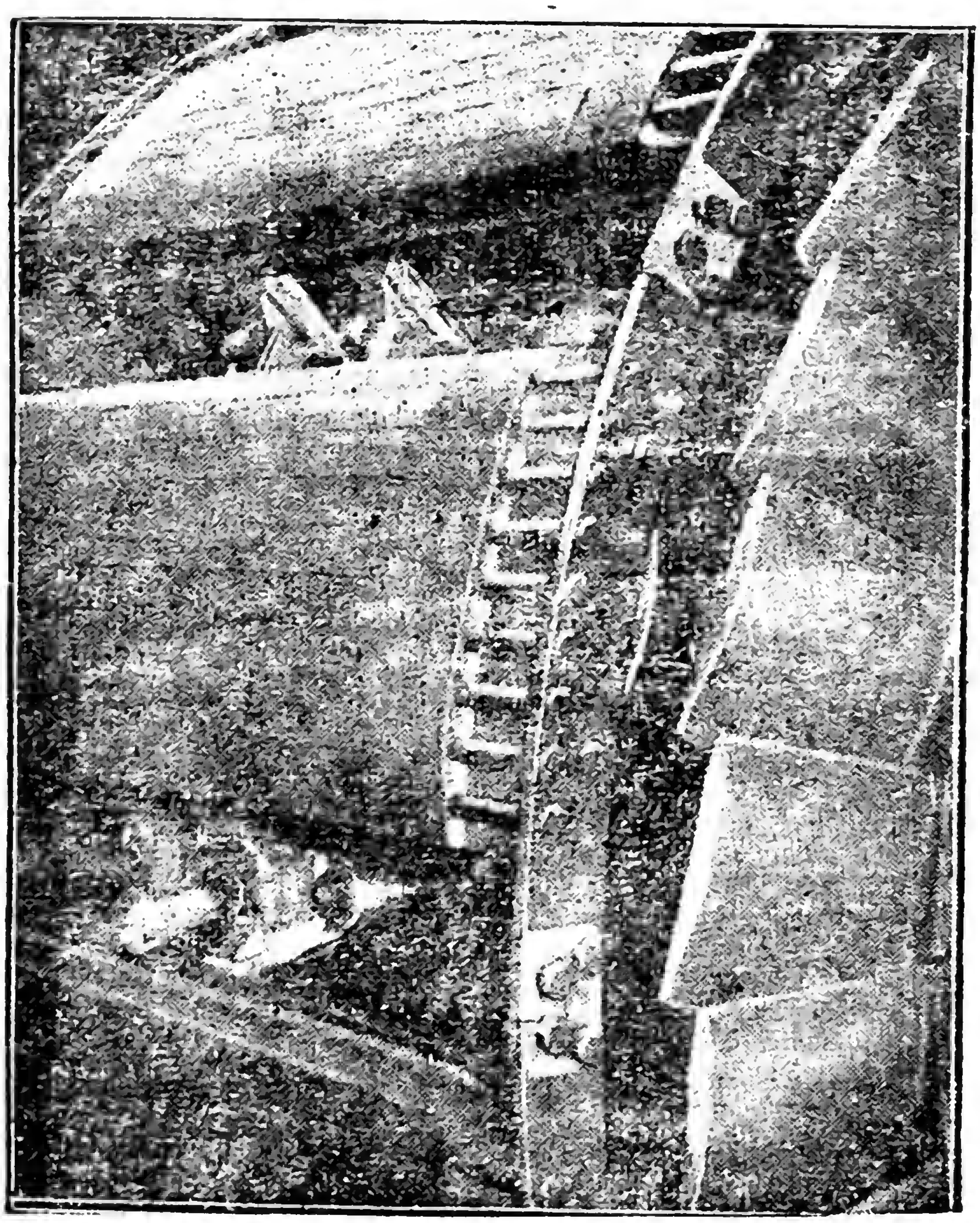
$$= 250{\cdot}2 \text{ as before.}$$

$$\therefore \quad \tan \phi_{sy} = \frac{205 \cdot 2}{150} \left(= \frac{DE}{BE} \right) = 1 \cdot 367, \text{ as before.}$$

$$\phi_{sy} = 53^\circ \cdot 48' \quad \therefore \quad \cos \phi_{sy} = 0 \cdot 59 \text{ (leading). (check)}$$

5. **Hunting:** Since a synchronous motor runs at the same speed from no load to full load, its torque is not proportional to speed but it is proportional to the angle of retard.

Suppose that a synchronous motor is loaded to a certain extent. If the load is reduced, the angle of retard δ must reduce. To do this the rotor must accelerate momentarily i. e. it must run slightly above synchronous speed only for a small interval of time. In trying to find the correct position the rotor may overshoot the mark so that δ now is less than what it ought to be. The rotor now must slow down to



(*Courtesy: Bruce Peebles and Co., Ltd.*)

Fig. 7. (a) Part of Rotor with Damper Winding and V-Shaped Bronze Clamps.

increase δ. In other words, the motor is "hunting" for the right position when the load on the shaft changes.

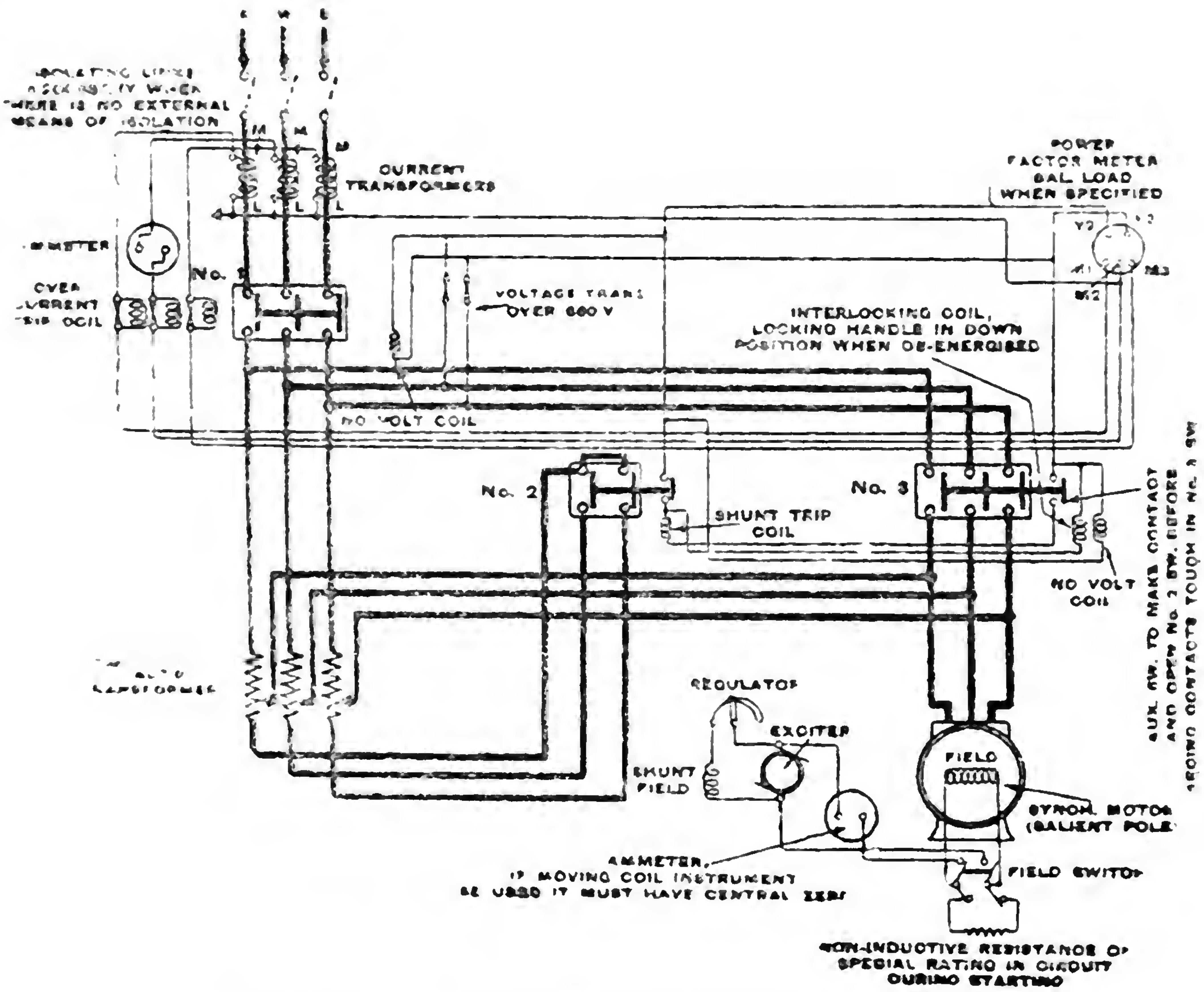
When this frequency of oscillations is equal to the natural frequency of the rotating mass, the oscillations increase, i. e. the swing of each successive oscillation increases in magnitude with the result that in a very short time the machine falls out of step and stops.

To dampen these oscillations, all synchronous motors are provided with *amortisseur* or *damper winding* on the pole-shoes. This consists of short circuited grids, similar to the rotor of a squirrel cage induction motor. These are embedded in slots or holes in the pole-shoes. See Fig. 7. The action of the damper winding is to produce induced e. m. f. in the grids or bars *when the rotor oscillates,* and to cause currents to flow in the short-circuited grid. The I^2R losses of these currents act as a brake and reduced the magnitude of oscillations very rapidly.

6. Methods of Starting: Since the production of torque in synchronous motors is only possible at synchronous speed all synchronous motors inherently are not self-starting. Hence three methods are adopted to start a synchronous motor, namely, (a) by a pony motor, (b) making use of damper winding as a squirrel-cage winding to start it as an induction motor, and (c) synchronous induction motor.

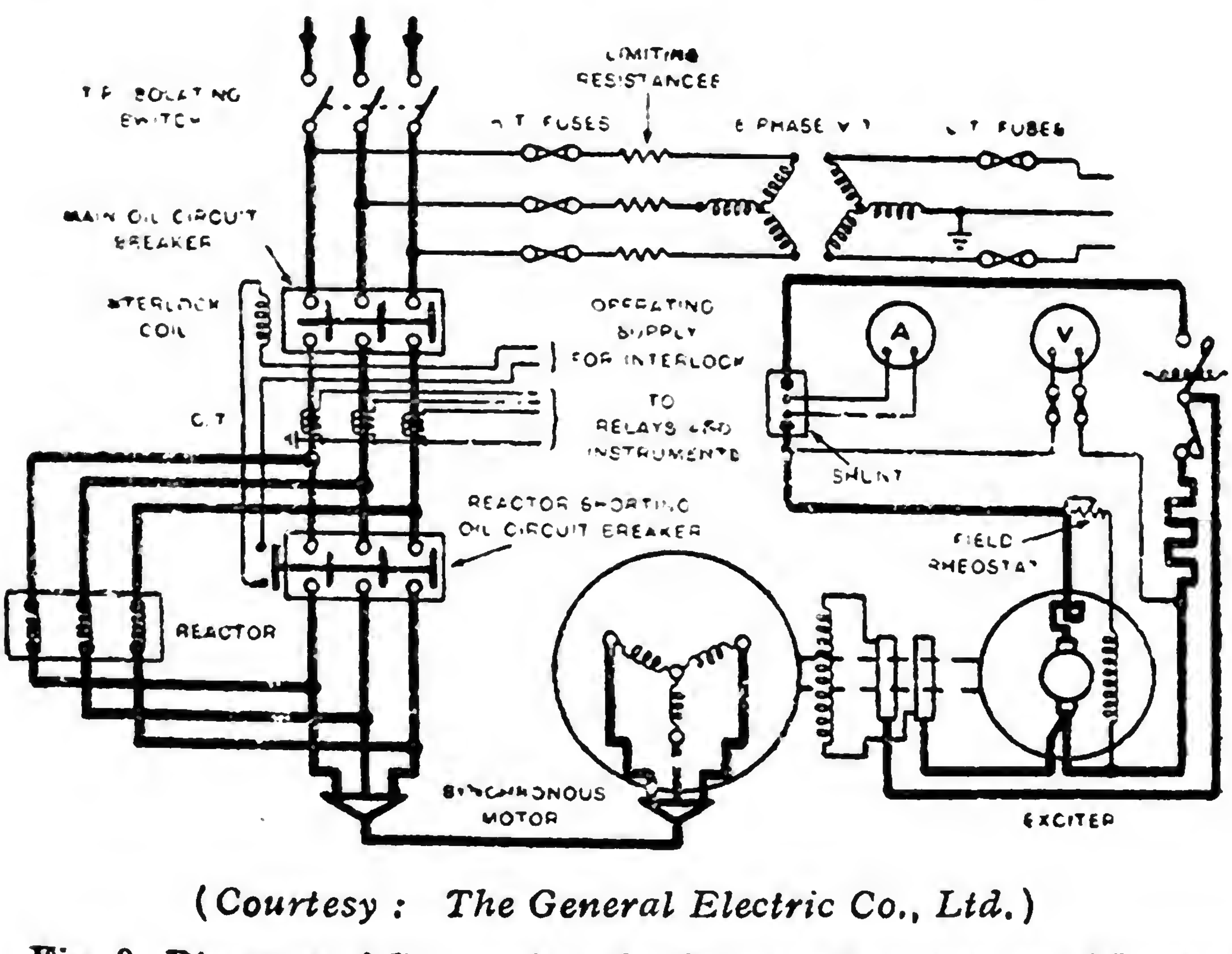
Method (a) consists in having a small induction motor attached at the end of the shaft of the main motor. This motor has the same number of poles as the main motor, or preferably one pole-pair less. The synchronising is done manually when the speed of the set is near synchronism. The torque produced by the pony motor is just enough to accelerate the mass of the main machine. If the number of poles of the pony motor is equal to that of the main motor, then it is advisable that the resistance of the rotor winding (of the pony motor) be as low as possible. This will make the slip small and therefore easier for the main motor to pull into step.

Method (b) is to make use of the damper winding as a *cage winding* to start as an induction motor. The supply is given to the armature winding in the same way as is done in the case of an induction motor, but the voltage is reduced per phase, either by an auto-



(*Courtesy : The General Electric Co., Ltd.*)

Fig. 8 Diagram of Connections for Starting Synchronous Moto rs by using Auto-Transformers



(*Courtesy : The General Electric Co., Ltd.*)

Fig. 9 Diagram of Connections for Starting Synchronous Motors by the Reactor Method

transformer or by a transformer. This is necessary to keep the starting current within reasonable limits. When the rotor is near synchronous speed, d. c. excitation is switched on, the two sets of poles, i. e. the stator and the rotor poles, get magnetically interlocked and the machine runs at synchronous speed. The full voltage to the armature is applied soon after the d. c. field current is switched on. Hence the damper winding has really a two-fold function. See Figs. 8 and 9.

Method (c) requires a machine where a slip-ring winding is provided on the rotor. This winding, eventually, is supplied with direct current to produce N– and S–poles on the rotor.

At starting, the rotor winding has a rheostat connected to it through the slip-rings in the same way as is done while starting a

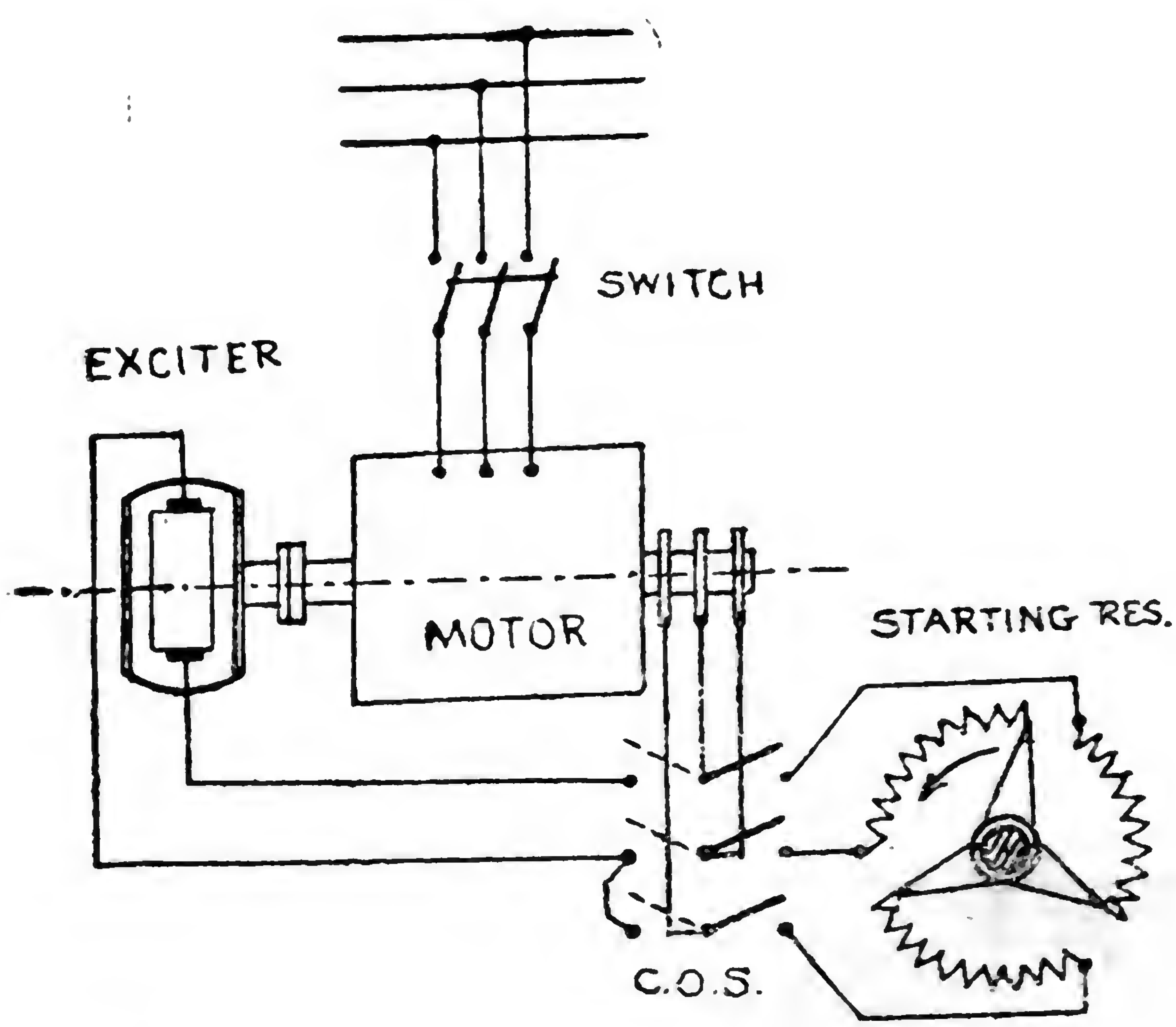


Fig. 10. Synchronous–Induction Motor.

wound rotor induction motor. When all the resistance is out the motor runs at a speed near to synchronism, i. e. it runs as an induction motor with a small slip. At this speed the slip-ring connections are switched over to d. c. supply from an exciter. This method gives a very good starting torque. Fig. 10 shows a schematic diagram of connections for starting and running conditions.

7. Losses and Efficiency: The losses in these machines are:

(a) *armature copper loss*; (b) *iron loss*; (c) *windage and friction loss* and (d) *field loss*.

(*a*) Armature copper loss $= 3 \times I^2R$ watts, where I = current per phase and R = resistance per phase. Usually all machines have 3 phases.

(*b*) and (*c*) Iron loss and windage and friction losses are considered to be constant from no load to full load. This is a mechanical loss i. e. lost torque.

(*d*) Field loss is the product of field amperes and volts across the field windings.

While calculating the *overall efficiency* of the machine, the field loss must be included. If field loss is omitted it gives the *armature efficiency* only.

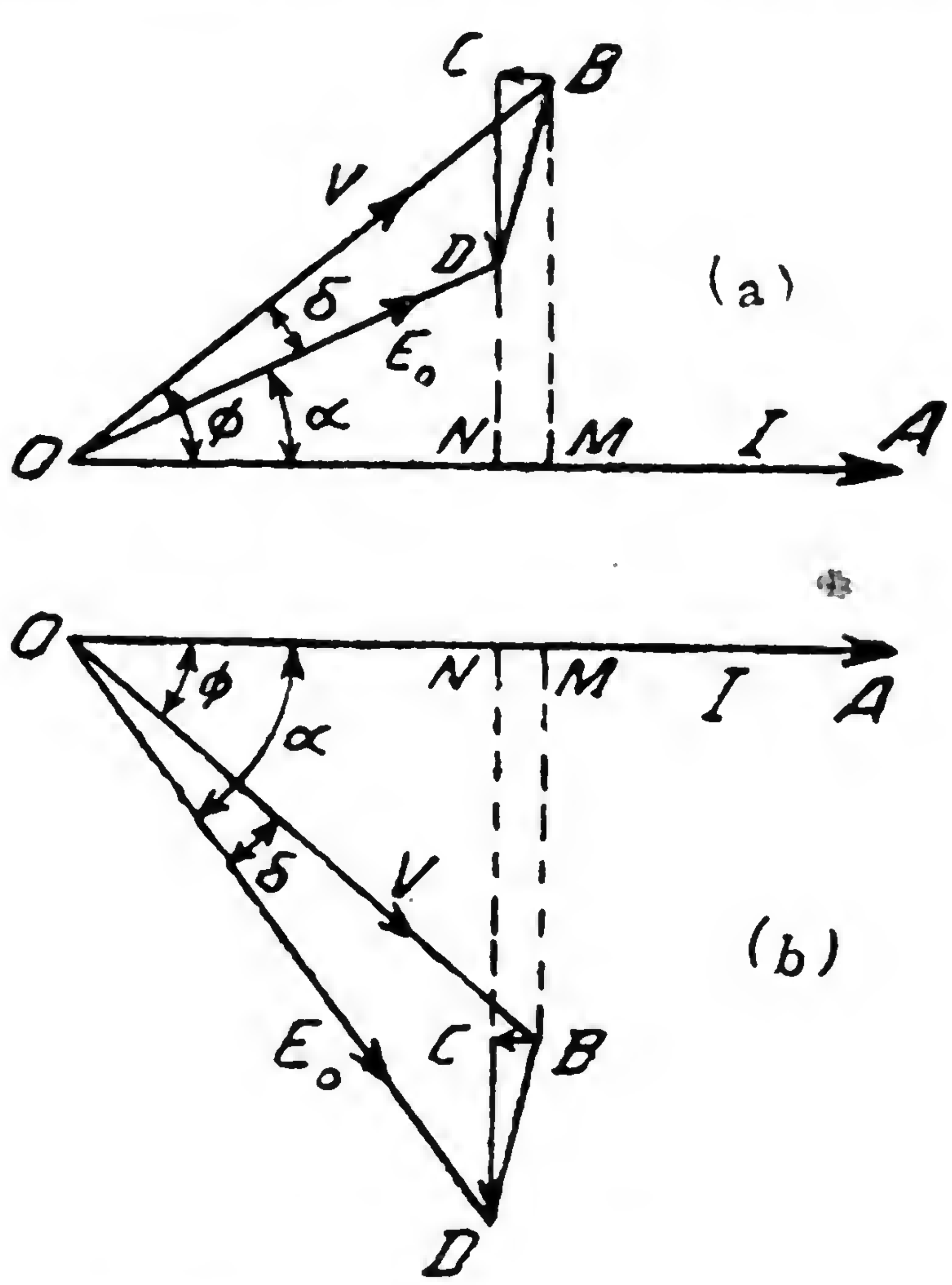


Fig. 11 Vector Diagram.

Fig. 11 (a) is alternative vector diagram of synchronous motors for lagging power factors. This may be used in place of the vector diagram of Fig. 3 (a), and for leading power factors the vector diagram of Fig. 11 (b) may be used instead of Fig. 3 (b).

In both the diagrams of Fig. 11.

OA is the current I
OB ,, ,, bus-bar voltage V
OD ,, ,, induced e. m. f. E_0
ϕ ,, ,, power factor angle
δ ,, ,, angle of retard.

CHAPTER XII

THE INDUCTION MOTOR

1. **Introductory:** A 3-phase induction motor may be treated as a form of a transformer, whose magnetic circuit has been cut into two parts. These parts are circular. The *stator* houses the primary winding and the *rotor* the secondary. There is a small air-gap between the two, and this causes the no—load current to be greater than in a transformer.

The stator frame consists of a symmetrical and substantial casting, having feet cast integral with it. The stator core is built up of high grade, low loss electrical sheet steel stampings, having either enclosed or semi-enclosed slots to reduced the effective length of the air-gap.

The rotors are of two types—(a) *Squirrel-cage* and (b) *wound rotor.* The former is made up of stampings which are keyed directly on to the shaft. The slots are partially enclosed and the winding consists of embedded copper bars to which the short-circuiting rings are brazed. The wound rotor also has slotted stampings and the winding is former wound. The number of poles produced must equal those of the stator. The ends of the rotor windings are connected to phosphor-bronze slip-rings at one end of the rotor shaft. The brushes which carry the current are held in box-type holders mounted on insulated steel rods securely bolted to the end-shield. The rotor teeth are sometimes skewed to minimise noise.

2. **Production of Rotating Field:** If a set of stationary coils, suitably wound, is supplied with polyphase voltage a uniformly rotating magnetic field is produced. In any m-phase system, the magnitude of the resultant flux is constant and is

$$\Phi = \frac{m}{2} \Phi_{max}$$

where Φ_{max} is the maximum value of pulsating flux in any one coil.

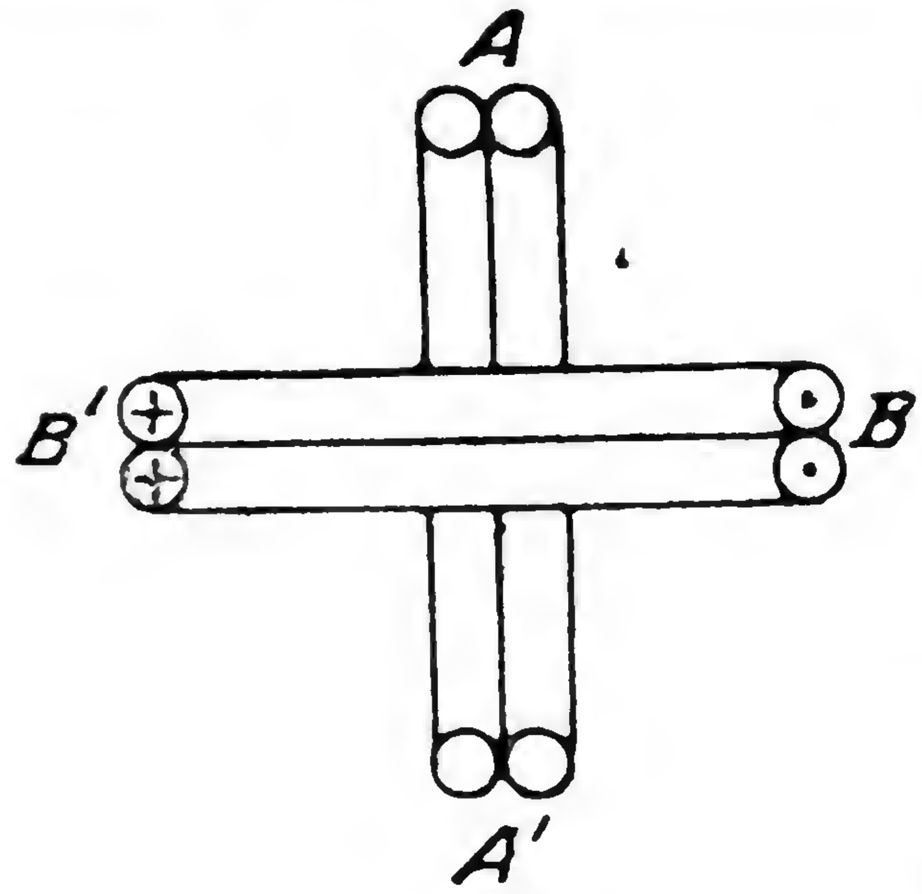


Fig. 1

Fig. 1 shows two coils **A** and **B** arranged with their axes at 90°. When connected to a 2-phase supply voltage, the resultant flux is of constant magnitude and rotates at supply frequency. Note that the coils are stationary, and the flux produced in each coil is pulsating in the direction of its axis.

The flux produced in each coil is proportional to the current in the coil. Hence Fig. 2 shows the variation of flux in each coil for one periodic time. Considering four instants at intervals of 1/6th periodic time, the fluxes produced in the two coils are drawn in Fig. 3. These fluxes are marked as Φ_A and Φ_B, and the resultant flux by Φ. Let the maximum flux be unity.

At instant 1, Φ_B is maximum and Φ_A is zero, and if the direction of current is as shown in Fig. 1 for instant 1, then Φ_B is downwards. The resultant flux is therefore equal to maximum flux in coil **B**. At instant 2, $\Phi_B = \frac{1}{2}$ maximum and $\Phi_A = 0{\cdot}866$ of maximum and since the angle between the two fluxes is 90°, $\Phi = \sqrt{[(0{\cdot}5)^2 + (0{\cdot}866)^2]} = 1$ as before. The resultant flux has now shifted by 60° i. e. in $\frac{T}{6}$ periodic time its angle has changed by $\left(\frac{2\pi}{6}\right)$ radians. The vector diagrams of fluxes are shown for the four instants in Fig. 3, and it is seen that the resultant flux is constant in magnitude and rotates at supply frequency.

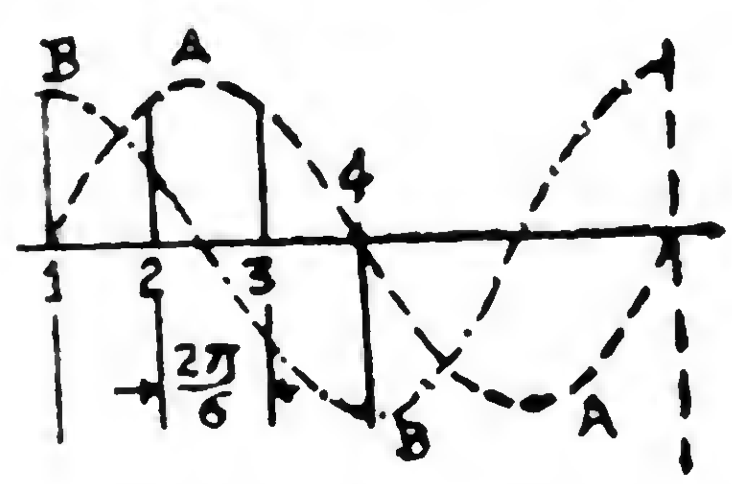


Fig. 2

Similarly, Fig. 4 (a) shows 3 coils whose axes are 120° to each other. The graphs of fluxes is shown in Fig. 4 (*b*). Again four instants are taken as indicated in Fig. 4 (b). At instant 1 flux in coil **A** is zero, flux in **B** is negative and is 0·866 of maximum and flux in **C** is positive and 0·866 of maximum. The angle between these two fluxes being 60°, the resultan flux Φ is equal to 1·5 time maximum or

$$\Phi = \sqrt{[(0{\cdot}866)^2 + (0{\cdot}866)^2 - 2 \times (0{\cdot}866)^2 \cos 120^\circ]}$$

now $\cos 120^\circ = -\cos 60^\circ = -0{\cdot}5$

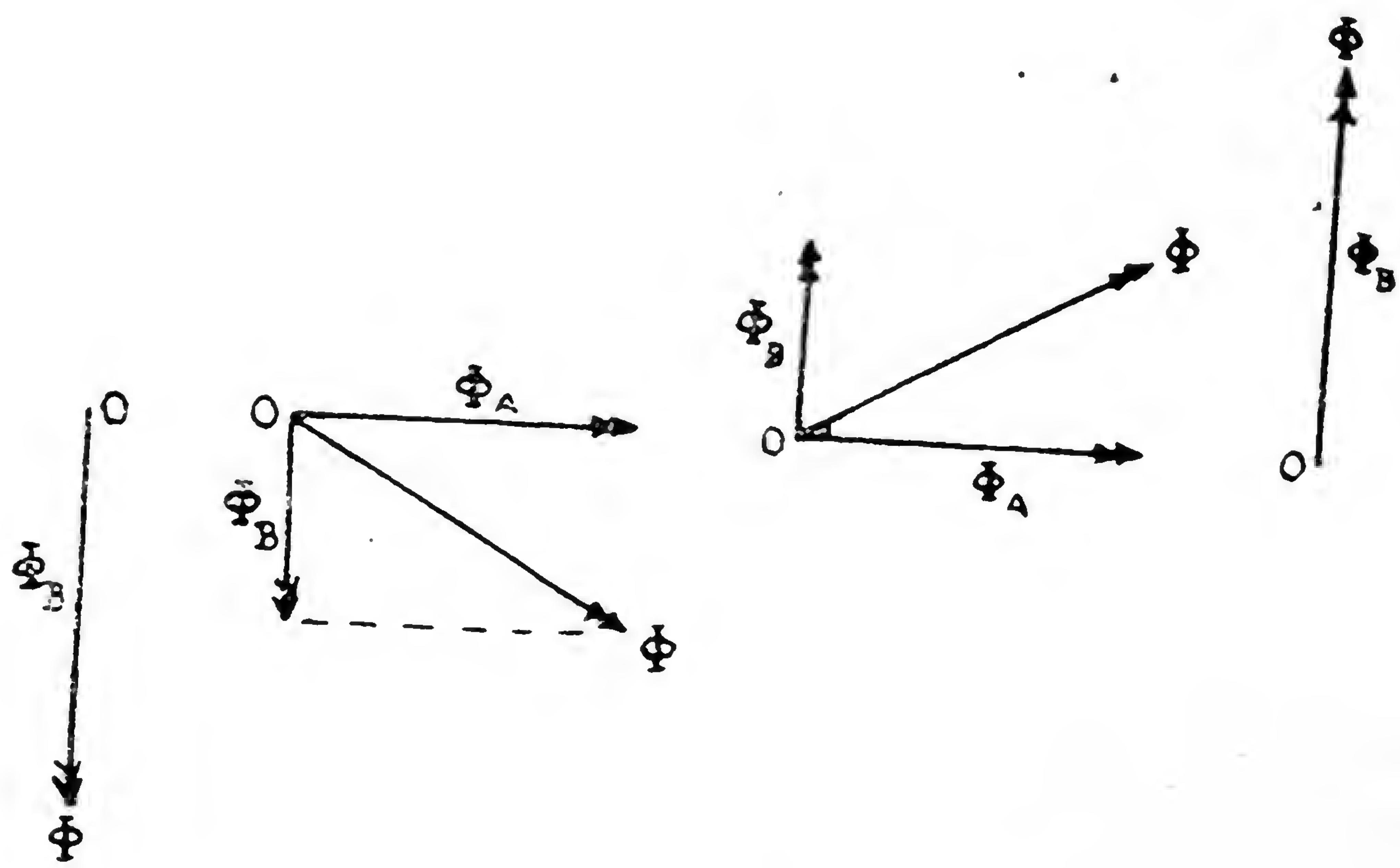


Fig. 3

$$\therefore\ \Phi = \sqrt{[(0{\cdot}866)^2 + (0{\cdot}866)^2 + 2 \times (0{\cdot}866)^2 \times 0{\cdot}5]} = \sqrt{2{\cdot}25}$$
$$= 1{\cdot}5$$

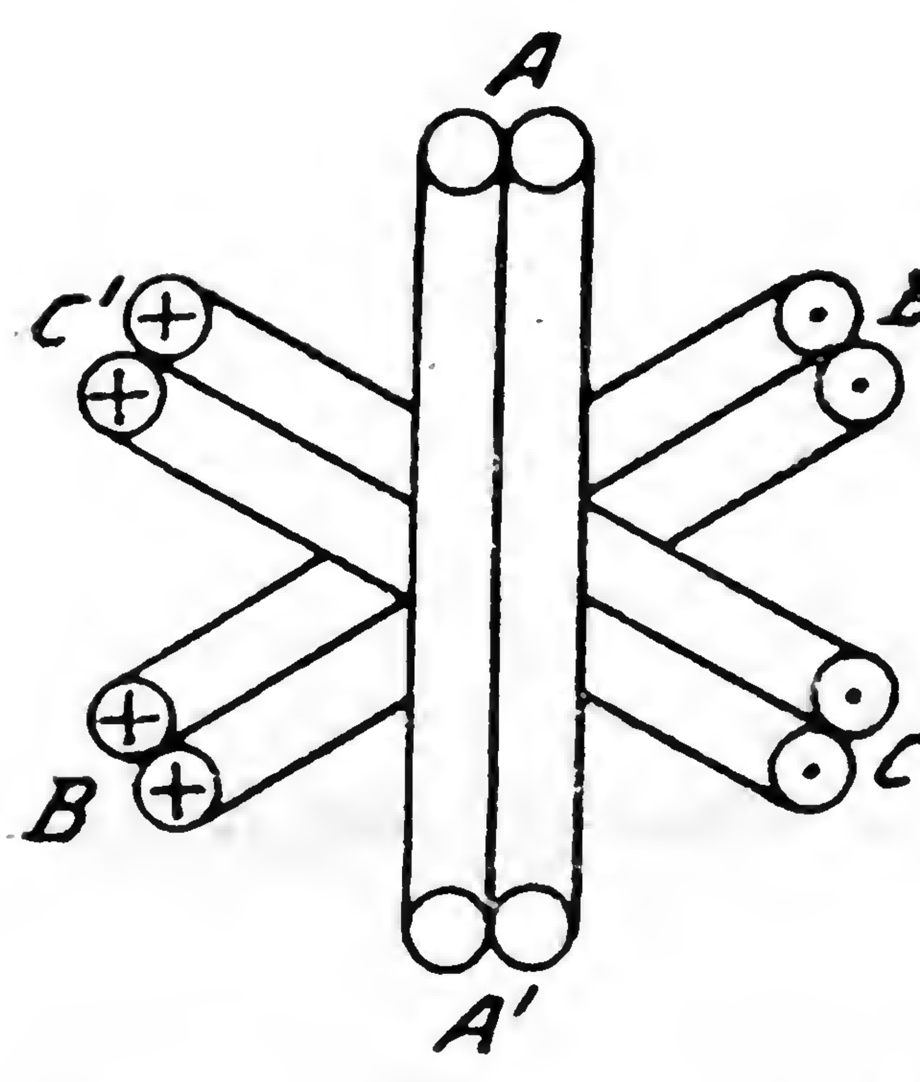


Fig. 4(a)

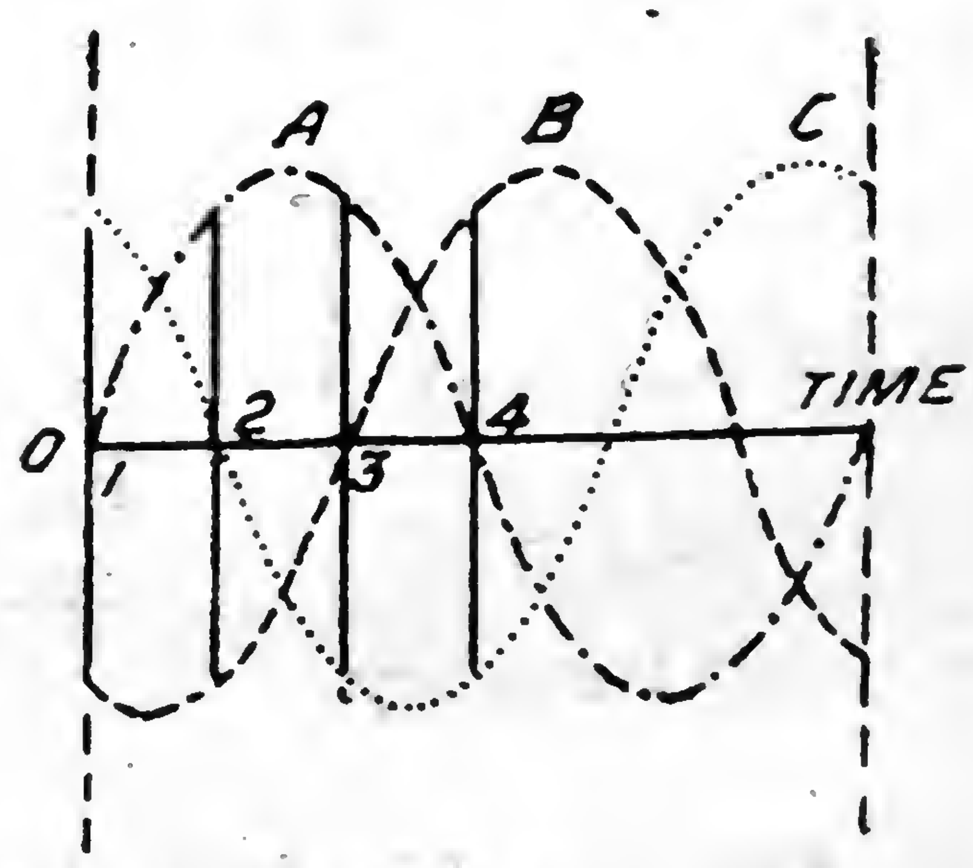


Fig. 4 (b)

Fig. 5 shows the vector diagrams of fluxes for the four instants and it is seen that the resultant flux has a constant magnitude of 1·5 times the maximum of flux in any one coil and that it rotates at supply frequency.

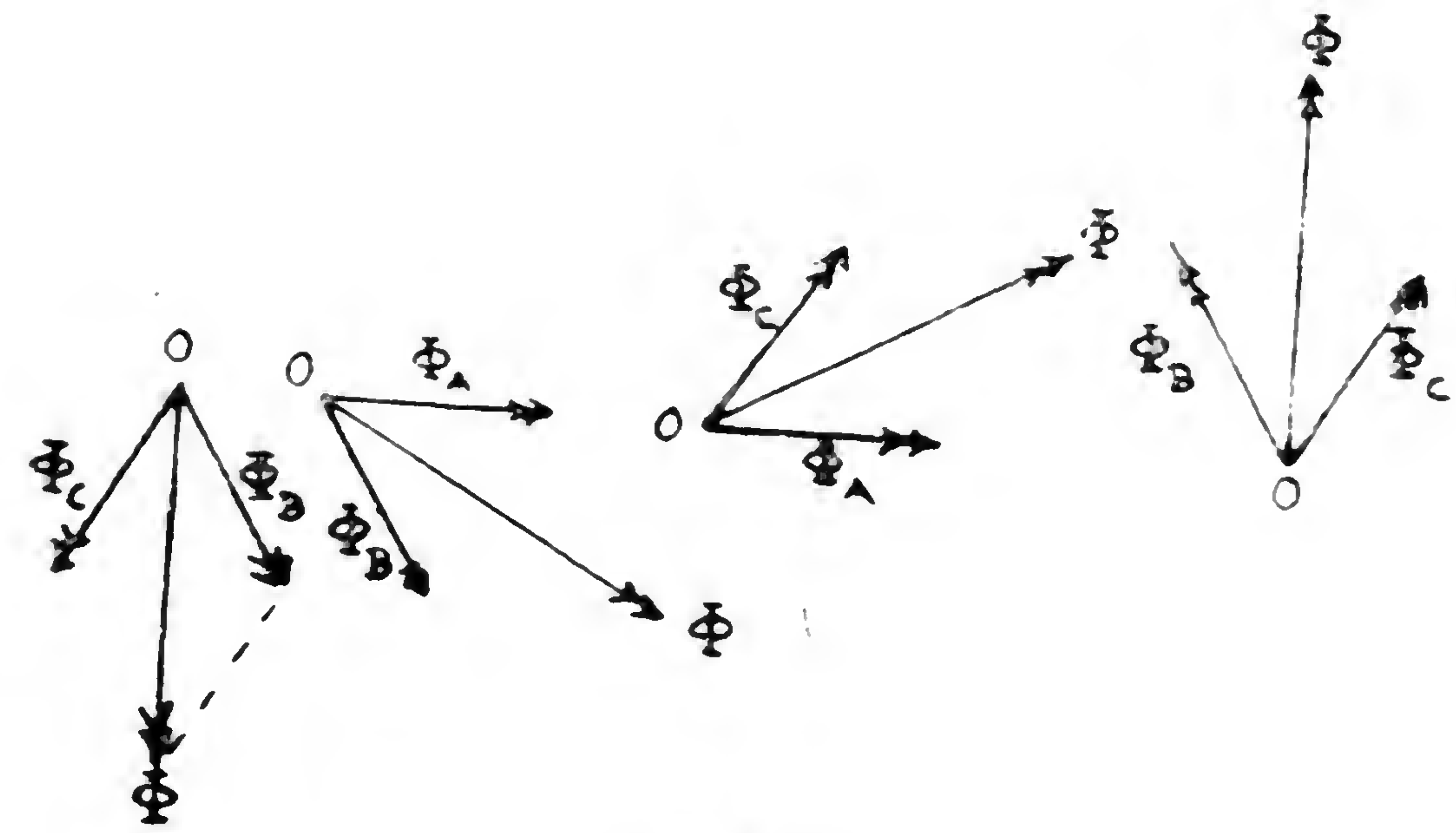


Fig. 5

3. Nature and Direction of Torque Produced in the Rotor :

At standstill, i. e. when the rotor is not rotating, the rotating flux sweeps across the rotor conductors at supply frequency. Rotor conductors over two pole-pitches are shown in Fig. 6 (a). The flux is shown as a sine wave moving from right to left and the conductors belonging to a turn bear the same numerals. The e. m. fs. produced in each turn depend upon the cosine of the angle it makes with the flux-axis. Assuming that the rotor circuit consists only of pure resistance, the turn **1** has maximum current in phase with its voltage. The other turns have less currents.

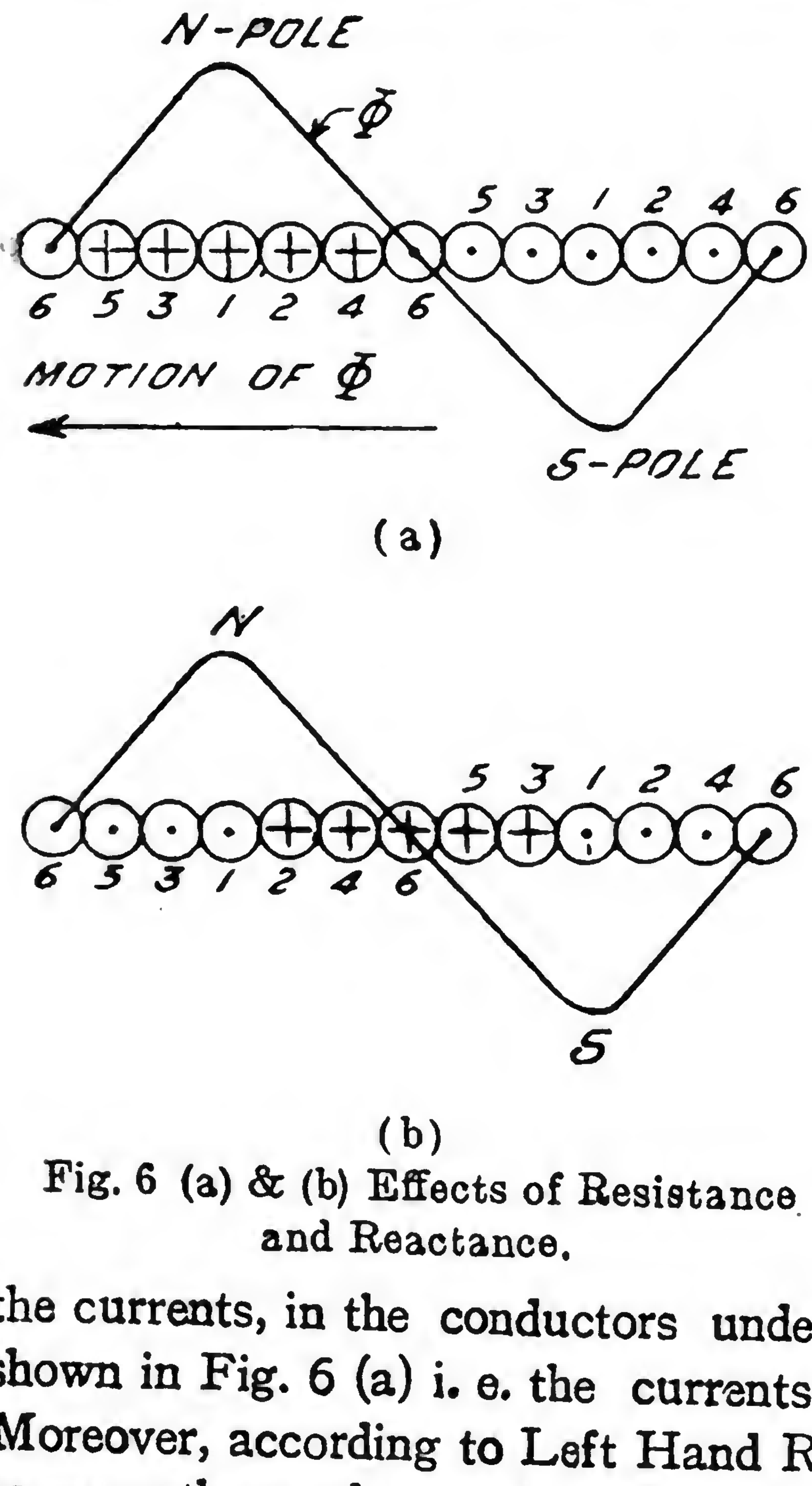


Fig. 6 (a) & (b) Effects of Resistance and Reactance.

According to Right Hand Rule, the induced e. m. fs. and the currents, in the conductors under the N–pole, have directions as shown in Fig. 6 (a) i. e. the currents flow into the plane of the paper. Moreover, according to Left Hand Rule, the mechanical force tending to move the conductors is in the direction of the rotating flux. Turn **6** has no force acting on it because there is no current in it and it is not situated in the magnetic field. Other coils exert smaller force since

they carry smaller currents and lie in fields of lesser density than coil **1**. However all torque directions are the same.

Fig. 6 (b) shows the same coils, but now they possess only inductances, hence the currents lag 90°. Applying Left Hand Rule, the torque directions of coils **2** and **4** are the same, but those of coils **3** and **5** are now in the reverse direction, and as before, coil **6** experiences no torque, though the current in it is maximum, because it is not situated in the magnetic field, i. e. the flux density at that point is zero. Further, the magnitude of torque on coils **2** and **4** is equal to those on coils **3** and **5**. The net result, therefore, is zero torque. This shows that at least at starting, reactance is undesirable, because the torque not only depends upon the magnitude of current but also on the *power factor of the rotor circuit.*

If p is the number of poles,

f the supply frequency in cycles per second and

n_s the flux speed in revolutions per minute

$$n_s = f \times \frac{2}{p} \times 60 \ r.\ p.\ m.$$

$$\text{or} \quad f = \frac{pn_s}{120} \ c.\ p.\ s. \qquad \dots \qquad \dots \qquad \dots \qquad (1)$$

n_s is called the synchronous speed and all synchronous machines run at their respective synchronous speeds. The induction motor is an asynchronous machine, therefore it runs at speeds less than its synchronous speed. For at synchronous speed the rotor of an induction motor has no induced e. m. f. and therefore no currents to produce any torque.

If n is the rotor speed in r. p. m., $(n_s - n)$ is the relative speed of the rotor with respect to the flux speed.

The fraction $(n_s - n)/n_s$ is called *fractional slip* i. e.

$$\frac{(n_s - n)}{n_s} = s \qquad \dots \qquad \dots \qquad \dots \qquad \dots \qquad (2)$$

When the rotor is not rotating n is zero and the slip is

$$s = \frac{n_s - 0}{n_s} = 1$$

The frequency of rotor e. m. f. and therefore rotor currents is

$$f_2 = \frac{(n_s - n)p}{120} \text{ c. p. s.} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

But since $s = \dfrac{n_s - n}{n_s}$

$$f_2 = \frac{s\, n_s p}{120}$$

$$f_2 = sf \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

When there is no load on the motor shaft the rotor speed is less than n_s and the fractional slip is about 0·01 or 1%. But when the motor is loaded the rotor slows down, thereby increasing its relative speed with the rotor flux. This increases the rotor e. m. f., the rotor current and the rotor frequency. The result is larger torque. In small and medium size machines the slip increases from 0·01 on no load to about 0·05 at full load. Thus only a small increase in slip is sufficient to develop full load torque in the rotor.

Hence the induction motor is a machine of fairly constant speed like the d. c. shunt motor.

If the rotor reactance at standstill is X_2 its value at slipe s becomes sX_2. For at no load the reactance becomes almost negligible and the rotor impedance is now all resistance. Further, if the rotor resistance is small the rotor current is large, so that the motor works with a larger torque which brings the speed near to synchronous speed, i. e. the slip is reduced.

4. Induced E. M. F. and Rotor Current: It is assumed that

(a) the resultant rotating flux Φ_{max} is sinusoidally distributed in the air-gap and is common to the stator and the rotor;

(b) the winding factor for both the stator and the rotor is unity;

(c) the number of stator and the rotor phases is the same and

(d) the induced E. M. F. of the stator winding is E_1 and equals the applied voltage V.

Hence

$$E_1 = 4{\cdot}44\, \Phi_{max} f T_1\, 10^{-8} \text{ volts per phase} \quad \ldots \quad \ldots \quad (5)$$

$$E_2 = 4{\cdot}44\, \Phi_{max} f_2 T_2\, 10^{-8} \text{ volts per phase} \quad \ldots \quad \ldots \quad (6)$$

$$= 4{\cdot}44\, \Phi_{max}\, sf T_2\, 10^{-8} \text{ volts per phase}$$

where Φ_{max} is the rotating resultant flux in lines or maxwells, T_1 and T_2 are the stator and rotor turns per phase respectively, f is the supply frequency, and $f_2 = sf$ is the frequency of the rotor e. m. f.

Both E_1 and E_2 lag Φ_{max} by 90°.

The rotor impedance per phase is $(R_2 + jX_2)$. Therefore the rotor current I_2 per phase is

$$I_2 = \frac{E_2}{\sqrt{(R_2^2+s^2X_2^2)}}$$

But $E_2 = \dfrac{T_2}{T_1} E_1$ at standstill, therefore at slip s

$$E_2 = \frac{T_2}{T_1} s E_1$$

Putting $\dfrac{T_2}{T_1} = k$, a constant

$$I_2 = \frac{k s E_1}{\sqrt{(R_2^2 + s^2 X^2_2)}} \quad \dots \quad \dots \quad \dots \quad (7)$$

or dividing throughout by s

$$I_2 = k \frac{E_1}{\sqrt{\left(\frac{R_2^2}{s^2} + X_2^2\right)}} \quad \dots \quad \dots \quad (8)$$

Eq. (7) shows that E_2 (or I_2) is of slip frequency and of reduced magnitude and that the rotor impedance is of constant resistance and variable reactance. But Eq. (8) states that the rotor e. m. f. and current are of line frequency. In other words, the rotor is stationary and the rotor impedance consists of a variable resistance and constant reactance. This justifies the vectorial addition of the primary and the secondary e. m. fs. and currents as in the case of static transformers.

The rotor current lags the rotor induced e. m. f. by an angle ϕ such that

$$\phi = \tan^{-1} \frac{sX_2}{R_2} \quad \dots \quad \dots \quad \dots \quad \dots \quad (9)$$

An assumption is made that the *equivalent rotor* has the same number of turns as the stator and the actual rotor quantities are replaced by *equivalent* quantities in terms of the stator (primary).

These equivalent quantities are expressed by a dash (′). For instance, R'_2, X'_2, E'_2, I'_2 etc. Hence the following relations hold

$$E'_2 = \frac{T_1}{T_2} E_2 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

$$E'_2 = s E_1 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

$$I'_2 = \frac{E'_2}{Z'_2} = \frac{s E_1}{\sqrt{(R'^2_2 + s^2 X'^2_2)}} \quad \ldots \quad \ldots \quad (12)$$

and the angle of lag between E'_2 and I'_2 is the same as that of Eq. (9).

5. **The Vector Diagram :** Fig. 7 shows the vector diagram of an induction motor. The rotor quantities shown in the diagram (the lower half) are *equivalent* values. Note the similarity of this diagram with that of the static transformer.

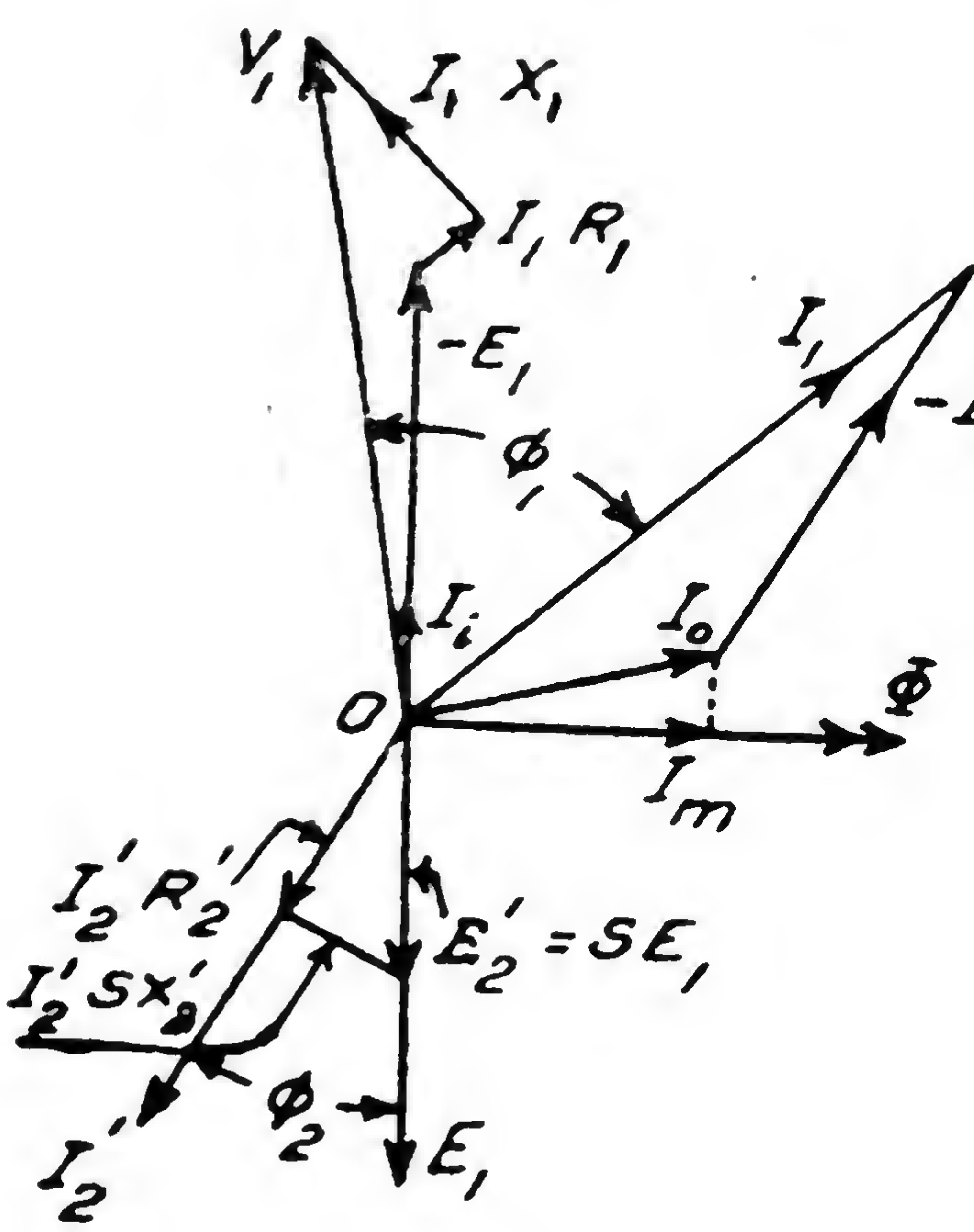


Fig. 7

The flux Φ is established by the magnetising component I_m. The core loss must also be supplied by another component I_i. When there is no current in the rotor circuit, the stator current will be I_0 such that vectorially

$$\dot{I}_0 = \dot{I}_m + \dot{I}_i.$$

But when the rotor current is I'_2 it produces an m. m. f. which opposes the primary m. m. f. And since the flux must be maintained at its original value, the stator must draw from the supply mains an additional current, equal and opposite to I'_2. This component of I_1 is shown as $-I'_2$ in the figure. The applied voltage V_1 overcomes not only the induced e. m. f. E_1 but also makes up for the resistance drop I_1R_1 and the reactance drop I_1X_1. Hence vectorially

$$V_1 = -E_1 + I_1R_1 + I_1X_1$$

Note that the power input P_1 to the stator is

$$P_1 = V_1 I_1 \cos \phi_1 \text{ (per phase)} \quad \ldots \quad \ldots \quad (13)$$

From the vector diagram,

$$\text{iron loss} = (-E_1)\, I_i$$

and $$\text{stator copper loss} = I_1^2 R_1$$

Subtracting this power from P_1 the remainder is evidently P_2, the power input to the rotor. But P_2 from the vector diagram (the lower half) is

$$P_2 = E_1 I'_2 \cos \phi_2$$

which transferred to the stator side (the upper half) is seen to be $(-E_1)(-I'_2) \cos \phi_2$. Therefore

$$P_1 = (-E_1) I_i + I_1^2 R_1 + (-E_1)(-I'_2) \cos \phi_2 \text{ (per phase)}$$

6. **Rotor Input, Rotor Copper Loss and Mechanical Power:** In the vector diagram E'_2 is made up of two components, one is the resistance drop $I'_2 R'_2$ and the other, at right angles to $I'_2 R'_2$, is the reactance drop $I'_2 sX'_2$. Since the angle between $I'_2 R'_2$ and E'_2 is ϕ_2

$$I'_2 R'_2 = E'_2 \cos \phi_2$$

The power input to the rotor being

$$P_2 = E_1 I'_2 \cos \phi_2$$

and according to equation (11), $\left(E_1 = \frac{E'_2}{s} \right)$, the expression for P_2 can be written as

$$P_2 = I'_2 (I'_2 R'_2)/s = \frac{I'^2_2 R'_2}{s} \qquad \ldots \quad \ldots \quad (14)$$

$$\text{i. e. } \textit{power input to rotor} = \frac{\textit{rotor copper loss}}{\textit{fractional slip}} \qquad \ldots \quad (14)$$

Subtracting the rotor copper loss from P_2 the remaining power must be that which is converted into mechanical power P_m

$$\therefore \qquad P_m = P_2 - sP_2 = P_2 (1 - s) \qquad \ldots \quad \ldots \quad (15)$$

This power, P_m, does not appear in the vector diagram. If the power input to the rotor is given the value of unity then

$$\frac{P_2}{\textit{rotor copper loss}} = \frac{1}{s} \qquad \ldots \quad \ldots \quad \ldots \quad (16)$$

$$\frac{P_2}{P_m} = \frac{1}{1-s} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (17)$$

and
$$\frac{P_m}{\textit{rotor copper loss}} = \frac{1-s}{s} \qquad \ldots \quad \ldots \quad \ldots \quad (18)$$

Summarising

$$P_2 : \textit{rotor copper loss} : P_m :: 1 : s : 1 - s.$$

7. Torque and Maximum Torque : If the rotor speed is n r. p. s. then the mechanical power of the rotor is

$$P_m = \frac{2\pi\, nT}{550} \times 746 \text{ watts,}$$

where T is the torque expressed in lb-ft. If n_s is the synchronous speed, also in r. p. s., then $n = n_s\,(1 - s)$, where s is the fractional slip. But according to Eq. (15).

$P_m = P_2\,(1 - s)$ therefore we may write

$$P_m = \frac{2\pi\, n_s\,(1 - s)\,T}{550} \times 746 = P_2\,(1 - s)$$

$$\therefore \quad T = \frac{P_2}{2\pi\, n_s} \times \frac{550}{746} = 0{\cdot}1173\,\frac{P_2}{n_s} \text{ lb-ft.}$$

Hence torque is proportional to rotor input P_2. Substituting the value of $P_2 = E_1\, I'_2 \cos\phi_2$, we have

$$T = K_1\, E_1\, I'_2 \cos\phi_2 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (19)$$

where $K_1 = \left(\frac{550}{746} \times \frac{1}{2\pi\, n_s}\right) = \left(0{\cdot}1173\,\frac{1}{n_s}\right).$

Now $I'_2 = \dfrac{sE_1}{\sqrt{(R'^2_2 + s^2X'^2_2)}}$; and $\cos\phi_2 = \dfrac{R'_2}{\sqrt{(R'^2_2 + s^2X'^2_2)}}$

substituting these values in Eq. (19), we have

$$T = K_1\,\frac{sE_1^2\, R'_2}{R'^2_2 + s^2X'^2_2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (20)$$

Eq. (20) shows that the torque of an induction motor is proportional to the square of the applied voltage. This property renders the motor very susceptible to fluctuations in its terminal voltage.

If Φ_{max} is constant then E_1 is constant, hence

$$T = K_2\,\frac{sR'_2}{R'^2_2 + s^2\, X'^2_2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (21)$$

where K_2 is another constant or $K_2 = K_1\, E_1^2$.

Dividing the numerator and the denominator of (21) by s^2

$$T = K_2\,\frac{\dfrac{R'_2}{s}}{\dfrac{R'^2_2}{s^2} + X'^2_2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (22)$$

Compare Eq. (22) with Eq. (8).

If Eq. (21) is differentiated with respect to s and the result equated to zero, it will give the condition for maximum torque e. g.

$$\frac{dT}{ds} = 0 = (R'^2_2 + s^2 X'^2_2) R'_2 - sR'_2 (2s X'^2_2)$$

$$R'^2_2 = s^2 X'^2_2 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (23)$$

Or

$$\frac{R'^2_2}{X'^2_2} = s^2$$

i. e.

$$\frac{R'_2}{X'_2} = \pm s \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (24)$$

The positive sign is used for the working range of the machine as a motor, and the negative sign for the working range when run as a generator.

Substituting in Eq. (21) the condition for maximum torque we get

$$T_{max} = K_2 \frac{s}{2 R'_2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (25)$$

Or

$$T_{max} = K_2 \frac{1}{2 X'_2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (26)$$

Equation (26) clearly shows that

(i) the maximum torque of a motor is constant,

(ii) the maximum torque is independent of speed or slip,

(iii) the maximum torque is independent of R'_2 and

(iv) the slip at with maximum torque occurs will depend upon the ratio $\frac{R'_2}{X'_2}$.

8. Torque-Speed Curve: Eq. (21) or (23) may be used for plotting the torque/speed characteristic of a motor, if R_2 and X_2 are known. By using Eq. (21) the torque/speed curve of a 3-phase, 4 pole, 50 cycle induction motor is shown in Fig. 8. $X_2 = 4$ ohms and $R_2 = 0{\cdot}4$ ohms. Values for slip s were assumed. Maximum torque occurs when slip is 10 %.

With slip-ring induction motors an external resistance of r ohms can be added in series with each rotor phase winding, so that the starting torque is higher, and the maximum torque occurs at higher values of slip. The equation for torque of a slip-ring motor becomes

$$T = K_2 \frac{s\,(R'_2 + r)}{(R'_2 + r)^2 + s^2\,X'^2_2} \qquad \dots \qquad \dots \qquad \dots \qquad (27)$$

If $\dfrac{R'_2 + r}{X'_2} = s$, the four torque/speed curves of Fig. 9 are for values when $R_2 = X_2$; $0{\cdot}5\,X_2$: $0{\cdot}25\,X_2$ and $0{\cdot}1\,X_2$.

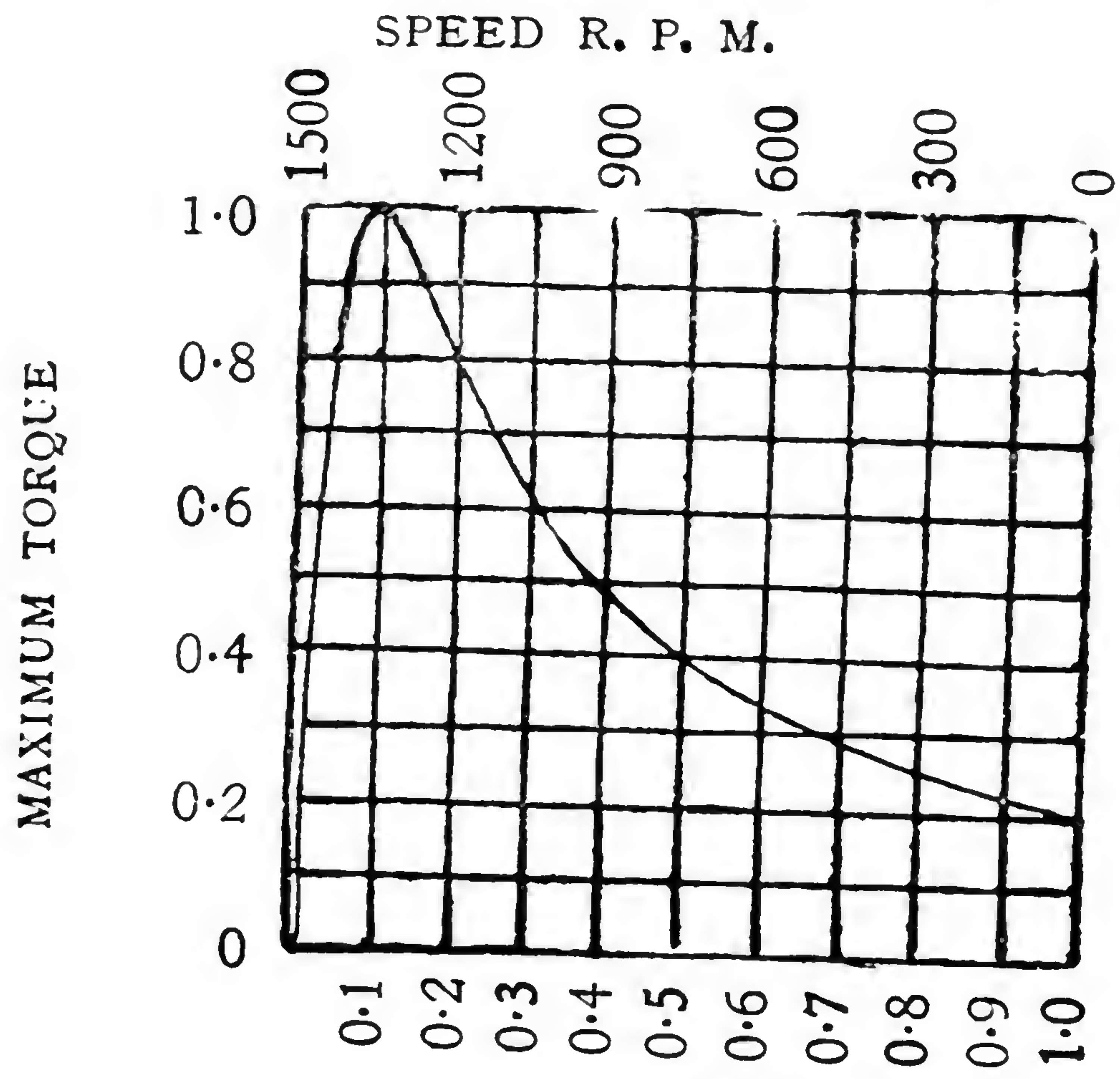


Fig. 8 Torque/Slip Curve.

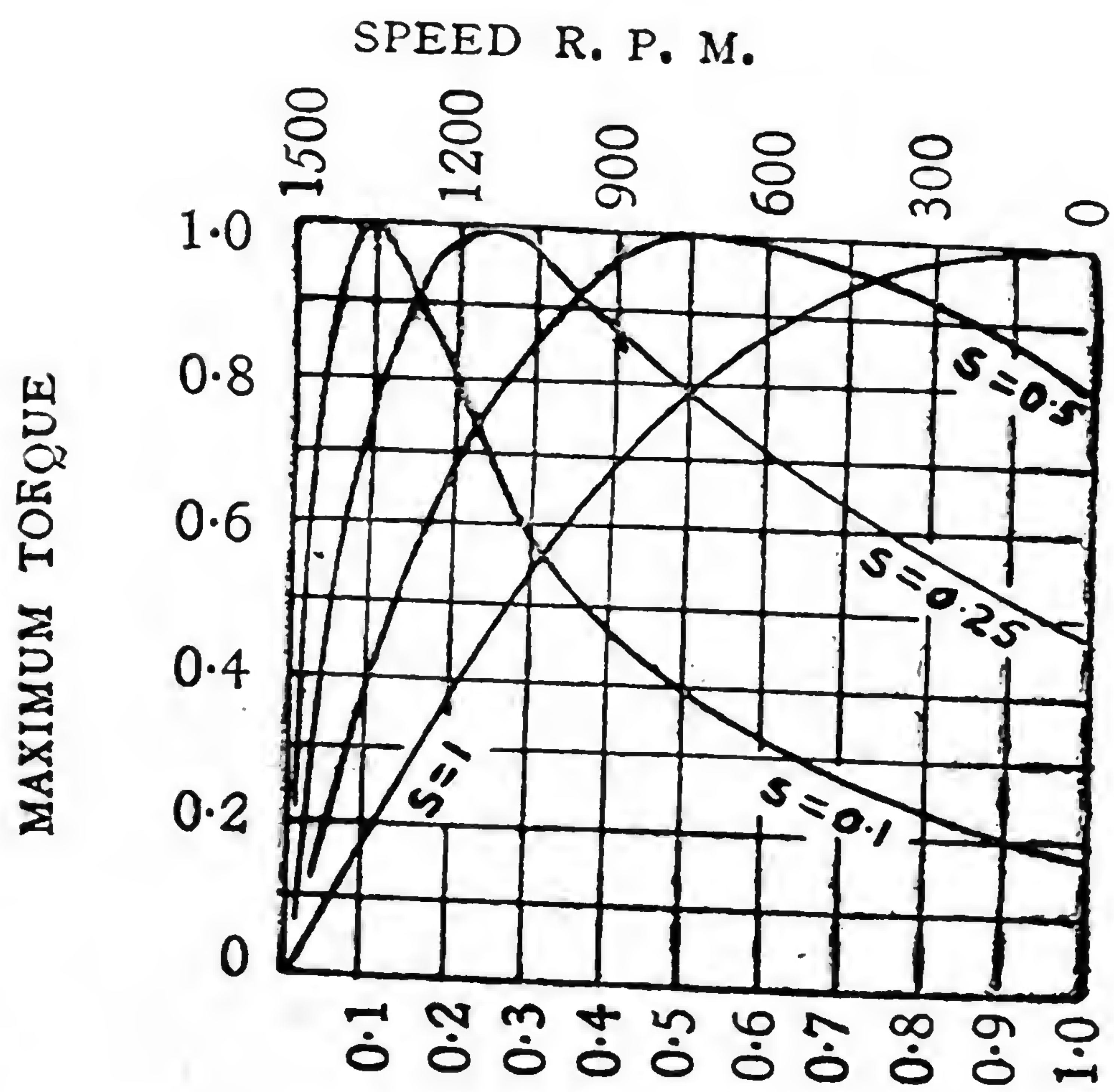


Fig. 9

The value of r can be so chosen that at starting the torque obtained may be any desired value between 20% and 100% of maximum torque. Such is not the case with the squirrel cage motors. For, R_2 is fixed in value and no adjustment is possible. The squirrel cage rotor must be so designed as to give maximum torque at some suitable speed or slip.

Example: The rotor of a 3-phase induction motor is star-connected and has an induced e. m. f. of 50 volts between slip-rings at standstill on open circuit, when the stator is connected to its normal supply voltage. The impedance at standstill is $(0{\cdot}5 + j\,3{\cdot}5)$ ohms per phase.

Calculate the current in each phase and the power factor at the instant of starting when (a) the rotor is connected to a star-connected starting rheostat having a resistance of 4 ohms per phase, and (b) when the slip-rings are short circuited.

Solution: Rotor induced e. m. f. per phase $= \dfrac{50}{\sqrt{3}} = 28{\cdot}9\ V.$

(a) The total impedance per phase

$$Z_2 = \sqrt{[(0{\cdot}5+4)^2 + (3{\cdot}5)^2]} = 5{\cdot}7 \text{ ohms.}$$

$\therefore$ current per phase $= \dfrac{28{\cdot}9}{5{\cdot}7} = 5$ *amperes*

and the power factor $= \dfrac{R_2}{Z_2} = \dfrac{4{\cdot}5}{5{\cdot}7} = 0{\cdot}76$ (lag.)

(b) The impedance per phase

$$Z_2 = \sqrt{[(0{\cdot}5)^2 + (3{\cdot}5)^2]} = 3{\cdot}535 \text{ ohms.}$$

$\therefore$ current per phase $= \dfrac{28{\cdot}9}{3{\cdot}535} = 8{\cdot}17$ *amperes*

and the power factor $= \dfrac{0{\cdot}5}{3{\cdot}535} = 0{\cdot}14$ (lag.)

Example: The rotor resistance and reactance per phase of a 4-pole, 50 cycle, 3-phase slip-ring induction motor is 0·4 and 4 ohms respectively. Calculate (i) the speed at which maximum torque will occur, and (ii) the ratio maximum torque/starting torque. What value per phase must an external resistance have so that the starting torque is half of maximum torque?

Solution: The synchronous speed $n_s = \dfrac{120f}{p}$ r. p. m.

$$= 1500 \text{ r. p. m.}$$

For the running condition,

torque, $T = k \dfrac{sR_2}{R^2_2 + s^2X^2_2}$ where s = fractional slip,

R_2 = rotor resistance per phase.

X_2 = rotor reactance per phase,

and k is a constant.

Maximum torque occurs when $R_2 = sX_2$

$\therefore$ maximum torque will occure when

$$s = \frac{R_2}{X_2} = \frac{0{\cdot}4}{4} = 0{\cdot}1.$$

The speed n, when the fractional slip $s = 0{\cdot}1$ will be

$$0{\cdot}1 = \frac{n_s - n}{n_s} = \frac{1500 - n}{1500}$$

$$\therefore \quad n = 1500 - 150 = 1350 \text{ r. p. m.}$$

Maximum torque when $s = 0{\cdot}1$ will be

$$T_{max} = k \frac{0{\cdot}1 \times 0{\cdot}4}{(0{\cdot}4)^2 + (0{\cdot}1 \times 4)^2} = 0{\cdot}125\, k$$

The starting torque, T_s, when $s = 1$ will be

$$T_s = k \frac{1 \times 0{\cdot}4}{(0{\cdot}4)^2 + (1 \times 4)^2} = 0{\cdot}0248\, k$$

$$\therefore \quad \frac{T_{max}}{T_s} = \frac{0{\cdot}125\, k}{0{\cdot}0248\, k} = 5{\cdot}1$$

To obtain starting torque $= \frac{1}{2} T_{max}$, a resistance r must be included in each rotor phase

$$\therefore \quad \tfrac{1}{2} T_{max} = 0{\cdot}5 \times 0{\cdot}125 k = k \frac{s\,(R_2 + r)}{(R_2 + r)^2 + (s\,X_2)^2}$$

But at starting $s = 1$, therefore substituting the values of R_2 and X_2, and cancelling k,

$$0{\cdot}5 \times 0{\cdot}125 = \frac{0{\cdot}4 + r}{(0{\cdot}4 + r)^2 + 4^2}$$

solving and arranging,

$$r^2 - 15{\cdot}2r + 9{\cdot}76 = 0,$$

$$\therefore \quad r = 14{\cdot}65\,\Omega \text{ or } 0{\cdot}67\,\Omega.$$

9. The Equivalent Circuit: Eq. (22) and the vector diagram shown in Fig. 7 are helpful to construct an equivalent circuit diagram

of an induction motor. The process is similar to that adopted in the case of a static transformer. In fact, the two are very much similar. Fig. 10 (a), (b) and (c) are the "step by step" developments.

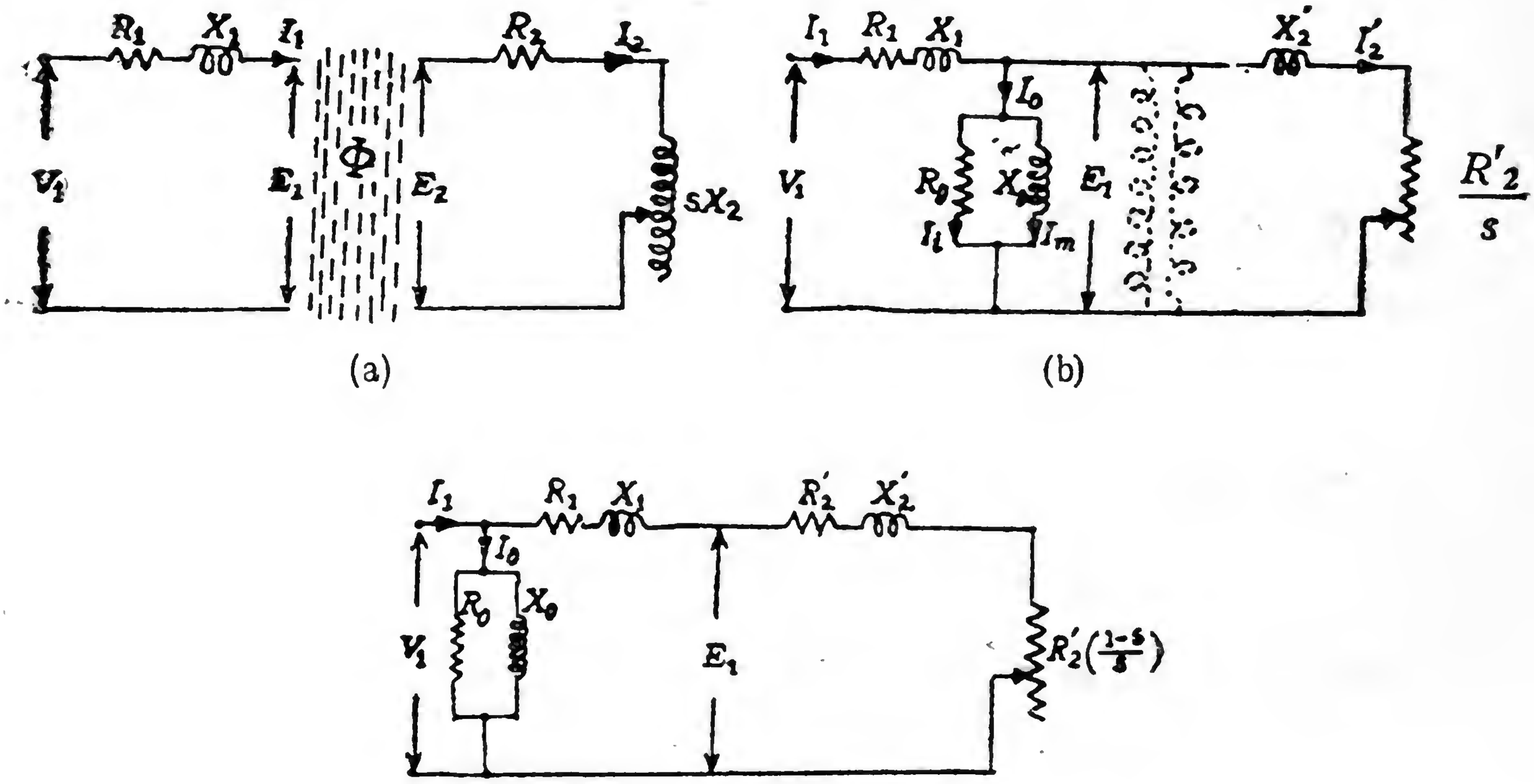


Fig. 10. Stages of Development.

Fig. 10 (a) shows the actual conditions. The primary impedance $(R_1 + j X_1)$ and the secondary impedance $(R_2 + j X_2)$ are separated by the magnetic circuit shown as Φ. The induced e. m. fs. are marked as E_1 and E_2. The currents that flow in the primary and the secondary are also marked as I_1 and I_2. Fig. 10 (b) shows a shunt impedance made up of R_0 and X_0. The former is to account for the current I_i shown in Fig. 7, and the latter to account for the magnetising current I_m. The secondary impedance has been expressed in terms of the equivalent primary and reducing the turns ratio to 1. Hence the dotted figure indicates an ideal transformer and the secondary is shown as having a constant reactance and a variable resistance according to Eq. (22). Finally, Fig. 10 (c) shows the shunt impedance being shifted and placed prior to the primary impedance. The current passes through R_1, X_1, R'_2 and X'_2, and through a *fictitious load resistance* $= R'_2 \frac{(1-s)}{s}$, which has a resistance varying from 0 to infinity. For

$$\frac{R'_2}{s} = R'_2 + R'_2 \left(\frac{1-s}{s}\right) \quad \ldots \quad \ldots \quad \ldots \quad (28)$$

We are now able to study the performance of the induction motor under several conditions by referring to Fig. 10 (c):

Firstly, on no-load, when $R'_2 \left(\frac{1-s}{s}\right)$ is equal to infinity i. e. $s = 0$, and the rotor runs very near to synchronous speed, the only current that flows will be I_0 through the shunt impedance (R_0 and X_0), which is now the predominating impedance. **The losses at no load are core loss, windage and friction losses.** However, I_0 does not provide the energy, so to say, for the last two losses, therefore a very small current has to flow through the rotor impedance to provide a small torque. At no-load the power factor is low since I_m is much larger than I_i. (See Fig. 7).

Secondly, under load conditions, the expression for the current is

$$I'_2 = \frac{V_1}{\sqrt{\left\{\left[R_1 + R'_2 + R'_2\left(\frac{1-s}{s}\right)\right]^2 + (X_1 + X'_2)^2\right\}}} \quad \ldots \quad (29)$$

and
$$\sin \phi_2 = \frac{X_1 + X'_2}{\sqrt{\left\{\left[R_1 + R'_2 + R'_2\left(\frac{1-s}{s}\right)\right]^2 + (X_1 + X'_2)^2\right\}}} \quad (30)$$

Therefore equation (29) reduces to

$$I'_2 = \frac{V_1}{(X_1 + X'_2)} \sin \phi_2 \quad \ldots \quad \ldots \quad \ldots \quad (31)$$

which is an equation of a circle whose diameter is $\frac{V_1}{X_1 + X'_2}$. This means that the locus of the current vector lies on the circumference of a circle. When $\phi_2 = 90°$, I'_2 will lie along the diameter. This is only possible when the resistance values of Eq. (29) vanish. Hence the *ideal short-circuit current* is given by the expression

$$I'_2 = \frac{V_1}{(X_1 + X'_2)} \quad \ldots \quad \ldots \quad \ldots \quad (32)$$

But $(R_1 + R'_2)$ is constant, therefore the *usual short-circuit current* is

$$I'_{2\,(s.c.)} = \frac{V_1}{\sqrt{[(R_1 + R'_2)^2 + (X_1 + X'_2)^2]}} \quad \ldots \quad (33)$$

The primary current is the vector sum of I'_2 and I_0 and since I_0 is assumed to be constant throughout, the locus of I_1 also lies on

the same circumference on which I'_2 lies. The corresponding power factor at usual short-circuit condition is

$$\cos \phi_{2\,(s.c.)} = \frac{R_1 + R'_2}{\sqrt{[(R_1+R'_2)^2+(X_2+X'_2)^2]}} \quad \dots \quad (34)$$

Note that at the condition of the usual short-circuit, the stator impedance drop (IZ) is fairly large, consequently the induced e. m. f. E_1 is almost half of its usual working range.

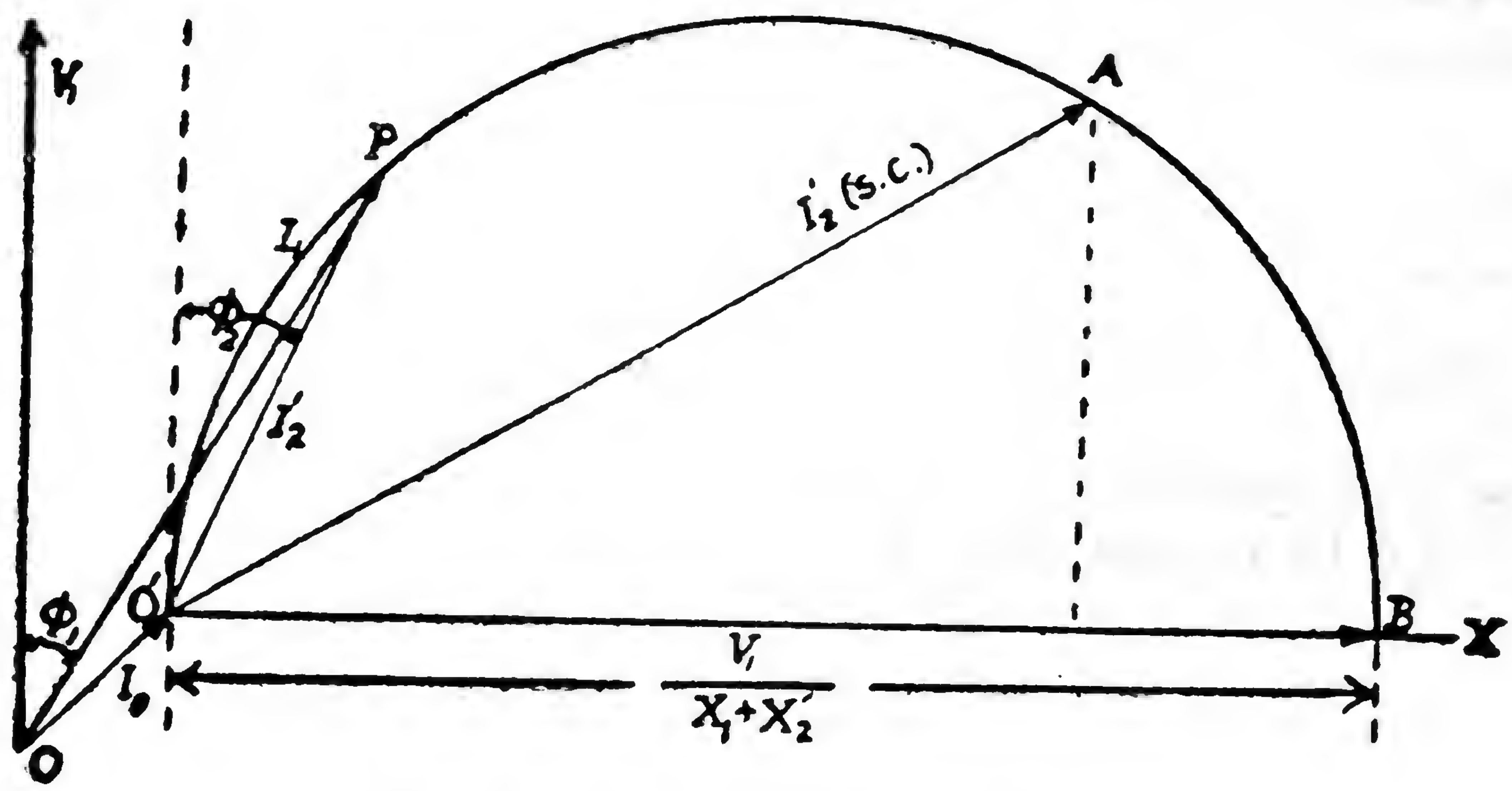


Fig. 11 Locus of Current Vector

Fig. 11 illustrates what has been discussed so far. All the quantities marked in the diagram are in terms of stator values. $O'B$ is the *ideal short-circuit* current of Eq. (32), while $I'_{2\,(sc)}$ is the current of Eq. (33).

10. The Circle Diagram : For the purpose of studying the various characteristics of an induction motor, the *circle-diagram*, which is a *vector locus* diagram of stator and rotor currents, is very useful and invaluable.

We will first construct a circle diagram from the test data and then study the performance of a motor whose particulars are :—

400 volts, 50 cycle, 3-phase, number of poles 4; full load output 7·5 h. p. The motor has wound-rotor having a ratio of primary to secondary turns = 2·62. The stator resistance per phase = 0·67 ohm, and the rotor resistance per phase = 0·185 ohm.

No-load readings :—400 V; 6 amp; 360 watts.

Short circuit test :—100 V; 12 amp; 720 watts, (rotor locked).

(*a*) *Preliminary Calculations* :—

The no load power factor is

$$\cos \phi_0 = \frac{W_0}{\sqrt{3} \times V \times I_0} = \frac{360}{\sqrt{3} \times 400 \times 6} = 0{\cdot}087$$

$\therefore \quad \phi_0 = 85^\circ$ (from Tables).

The primary short-circuit current, $I_{1\,(sc)}$, when full voltage of 400 is applied is

$$I_{1\,(sc)} = \frac{400}{100} \times 12 = 48 \text{ amperes.}$$

The corresponding power loss is

$$W_{(sc)} = 720 \left(\frac{48}{12}\right)^2 = 11520 \text{ watts,}$$

and the power factor on short-circuit is

$$\cos \phi_{(sc)} = \frac{11520}{\sqrt{3} \times 400 \times 48} = 0{\cdot}347$$

$\therefore \quad \phi_{(sc)} = 69^\circ - 40'$.

The measurement of resistance is done by passing a direct current. But for alternating work the value of resistance increases due to *skin effect*. Adding, therefore, 10% to the d. c. value of resistance the stator resistance per phase is $0{\cdot}67 \times 1{\cdot}1 = 0{\cdot}737$ ohm, likewise, the rotor resistance per phase is $0{\cdot}185 \times 1{\cdot}1 = 0{\cdot}2035$ ohm. Again, since the ratio of turns is 2·62, the ratio

$$\frac{\textit{rotor copper loss}}{\textit{stator copper loss}} = (2{\cdot}62)^2 \times \frac{0{\cdot}2035}{0{\cdot}737} = 1{\cdot}88$$

Therefore the ratio of

$$\frac{\textit{rotor copper loss}}{\textit{total copper loss}} = \frac{1{\cdot}88}{2{\cdot}88} = 0{\cdot}656.$$

(*b*) *Construction of Circle Diagram* :—

Choosing a scale of $1'' = 10$ amperes the no load current I_0 is drawn at an angle of 85° to the *y*-axis, which represents the voltage vector, see Fig. 12 and $I_{1\,(s.c.)}$ is drawn at an angle of $69^\circ - 40'$.

$$I_0 = OA_0 = 0{\cdot}6''$$

$$I_{1\,(s.c.)} = OA_S = 4{\cdot}8''$$

Join A_0 to A_S then $A_0 A_S = I'_2$, the secondary equivalent current. Two horizontal lines OX and A_0X_1 are drawn. Bisect A_0A_S, the perpendicular bisector meeting the horizontal line A_0X_1 at C. Then C is the centre of the semicircle which will pass through A_0 and A_S. This semicircle is the locus of the extremities of the primary and the secondary equivalent currents I_1 and I'_2.

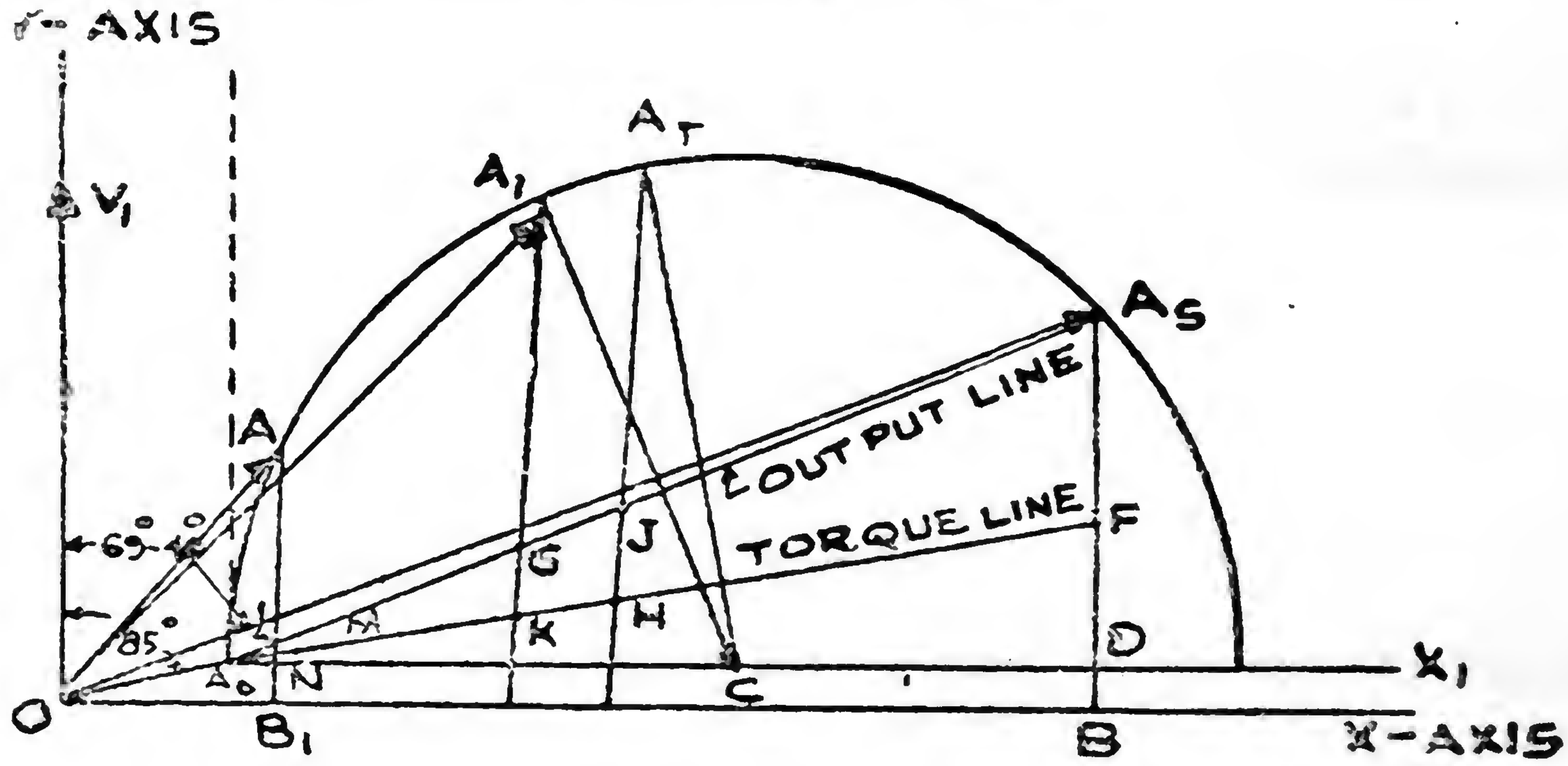


Fig. 12 The Circle Diagram.

Drop a perpendicular A_SB from A_S on the x-axis, cutting the line A_0X_1 at D. Divide A_SD in the ratio of 1·88 : 1 at F. Therefore A_SF represents the rotor copper loss and FD the stator copper loss at short-circuit condition. [The short circuit condition is when the rotor is *locked* i. e. there is no mechanical power output, the whole power is lost in the copper of the stator and rotor.].

Join F to A_0, then the line A_0F is the "*torque line*". $A_0 A_S$ is called the "*output line*". $A_0 A_S$ is also the rotor equivalent current at short-circuit condition.

According to calculation, the short-circuit loss is 11520 watts. Therefore A_SD represents 11520 watts. This fixes the power scale as follows :—

A_SD measures to be 1·66 inch, so that power seale is

$$I'' = \frac{11520}{1{\cdot}66} = 6920 \text{ watts.}$$

From C draw perpendiculars to A_0A_S and A_0F, meeting the semicircle at A_1 and A_T respectively. From A_1 and A_T drop perpendiculars on to OX meeting A_0A_S at G and J, and meeting A_0F at K and H respectively. Then A_1G represents maximum output in watts, A_TH the maximum torque in "*synchronous watts*".

An output of 7·5 h. p. = 7·5 × 746 = 5595 watts, which on the scale of watts represents a length of 0·835 inch. Therefore measuring a vertical distance of 0·835 inch from the line $A_0 A_S$ to the semicircle the point A is obtained, i. e. $AL = 0{\cdot}805'' = 5595$ watts = 7·5 h. p. Produce AL to B_1 cutting A_0F at M and $A_0 X_1$ at N and meeting OX at B_1. Join O at A. Then OA is the full load stator current, AM is the full load torque (synchronous watts) and the full load power factor angle is VOA.

Also AM is the input to rotor,
LM is the rotor copper loss,
MN is the stator copper loss,
NB_1 is the no load loss due to friction, windage, bearing and that due to the core. These are assumed to be constant throughout.
AB_1 is the input to the motor, and
AL is the output.

All the above quantities are in watts, and measurable on the scale of watts.

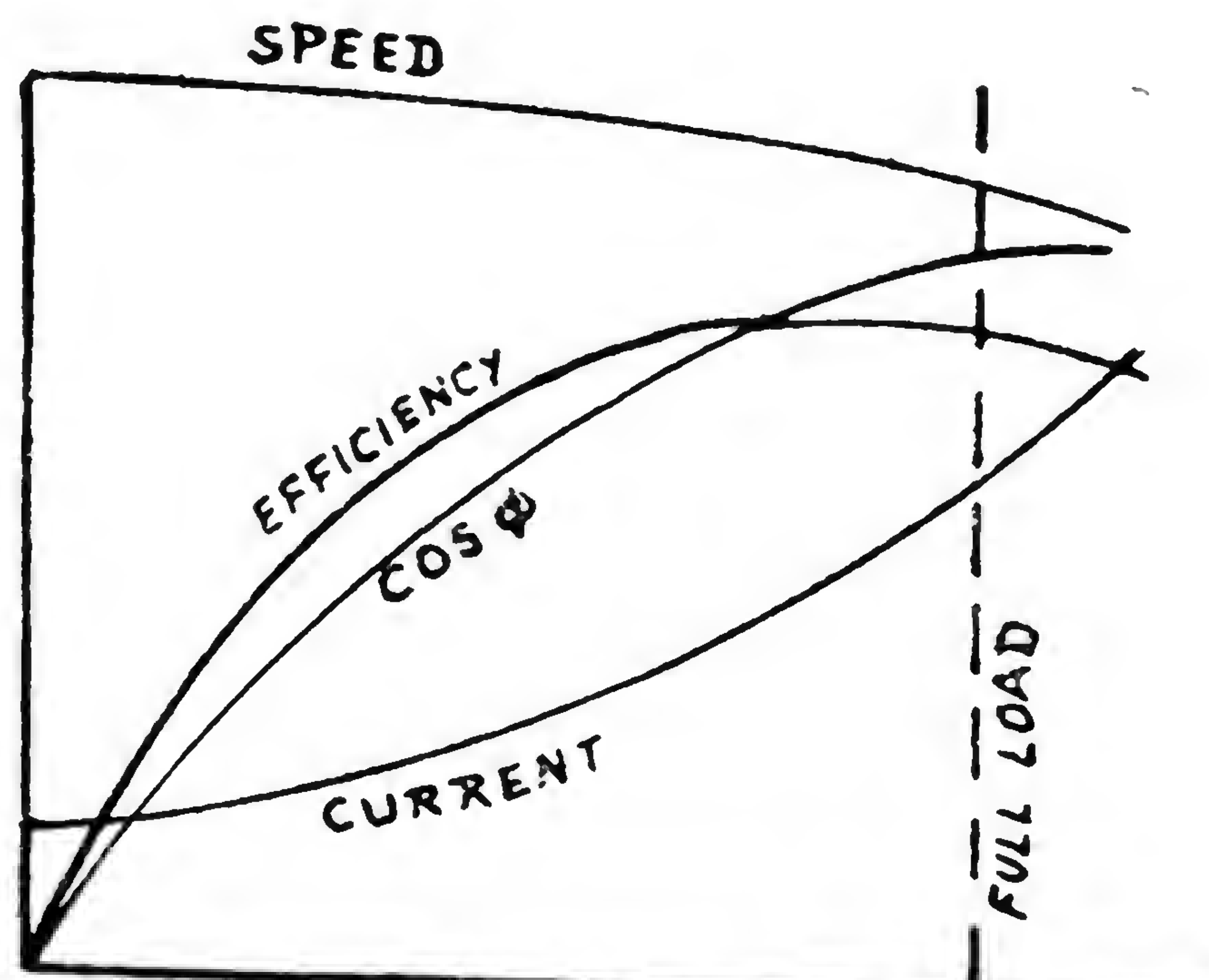


Fig. 13. Performance Curves.

The following results are obtained by measuring the various lengths and using the proper scale relation :—

(*a*) Full load current = OA = 12·8 amperes.

(*b*) Full load slip $= \dfrac{LM}{AM} = 0{\cdot}04.$

(*c*) Full load power factor = 0·79 (lag.)

(*d*) $\dfrac{\text{Maximum torque}}{\text{Full load torque}} = \dfrac{A_T H}{AM} = 2{\cdot}25.$

(*e*) Maximum power output $= A_1G = 13{\cdot}4$ h. p.

(*f*) Full load efficiency $= \dfrac{AL}{AB_1} = 83\,\%$.

The performance of a typical induction motor is shown by curves in Fig. 13. On light load the power factor is low since the major portion of the current is reactive magnetising current. But as the load increases the power factor increases to a fairly high value.

The efficiency curve is almost identical with that of a d. c. motor. The efficiency becomes maximum near full load output. The speed falls very slightly from no load to full load and the curve is similar to that of a shunt motor. The current value rises in proportion to the load upto a certain value, after which it rises very rapidly.

11. Starting Devices ; Small induction motors, upto 3 h. p. capacity, may directly be switched on to the supply mains, but those of higher capacity must use some type of starting device, or starters as they are commonly called. The function of these starters is to restrict the initial rush of current, which, in the case of induction motors, is about 5 times the full load current.

(*a*) *Stator Rheostat Starter* : Fig. 14 shows the diagram of connection. Reduced voltage is impressed on each stator phase due to the resistance R of the rheostat. Hence the initial current drawn from the supply mains will be less than if the machine were to be switched directly on to the supply mains.

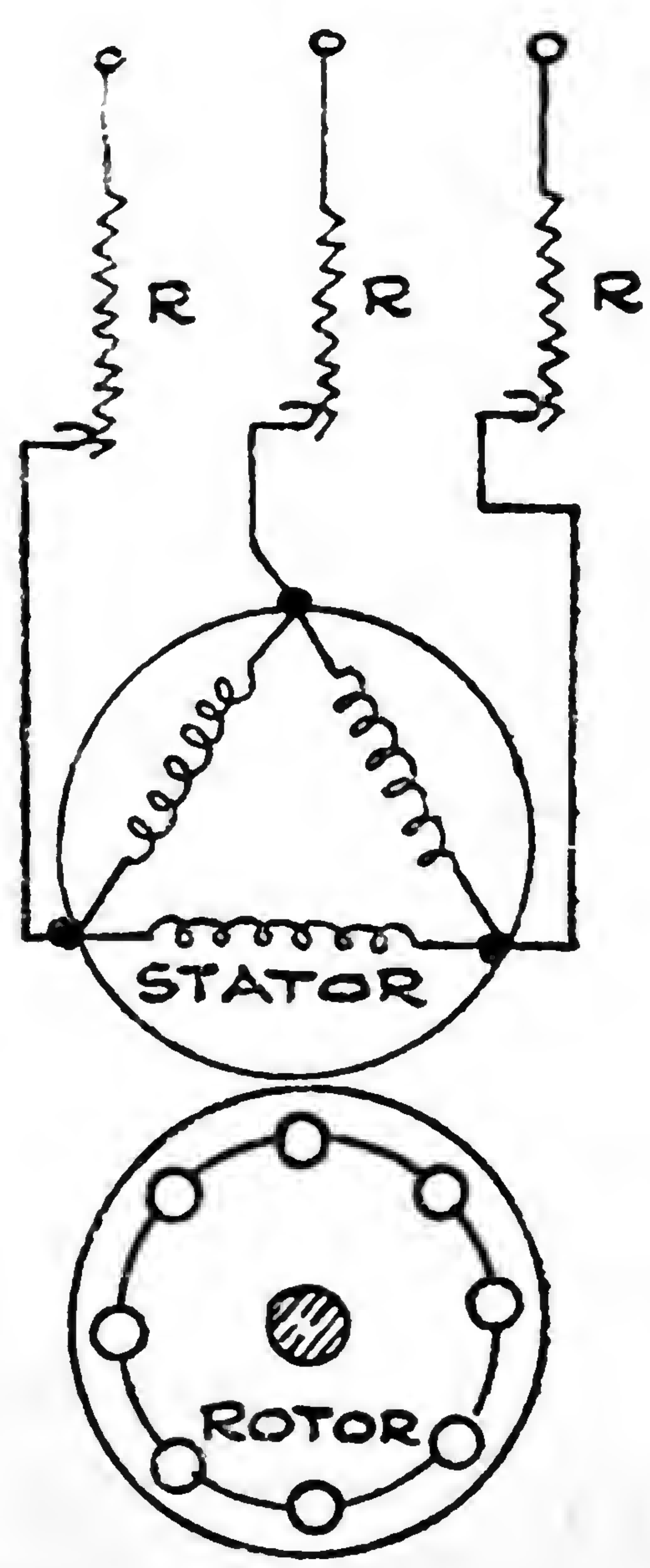


Fig. 14
Stator Resistance Starter

Let I_{sc} be the short-circuit current, i. e. the starting current when no device is used. If R is such as to limit the starting current to $\frac{1}{2}\,I_{sc}$, then the starting current $I_s = \frac{1}{2}\,I_{sc}$. Let the full load current of a motor be

$$I_{fl} = \tfrac{1}{5}\,I_{sc}$$

According to Eq. (19), the torque $T \propto P_2$ the rotor input, and

$$P_2 = \frac{\textit{rotor copper loss}}{\textit{fractional slip}} = \frac{I'^2_2\,R'_2}{s}$$

$\therefore \quad T_s \propto \dfrac{I^2{}_s R'_2}{s}$; (the stator current at starting is I_s which bears a fixed ratio to the rotor current at starting).

Now, full load torque is

$$T_{fl} \propto \frac{I^2{}_{fl} R'_2}{s_{fl}} \quad \text{where } s_{fl} \text{ is the slip at full load.}$$

And starting torque T_s is

$$T_s \propto I^2{}_s R'_2 = (\tfrac{1}{2} I_{sc})^2 R'_2, \text{ since } s = 1.$$

$$\therefore \quad \frac{T_s}{T_{fl}} = (\tfrac{1}{2})^2 \left(\frac{I_{sc}}{I_{fl}}\right)^2 \times s_{fl}$$

If we call the fraction $\frac{1}{2}$ as x, then

$$\frac{T_s}{T_{fl}} = x^2 \left(\frac{I_{sc}}{I_{fl}}\right)^2 \times s_{fl} \quad \ldots \quad \ldots \quad (35)$$

Further, this starting torque bears to the torque obtainable by direct switching a ratio

$$\frac{\textit{starting torque by stator rheostat}}{\textit{starting torque by direct switching}} = \frac{x^2 I^2{}_{sc} R'_2}{I^2{}_{sc} R'_2} = x^2$$

(*b*) *Star-Delta Starter*: Any three equal impedances connected in star have an impedance three times as great as when they are connected in delta. Hence the three phases of the stator are connected in star at the instant of starting and they are connected in delta after the motor has attained normal speed. The starter must have three positions—(1) off, (2) start and (3) run.

For the purpose of making possible these two different connections by the starter, all the six ends of the three phase windings must be brought out to the starter. So that nine wires, or leads, must be connected to the starter—6 from the windings and 3 from the supply mains. Fig. 15 shows one type of a star-delta starter.

Since at the starting instant the voltage across each phase is $\dfrac{V}{\sqrt{3}}$, the starting current will be, $I_s = \dfrac{I_{sc}}{\sqrt{3}}$.

Hence with the same procedure that was adopted in (*a*) the ratio of starting to full load torque according to Eq. 35, is

$$\frac{T_s}{T_{fl}} = \left(\frac{1}{\sqrt{3}}\right)^2 \left(\frac{I_{sc}}{I_{fl}}\right)^2 \times s_{fl}$$

This shows that the starting torque is poor since it is $\frac{1}{3}$ of the torque available by direct switching. Therefore motors which are started by star-delta starters must not have any load at the time of starting.

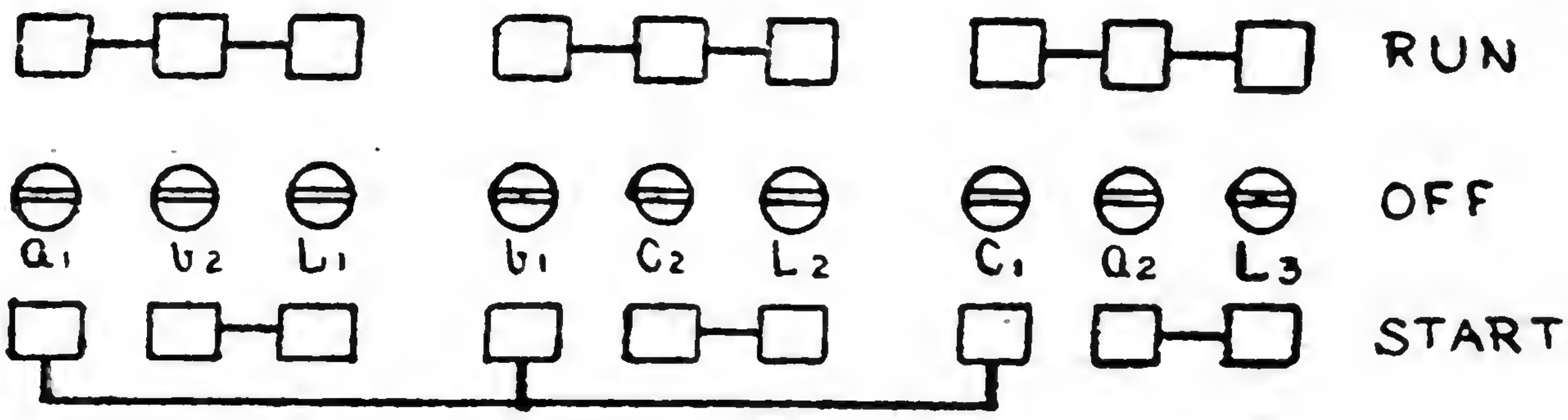


Fig. 15 Star-Delta Starter

(c) *Auto-Transformer Starter*: Fig. 16 shows a 3-phase auto-transformer which reduces the voltage to each stator phase to

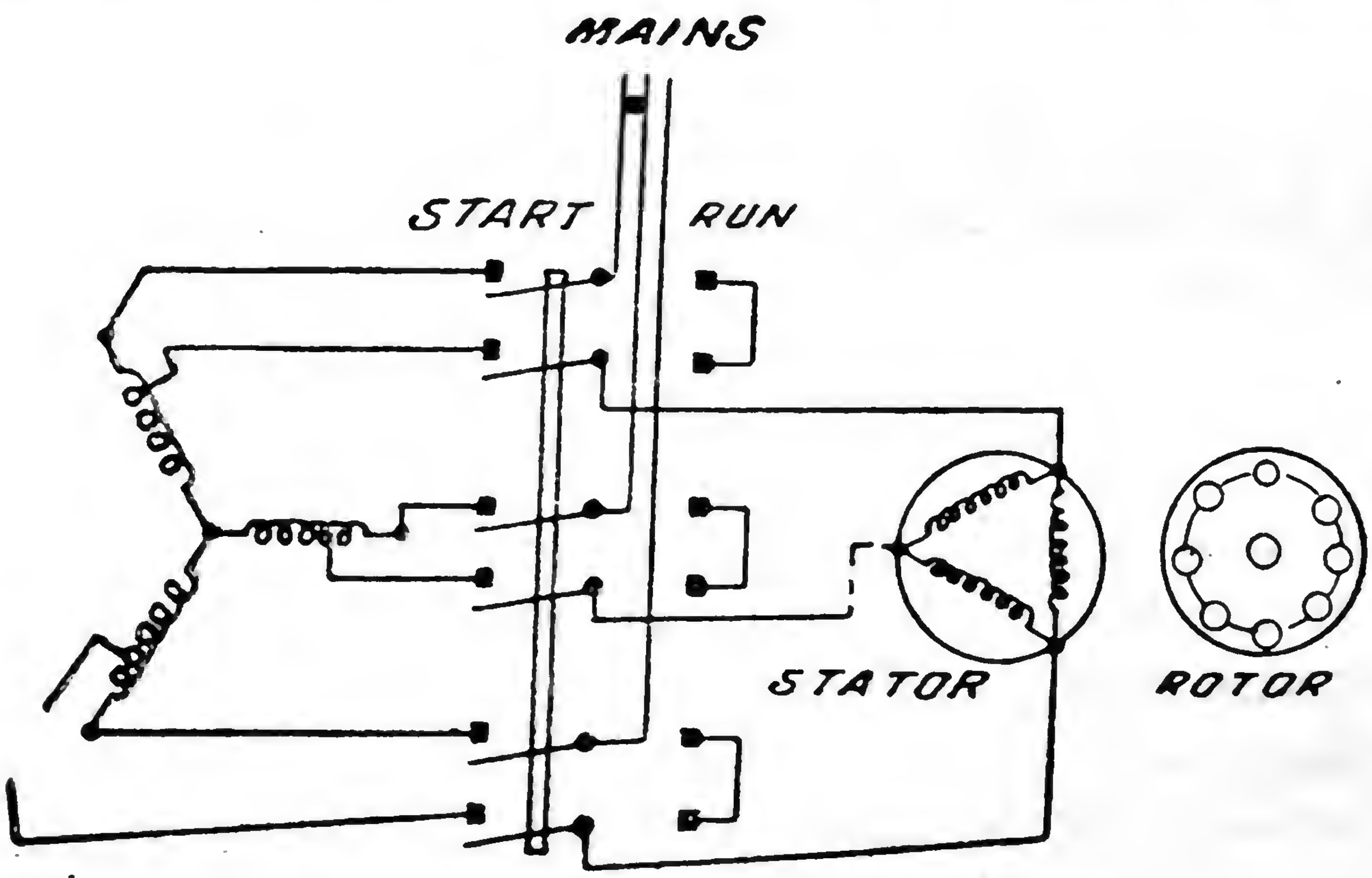


Fig. 16 Auto-Transformer Starter

a fraction x of the supply voltage, the value of x is usually somewhere between 50% and 60%. Therefore the starting current per phase is $I_s = xI_{sc}$

$$\text{Hence} \quad \frac{T_s}{T_{fl}} = x^2 \left(\frac{I_{sc}}{I_{fl}}\right)^2 \times s_{fl}$$

This method draws a lesser amount of current from the supply mains as compared with that of (*a*), and can be calculated with the help of Fig. 17. If the tapping is at *c*, the voltage between *ac* is xV and that between *ab* is V. If the starting current $I_s = xI_{sc}$ the current that will pass will be xI_s. Therefore the line current will be $I_L = x^2 I_{sc}$. This value of

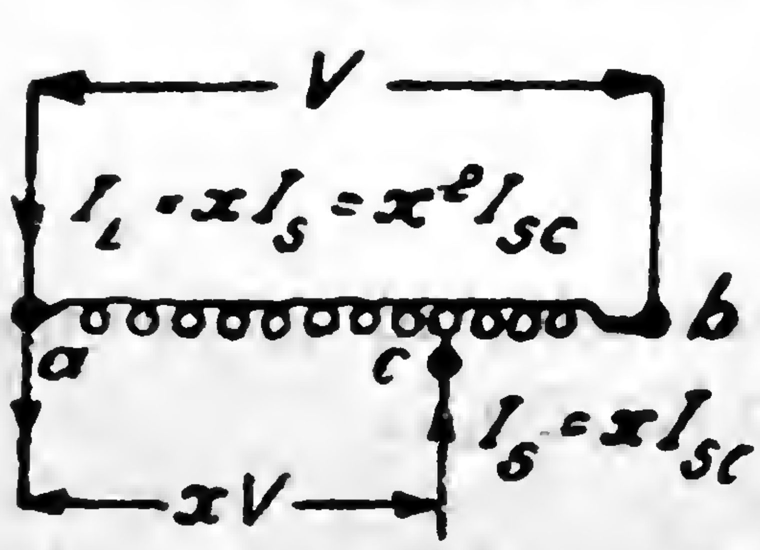


Fig. 17

the line current does not include the magnetising and loss current of the auto-transformer.

Example: The full load speed of a 50 cycle, 3-phase cage motor is 1440 r. p. m. The ratio of short-circuit current to full load current is 5. Calculate the starting torque as percentage of full load torque when

(i) switched on directly,

(ii) started by a star-delta switch, and

(iii) started by an auto-transformer starter with 60% tapping.

Solution: Since the speed is 1440 r. p. m. the motor must be having 4 poles, therefore the synchronous speed is 1500 r. p. m. and the fractional slip at full load is

$$s = \frac{1500 - 1440}{1500} = 0{\cdot}04$$

(i) When the motor is directly switched on, the starting torque $T_s \propto I^2_{sc} R'_2$, and the full load torque

$$T_{fl} = \frac{I^2_{fl} R'_2}{s} \qquad \therefore \quad \frac{T_s}{T_{fl}} = \frac{I^2_{sc} R'_2}{I^2_{fl} R'_2} \times s$$

But $I_{sc} = 5\, I_{fl}$, and $s = 0{\cdot}04$, therefore

$$\frac{T_s}{T_{fl}} = 25 \times 0{\cdot}04 = 1$$

In this case starting torque = full load torque.

(ii) When the motor is started by a star-delta starter, the starting current is $\left(\frac{1}{\sqrt{3}} I_{sc}\right)$. Therefore the starting torque T_s is

$$T_s \propto \left(\frac{I_{sc}}{\sqrt{3}}\right)^2 R'_2$$

Hence $$\frac{T_s}{T_{fl}} = \left(\frac{5\, I_{fl}}{\sqrt{3}}\right)^2 R'_2 \div \frac{(I_{fl})^2 R'_2}{s_{fl}}$$

Substituting the value of s and cancelling,

$$\frac{T_s}{T_{fl}} = \left(\frac{5}{\sqrt{3}}\right)^2 \times 0{\cdot}04 = 0{\cdot}33$$

In this case the starting torque is $\frac{1}{3}$ the full load torque.

(iii) Since the slip is 0·04 at full load and $I_{sc} = 5\, I_{fl}$

$$\frac{T_s}{T_{fl}} = (0{\cdot}6)^2 \left(\frac{5\, I_{fl}}{I_{fl}}\right)^2 \times 0{\cdot}04 = 0{\cdot}36$$

i. e. the starting torque is 36 % of full load torque with 60 % tapping.

All the above three methods of starting are for squirrel-cage induction motors. For slip-ring induction motors the method of starting is by inserting an external resistance in each phase of the rotor winding.

(*d*) *Rotor Resistance Starter*: With this type of starter the starting torque available can be a maximum by making the resistance of rotor winding *plus* the external resistance equal to the combined reactance ($X_1 + X'_2$) as per Eq. (24 a). Fig. 18 shows a rotor resistance starter. The stator is directly connected to the supply through a switch containing the usual *over-load* and *hold-on* coils. The external resistance is divided into three units and each unit is in series with a rotor phase winding through the slip-rings. The resistance is gradually cut out by a handle which also forms the star point of the external resistance units.

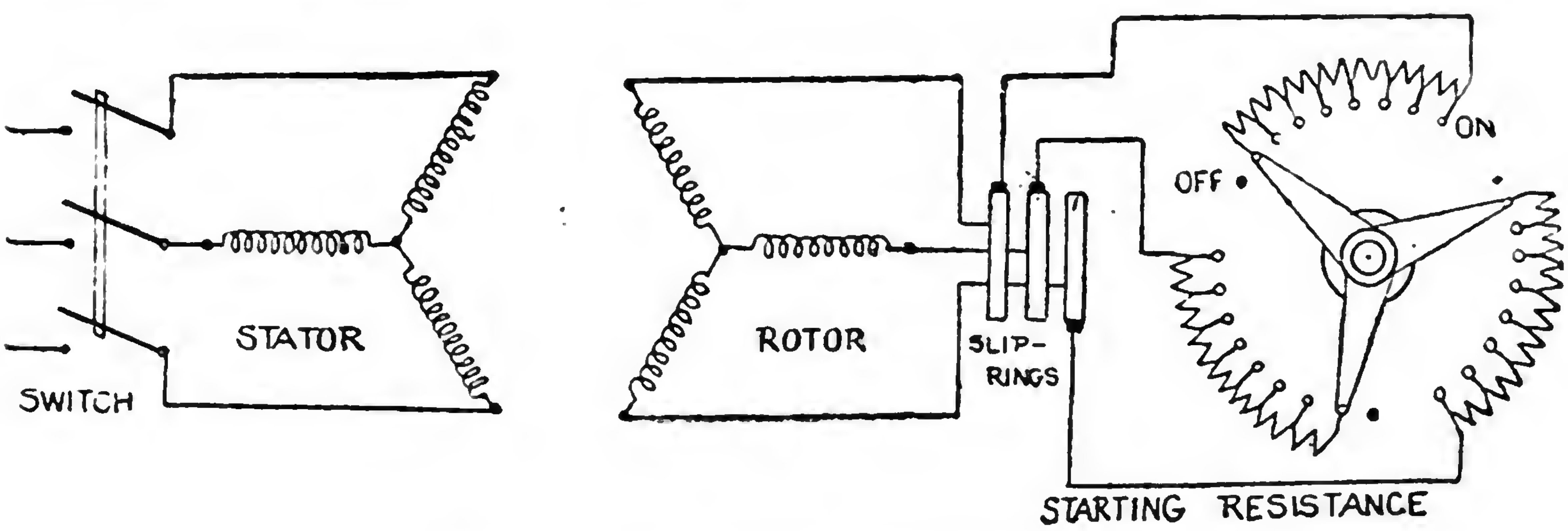


Fig. 18 Rotor Resistance Starter.

12. The Double-Cage Rotor: It has been the aim of designing engineers, from early days, to combine in one machine two characteristics (a) good starting torque and (b) excellent running performance. To realise (a), it is necessary, as has been shown, to have a high resistance for the rotor phases during starting periods, and to realise (b), the rotor resistance must be low to obtain excellent running performance. This ensures small fractional slip and high efficiency.

The desired characteristic is obtained by housing two independent squirrel-cage windings on the rotor, each having its own set of slots. The resistance of one winding is high and its inductance low, while the other has low resistance and high inductance. Fig. 19 shows a cross-sectional view of the rotor and bars of the two windings, while

Fig. 20 shows a typical slot arrangement. The slot is deep and has a slit in between two bars. The top bar **X** belongs to a winding having high resistance, while the bottom bar **Y** belongs to the other winding. The use of brass for bars makes the resistance of winding **X** high, while the resistance of winding **Y** is kept low by using copper for its bars and having a large cross-sectional area.

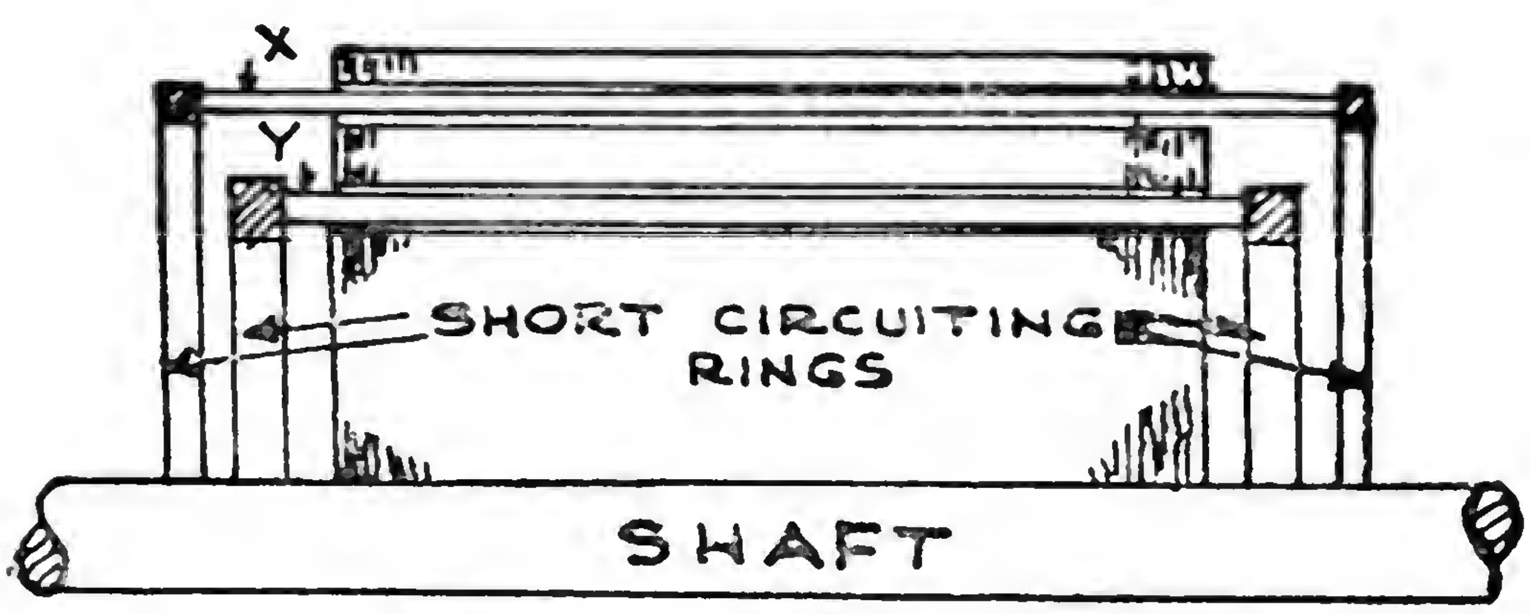


Fig. 19 Double Cage Slots and Conductors.

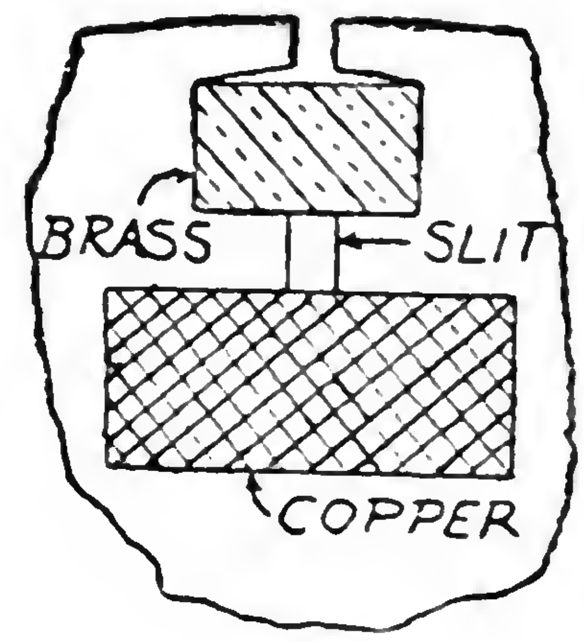


Fig. 20

By placing the high resistance winding at the top of the slot, the leakage flux per ampere linking with it is small, hence its inductance is low, while the leakage flux linking with the lower winding is large and hence the self-inductance of the lower winding is high.

On starting, when the rotor frequency is equal to the line frequency, the reactance ($2\pi f L$) of the lower winding is very high, therefore comparatively only a very small current will flow through it, while a large current will flow through the top winding. But it has a higher resistance. This, then, is the cause of production of high starting torque. When the motor speeds up and attains speed approaching synchronous speed, the rotor frequency is so small that the reactance of both windings become negligible. The current, therefore, automatically divides between the two windings, the greater amount of current flowing through the low resistance winding **Y**. The torque at any instant is the arithmetic sum of torques due to each winding. With good design the net torque is always high as shown in Fig. 21.

The equivalent circuit diagram of a double-cage induction motor is shown in Fig. 22. The two rotor windings **X** and **Y** are shown in parallel according to the conception of an equivalent circuit.

The double-cage induction motor is ideal for compressors.

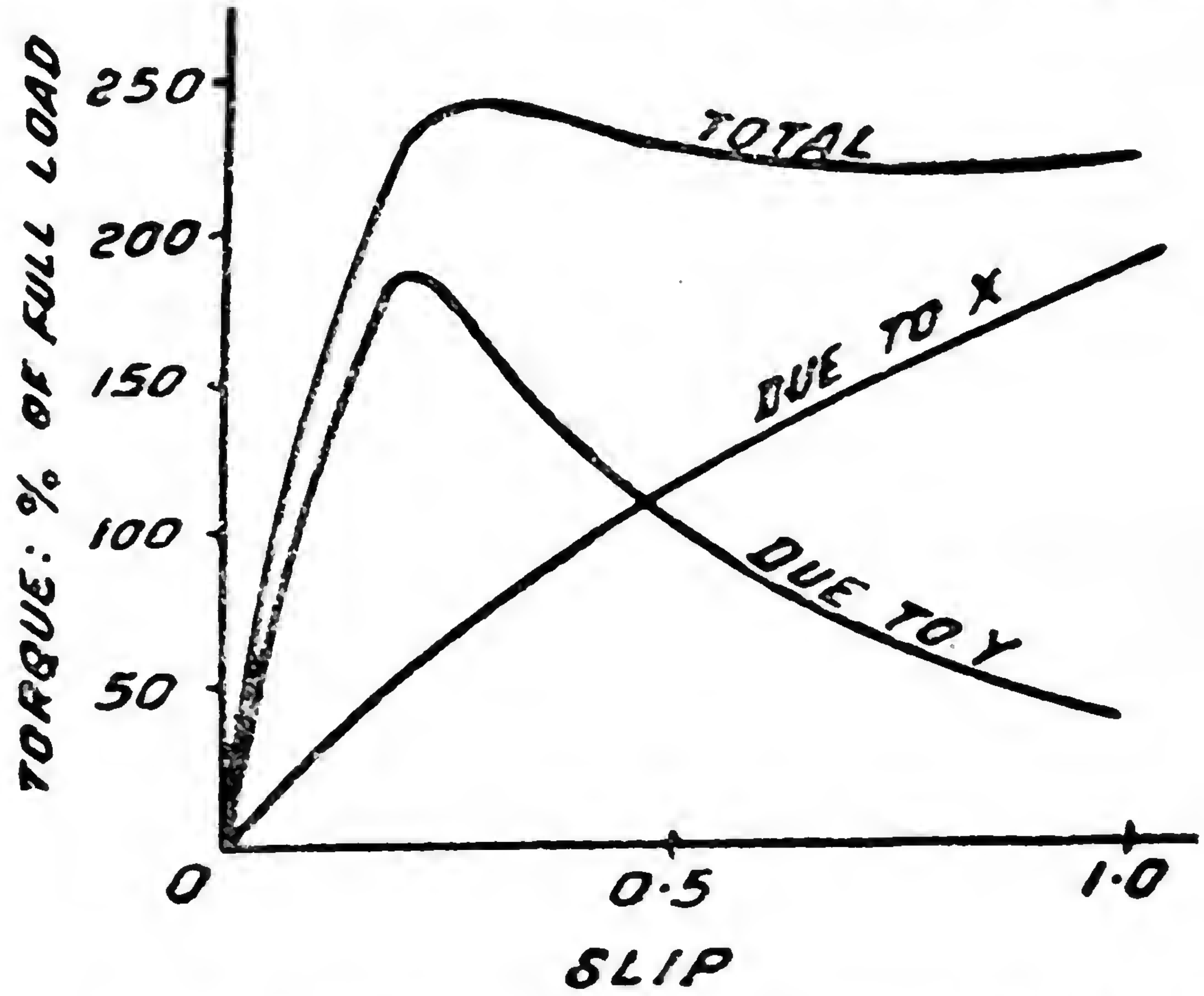


Fig. 21. Torque Slip Curves. Double-Cage Motor

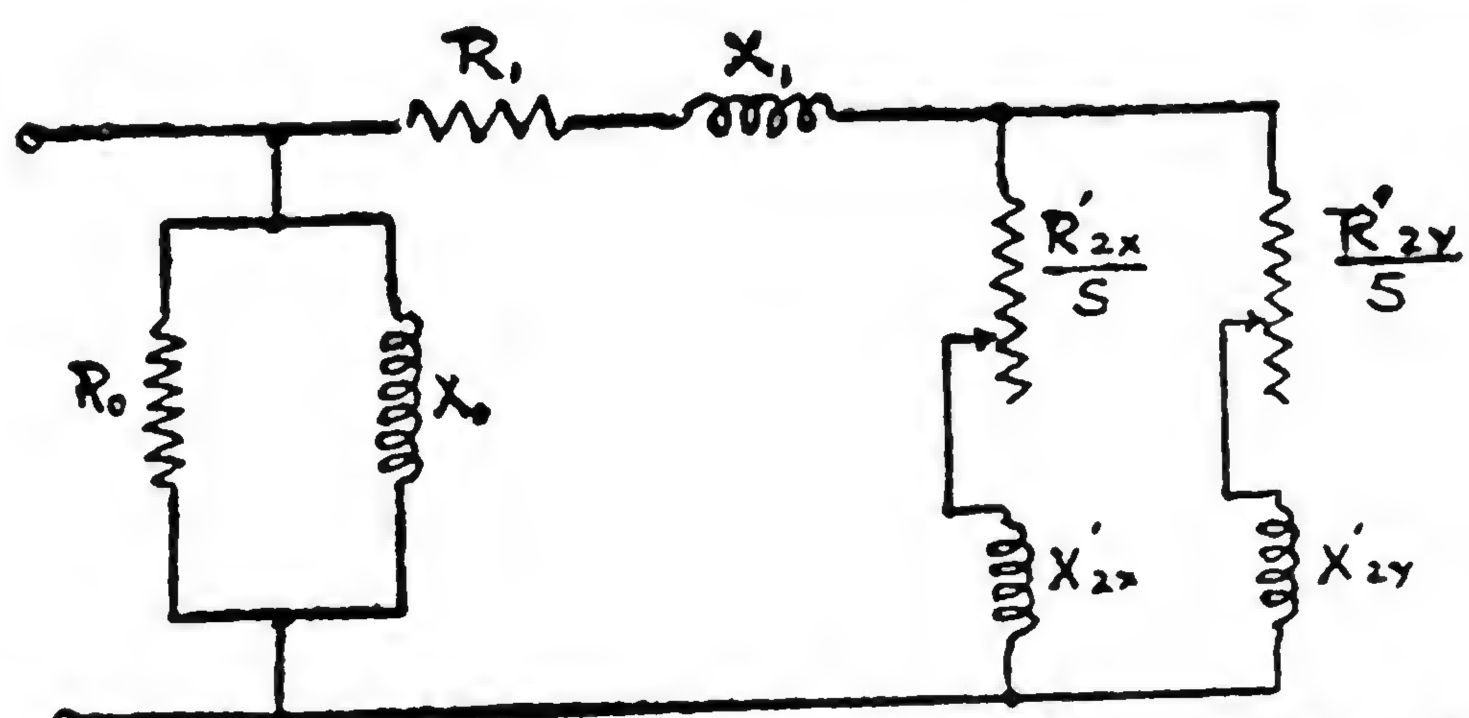


Fig. 22. Equivalent Circuit Double-Cage Motor

13. Measurement of Slip: There are several methods of measuring slip of an induction motor, but only a few will be described here.

I. *By measuring the speed of the motor by a tachometer.* If the machine has p number of poles and the supply frequency is f. c. p. s, the synchronous speed n_s is given by the expression

$$n_s = \frac{f \times 120}{p} \text{ r. p. m.}$$

Then if n r. p. m. is the measured speed by the tachometer, the fractional slip is given by

$$s = \frac{n_s - n}{n_s}$$

or $$\text{per cent slip} = \frac{n_s - n}{n_s} \times 100$$

For example, if the speed of a 4-pole, 50 c. p. s. induction motor is 1440, as measured by a tachometer, the per cent slip can be calculated as follows :—

The synchronous speed, $n_s = \dfrac{50 \times 120}{4} = 1500$ r. p. m.

$\therefore$ per cent slip $= \dfrac{1500 - 1440}{1500} \times 100 = 4\%$

II. *By a Stroboscopic Disk.* This is an excellent and accurate method of measuring small slip. The disk has white sectors painted on it with a black background. The number of sectors are equal to the number of poles of the motor. The disk is fitted at one end of the shaft of the rotor, their centres coinciding, and is illuminated either by an arc lamp or by a neon lamp. In the latter case, however, the disk is enclosed in a box which contains the neon lamp. A small slit in the box is provided at one end for making necessary observations, while at the opposite end the box is provided with a hole large enough to accomodate the shaft of the motor in such a manner that it will rotate freely.

The arc or the neon lamp, must be supplied with voltage having the same frequency as that of the motor. These lamps glow every time the value of the voltage wave across their terminals exceeds a certain value. For instance, if the supply frequency is 50 c. p. s. they will become bright and dark 100 times in one second. But the human eye is unable to detect this phenomenon, which is essential for the measurement of slip by this method. The effect of the arc or the neon lamp on the sectors of the disk is as follows :—

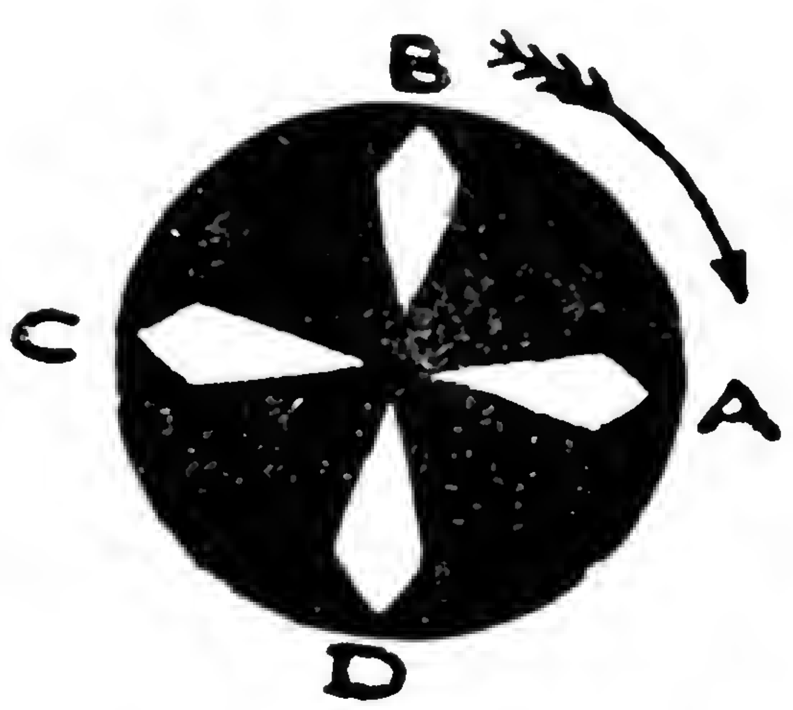


Fig. 23
Stroboscopic Disk

Taking the case of a 4-pole, 50 c. p. s. motor, the disk is marked with 4 white sections *A*, *B*, *C* and *D* as shown in Fig. 23. The synchronous speed will be

$$n_s = \frac{f}{p/2} = \frac{50}{4/2} = 25 \text{ rev. per second.}$$

The lamp will illuminate the disk, or rather the 4 sectors, every $\frac{1}{100}$th of a second. Now supposing the motor were to run at

synchronous speed, $25 \times 4 = 100$ sectors will pass a certain fixed point per second i. e. if the direction of rotation of the shaft is clockwise, sector A will occupy the position of sector D after an interval of $\frac{1}{100}$th second. And since the lamp glows after an interval of $\frac{1}{100}$th of a second, the sectors *appear to be stationary* to an observer, because every time the lamp glows the sectors seem to occupy the same positions. This is a mere optical illusion. These sectors which *appear to be stationary* may be termed as "apparent sectors."

But an induction motor always runs at speeds less than the synchronous speed. Under these circumstances the sectors do not occupy the same positions after an interval of $\frac{1}{100}$th of a second, *i. e.* consider sector A, it will move a distance a little less than the arc AD, Fig. 23. To an observer the apparent sectors will be seen to be moving in anti-clockwise direction, i. e. in the reverse direction to that of the direction of rotation of the shaft.

The measurement of slip is made by observing with a stop-watch the time, in seconds, an "apparent sector" makes a certain number of revolutions. Let N be the number of revolutions of an apparent sector in t seconds, then $\frac{N}{t}$ is the number of revolutions lost per second. If n_s is the synchronous speed *per second* of the machine, the rotor speed per second is

$$n = \left(n_s - \frac{N}{t}\right) \textit{ revolution per second.}$$

Example: Find the speed in r. p. m. of a 4-pole, 50 c p. s. induction motor if an "apparent sector" on the stroboscopic disk makes 10 revolutions in 25 seconds.

Solution: Synchronous speed per second

$$n_s = \frac{f \times 2}{p} = \frac{50 \times 2}{4} = 25 \text{ r. p. s.}$$

$$\frac{N}{t} = \frac{10}{25} \text{ r. p. s.}$$

$$\therefore \text{ rotor speed per second} = 25 - \frac{10}{25} = 24{\cdot}6 \text{ r. p. s.}$$

$\therefore$ rotor speed per minute $= 24{\cdot}6 \times 60 = 1476$ *r. p. m.*

III. *By the use of an instrument in the rotor circuit.* If an ammeter is placed in one of the rotor leads of a wound induction motor, the instrument pointer will oscillate to and fro. The number of sweeps depends upon the frequency of the induced e. m. fs. The rotor frequency is usually one to two cycles per second.

If the instrument is of moving iron type, i. e. one used on a. c. circuits, the number of sweeps is two per cycle. If however, a d. c. instrument, i. e. a moving coil instrument, is used the number of sweeps is only one per cycle. Hence it is easier to count the lesser number of sweeps of the d. c. instrument.

For example, consider a 50 cycle induction motor. If the number of sweeps of a d. c. instrument pointer are 45 in 30 seconds, the frequency of rotor e. m. f. f_2 is $\frac{45}{30}$ c. p. s. Using Eq. (2),

$$\text{slip } s = \frac{f_2}{f} = \frac{45}{30} \times \frac{1}{50} = 0{\cdot}03.$$

If the motor has 4 poles, its synchronous speed is 1500 r. p. m. and the rotor speed n is found from

$$0{\cdot}03 = \frac{1500 - n}{1500}$$

$$\therefore \quad n = 1500 - 45 = 1455 \text{ r. p. m.}$$

If the motor has 6 poles, $n_s = 1000$ r. p. m.

$$\therefore \quad n = n_s - sn_s$$

$$= 1000 - 0{\cdot}03 \times 1000 = 970 \text{ r. p. m.}$$

If the motor is a squirrel-cage type, a very sensitive moving-coil millivoltmeter may be used. The two leads from the voltmeter are placed or touched at each end of the rotor shaft. The beats of the instruments are then counted and their time observed by a stop-watch.

The reason for the indication on the instrument in this case is that there is usually a weak magnetic flux passing right through the centre of the rotor and cutting the shaft which has a small e. m. f. induced in it.

14. Speed Control of Induction Motors : The speed of an induction motor may be stated by the equation

$$n = \frac{120\,f}{p}(1 - s) \quad \cdots \quad \cdots \quad \cdots \quad (36)$$

where n = rotor speed in r. p. m.

f = frequency of supply,

p = number of poles, and

s = fractional slip.

This shows that a change in any one of the three quantities will affect its speed, namely,

(*a*) change of frequency,

(*b*) change of number of poles, and

(*c*) change of slip.

The change in frequency is not possible ordinarily, since the Central Stations operate at a fixed frequency. However, in the electric propulsion of ships where the ship's alternators supply power to its own motors, a change of frequency is possible by (i) changing the speed of the alternators or (ii) having two alternators working at different frequencies, say 40 and 60 c. p. s.

The method of operational sequence would be as follows—

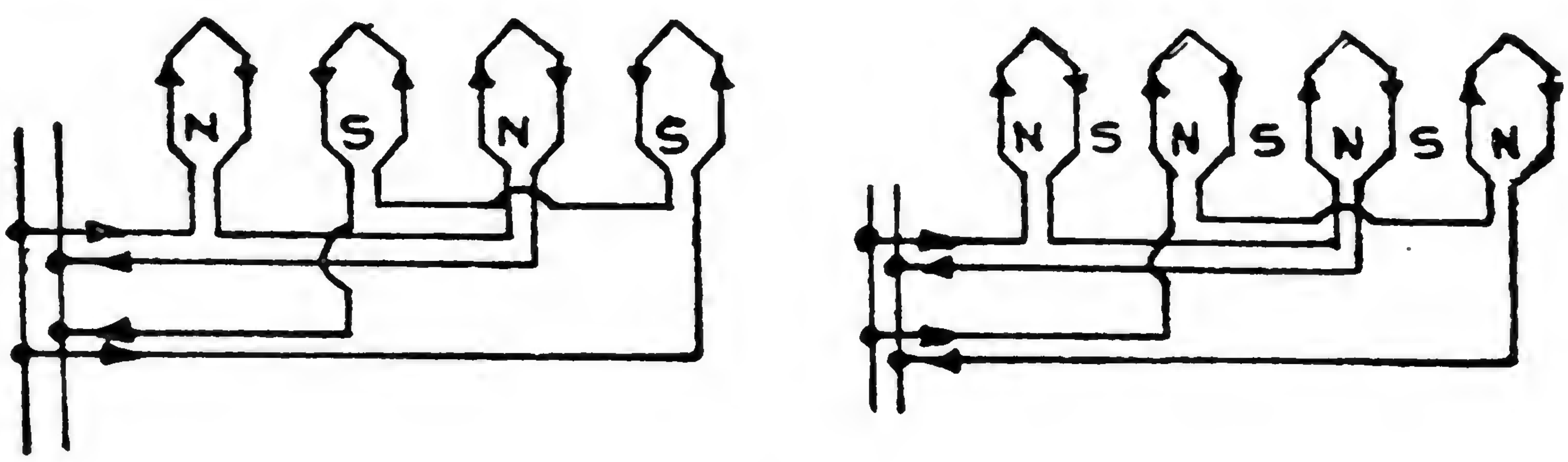
Since the speed of the propeller screws is very low, a motor with a large number of poles, say 36, drives the propeller screws at normal, or crusing speed. At 40 c. p. s. its synchronous speed

$$n_s = \frac{120 \times 40}{36} = 133 \text{ r. p. m.}$$

If the speed has to be increased in an emergency, the alternator running at 60 c. p. s. will give power to the same motor and will increase its synchronous speed to 200 r. p. m. But for going faster the propellers will require greater power. This increased power can be supplied by the 40 c. p. s. alternator to another motor having 24 poles. The synchronous speed of the second motor will also be 200 r. p. m.

Change in the number of poles is effected by making change in the stator winding connections with the help of suitable switching arrangement. When the desired speed ratio is 2 : 1, the "Consequent

Pole" method is adopted and is shown diagramatically in Fig. 24. Two distinct windings are on the stator producing the same number of poles, one winding creates two N-poles, and the other two S-poles as shown in Fig. 24 (a). This gives four poles, and the synchronous speed at 50 c. p. s. is 1500 r. p. m.



(a) Fig. 24 (b)

If the connection of one of the windings is reversed, there will be in all four N-poles or four S-poles, depending upon which of the winding connection is reversed. But in between these poles other four poles of opposite polarity will be created. Thus the stator now has 8 poles and its synchronous speed is 750 r. p. m.

The 2 : 1 change in speed is rather drastic. Hence motors are manufactured with two windings one is so wound that it creates 8 poles and the other creates 10 poles, the synchronous speeds being 750 and 600 respectively. But the disadvantage is that at either speed only 50% of stator copper is utilised, since only one winding is in use at a time.

In the case of squirrel-cage motors, change in the number of poles on the stator does not affect the existing arrangement on the rotor. But in the case of wound rotors, the number of poles on the stator must equal the number of poles on the rotor. Otherwise there will be a greater reduction in torque due to some rotor conductors developing a negative torque.

Change in slip is effected by introducing an external resistance in the rotor circuit of wound rotors, as is shown earlier in this Chapter. See Fig. 18. But this is done at the sacrifice of efficiency and besides the speed regulation is poor.

These disadvantages are more or less overcome by supplying counter-e. m. fs. to the rotor at slip frequency or at supply frequency.

The former method requires auxiliary commutating machine which injects e. m. fs. into the rotor at rotor frequency through the slip-rings, while the latter requires a commutator for the rotor. The student, if he desires, can refer to the description of Scherbius method of speed control in a Standard Hand-book. The next Section deals with another interesting method of speed control.

15. The Cascade or Tandem Connection : Different speeds are possible if two induction motors are connected with their shafts coupled and also connected electrically, as shown in Fig. 25, where the two machines are wound rotor type.

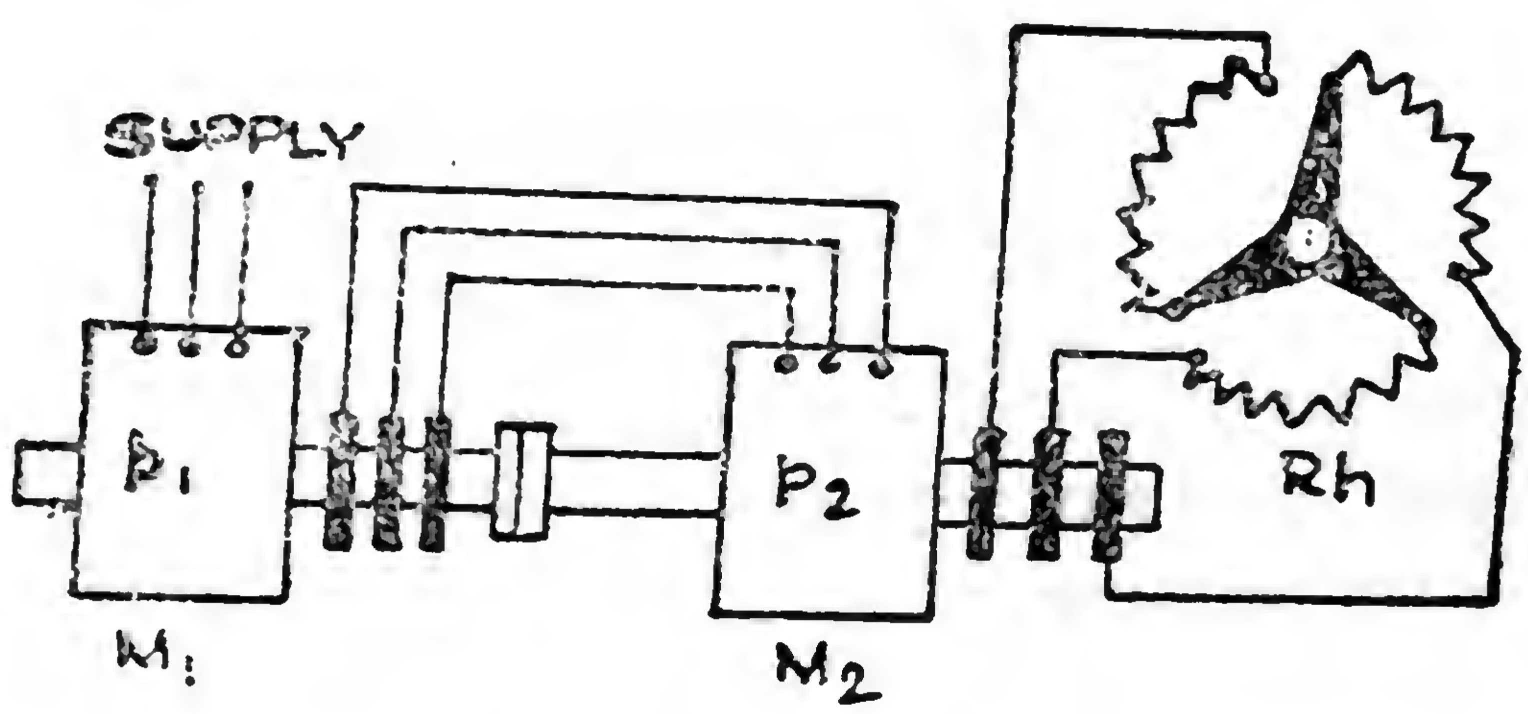


Fig. 25

The stator of M_1 receives power from the supply mains. The rotor of M_1 supplies electrical power to the stator of M_2 and the rotor of M_2 is short circuited after the starting rheostat is brought to the "run" position. The direction of torques of both the rotors is the same.

Let the frequency of supply voltage be f_1 and let the machines M_1 and M_2 have p_1 and p_2 number of poles respectively. If the two machines, M_1 and M_2, run with slip of s_1 and s_2 respectively, the following relations are true:—

$$\text{Speed in r. p. m. of } M_1 = n_1 = \frac{120 f_1}{p_1}(1 - s_1)$$

$$\text{Speed in r. p. m. of } M_2 = n_2 = \frac{120 f_2}{p_2}(1 - s_2)$$

But the shafts are mechanically coupled, therefore

$$n_1 = n_2$$

also $f_2 = s_1 f_1$. Hence substituting the value of f_2

$$n_2 = \frac{120\, s_1 f_1}{p_2}(1 - s_2).$$

Equating the expression for n_1 and n_2 and solving for s_1

$$\frac{120 f_1}{p_1}(1 - s_1) = \frac{120\, s_1 f_1}{p_2}(1 - s_2)$$

$$\therefore \quad s_1 = \frac{p_2}{p_1 - p_1 s_2 + p_2}$$

$$= \frac{p_2}{p_1(1-s_2) + p_2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (37)$$

But when the rheostat is short-circuited, s_2 approaches zero. The above expression then reduces to

$$s_1 = \frac{p_2}{p_1 + p_2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (38)$$

But $s_1 = \dfrac{n_{s1} - n_1}{n_{s1}}$, where n_{s1} is the synchronous speed of M_1.

Therefore $\dfrac{n_{s1} - n_1}{n_{s1}} = \dfrac{p_2}{p_1 + p_2}$

solving for n_1 we have, $n_1 = n_{s1} \dfrac{p_1}{p_1 + p_2}$

However, $n_s p_1 = 120 f_1$, so that substituting the value of $n_{s1} p_1$ in the above expression it becomes

$$n_1 = \frac{120 f_1}{p_1 + p_2}\ r.p.m. \quad \ldots \quad \ldots \quad \ldots \quad (39)$$

Eq. (39) shows that the speed of the set is that of a single machine having the number of poles equal to the sum of the numbers of poles of the two machines. Hence the set can give four different speeds, as the following example shows.

Example: Find approximately the various speeds obtainable from two induction motors connected in cascade. One machine has 12 poles and the other 8, while the supply frequency is 50 c. p. s.

Solution: (a) If the machine having 12 poles works alone, its no load speed will be nearly $n = \dfrac{120 \times 50}{12} = 500\ r.p.m.$

(b) If the 8-pole machine is made to run alone its approximate speed will be $n = \dfrac{120 \times 50}{8} = 750\ r.p.m.$

(c) When both the machines work, as shown in Fig. 25, and the direction of their torques is the same, then the speed, as given by

Eq. 39, will be $n = \dfrac{120 \times 50}{12+8} = 300\ r.p.m.$

If the two torques are in the same direction the set is said to be connected in "*cumulative*" *cascade.* If the two torques are opposing, the set is said to be connected in "*differential*" *cascade,* the expression for the latter case is

$$n_1 = \frac{120 f_1}{p_1 - p_2} \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad (40)$$

by the same reasoning as that by which Eq. (39) is derived.

Hence,

(d) the speed of "differential" cascade connection will be

$$n = \frac{120 f_1}{12-8} = \frac{120 \times 50}{4} = 1500\ r.p.m.$$

Therefore the possible speeds with such a set are 1500, 750, 500 and 300 r. p. m.

In order to limit the magnitude of the slip-ring currents, the ratio of transformation of both the motors should be preferably unity. If this is not the case and if the rotors have equal current carrying capacity, the two machines can be connected as shown in Fig. 26, where the rotor of M_1 supplies current to the rotor of M_2. The starting rheostat is connected across the stator of M_2. It must be noted, however, that the two connection diagrams of Figs 25 and 26 are electrically identical.

The connection diagram of Fig. 26 suggests that the two sets of slip-rings can be dispensed with altogether. The rotors can be built up on one shaft and the connections between the two rotors can pass along the shaft. Or, if the two shafts are different, and since they have to be coupled, the connecting wires can be made to pass through by making the shafts hollow. But it is a better

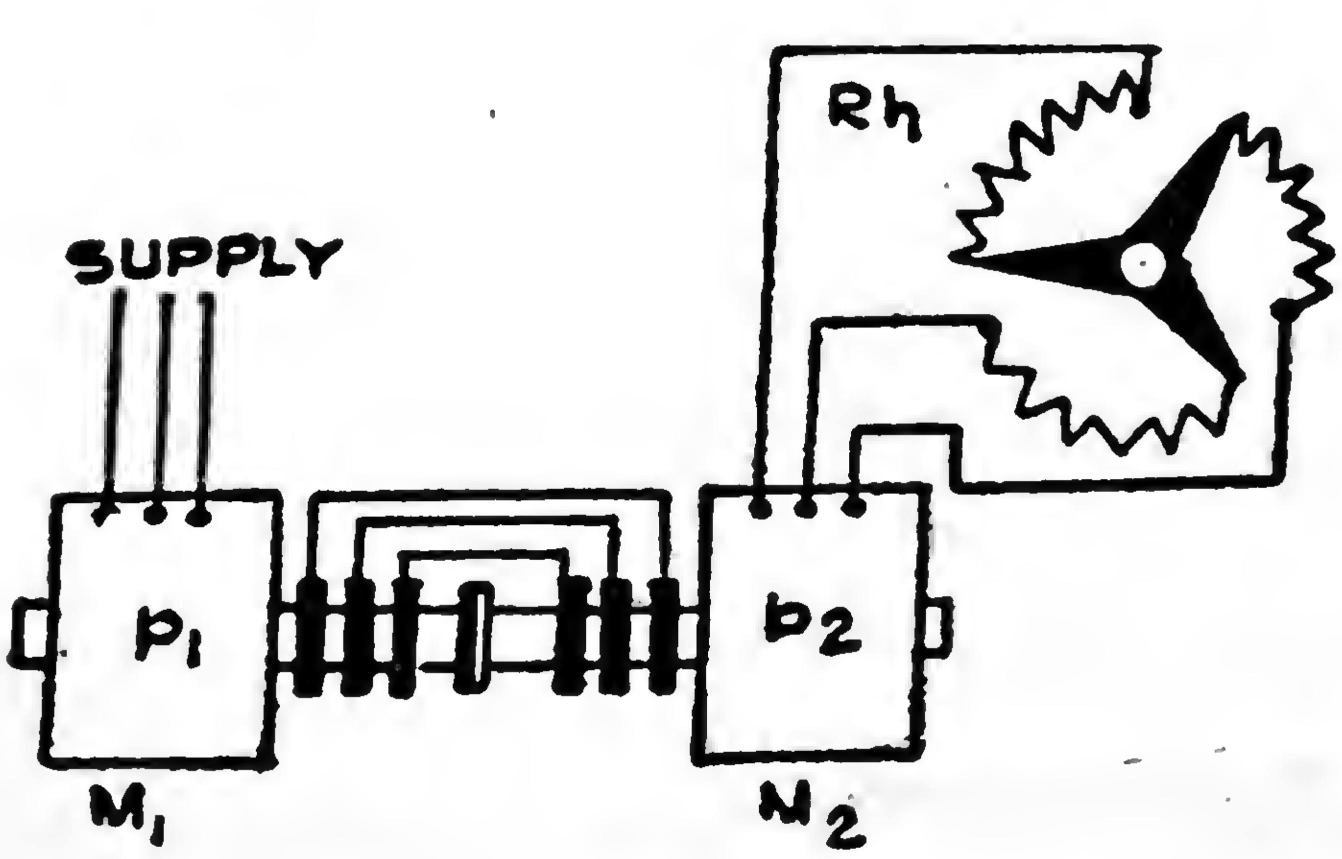


Fig. 26

practice to retain the slip-rings in case either machine is required to operate singly.

The ratio of load sharing by the two machines, when operating in cascade, can be derived as follows:—

Let the power received by the stator of M_1 from the supply mains be W watts. Neglecting all the losses that occur in the two machines, W is transferred from the stator of M_1 to its rotor, where part of it, say W_1, is converted into mechanical power which is given to the shaft. The remaining power, say W_2, is transferred (electrically) to the stator of M_2. Therefore W_2 is the mechanical power given to the shaft by the rotor of M_2, since there are no losses.

If the slip of M_1 is s_1, $W_1 = W(1-s_1)$ and $W_2 = s_1 W$

$$\text{But by Eq. (38), } s_1 = \frac{p_2}{p_1+p_2}$$

$$\therefore \quad W_1 = W\left(1 - \frac{p_2}{p_1+p_2}\right) = W\frac{p_1}{p_1+p_2}$$

and $$W_2 = W\frac{p_2}{p_1+p_2}$$

Therefore the ratio of powers will be

$$\frac{W_1}{W_2} = \frac{p_1}{p_2} \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad (41)$$

or $$\frac{W_1}{W_2} = \frac{1-s_1}{s_1} \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad (42)$$

16. Induction Regulators: In construction, these resemble a wound induction motor but without any slip-rings. The stator winding is exactly like that of an induction motor. The rotor winding ends are all brought out on a terminal board. These units are manufactured either for single or three phase working.

The rotor does not rotate but it is merely made to shift its position through any desired angle. The maximum shift is 180° (electrical). The movement is effected either by a hand-wheel or by a small reversible motor through a worm and worm-gear. For small units the shaft is horizontal but in the case of large units the shaft is arranged vertically for convenience. The rotor is connected to the supply lines and is called the primary. The secondary winding

forms a link between the supply lines and the lines whose voltage is to be regulated. See Fig. 27.

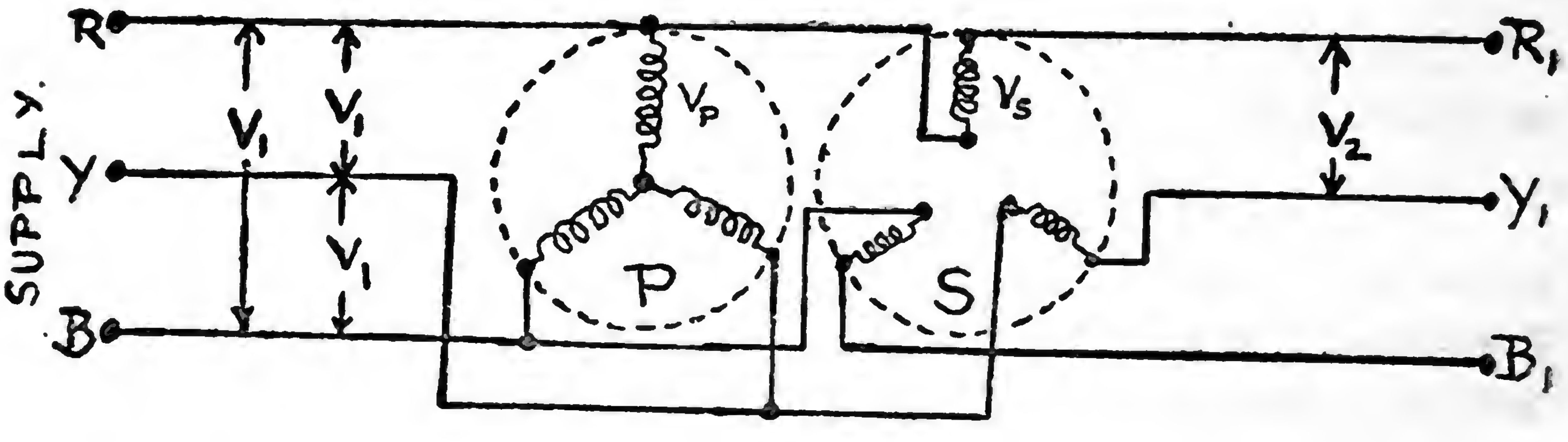
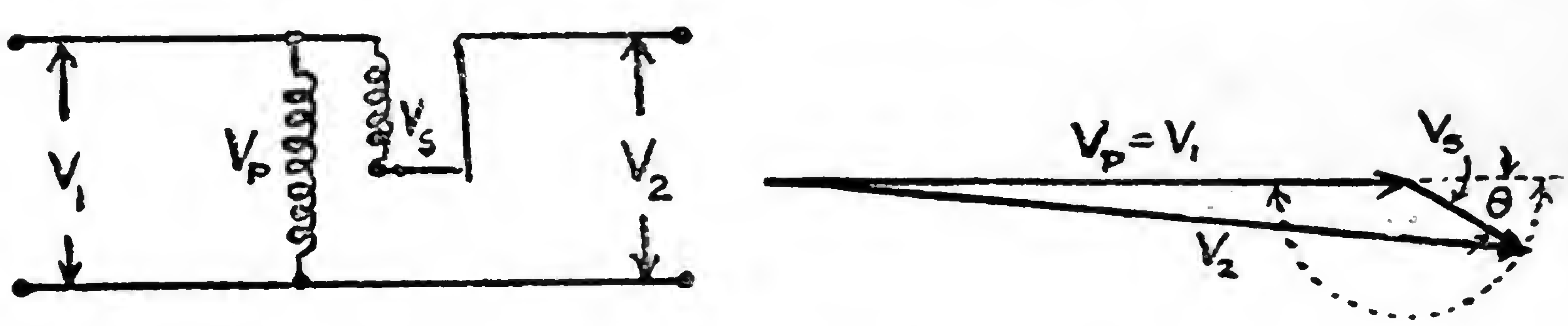


Fig. 27

The action of an induction regulator can best be explained by considering a single-phase unit shown in Fig. 28 (a), where V_p is



(a) Fig. 28 (b)

the primary and V_s the secondary voltage of the two windings, and V_1 and V_2 are the voltages of supply lines and the regulated lines respectively.

Since the rotor can move through 180°, the desired relative position of the secondary with respect to the primary is obtained at will by the movement of the hand-wheel. Hence the vector V_s will move through a semi-circle, as shown in Fig. 28 (b). V_2 will be maximum when the angle $\theta = 0°$, i. e.

$$V_2 = V_p + V_s \text{ (arithmetic sum)} \quad \ldots \quad \ldots \quad \text{(i)}$$

while V_2 will be minimum when $\theta = 180°$, i. e.

$$V_2 = V_p - V_s \text{ (arithmetic difference)} \ldots \quad \ldots \quad \text{(ii)}$$

Eq. (i) shows the condition for maximum *boost* and Eq. (ii) for maximum *buck*. For any intermediate position

$$V_2 = V_p + V_s \text{ (vectorially)} \quad \ldots \quad \ldots \quad \text{(iii)}$$

Except at the positions when $\theta = 0°$ and $\theta = 180°$, V_2 gets a *phase shift* with respect to V_1. This shift is of no consequence when the regulated lines $R_1 Y_1 B_1$ supply power to an isolated

feeder or a factory. But where $R_1\ Y_1 B_1$ form part of parallel feeders or inter-connected net-work, this phase shift will cause large circulating currents and unbalanced loading of lines. Phase shift can be avoided by using "double" or "twin" induction regulators, as shown in Fig. 28 (c). It will be observed that there are two secon-

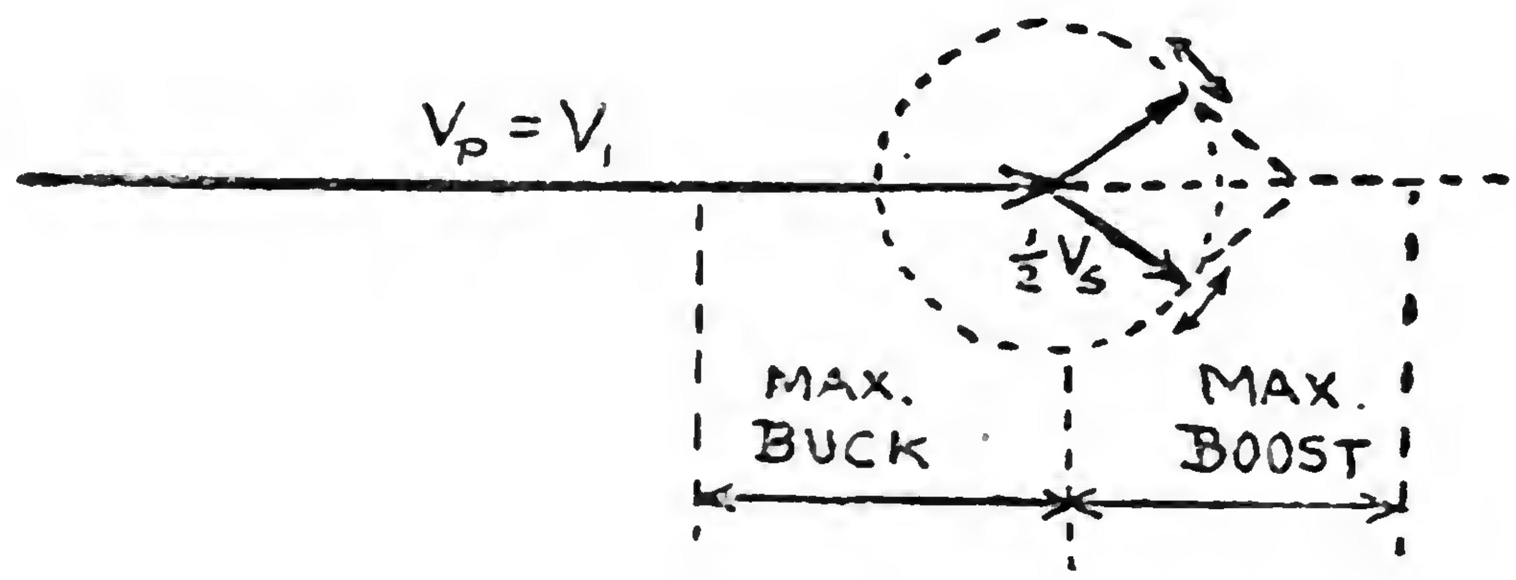


Fig. 28 (c)

dary elements (per phase) which move in opposite directions simultaneously, so that the resultant voltage is always in phase with the supply voltage. The rating of each secondary, in this case, need only be half of the total maximum-boost.

17. The Single-phase Induction Motor: The magnetic field produced by the stator coils is pulsating, though varying sinusoidally with time. Ferraris pointed out that such a field can be resolved into two equal fields but rotating in opposite directions with equal angular velocities. The maximum value of each component is equal to half the maximum of the pulsating field.

If the initial time is such that the rotating vectors of the two component fields are along the y–axis in the positive direction, the

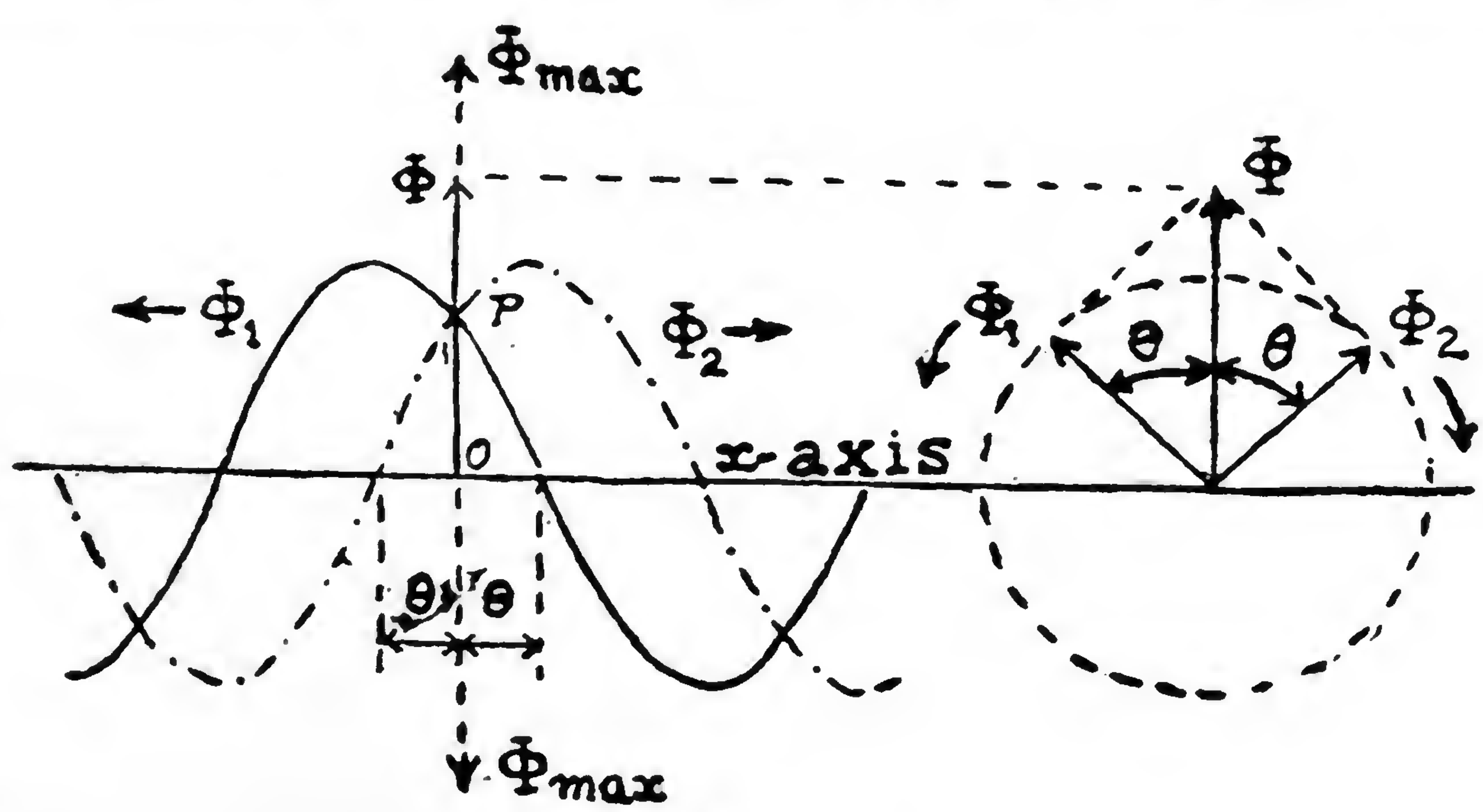


Fig. 29

two component waves Φ_1 and Φ_2 coincide. The resultant of these two is Φ_{max}. After a short interval of time the two vectors rotate through an angle θ in their respective directions and the waves are shown to occupy the positions in Fig. 29. These waves intersect at P on the y-axis and as the waves travel P moves along the y-axis. Hence the resultant of these two component waves at any instant is equal to 2 OP.

$$\Phi_1 = OP = \Phi_{1\,(max)} \cos(\omega t - \theta) \quad \dots \quad \dots \quad \dots \quad (\text{i})$$

$$\Phi_2 = OP = \Phi_{2\,(max)} \cos(\omega t + \theta) \quad \dots \quad \dots \quad \dots \quad (\text{ii})$$

and $$\Phi_{1\,(max)} = \Phi_{2\,(max)}$$

By expanding and adding (i) and (ii),

$$\Phi_1 + \Phi_2 = 2\,\Phi_{1\,(max)} \cos\theta \cos\omega t$$

$$2\,OP = \Phi_{max} \cos\theta \cos\omega t \quad \dots \quad \dots \quad \dots \quad \dots \quad (\text{iii})$$

which is the equation of the pulsating field and proves Ferraris's statement. Thus a single-phase induction motor is inherently not self-starting.

The existence of these two fluxes rotating in opposite directions can be verified by supplying a fractional horse power single-phase induction motor with rated voltage. The motor does not start, but if the shaft is turned by hand, say in clockwise direction, the rotor picks up speed. This means that the rotor conductors are rotating in the direction of that field which rotates in clockwise direction. When the motor is braked and stopped without switching off the supply, the rotor remains at rest. If now the shaft is turned by hand in anti-clockwise direction, the motor picks up speed in that direction. This means that the rotor conductors are now rotating in the direction of the other field.

This behaviour of the motor is due to the presence of two opposing torques due to the two fields. When the rotor is at rest, (i. e. slip = 1) the two torques are equal but opposite in direction. Hence the net torque is zero and therefore the rotor remains at rest. Fig. 30 shows the torque variations due to the two fields. If the rotor is made to speed up in one direction, say in that direction in which T_1 increases, T_1 exceeds the opposing torque T_2 and the motor begins to accelerate. T_2 goes on diminishing until at the working speed it is negligibly small. Hence the single-phase induction motor rotates in the direction in which it is made to run.

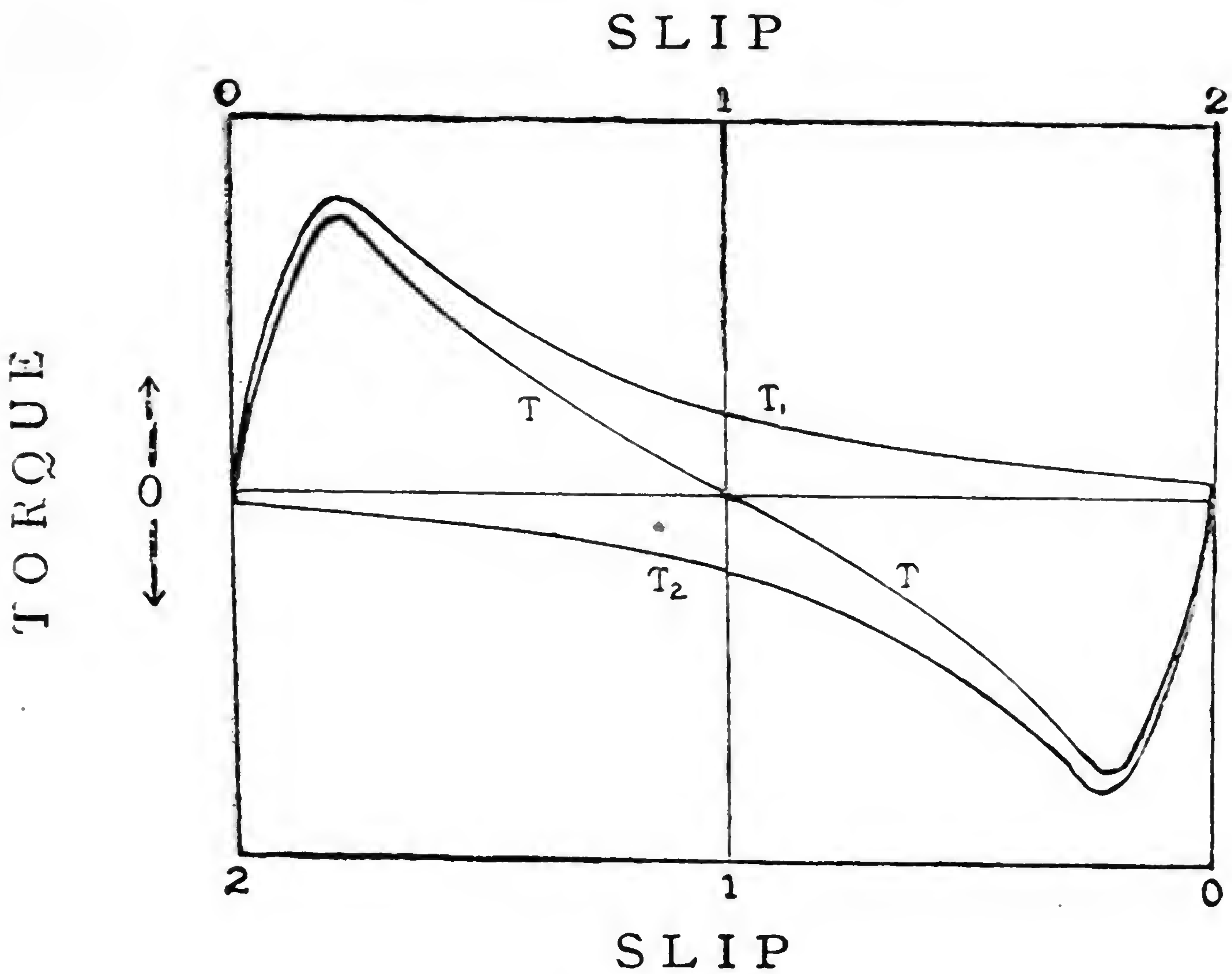


Fig. 30

18. Methods of Starting Single-phase Motors : Since this motor is not self-starting, means must be provided to create an initial torque. But the initial torque is only possible if a rotating flux is created in the stator. We have shown in Section 3 that a rotating flux is the result when there is a phase difference of 90° (elec.) between the currents of two stationary coils. Or if the stator possesses two fluxes having a large phase difference the result is a rotating flux.

(a) The *Split-phase motor* has two sets of stator windings in parallel. One is the main winding *M* and the other the auxiliary or starting winding *S*. These two are spaced 90° (elec.) degrees from each other and the impedances of the two circuits are so made up that the phase difference between their currents is nearly 90° (elec.). This is called splitting the phase.

(i) The ordinary split-phase motor circuit diagrams are shown is Figs. 31 (a) and (b). In (a) the winding *S* is in series with a resistance *R*. The current I_M in the main winding lags behind the applied voltage *V* by a large angle, while the current I_S in the auxiliary winding lags behind *V* by a small angle. Thus the phase difference is fairly large though not 90°. This is sufficient to produce

a resultant rotating flux in the stator. Once the rotor starts and picks up speed, the starting winding circuit is broken by the centri-

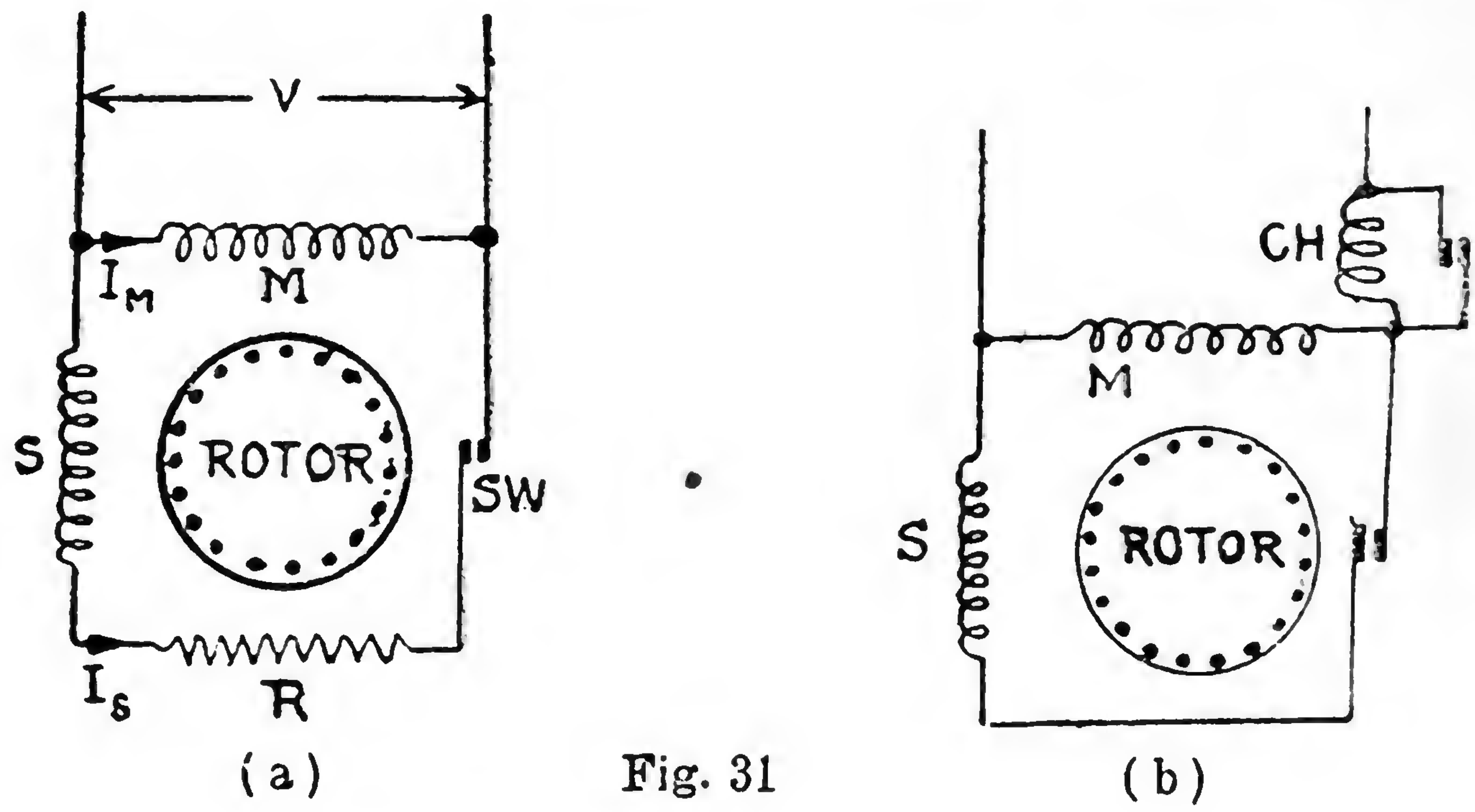


Fig. 31

fugal switch *SW*. Thereafter the motor runs due to the torque of the main winding.

In Fig, 31 (b) *M* has a choke *CH* in series with it during the starting period only. The winding *S* is of small section wire and therefore its resistance is fairly high. Once the motor picks up speed, both the auxiliary winding and choke are out of circuit.

(ii) The circuit of *Capacity-start motor* is shown in Fig. 32. The split phasing is done by the use of capacitance instead of resistance or inductance. The phase difference between the currents in the two parallel circuits is almost 90° (elec.). The starting torque is therefore much greater than that possible with the ordinary split-phase motor. Besides this the power factor of this motor is nearly unity if the capacitor remains in circuit throughout.

A greater advantage is gained if there are two capacitors in parallel in the circuit of the auxiliary winding as shown in Fig. 32 (b).

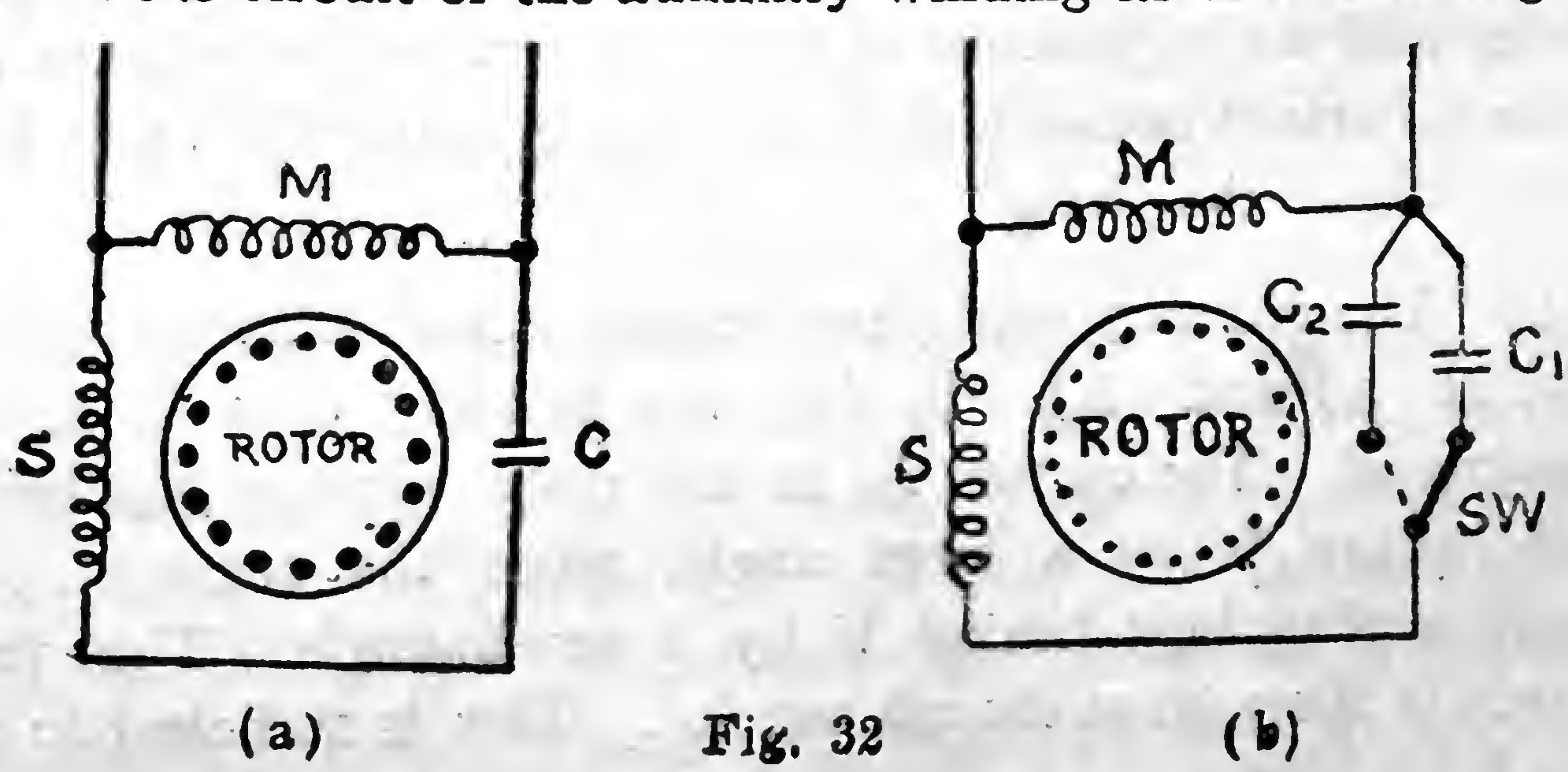


Fig. 32

C_1 is of a larger capacitance than C_2 and is in circuit during the starting period only. The centrifugal switch SW connects C_2 during the operating period and C_1 is out of the circuit. This motor is virtually a 2-phase motor. The capacitor motor has become very popular during recent years due to the low cost and reliability of modern capacitor units.

(b) The *shaded-pole motor* operates, on a similar principle, but in this case the motor has salient poles. Fig. 33 shows a sketch of the motor. Only one of the four poles of the motor is shown and the pole winding is omitted. The winding on the poles is excited by the current from the supply mains. There is no auxiliary winding for this type of motor. Each pole is split into two parts and the smaller part has a stout copper band round it.

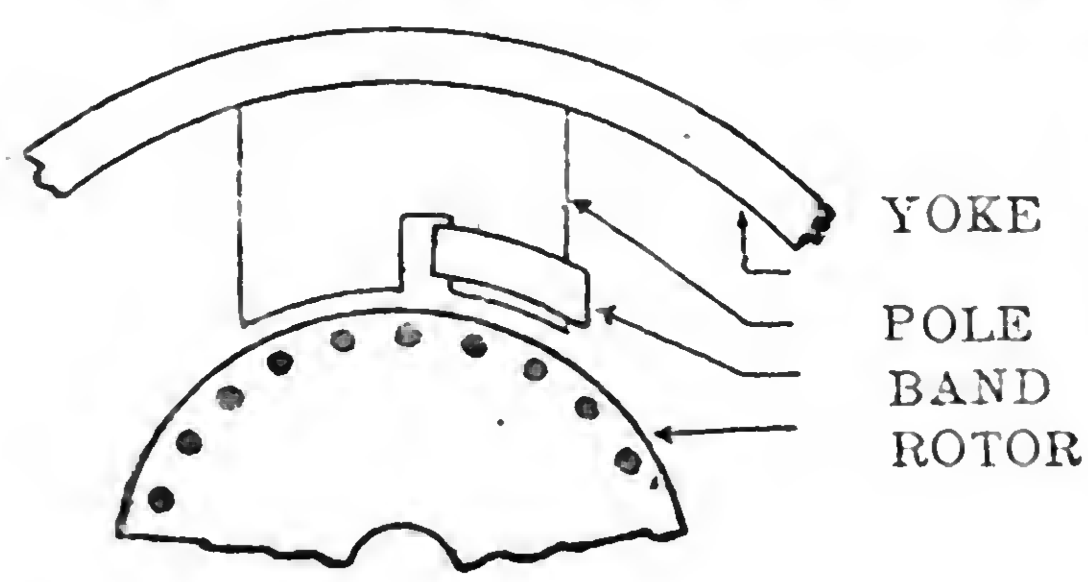


Fig. 33

The pole winding produces the main flux which is pulsating but its magnitude varies sinusoidally. The copper band on the pole acts as a short-circuited secondary and the induced current in it produces a pulsating flux of its own. The magnitudes of the two fluxes are not equal and the phase difference between them is much less than $90°$ (elec.). This produces a weak torque. Therefore these motors are suitable where starting conditions are not severe and the output is less than 1 h. p.

19. The Schrage Motor: The most successful polyphase variable speed motor for general industrial use is the Schrage motor which can be built for a wide range of speed with a reasonably good all-round performance and with a speed/load characteristic similar to that of a shunt d. c. motor. This motor is produced in capacities from about 5 h. p. up to a few hundred h. p. and the permissible speeds are not unduly low for ordinary industrial requirements. If a polyphase low tension supply is available such a motor is entirely self-contained and does not require any auxiliary apparatus such as transformers, induction regulators etc. which are necessary for certain other types of variable speed a. c. motors.

In the Schrage type commutator motor, the primary winding of the 2-phase or 3-phase type, connected through slip-rings and brushes

to the source of supply, is embodied in the rotor of the machine, the secondary winding being accommodated in the stator. So far the machine is such that it could be operated as a normal wound-rotor induction motor with the exception that the usual functions of stator and rotor are reversed. To provide the variable speed feature, a third winding connected to the commutator, is wound on the rotor in the same slots as the primary winding. This is the *regulating winding* and in this winding, functioning as the secondary winding of a transformer, voltages are induced at supply frequency. Usually there is one turn of this regulating winding between each pair of bars on the commutator. Double brush rockers are employed, there being three double brush arms for each pair of poles on the motor. A phase of the secondary winding is connected between a front and a back brush arm. These arms occupy the same commutator bar when the brushes are in the *neutral position.* Fig. 34 shows both the external

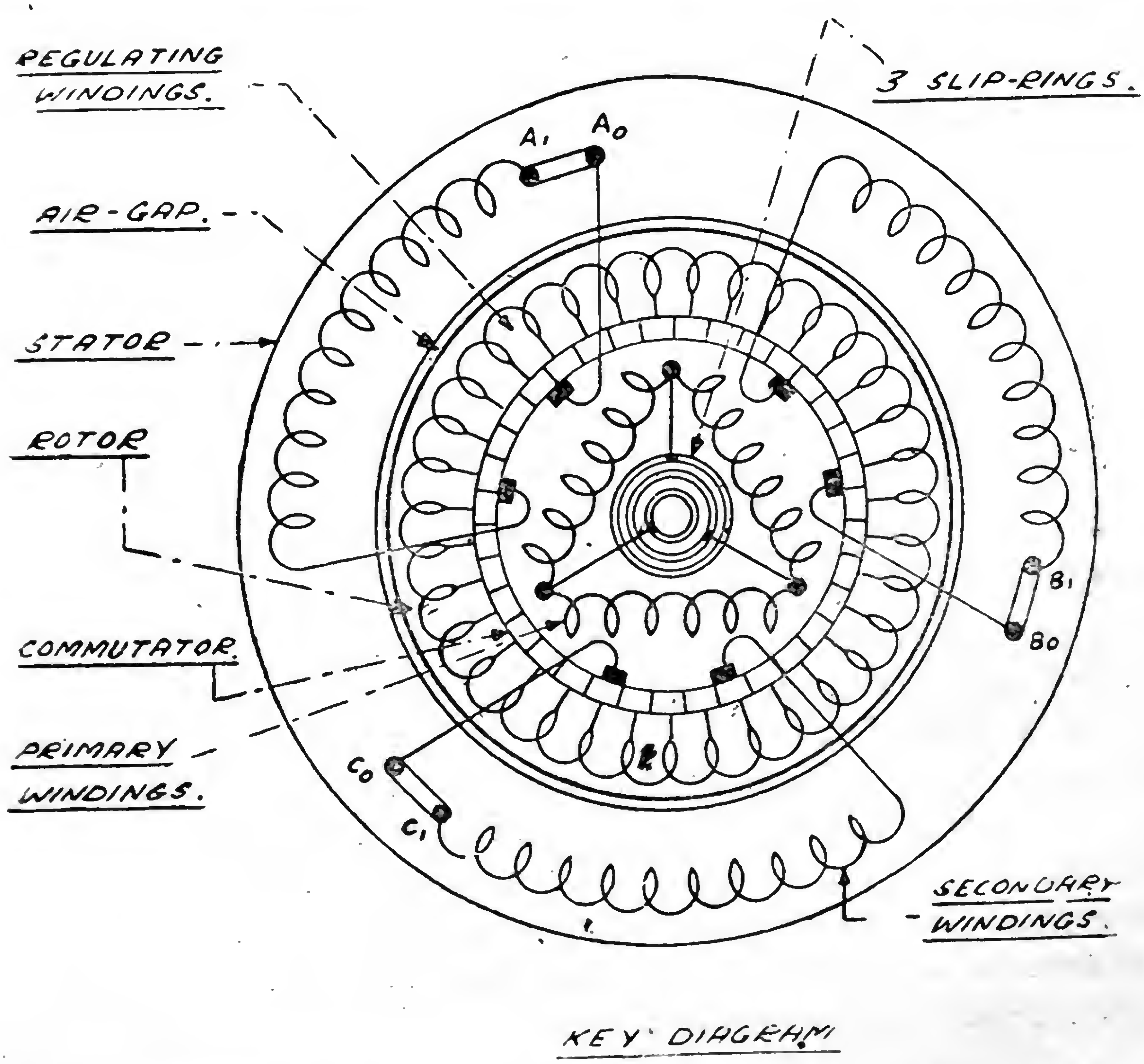


Fig. 34

(*Courtesy: Mather & Platt*)

and internal connections of this type of motor, and Fig. 35 (a) shows the brushes of one phase in the neutral position *NN*.

At standstill, when the brushes are in the neutral position, each phase of the secondary winding is short-circuited by the contact of its front and its back brush arms with a common commutator bar. If the front and back brush arms are not on the same commutator bar it follows that there is a certain proportion of the regulating winding in circuit between the two brushes belonging to the same phase of the secondary winding. This being the case, there is injected into its secondary circuit a voltage proportional to the number of turns of regulating winding between the front and back brush arms belonging to one phase of the secondary winding, and also proportional

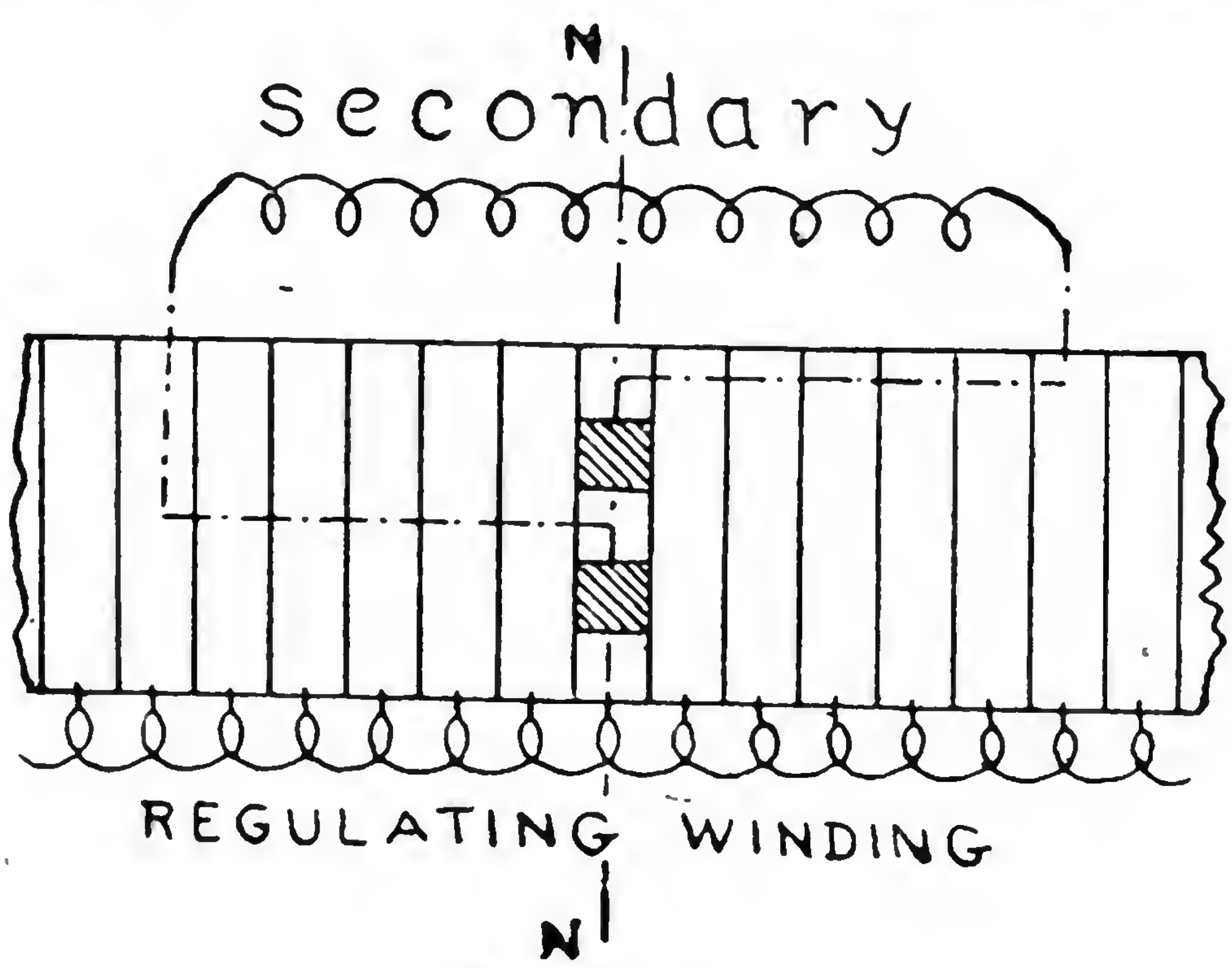


Fig. 35 (a)

to the effective ratio of number of turns in the primary and regulating windings. This voltage is at primary frequency which, in the case of a machine at standstill, is equal to secondary frequency. The value of this injected voltage in a given motor may be increased or decreased by varying the number of turns of the regulating winding in circuit with the secondary phase, viz. by altering the amount of brush opening between the front and back brush arms of the secondary phases.

Further, the relative phase angle of this injected voltage with respect to the secondary voltage may be altered by changing the position on the commutator of the two brushes collecting for a "phase" of the secondary winding. As the motor starts rotating, the frequency in the secondary winding falls below that of the primary

supply, but the frequency of the voltage injected into the secondary winding is coincident with that of the voltage generated in the secondary winding. This is due to the frequency changer characteristics of the commutator arrangement.

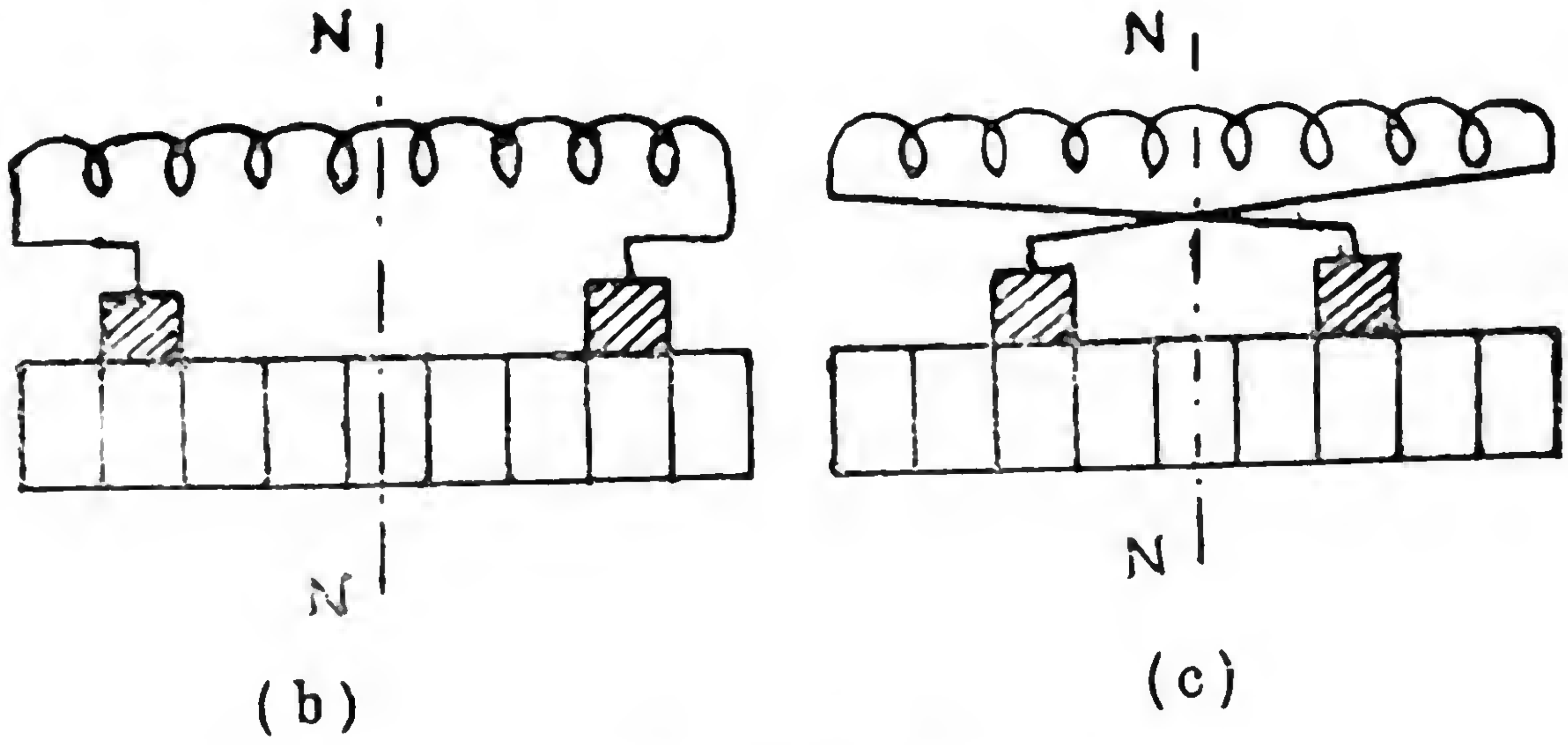


Fig. 35

The speed of an ordinary induction motor depends on the total e m.f. acting in the secondary circuit. Now consider the Schrage motor in operation with the brushes in the neutral position. Obviously, this motor performs as a wound rotor induction motor with secondary winding short circuited, and it runs at normal speed. If the brush arms are moved apart to a situation on the commutator, as shown in Fig. 35 (b), the regulating winding injects into the secondary circuit an e. m. f. having a component in direct opposition to the induced e. m. f. of the secondary. The result is that the speed is reduced. On the other hand if the brushes are moved to a situation on the commutator as shown in Fig. 35 (c), the injected voltage has a component in phase with the induced e. m. f. of the secondary. Thus the total e. m. f. acting in the secondary is increased and the speed will be above normal. In addition, if the injected e. m. f. has a quadrature component which leads the induced e. m. f. of the secondary circuit, the power factor of the motor will be improved. Thus in this motor both speed control and power factor correction are available without the use of any external apparatus.

CHAPTER XIII

CONVERTING APPARATUS

1. Introductory: In many special cases the direct current is either essential or desirable. These are

(a) Plants for Electro-plating
(b) Electric Traction
(c) Electrolysis Plants
(d) Rolling Mills
(e) Telephone Exchanges
(f) Radio Transmitting Stations, etc.

Since the universal practice is to generate, transmit and distribute 3-phase a. c. power, it becomes necessary to use some kind of apparatus to convert a. c. into d. c. power. These are

(a) **A Motor-Generator Set**, consisting of an a. c. motor coupled to a d. c. generator. The resultant d. c. voltage is independent of the a. c. supply voltage. If the a. c. motor happens to be a synchronous motor, advantage may be taken to improve the power factor. It requires a larger floor space and is costlier than either a rotary converter or a motor converter.

(b) **A Rotary Converter** has a d. c. armature and a d. c. field system. The armature has a commutator at one end of its shaft and slip-rings at the other end. The a. c. power is fed to the armature through the slip-rings which are connected to suitable tapping points on the armature. Since there is a definite relation between the a. c. and the d. c. voltages of a rotary converter, a transformer is necessary to ensure a d. c. voltage which is *standard*.

(c) **Mercury Arc Rectifier**: This has no moving parts. The operation depends upon the value action of an arc in mercury vapour. Transformer is necessary to obtain a standard d. c. voltage.

(d) **Motor Converter**: A wound induction motor is coupled electrically and mechanically to a rotary converter. The electrical coupling consists in the wound rotor (of the induction motor) being connected, through a hollow shaft, to tapping points on the rotary armature.

(e) **Hot Cathode Rectifiers** and (f) **Metal Rectifiers** are used for small power output.

A. The Rotary Converter

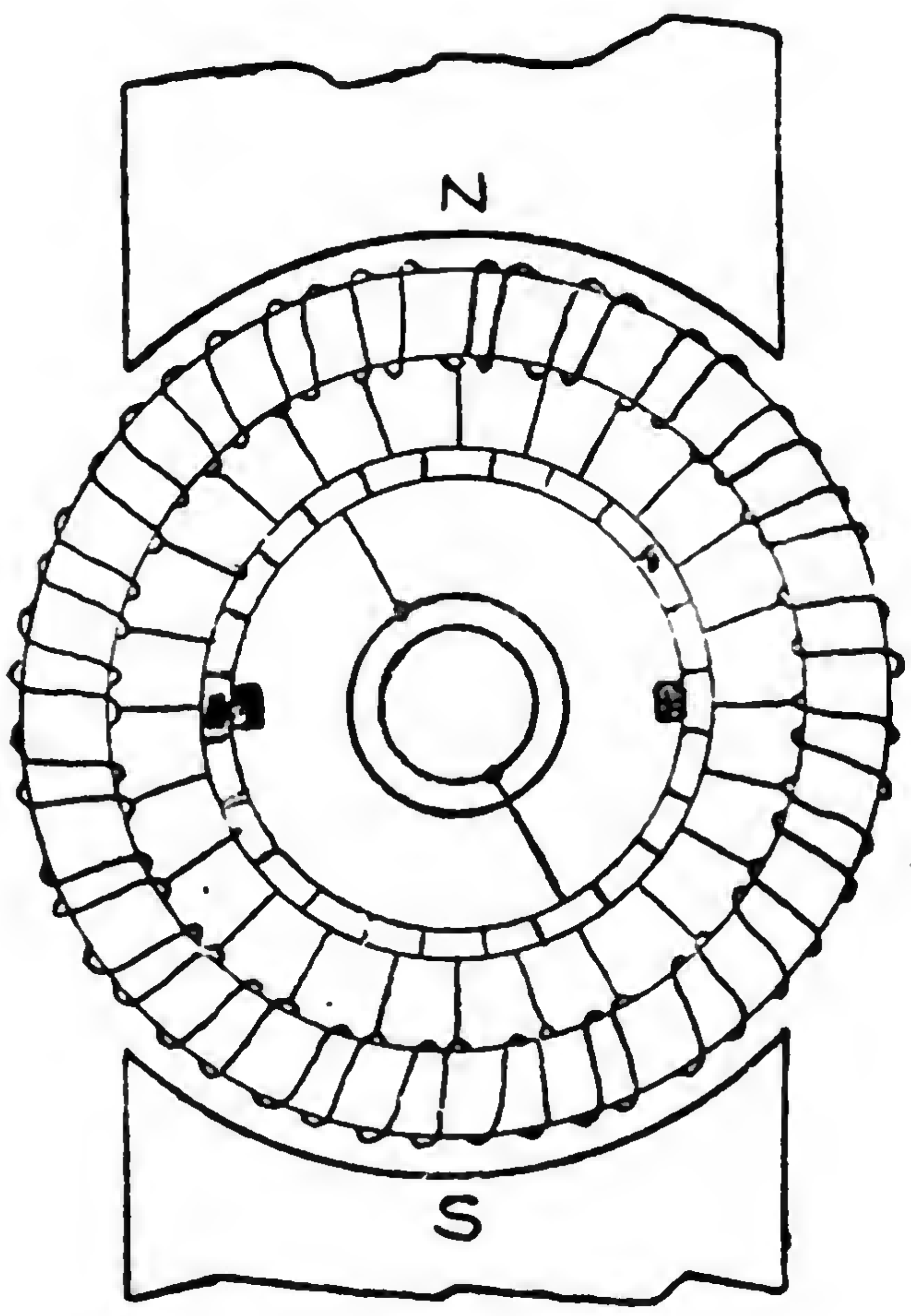
2. Voltage Ratios: It is here assumed that

(a) the d. c. output power = a. c. input power;

(b) maximum voltage is generated when a group of coils is under the centre of poles and that no voltage is induced when midway between the poles.

(c) maximum voltage and maximum current occur at the same instants on the a. c. side; i. e. assuming a unity power factor.

Fig. 1 shows a 2-pole, ring-type armature having 24 coils. 2 slip-rings and 2 d. c. brush positions on the commutator. When the armature rotates, the induced e. m. f. generated in the coils is alternating in nature. The phase difference between voltages induced in successive coils is $\frac{360}{24} = 15°$. The vector diagram of these voltages



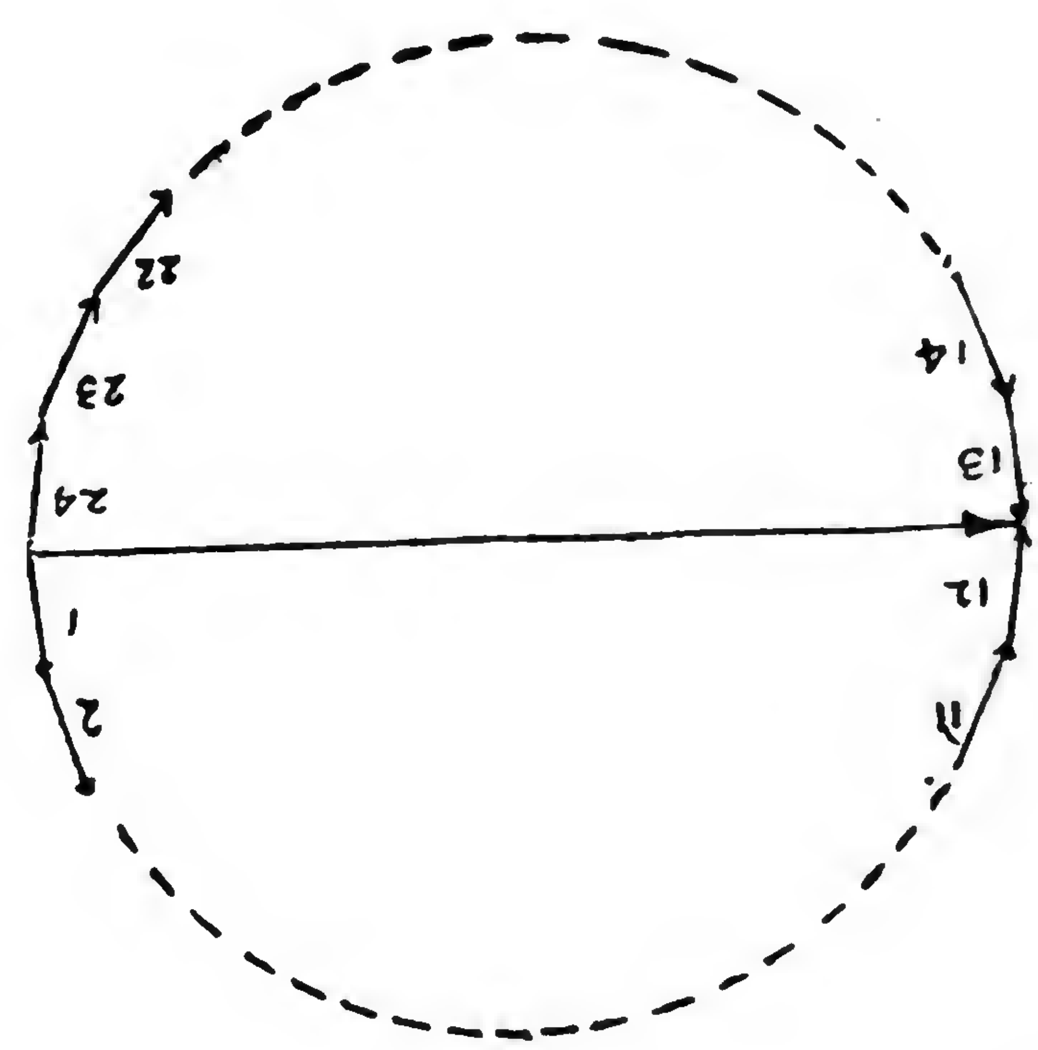


Fig. 2

Fig. 1. Single-Phase Rotary.

is shown in Fig. 2, where the resultant of the vectors of 12 coils is the diameter. The slip-rings are connected to two diametrically opposite points in the winding for a 2-pole single-phase rotary. These tappings pass under the d. c. brushes at the point of maximum a. c. voltage. Hence for a single-phase rotary

$$\textit{d. c. voltage} = \textit{maximum value of a. c. voltage}$$

$$V_{dc} = V_{ac\ max}.$$

a. c. r. m. s. value is $V_{ac} = \frac{V_{dc}}{\sqrt{2}} = 0{\cdot}707\ V_{dc}$ ... ... (1)

Fig. 3 shows a 2-pole, 3-phase rotary, the slip-ring tappings being at 3 points spread 120° apart. If the three tappings are ***a***, ***b*** and ***c*** shown in Fig. 4, then ***ab***, ***bc*** and ***ca*** are the three phase e. m. fs. and ***de*** is the d. c. voltage, while the circumference may be considered as a winding of the armature.

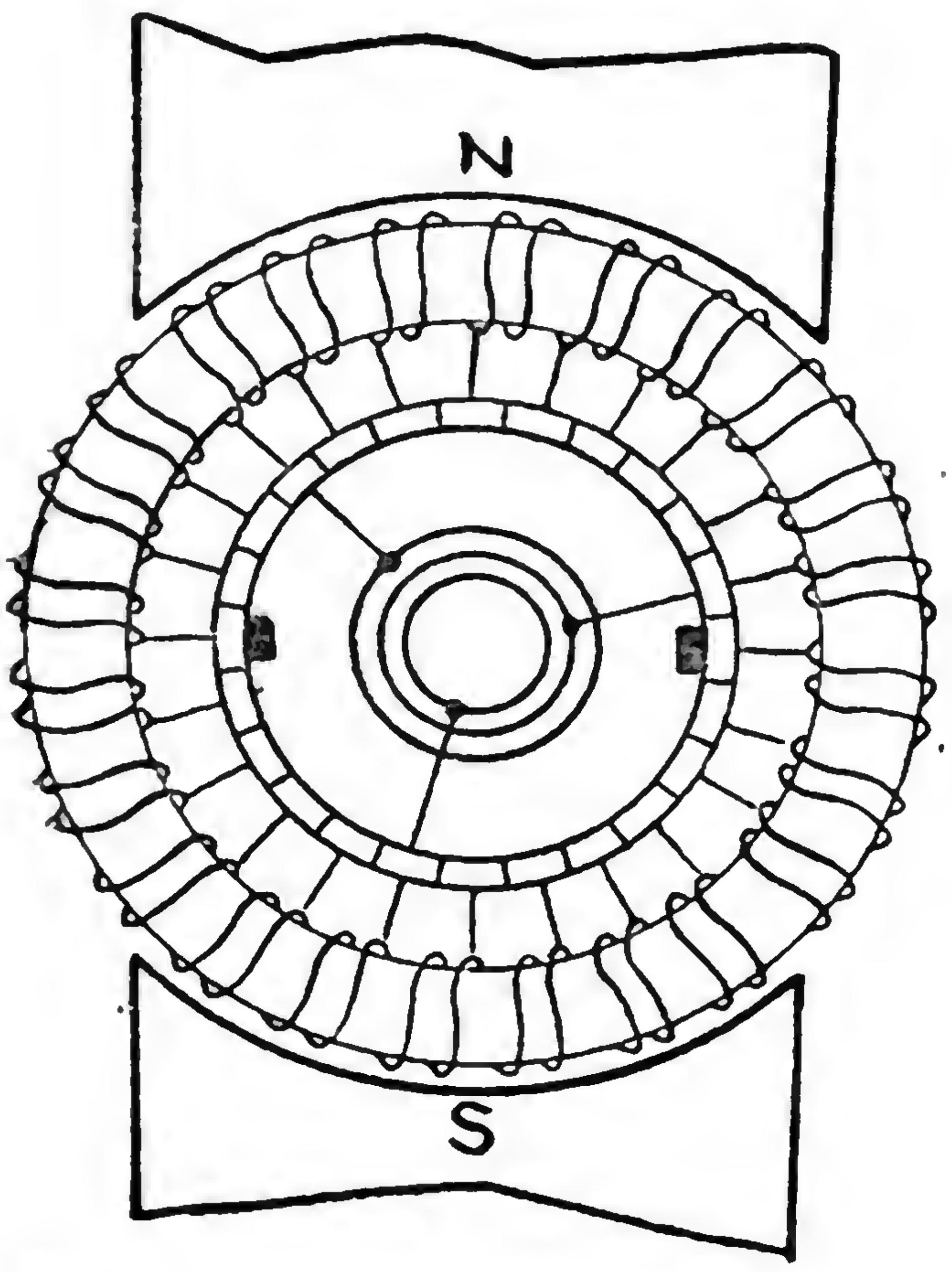


Fig. 3. Three-Phase Rotary.

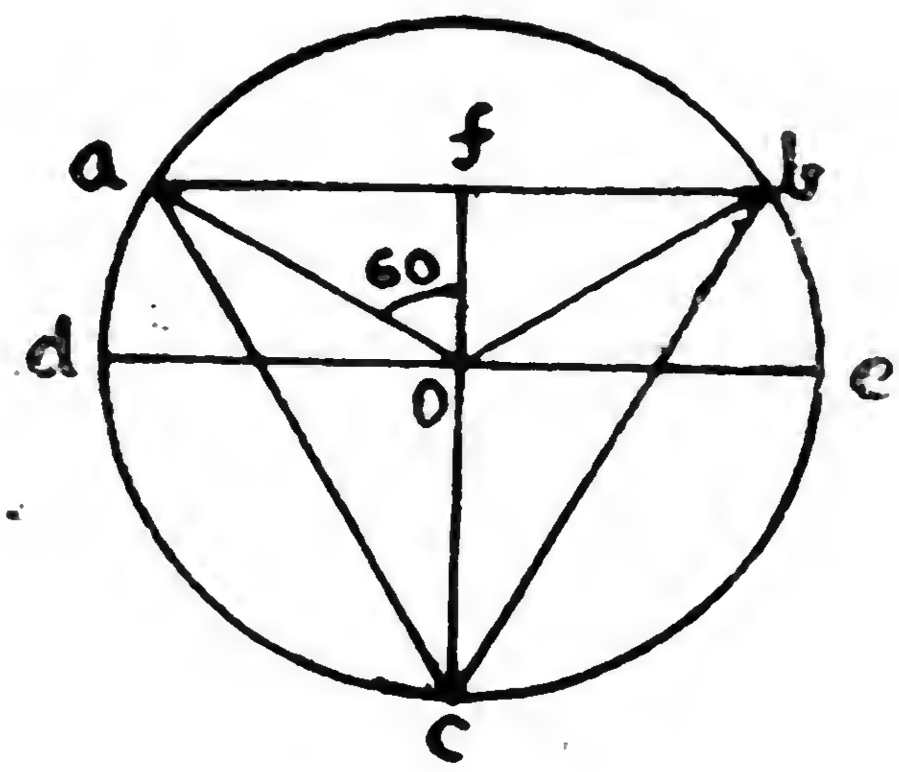


Fig. 4

ab = ***bc*** = ***cd***. ***O*** is the mid-point of ***de***

ab = 2 ***af*** = 2 ***oa*** sin 60.

ab = ***de*** sin 60.

Hence *maximum phase voltage* = V_{dc} *sin 60*

$$\therefore \quad V_{ac}\,(r.\,m.\,s.) = V_{dc}\,\frac{\sqrt{3}}{2\sqrt{2}} = 0{\cdot}612\ V_{dc} \quad \ldots \quad \ldots \quad (2)$$

If N is the number of slip-rings or phases, the angle between two consecutive tapping points is $\frac{2\pi}{N}$. For a 3-phase rotary the angle is $\frac{2\pi}{3} = 120°$, where $N = 3$. Hence in general

$$V_{ac} = \frac{V_{dc}}{\sqrt{2}} \sin\frac{\pi}{N} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

$$\frac{V_{ac}}{V_{dc}} = \frac{\sin\frac{\pi}{N}}{\sqrt{2}}$$

3. **Current Ratios:** Since it is assumed that the d. c. output is equal to a. c. input, i. e. the rotary is assumed to have no losses,

$$N \times V_{ac} \times I_{ac} \cos\phi = V_{dc} \times I_{dc} \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

where V_{ac} is voltage per phase, I_{ac} is the current in each phase winding and N is the number of phases. Hence from Eq. (4)

$$I_{ac} = I_{dc}\left(\frac{V_{dc}}{V_{ac}}\right)\frac{1}{N\cos\phi}$$

substituting the value of $\left(\frac{V_{dc}}{V_{ac}}\right)$ from Eq. (3),

$$I_{ac} = I_{dc}\frac{\sqrt{2}}{N\sin\frac{\pi}{N}} \times \frac{1}{\cos\phi} \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

The line current I_l is the current flowing into each slip-ring on the a. c. side. This current is the vector difference of the currents in the two adjoining phase windings. Fig. 5 shows an N-phase rotary

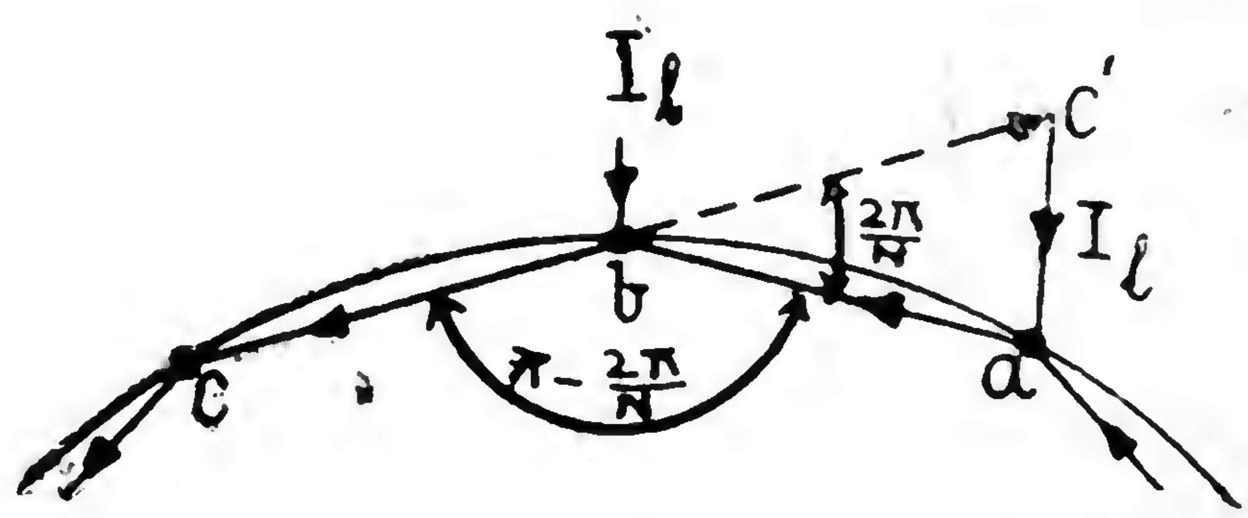


Fig. 5

phase currents ab and bc in two adjoining phase windings bc' is bc reversed, so that $c'a$ is slip-ring current I_l. From the geometry of the figure

$$I_l = c'a = ab \times 2\sin\frac{\pi}{N}$$

$$= I_{ac} \times 2\sin\frac{\pi}{N} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (5a)$$

Substituting the value of I_{ac} from Eq. (5) in the above

$$I_l = I_{dc}\left[\frac{\sqrt{2}}{N\sin\frac{\pi}{N}} \times \frac{1}{\cos\phi}\right] \times 2\sin\frac{\pi}{N}$$

$$I_l = I_{dc}\frac{2\sqrt{2}}{N\cos\phi} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

The numerical values of the ratio of $\frac{V_{ac}}{V_{dc}}$ for various values of N are given in the following Table. Note that N really stands for the number of tapping points on the armature.

D. C. voltage	1-phase N=2	2-phase N=2	3-phase N=3	4-phase N=4	6-phase N=6	12-phase N=12
	$\frac{\sin(\pi/2)}{\sqrt{2}}$	$\frac{\sin(\pi/2)}{\sqrt{2}}$	$\frac{\sin(\pi/3)}{\sqrt{2}}$	$\frac{\sin(\pi/4)}{\sqrt{2}}$	$\frac{\sin(\pi/6)}{\sqrt{2}}$	$\frac{\sin(\pi 12)}{\sqrt{2}}$
1	0·707	0·707	0·612	C·5	0·354	0·183

Similarly, the ratio $I_l / I_{dc} = \frac{2\sqrt{2}}{N \cos\phi}$, and assuming $\cos\phi = 1$ the following Table gives the values of this ratio for different values of N

D. C. current	1-phase	2-phase	3-phase	4-phase	6-phase	12-phase
1	1·414	0·707	0·943	0·707	0·472	0·236

Example : The d. c. output of a 6-phase rotary converter is 200 kW at 500 volts and the efficiency is 90%. The primary of the transformer is delta connected and is supplied from a 3-phase, 3300-volt line. The secondary of the transformer is in double-star. Assuming unity power factor, calculate (a) the voltage between adjecent slip-rings, (b) the slip-ring current, (c) line current on the high voltage side and (d) the transformer turns ratio (per phase).

Solution : Number of tappings on the rotary = 6 i. e. $N = 6$

(a) Using Eq. (3) and $\cos\phi = 1$,

$$V_{ac} = V_{dc} \frac{\sin\frac{\pi}{N}}{\sqrt{2}} = 500 \frac{\sin\frac{180}{6}}{\sqrt{2}} = 177 \textit{ volts.}$$

(b) $I_{dc} = \frac{200 \times 1000}{500} = 400$ A.

Using Eq. (6) and efficiency 90%,

$$I_l = I_{dc} \frac{2\sqrt{2}}{N\cos\phi} \times \frac{1}{\textit{efficiency}}$$

$$I_l = 400 \frac{2\sqrt{2}}{6 \times 1} \times \frac{1}{0 \cdot 9} = 209 \cdot 5 \ A.$$

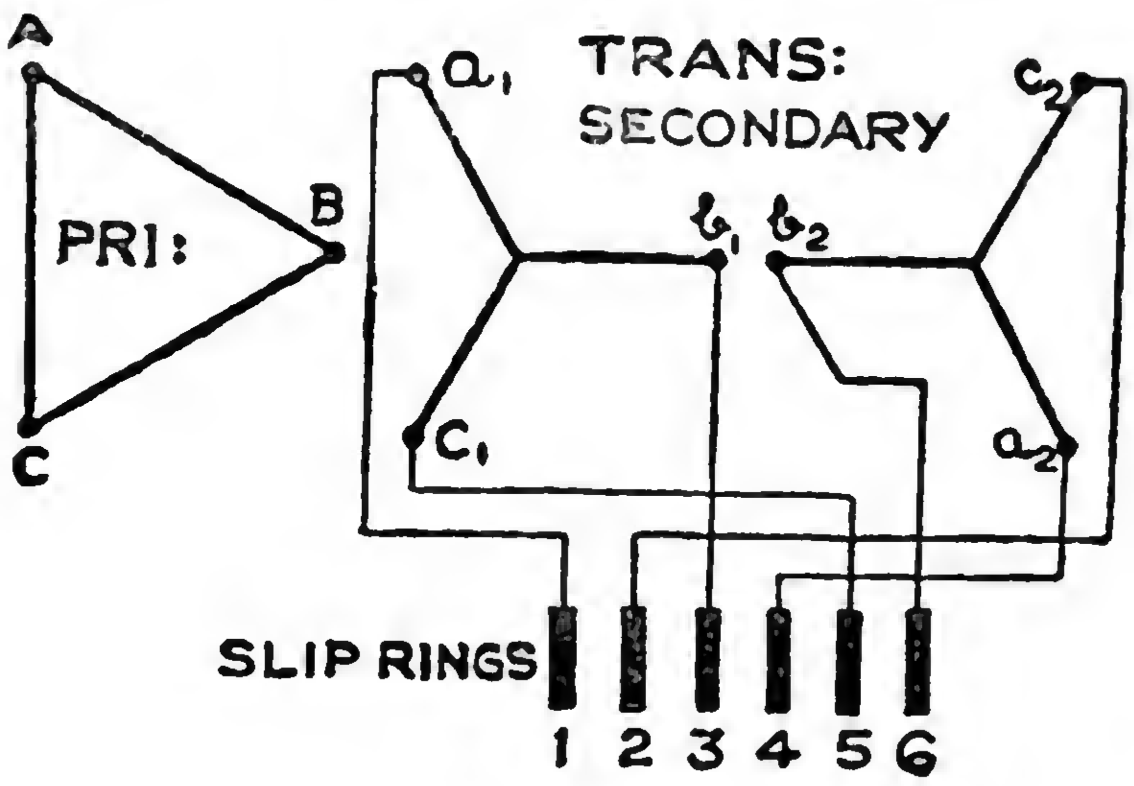


Fig. 6

(c) $\dfrac{output\ (d.\ c.)}{input\ (a.\ c.)} = efficiency.$

$$\frac{200 \times 1000}{\sqrt{3} \times 3300\, I \times \cos\phi} = \frac{90}{100}$$

$\therefore$ h. v. line current $I = \dfrac{200 \times 1000}{\sqrt{3} \times 3300} \times \dfrac{100}{90}$

$$I = 39\ A.$$

(d) Primary phase voltage = 3300 V.

Secondary phase voltage = 2 × 177 V, there being two sections per each secondary phase.

$$\therefore \quad \frac{Primary\ turns}{secondary\ turns} = \frac{3300}{2 \times 177} = 9 \cdot 322.$$

4. Armature Heating of Rotary Converters: In the armature of a rotary converter the alternating current is a **motoring current**, while the direct current is a **generating current**. These two currents therefore flow in opposite directions. Hence the net or resultant current is the difference between the two, there being considerable degree of neutralisation. This results in

(1) great reduction in armature reaction,

(2) armature heating is much less than in either a motor or a generator of equal rating, and

(3) much better commutation.

As a result of neutralisation of currents, the armature torque is very small, just enough to overcome frictional and iron losses. The heating of the armature, however, is not uniform, because the I^2R

loss in a conductor at a tapping point is maximum, and minimum in a conductor which is midway between two tapping points. The two following figures show the curves of resultant current i and the curves of i^2. Fig. 7 (a) is for the conductor midway between two tapping

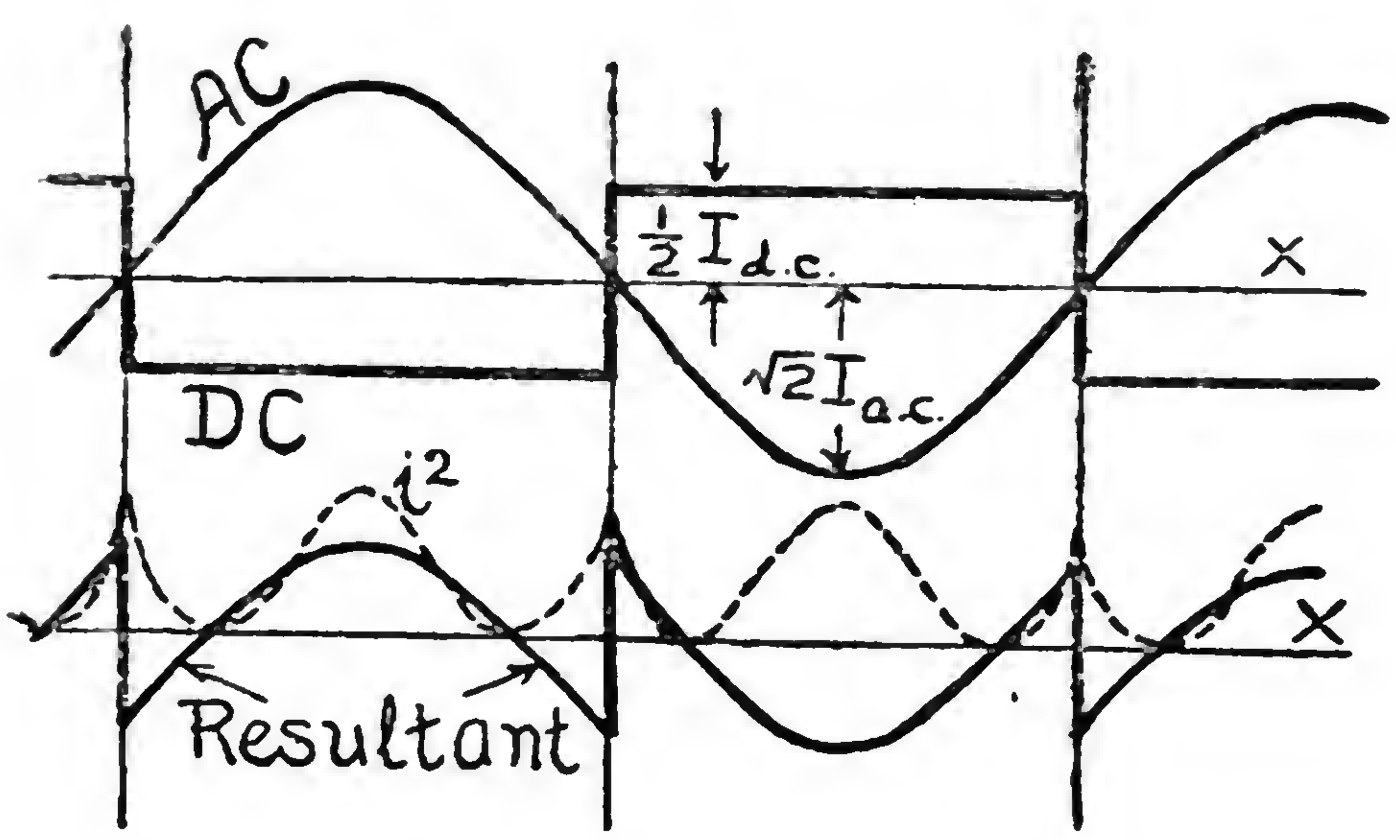


Fig. 7. (a)

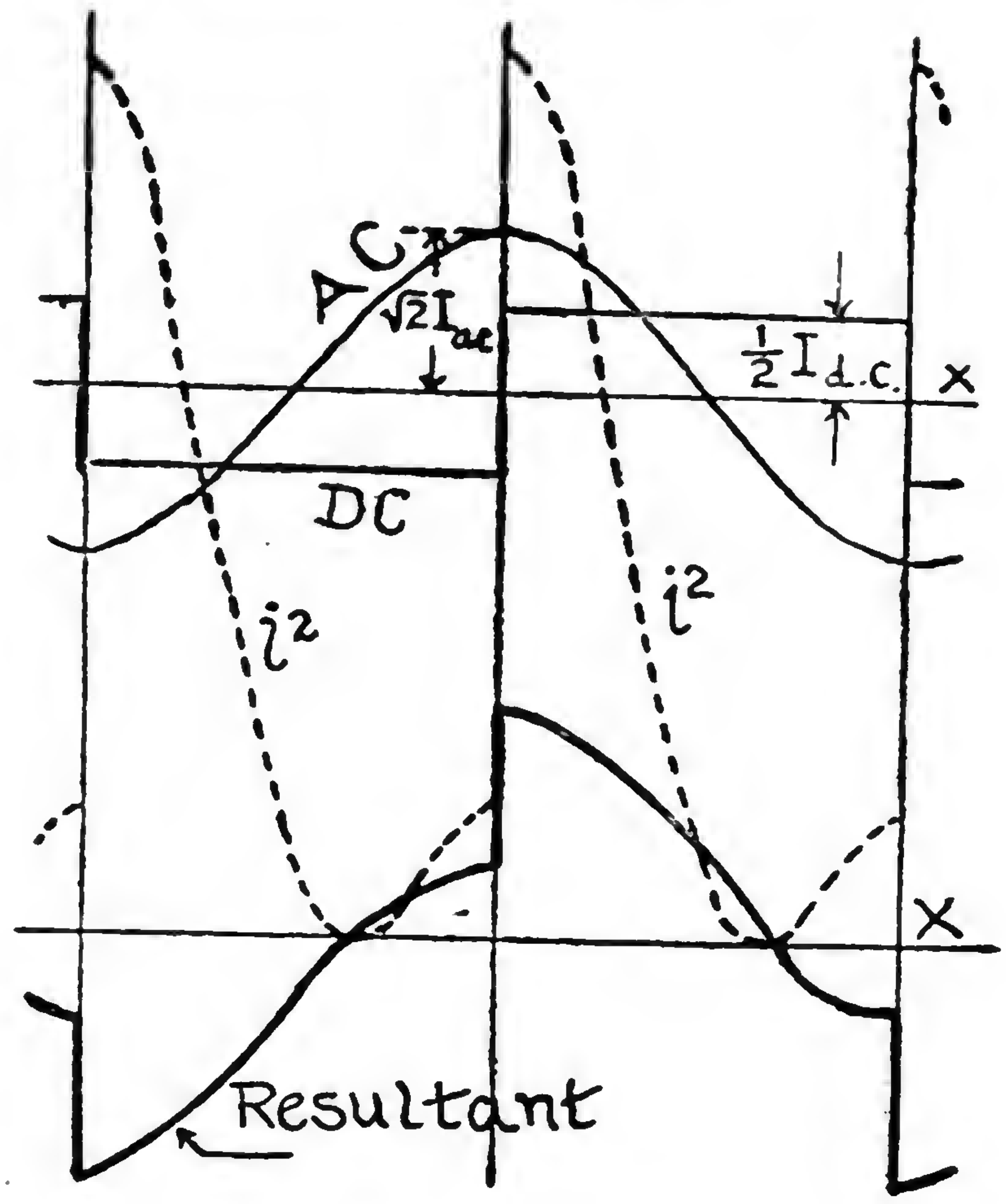


Fig. 7. (b)

points and Fig. 7 (b) for a conductor at a tapping point. These two figures are for the condition of unity power factor. As the power

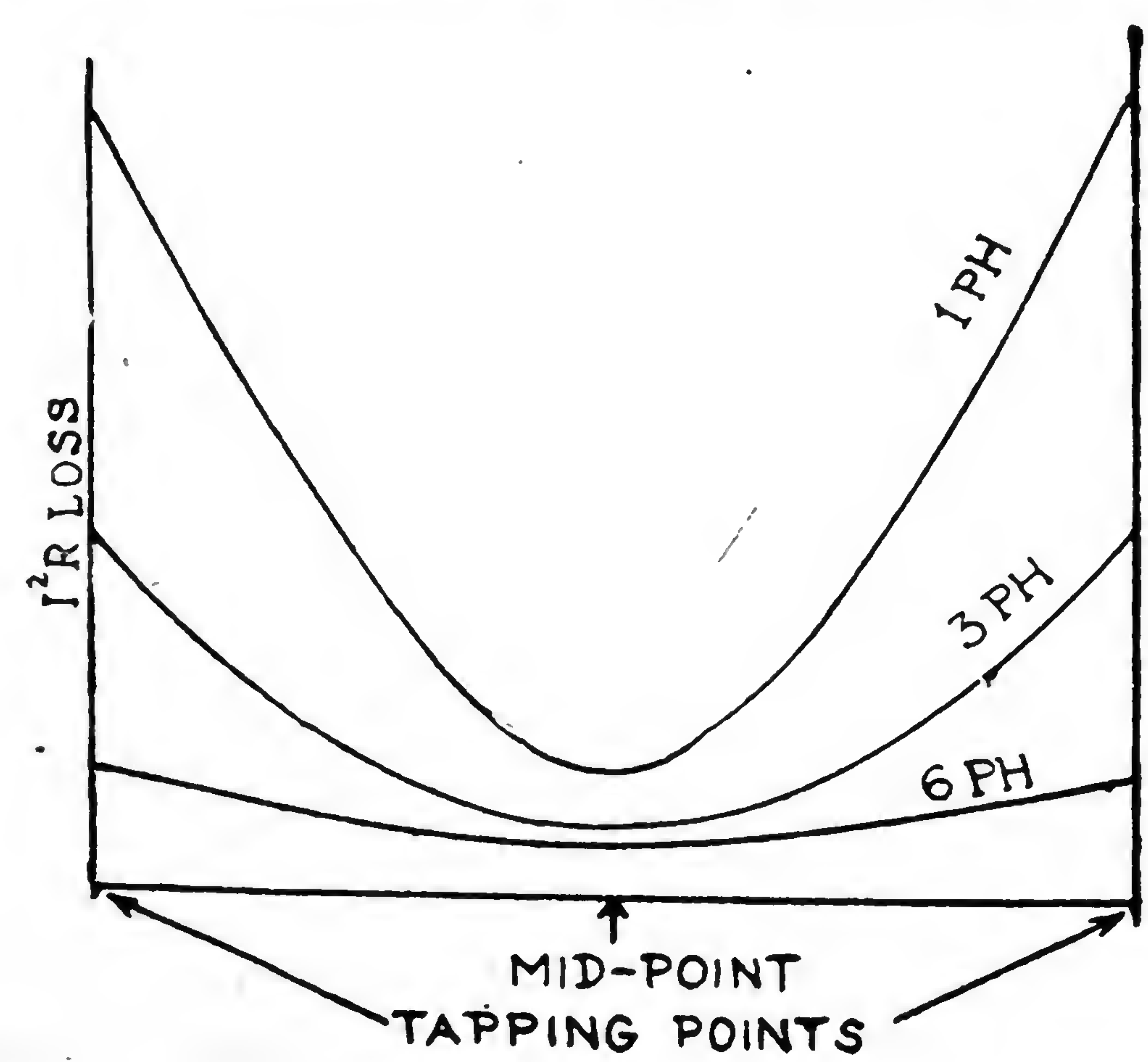


Fig. 8

factor worsens the heat loss increases considerably. Hence a rotary converter should work at as near unity power factor as possible.

If the distance between two tapping points is reduced the heat loss is reduced. This means that the more the number of phases a rotary has the more uniform is the heating loss between a conductor near a tapping point and a conductor midway between two tapping points as shown in Fig. 8.

Except in the case of a single-phase rotary, the heat loss ($I^2 R$) of a rotary is always less than that of a d. c. machine of equal output. The ratio of the losses in the two cases is

$$\frac{\textit{heat loss in rotary}}{\textit{heat loss in d. c. machine}}$$

$$= 1 - \frac{16}{\pi^2} + \frac{8}{N^2 \sin^2 \frac{\pi}{N}} \times \frac{1}{\cos^2 \phi} \quad \cdots \quad \cdots \quad (7)$$

where N is the number of phases and $\cos \phi$ is the power factor at which the rotary works.

Assuming the power factor to be unity

$$\frac{\textit{current in rotary}}{\textit{current in d. c. machine}}$$

$$= \frac{1}{\sqrt{\left[1 - \frac{16}{\pi^2} + \frac{8}{N^2 \sin^2 \frac{\pi}{N}}\right]}} \quad \cdots \quad \cdots \quad (8)$$

Thus on a basis of heat limitation, the comparative output are given below.

D. C. Machine	Rotary		
	1–Phase	3–Phase	6–Phase
100	85	133	192

5. Types of Armature Windings:—Rotary converters whose output is less than 100 kW, have wave-wound armatures as a rule. The number of tapping points per slip-ring is one in the case of wave wound armatures. But when the armature is lap wound the number of tapping points per slip-ring is equal to the number of pole-pairs the machine has.

6. Transformer Connections: A 3-phase rotary converter must have a 3-phase supply which can be obtained either from a 3-phase transformer or from three single-phase transformers. If the rotary supplies a 3-wire d. c. load the secondaries of the transformer should be connected in zig-zag if each secondary is in two equal sections. When the neutral of the d. c. system is connected to the neutral point of the zig-zag, the d. c. out-of-balance current is not capable of producing unidirectional magnetisation. This is because the d. c. ampere-turn of one section neutralises the d. c. ampere-turns of the other section of the same phase. See Fig. 9. This prevents

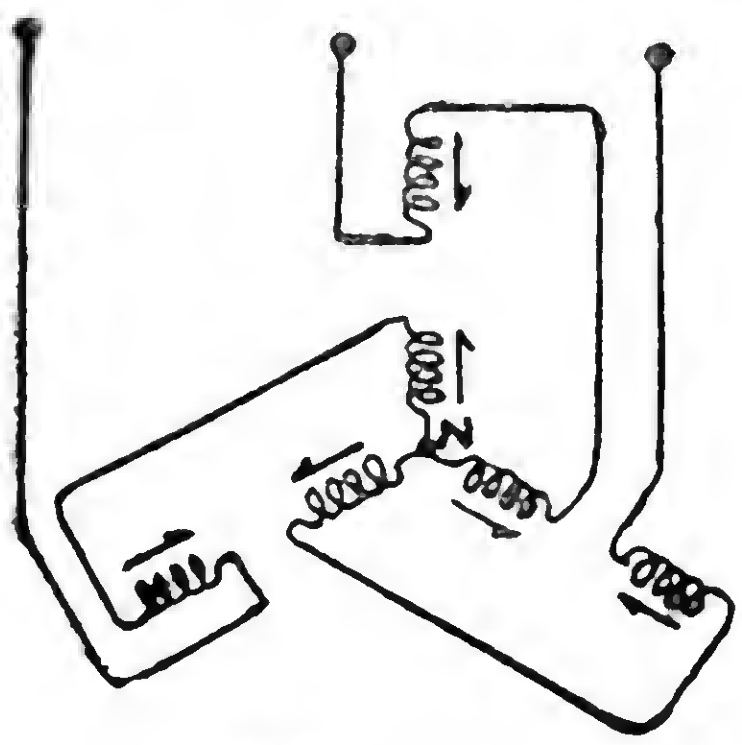


Fig. 9

(a) high flux density

(b) increased iron losses and

(c) higher temperature rise.

If the rotary supplies a 2-wire d. c. load the secondaries may be connected either in star or delta and the primaries also likewise, depending upon the individual cases.

For supplying power to a 6-phase rotary converter the most popular is the diametral connection, where ordinary 3-phase units or three units of single-phase transformers are used. Fig. 10 shows the diametral connection. If the rotary supplies a 3-wire d. c. load, the secondaries must have a midpoint tapping point. The three points together form the neutral point for the d. c. system.

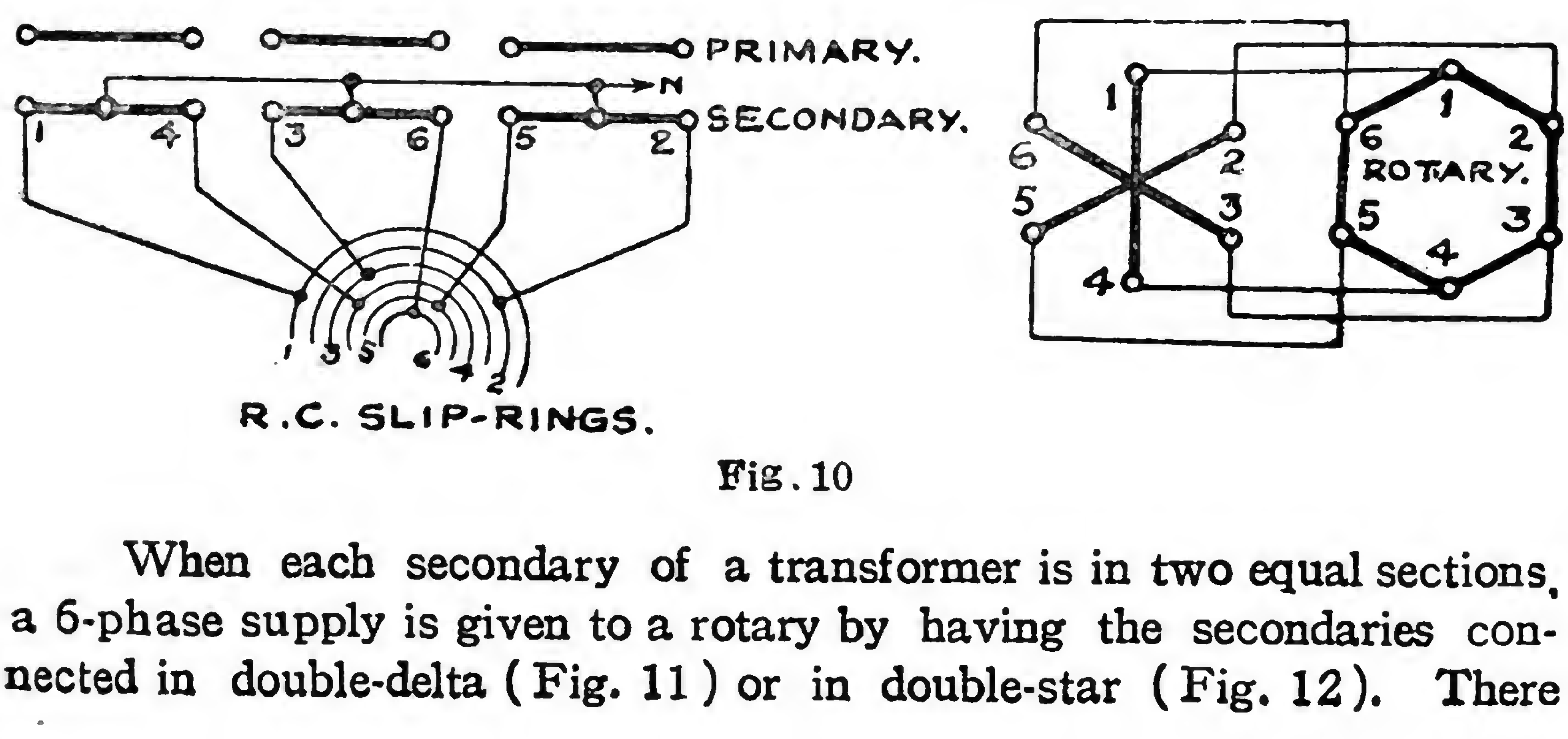


Fig. 10

When each secondary of a transformer is in two equal sections, a 6-phase supply is given to a rotary by having the secondaries connected in double-delta (Fig. 11) or in double-star (Fig. 12). There

are then 12 terminals on the secondary side of the transformer to make the necessary connections. The primaries of the transformer may be connected either in delta or in star.

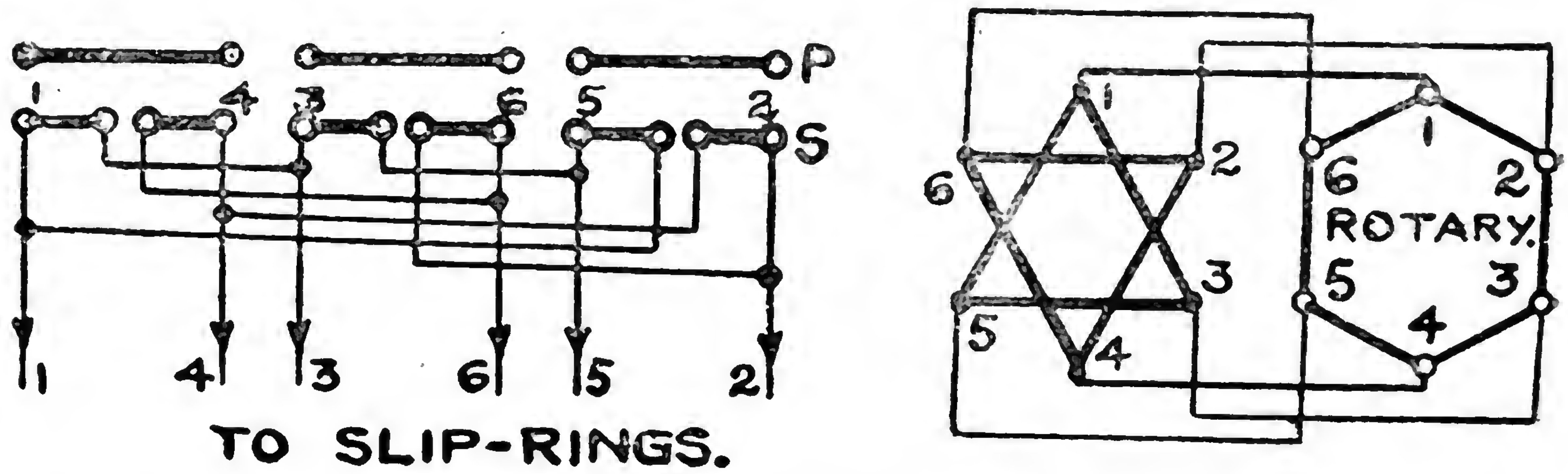


Fig. 11

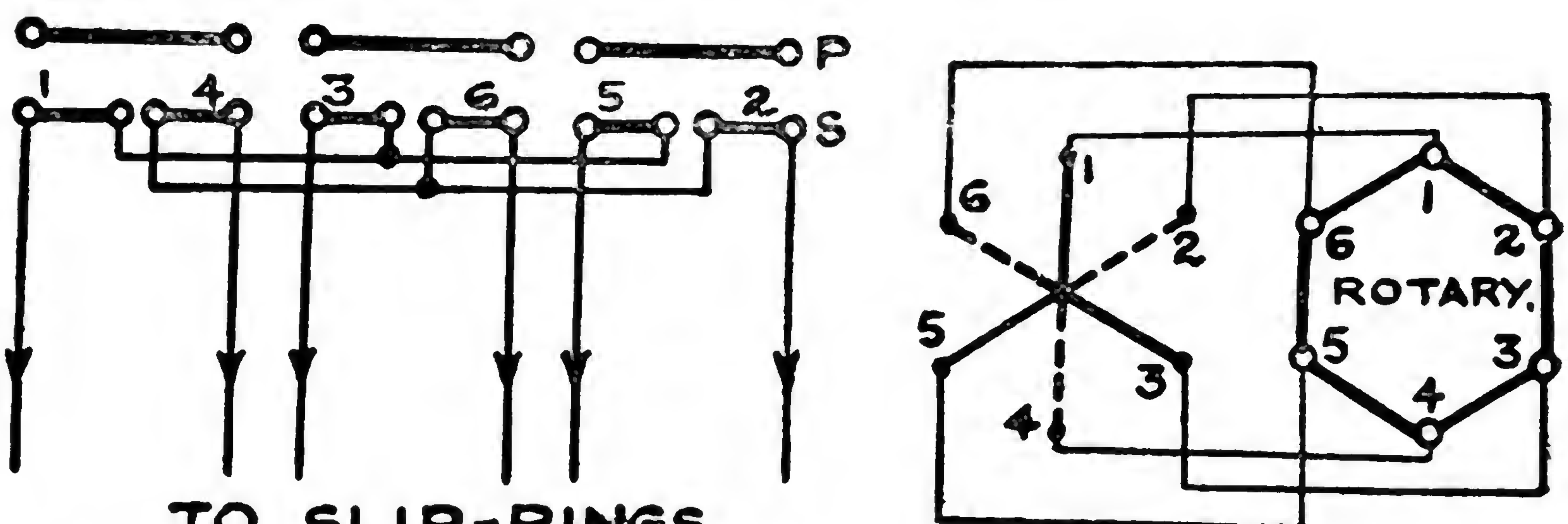


Fig. 12

. **Methods of Starting of Rotary Converters :** These are

(i) starting by an auxiliary motor ;

(ii) utilising the damper winding as a cage winding, to start the machine as a squirrel cage induction motor;

(iii) If an independent d. c. supply is available, the rotary is started as a d. c. shunt motor.

Method (*i*) : A small induction motor is directly coupled to the rotary. It drives the rotary and brings it near synchronous speed. The rotary is then synchronised like an alternator, the auxiliary motor then being cut out. Fig. 13 shows the diagram of connection.

Method (*ii*) : The damper winding on the pole-shoes of a rotary is used as a cage winding, so that the machine starts as an inverted induction motor. The rotating armature is the primary and gets a reduced voltage. The damper winding, which is stationary, is the secondary.

The reduced voltage is available either from an auto-transformer or from the main transformer having taps. The resistance of the damper winding being low, the starting torque is small but quite sufficient to bring the machine near to synchronous speed. The d. c. supply is not given to the pole windings of the rotary until it is running at synchronous speed.

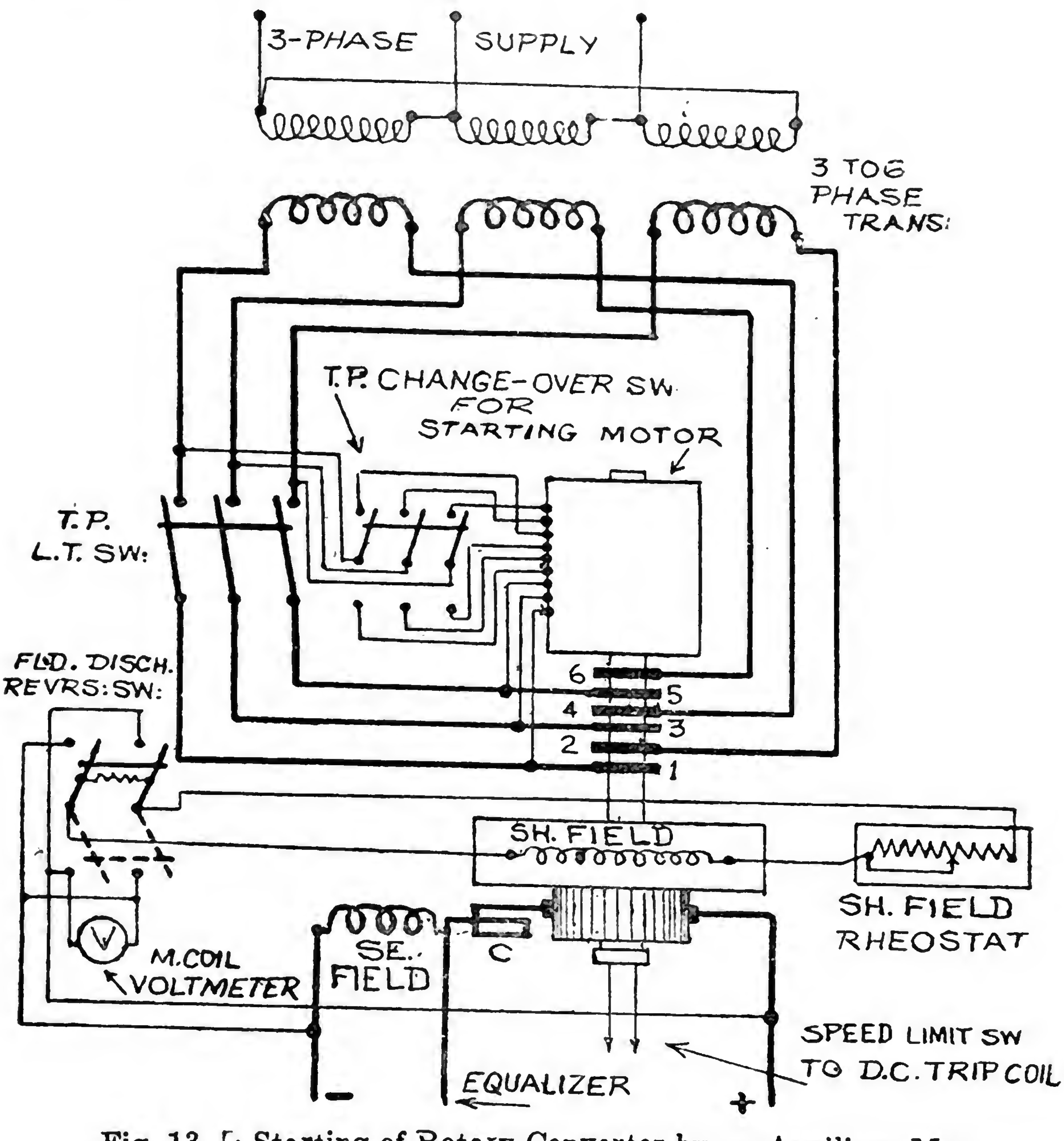
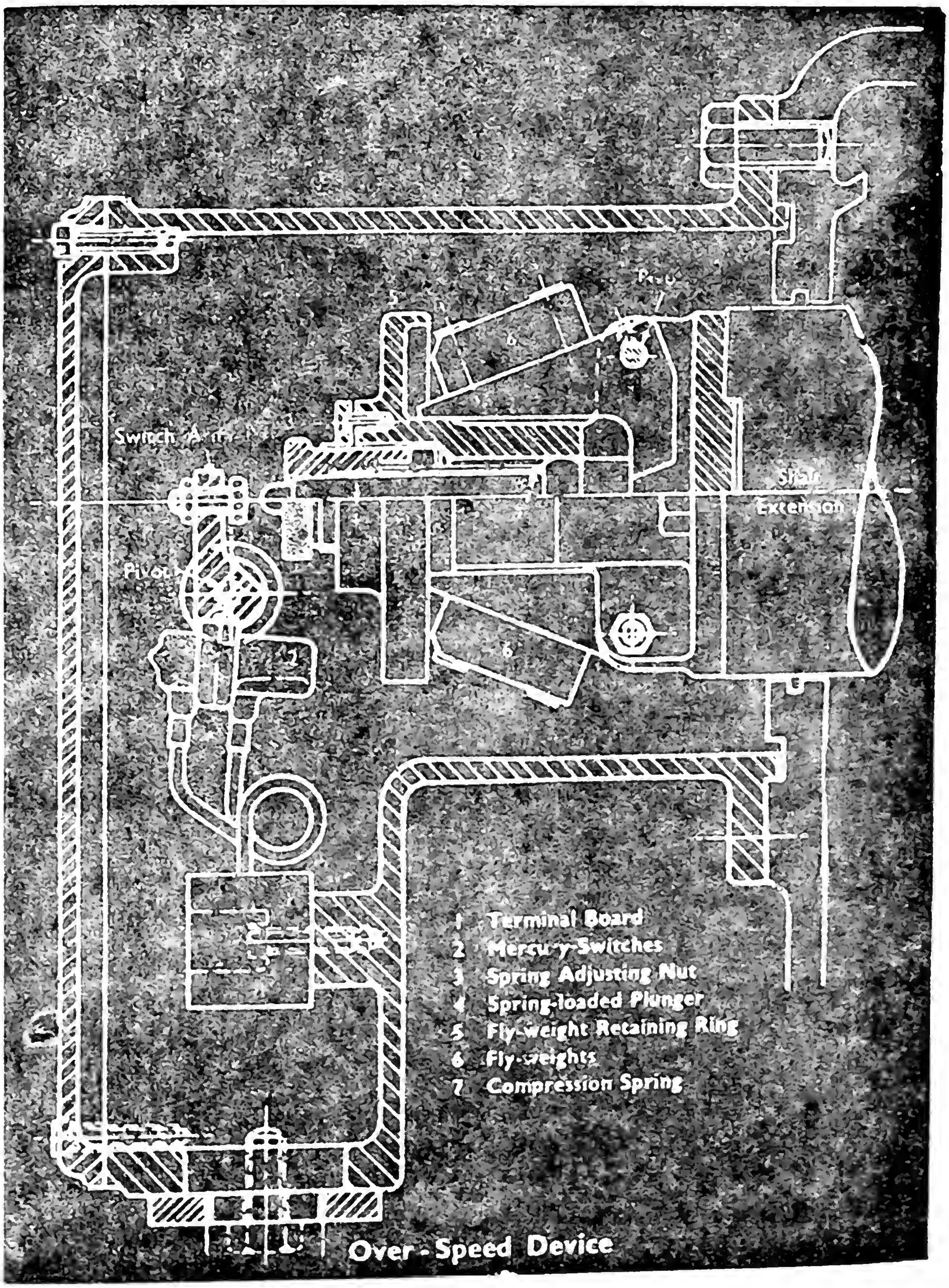


Fig. 13. [· Starting of Rotary Converter by an Auxiliary Motor Connection Diagram.

At the instant of switching on the a. c. supply, the armature poles rotate at synchronous speed in space and as the machine speeds up, the rotation of these poles gradually slows down and at synchronous speed they are stationary in space. If a central-zero moving-coil voltmeter is connected across the d. c. brushes, its pointer oscillates

on both sides of the zero position and the oscillations become less as the synchronous speed is reached. This shows that the d. c. brush voltage is not direct but alternating in nature, and that the residual magnetism of the field cores is destroyed. This is most unfortunate because the polarity of the d. c. side poles is uncertain and will depend upon the position of the armature *at the instant* the field coils are excited. After the field switch is closed the central zero voltmeter



(*By courtesy : Bruce Peebles and Co., Ltd.*)
Fig. 14. Speed Limiting Device.

will indicate whether the brush polarity is right or wrong. If the polarity is wrong, the field switch, which must be a double-pole double-throw type, is thrown over and back. This action is called *slipping a pole.* Thus the stator N–poles get locked with the rotor S–poles. This method requires no synchronising gear.

Method (*iii*) : The machine is connected up as a d. c. shunt motor. If there is a series field winding it is disconnected during the starting period. When the machine is running as a d. c. motor it is brought to normal speed and then synchronised on the a. c. side like an alternator.

If an inverted rotary runs alone, i. e. not in parallel with other machines, and supplies an a. c. load which has a large lagging power factor, it may run at a dangerously high speed. This is likely to happen owing to the demagnetising effect of wattless component of current in armature. Hence an exciter is used for supplying current to the field windings. Also a speed limiting device is fixed at one end of the shaft. See Fig. 14.

8. **Hunting :** Rotary converters are liable to give hunting trouble. In a synchronous motor there is no commutator and the armature poles rotate at synchronous speed, while in a rotary there is a commutator and the armature poles are stationary under perfect running conditions. If hunting sets in the armature poles oscillate to and fro about their ideal position, setting up e. m. fs. in coils undergoing commutation. This causes sparking at the brushes. Therefore damper winding should be provided on the pole-shoes.

9. **Armature Reaction :** Since a rotary converter carries a "generating" as well as a "motoring" current at all times, the effects of these two currents neutralise each other, particularly when the power factor is unity. Thus there is very little armature reaction. Therefore interpoles provided on a rotary are much smaller than those used on a d. c. machine.

10. **Power Factor :** The power factor of a rotary converter depends upon its field current. But a rotary cannot run at any power factor since the heat loss increases as the power factor departs from unity. Hence a rotary converter should operate as near unity power factor as possible.

11. D. C. Voltage Control: Since the field excitation does not alter the d. c. side voltage of a rotary coverter, methods are adopted to effect some voltage regulation. These methods are

(i) Reactance Control
(ii) Induction Regulator Control and
(iii) Booster Control.

Method (*i*): On account of its simplicity and relative cheapness, the Reactance Control method is widely used though there is some sacrifice in efficiency and increase in the size and cost of the machine.

Reactors or choke coils, having negligible resistance, are placed between the secondary terminals of transformer and the slip-rings, The voltage consumed in the reactor is in quadrature with the current in the slip-rings. Figs. 15 (b) and (c) show the vector diagram for lagging and leading power factor respectively. In the diagram

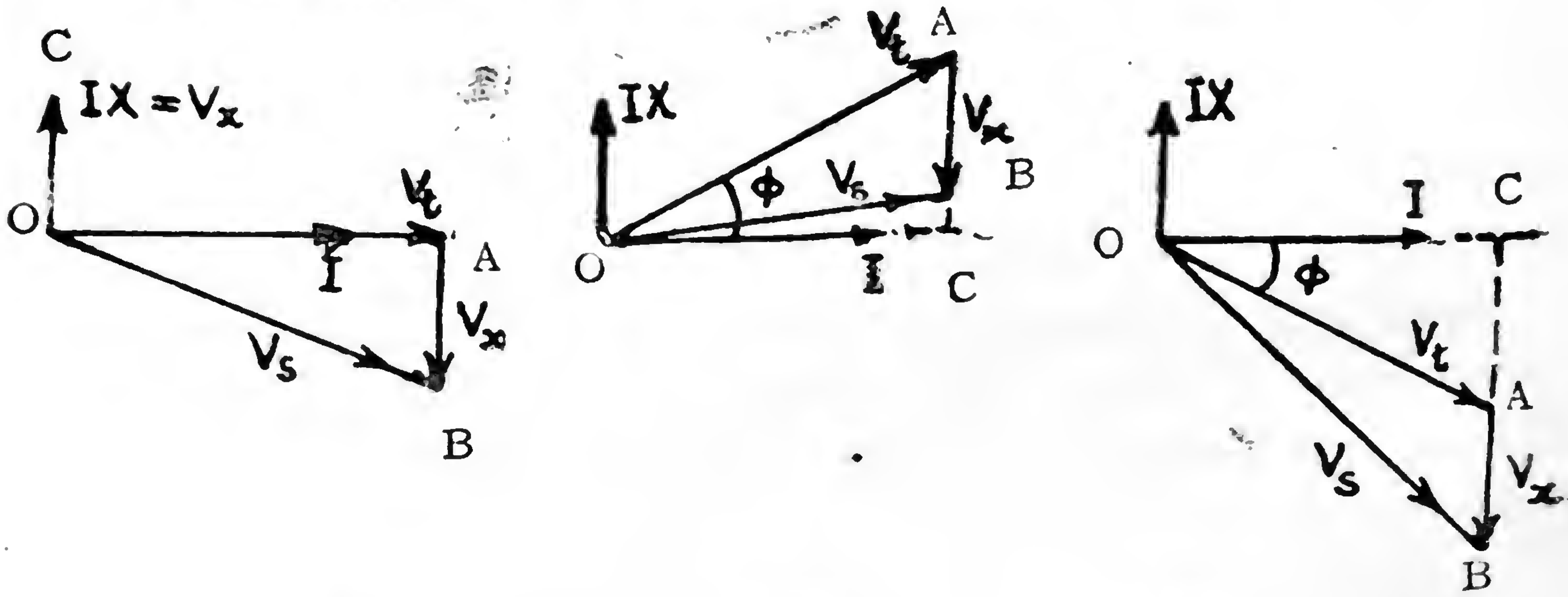
V_t is the voltage at the secondary terminals of the transformer and assumed to be constant.

V_x is the voltage consumed in the reactance. This is the product of the current in the slip-ring line and the reactance of the reactor, and makes an angle of 90° with I.

V_s is the voltage at the slip-rings and is the resultant or the vector sum of V_t and V_x.

I is the current in each slip-ring and

ϕ is the power angle between I and V_s.



(a) (b) (c)
Figs. 15

When the field excitation of the rotary is varied I can be made to lag or lead V_s. This alters the magnitude and the direction of V_x and conequently the magnitude of V_s and the d. c. voltage.

Method (*ii*): A voltage regulation of ± 5 to 10% is possible by using induction regulators. The voltage variation is gradual, very simple to operate but the cost of the equipment is high. Fig. 16 shows the connection diagram of only one phase of a 3-phase induction regulator controlling the voltage of a 6-phase rotary.

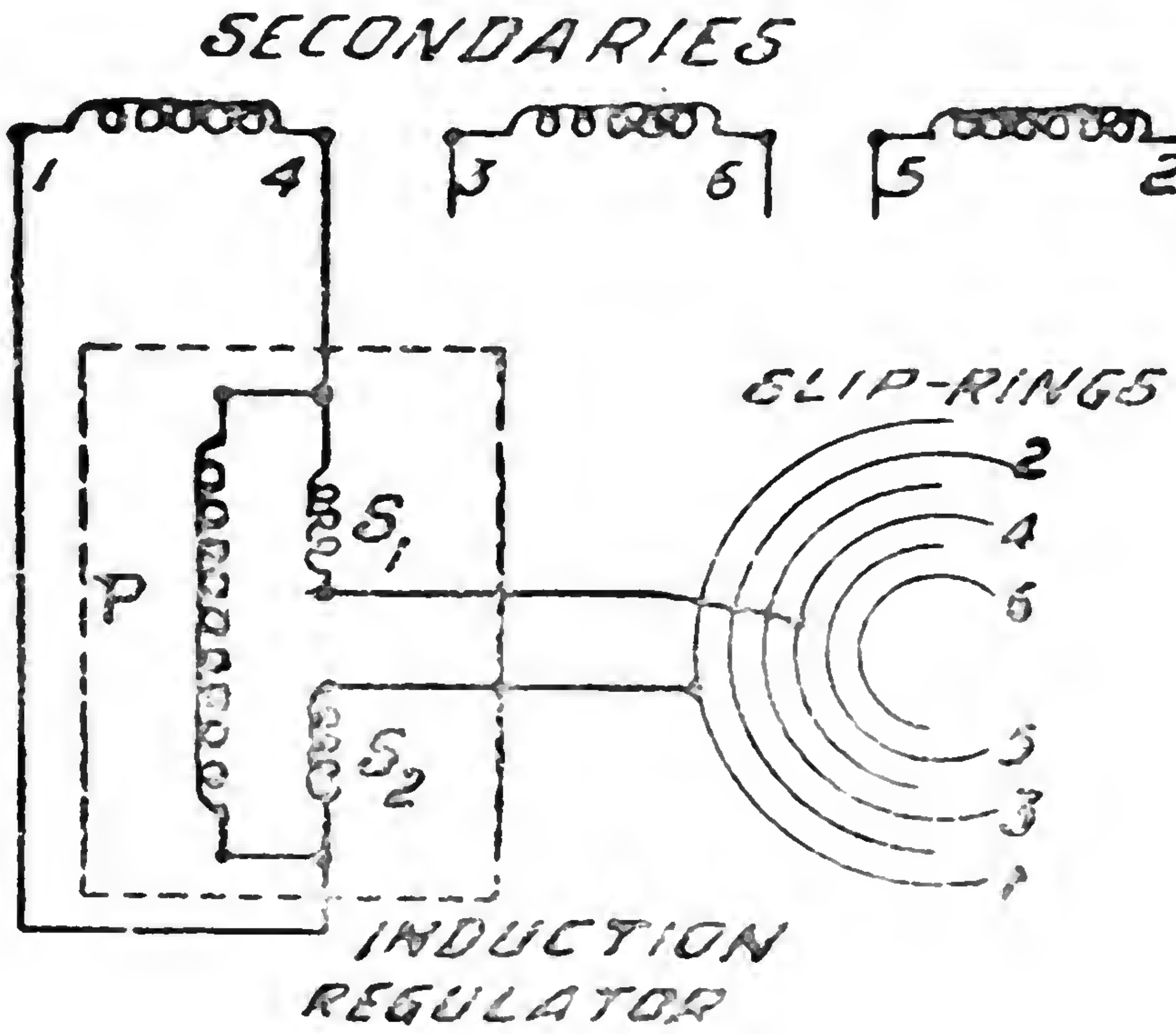


Fig. 16

Method (*iii*): A small low-voltage a. c. generator, having the same number of phases and the same number of poles as the rotary is mounted on the shaft of a rotary converter. Each phase of this a. c. generator is in series with the converter tapping points and the slip-rings, and the field windings are across the d. c. mains. This machine acts as a booster and its voltage is added to the a. c. voltage. This compensates for the natural drop of volts in the rotary. Fig. 17 shows the arrangement.

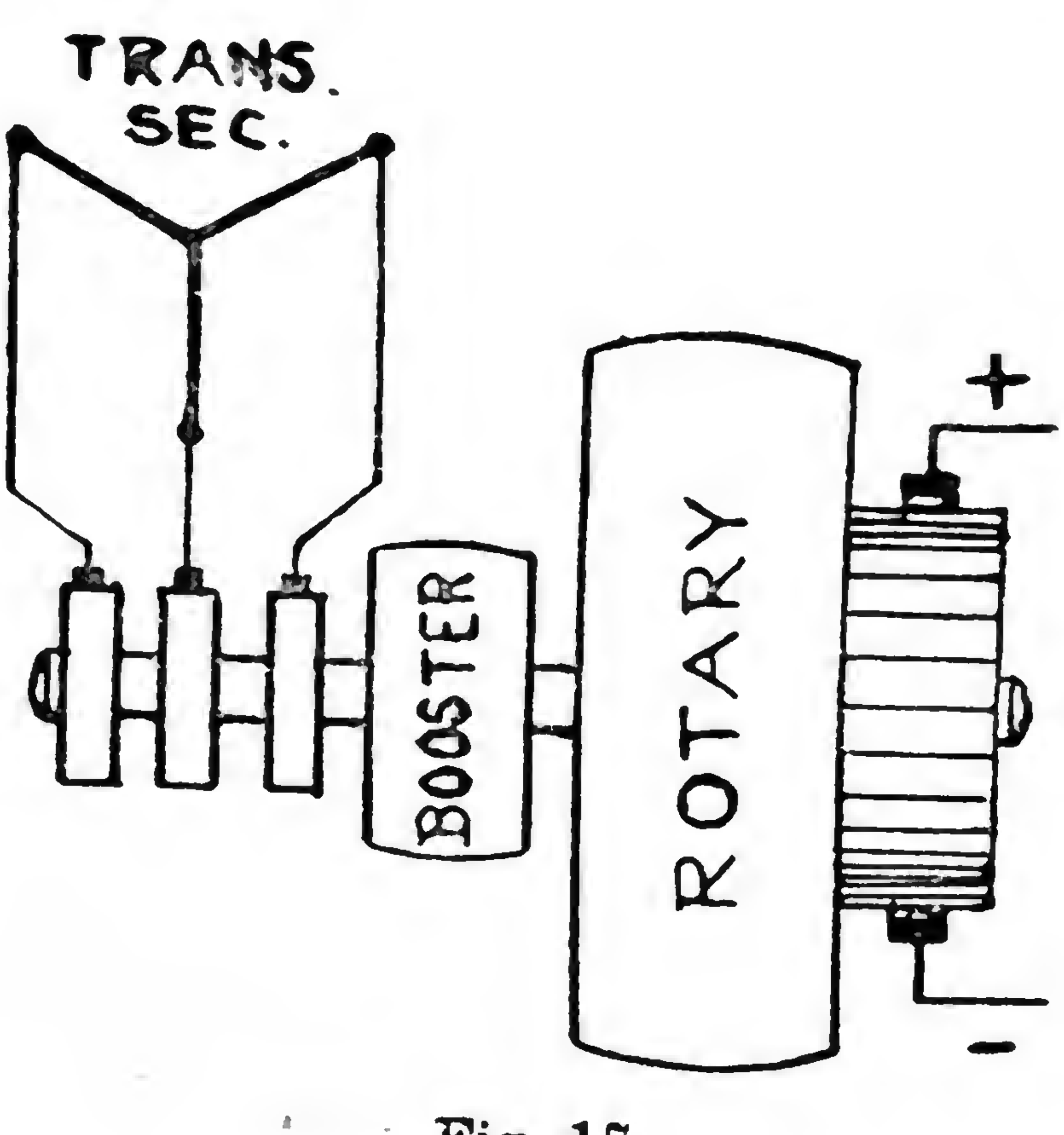


Fig. 17

The chief advantage of this method is that the voltage can be varied independently of power factor, therefore the power factor can be kept near unity. Overcompounding is possible, but this arrangement causes commutation difficulties if the rotary has interpoles. Because the interpole flux may become either too strong or too weak as the d. c. current fluctuates. This can be partially remedied by using diverters across the interpoles during heavy d. c. loads.

Example: A 6-phase rotary converter has a reactance of 0·15 ohm in each slip-ring lead. Calculate the d. c. voltage when the d. c. load is 500 amperes and the power factor is (i) 0·9 lag; (ii) unity and (iii) 0·9 leading. The machine voltage per phase is constant at 353 volts.

Solution:

By Eq. (6), the slip-ring current $I = \frac{2\sqrt{2}}{N} \times I_{dc} \times \frac{1}{\cos\phi}$.

$$\therefore \quad I = \frac{2\sqrt{2}}{6} \times 500 \times \frac{1}{0{\cdot}9} = 262 \text{ A.}$$

One secondary phase of the transformer is connected to two opposite tapping points on the rotary armature. Therefore the drop per phase due to reactance is $2\,IX$ volts.

$$2\,IX = 2 \times 262 \times 0{\cdot}15 = 78{\cdot}6 \text{ volts} = V_x$$

(i) From 15 (b), V_s is the vector difference of V_t and V_x
$V_s = \text{OB}$, $\text{OA} = V_t$ and $\text{AB} = V_x = 78{\cdot}6$; $\text{OC} = V_t \cos\phi$; $\text{BC} = V_t \sin\phi$.

$$\therefore \quad (\text{OB})^2 = (\text{OC})^2 + (\text{BC})^2$$

$$V_s^2 = (V_t \cos\phi)^2 + (V_t \sin\phi - V_x)^2$$

$$\therefore \quad V_s^2 = (353 \times 0{\cdot}9)^2 + (353 \times 0{\cdot}436 - 78{\cdot}6)^2$$

$$\therefore \quad V_s = 326{\cdot}5 \text{ volts.}$$

$$\therefore \quad V_{dc} = \sqrt{2} \times 326{\cdot}5 = 462 \text{ volts.}$$

(iii) When the power factor is leading, we use Fig. 15 (c).

The current is the same namely 262 A. $2IX = 78{\cdot}6$ volts.
$\text{OC} = V_t \cos\phi = (353 \times 0{\cdot}9)$: $\text{CA} = V_t \sin\phi = (353 \times 0{\cdot}436)$;
$\text{AB} = V_x = 78{\cdot}6$ volts.

$$\therefore \quad \text{OB} = V_s = [(\text{OC})^2 + (\text{CA} + \text{AB})^2]^{\frac{1}{2}}$$

$$= [(317{\cdot}7)^2 + (231{\cdot}5)^2]^{\frac{1}{2}} = 393 \text{ volts}$$

$$\therefore \quad V_{dc} = \sqrt{2} \times 393 = 556 \text{ volts.}$$

(ii) As before $I = 262\,A$, $2IX = 78{\cdot}6$ volts. Using the vector diagram of Fig. 15 (a), OB $= V_s$; OA $= V_t$ and AB $= V_x$

$$\therefore \quad V_s = [\,(353)^2 + (78{\cdot}6)^2\,]^{\frac{1}{2}} = 361{\cdot}6 \text{ volts}$$

$$\therefore \quad V_{dc} = \sqrt{2} \times 361{\cdot}6 = 511 \textit{ volts.}$$

12. Inverted Running of Rotary Converter : When a converter gets its power from a d. c. source and delivers a. c. power to a load it is known as an *inverted rotary converter.* If the commutating poles are adjusted for operation as a *straight rotary* only, the commutation will not be quite satisfactory if the machine is run as an inverted rotary. Therefore some compensating device is applied to ensure satisfactory operation when running inverted.

When the machine is operating alone to provide an entirely independent a. c. supply, and consequently is not running in parallel with any generating plant on the a. c. side, the operating conditions are more difficult, such as :—

(a) Voltage variation is not possible by variation of field strength, either on no load or on load.

(b) Voltage variation or voltage control is not possible by the use of reactance in slip-ring leads, since usually the nature of the power factor of the load is lagging.

(c) A wattless current, for a load of lagging power factor flowing in the armature of an inverted rotary, has a direct demagnetising effect on the main field. This results in increased speed of the machine and consequent increase in frequency of a. c. voltage.

Using induction regulators may be found to be most suitable for voltage control, and to check the variation in frequency (or speed) either simple compounding will help or a separate exciter for the shunt field winding which counteract the weaking of the main field strength. The exciter is so designed that a slight rise in its speed will give a large rise in its voltage. This exciter is mounted on the shaft of the rotary.

In conclusion, an inverted rotary is unsuited to supply a. c. power for normal industrial service since complicated additional auxiliary apparatus is necessary. On the other hand a motor-generator set with a synchronous type motor gives every satisfaction for such a duty.

B. Mercury Arc Rectifiers

13. Introductory: In mercury arc rectifiers the positive electrodes are of *iron* or *graphite* and the negative electrode is a *pool of mercury.* The operation of these rectifiers depends upon the behaviour of an electric arc in vacuum between these electrodes, namely that the current passes only from iron to mercury. This is known as a *valve action.* Hence the iron electrodes are the anodes and the mercury pool is the cathode.

Fig. 18. shows a single-phase mercury arc rectifier in its simplest form. There are two iron anodes A and B and at the bottom of the glass bulb the mercury pool, the cathode. The positive of the d. c. side starts from the cathode and the negative terminates at the neutral point N of the secondary winding *ab* of the main transformer. For half the periodic time A is positive so that the current flows from A to the mercury pool. In the other half periodic time B is positive and the current flows from B to the mercury pool. Thus during half cycles the electrons pass from the cathode to that anode which happens to be at a higher potential. This gives full-wave rectification.

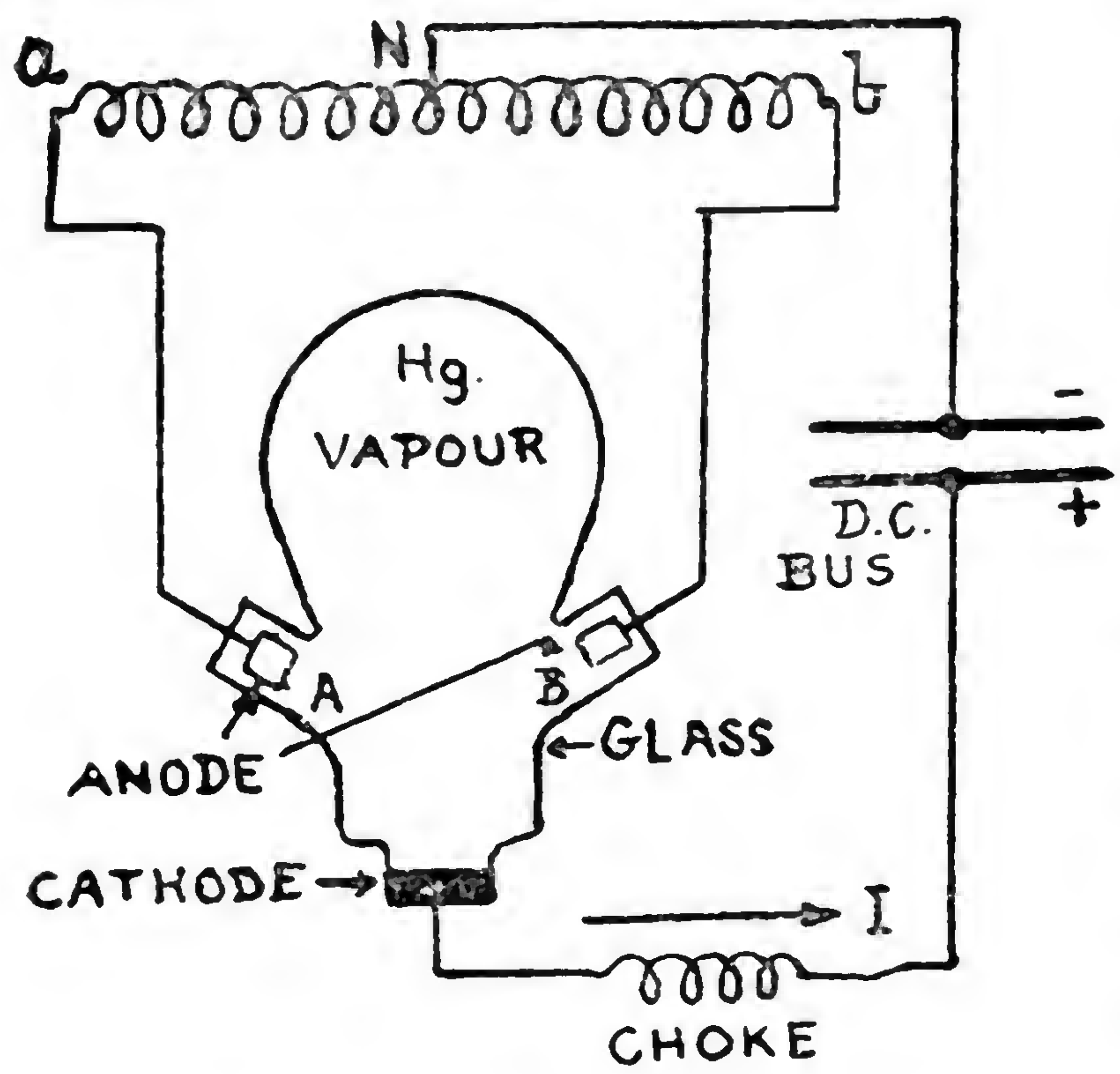
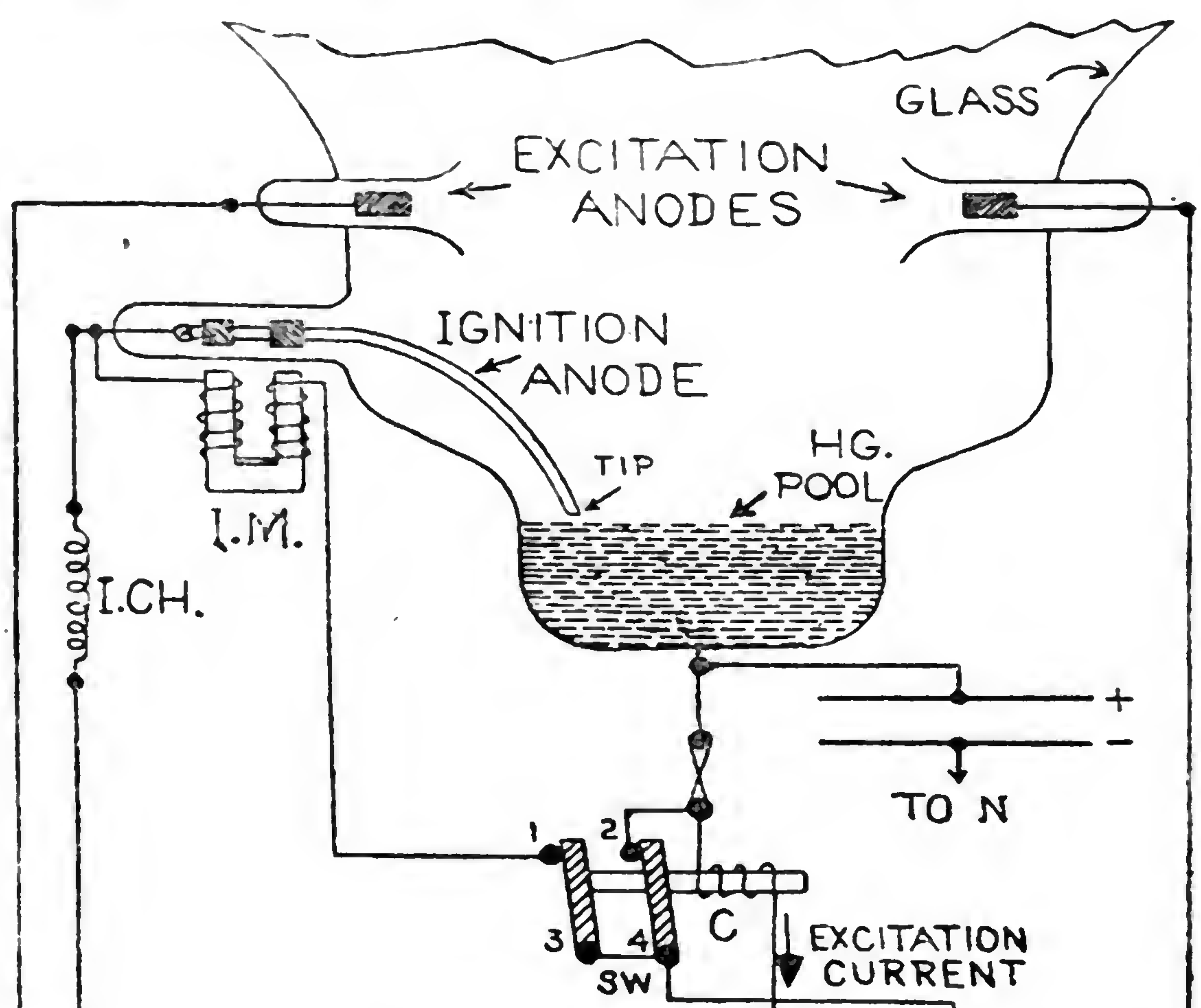


Fig. 18

14. Operation: The arc is initially struck by an *ignition electrode* which is connected to an auxiliary source of supply. A white spot, called the *cathode spot,* appears at the cathode. At the high temperature of this spot electrons are liberated and they travel to the anode at high speed. The electrons collied with mercury vapour atoms and ionise them. The result is that ions (positive) and electrons (negative) are produced. The former travel to the cathode where they deliver the necessary energy to maintain the cathode spot. This energy transfer entails a drop of voltage at the cathode. The arc will continue until the anode voltage falls below the ionisation voltage of mercury (about 10 volts).

The mercury vapour produced by the arc travels to the upper part of the container and condenses there. This is the reason why the glass bulb has a large dome. The liquid mercury after condensing runs back into the pool. Thus there is no wastage. The main anodes are sealed into recepticles of *L*-shaped arms, and at lower levels there are smaller arms for auxiliary anodes—two *excitation anodes* and one *ignition anode.*

Fig. 19 shows the ignition and the excitation circuits which are supplied from an auxiliary transformer. One secondary winding of this transformer supplies the ignition circuit and the other secondary winding, having a centre-tapped connection, supplies the excitation circuit. The ignition magnet *IM* when energised attracts the flexible ignition electrode causing it to dip into the mercury pool. The current rush is limited by the ignition choke *I. CH.* The magnet now gets de-energised and the electrode is released, breaking the current as the



AT—Auxiliary Transformer,
I. CH—Ignition Choke,
I. M—Ignition Magnet,
C—Coil of Relay (to open the ignition circuit.)

Fig. 19 Ignition and Excitation Circuits.

tip surfaces. This causes the arc and the hot spot momentarily. When the initial arc is struck, one of the excitation anodes, which is positive at that instant, takes up the arc. The high reactance of the centre-tapped secondary winding of the auxiliary transformer limits the current and prevents its collapse until the other excitation anode becomes sufficiently positive. The rectified current flows through the operating coil *C*, thus opening the ignition circuit. The main anodes are not shown in Fig. 19.

The only loss in the bulb or the container is a voltage drop, called the *arc drop*. Its value is between 20 to 30 volts and partly depends upon the length of the arc. The bulb efficiency therefore depends entirely upon the operating d. c. voltage. The higher the operating voltage the greater is the bulb efficiency. For instance, if the d. c. voltage is 220 volts and the arc drop 25 volts the bulb efficiency is

$$\% \eta = \frac{220}{220 + 25} \times 100 = 89{\cdot}8\ \%$$

while if the d. c. voltage is 1500 volts the efficiency is

$$\% \eta = \frac{1500}{1500 + 25} \times 100 = 98{\cdot}3\ \%$$

In the case of single-phase rectifiers, if only one anode is provided the rectification is said to be *half-wave*. The d. c. voltage in this case happens to be a series of impulses. When two anodes are provided the result is *full-wave* rectification. Fig. 20 shows half-wave and Fig. 21 shows full-wave rectified voltage. In half-wave rectification the d. c. voltage (and the current) is a series of impulses, each

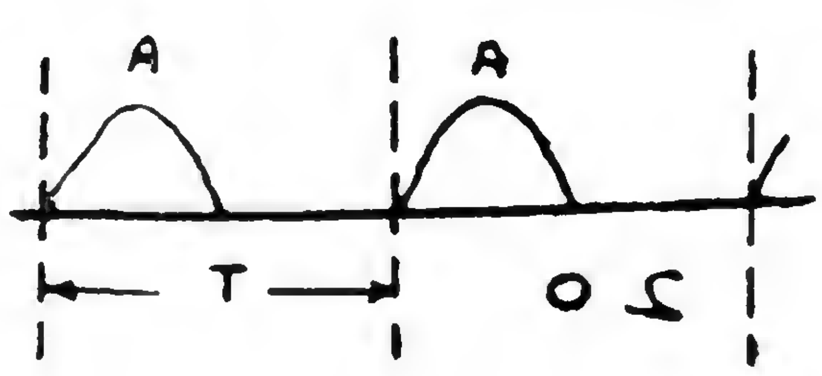


Fig. 20

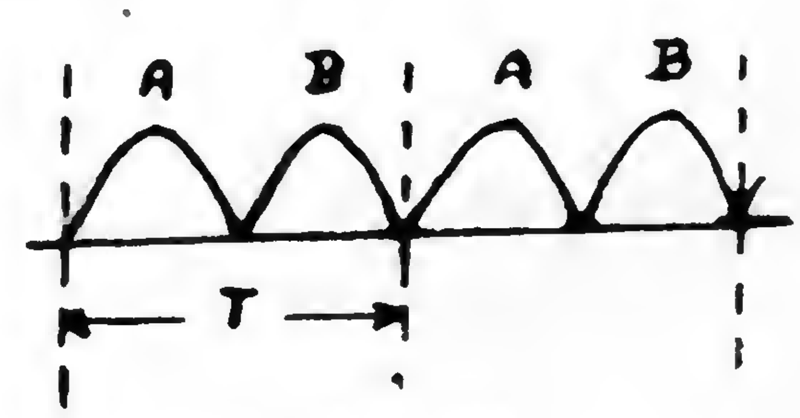


Fig. 21

impulse lasting for half the periodic time (T/2) and the d. c. voltage shape is similar to the a. c. half waves. All the negative half waves of the a. c. voltage are suppressed. In full-wave rectification the negative half waves are rectified by the other anode. So that the d. c. voltage is no more a series of impulses but varies in magnitude from instant to instant and is unidirectional.

In a 3-phase rectifier there are three anodes, each anode is active, i. e. carries the current, for $2\pi/3$ radians or 1/3 periodic time. The arc exists between that anode which happens to be at a higher potential and the cathode. So that there is the usual mutual 3-phase displacement of 120 degrees. The resultant d. c. rectification is shown in Fig. 23 with full lines. Thus a 3-phase rectifier may be considered as a unit of

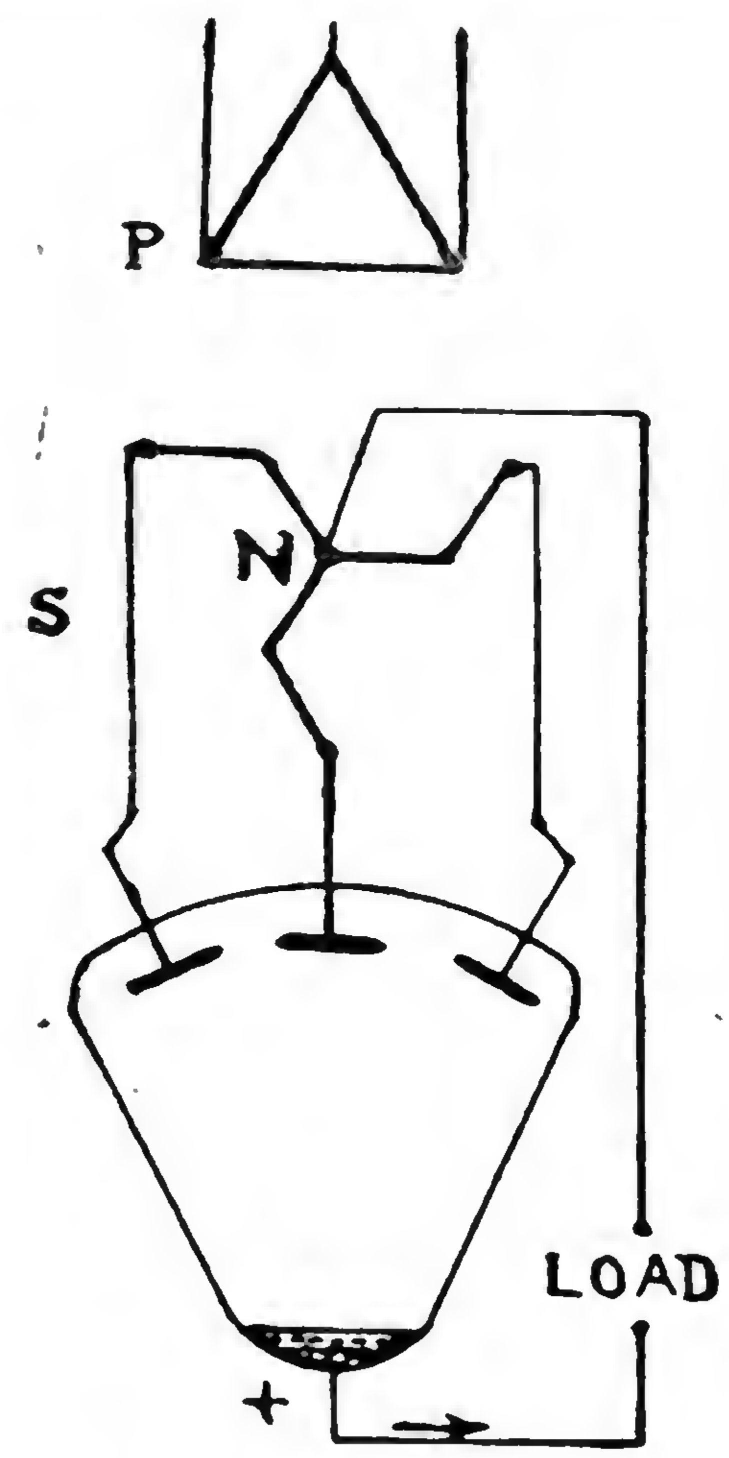


Fig. 22

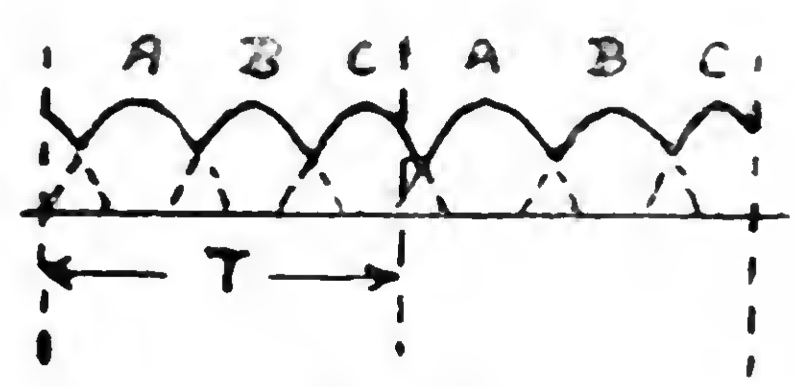


Fig. 23

three half-wave rectifiers. In the case of 3-anode rectifiers the secondary side of the transformer should be connected in zig-zag for reasons mentioned in Section 6 of this Chapter. The connection is shown in Fig. 22.

15. 6-Phase Rectifiers: The transformation from 3-phase to 6-phase supply is done by a number of different types of transformer connections on the secondary side. Fig. 24 shows the resultant d. c. voltage from a 6-anode rectifier. It is a full-wave rectification.

6-phase rectification may be effected by one unit having 6 anodes and one cathode in a container or by six containers each housing one anode and one cathode. Fig. 25 shows the six secondary phases split into two 3-phase star systems I and II displaced 180° relative to one another. The three anodes of each system fire successively, so that at any instant two of the six anodes are carrying the load current, one anode from each group carries half the direct current for a period of

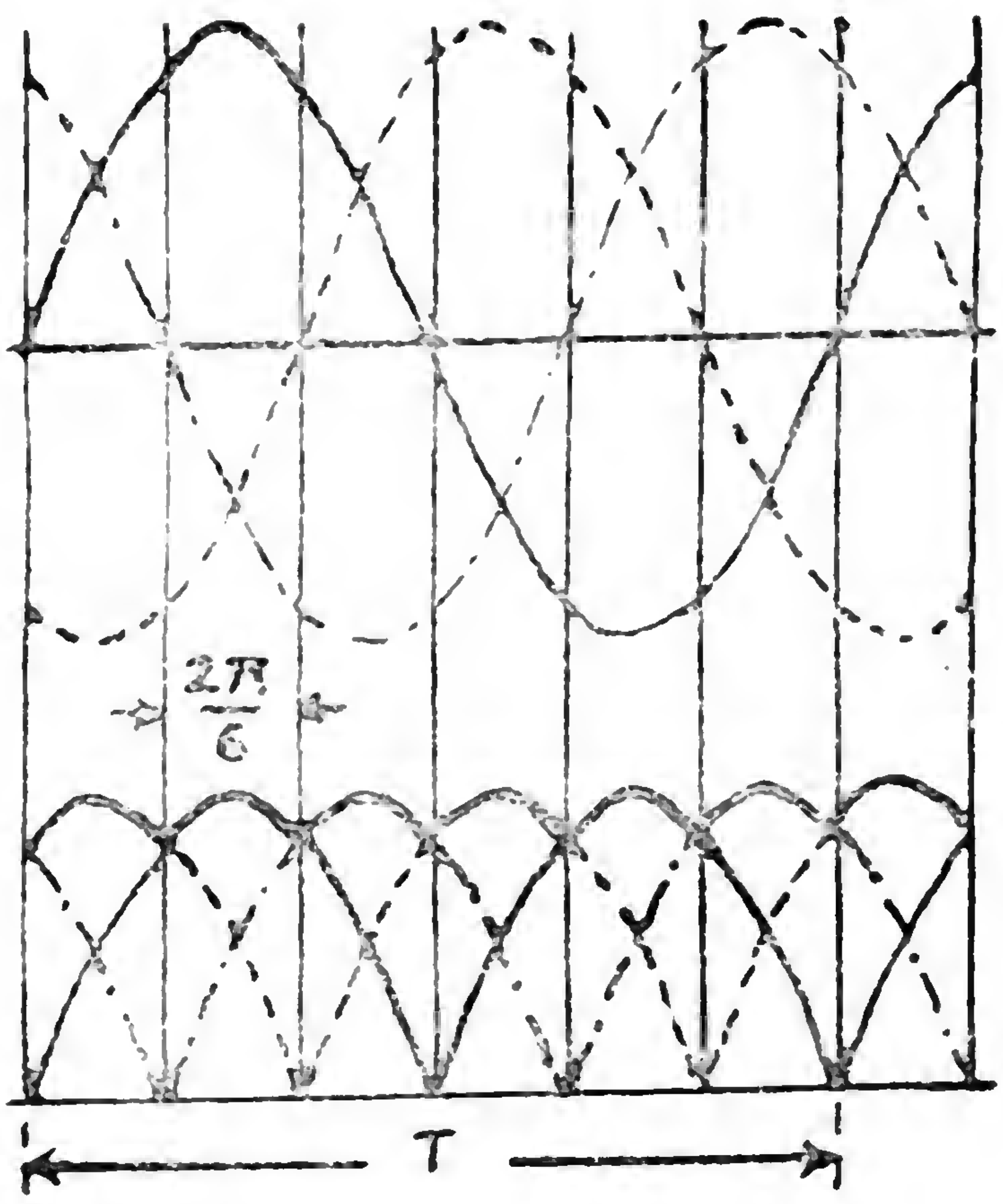


Fig 24

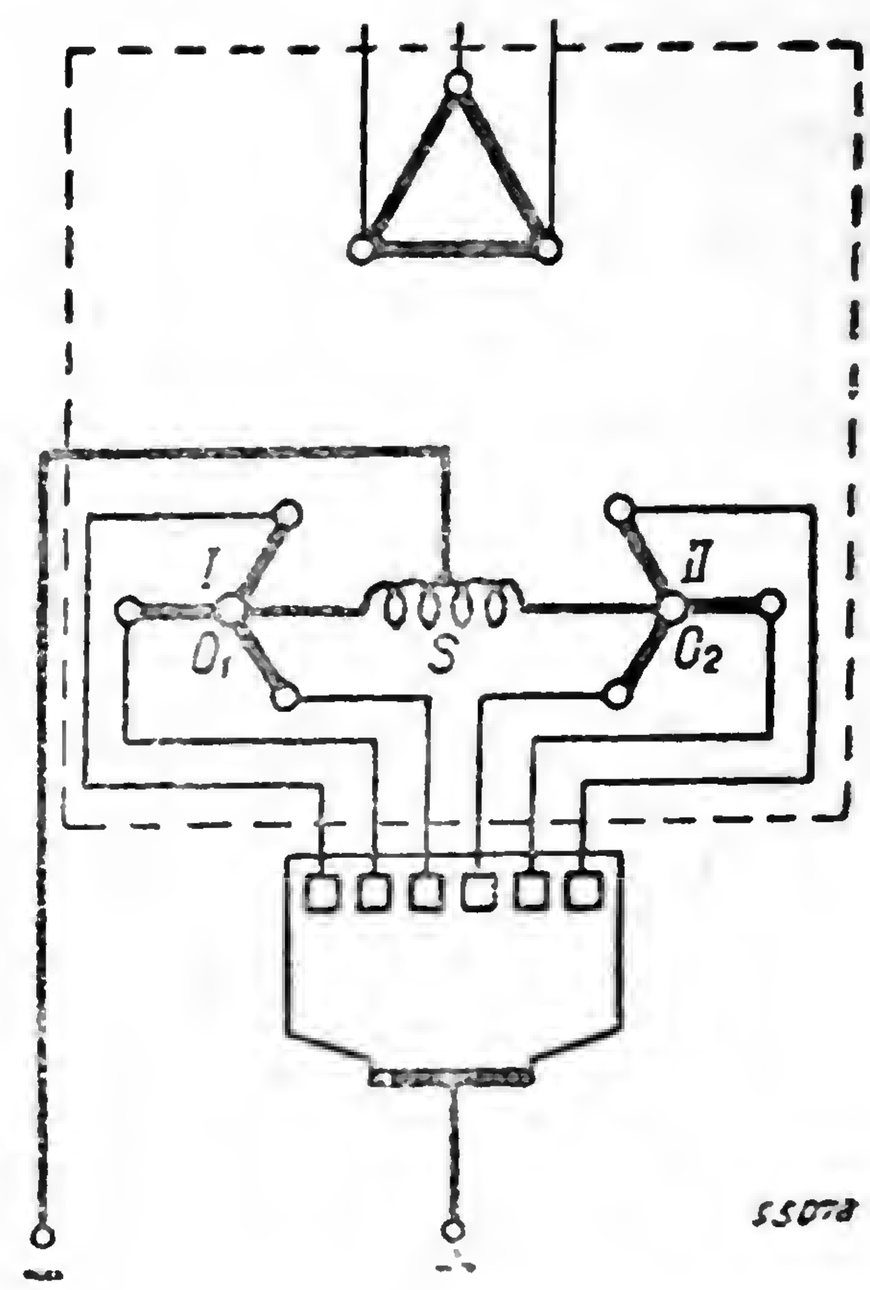


Fig. 25

120°. To maintain equal voltage and current between the two systems an *interphase transformer* is connected as shown in the figure. The d. c. negative is connected to the mid-point **S** of the interphase transformer.

Fig. 26 shows the 6-phase *fork* connection, which is in principle equivalent to a 6-phase zig-zag connection. This type of connection is preferred for small and medium sized plants. The arrangement of the windings is simple and the voltage variation from no load to full load is about 6%. The connection on the primary side may be either delta or star. No interphase transformer is employed and the d. c. negative is at the point **O** shown in the figure.

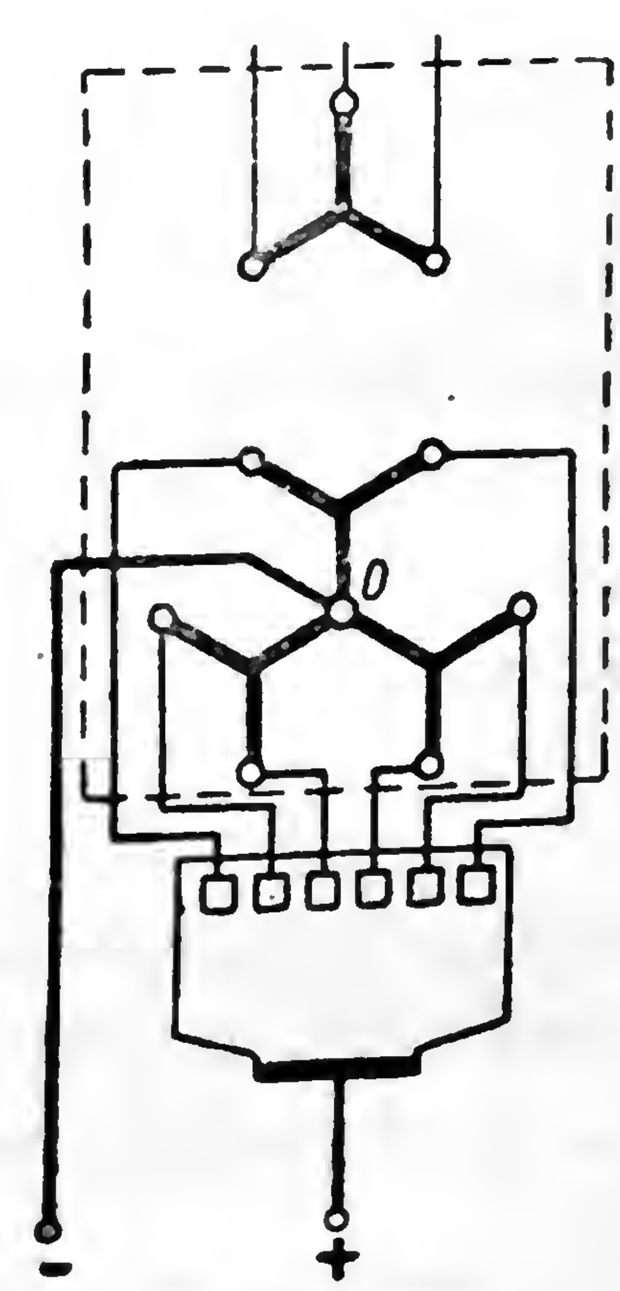


Fig. 26

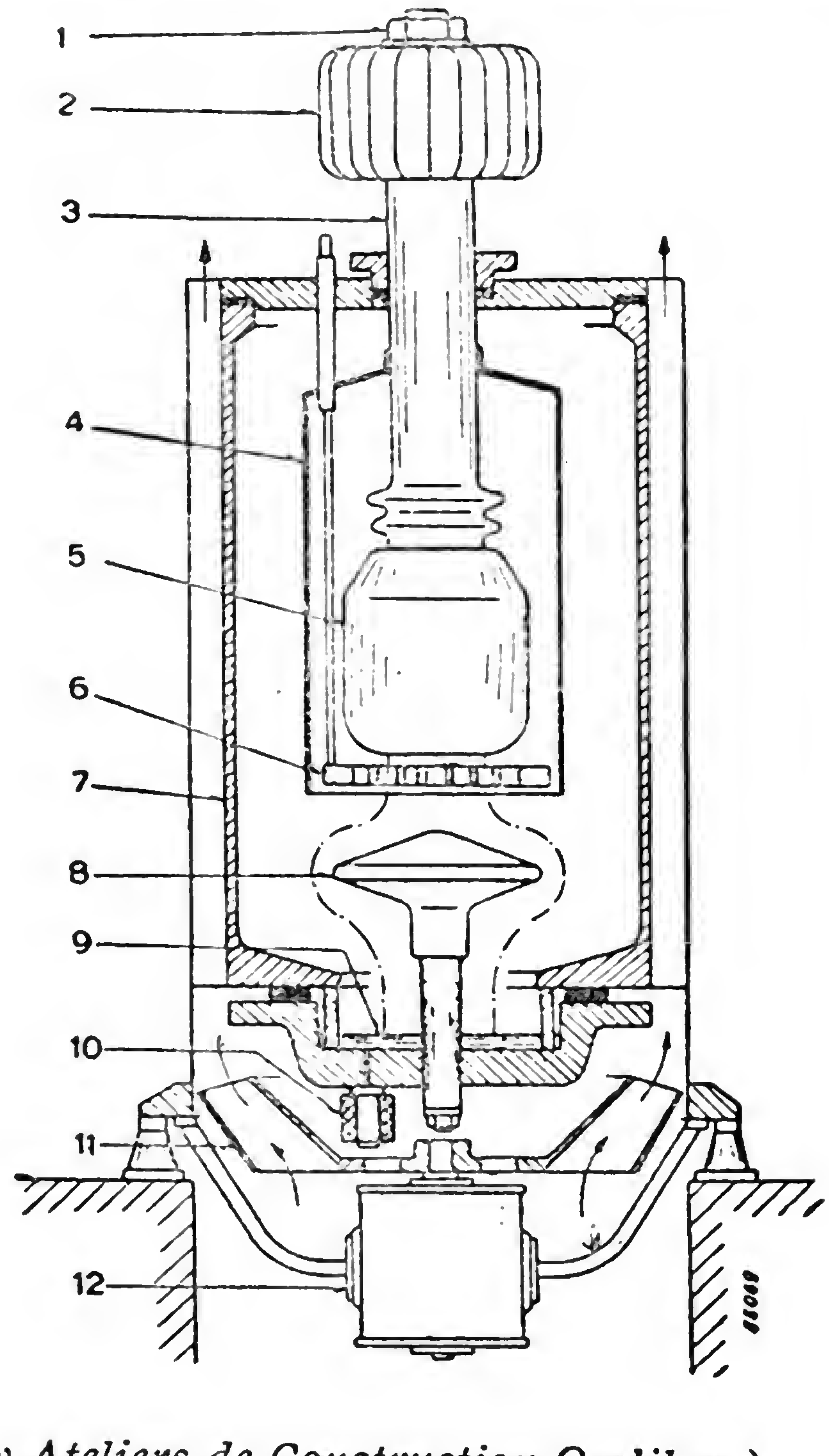
16. Faults and Constructional Features: In order to understand the justification of the presence of each fixture, it is well to enumerate the faults or rather the causes of failure in mercury arc rectifiers. These are listed below:—

(*a*) Improper vacuum; (*b*) condensed mercury on anodes; (*c*) temperature rise above 50° to 60° C; (*d*) short-circuit between anodes and (*e*) possible contact between the arc flame and the anodes.

Rectifiers upto 500 kW capacity have glass containers. Six anodes are used for the larger sizes and three anodes for the smaller ones. The anodes are located in glass tube extensions which are bent at right angles to avoid short-circuit between anodes. The various connecting wires are sealed directly into the glass. These units are fitted with full compliment of *protective devices*, a motor driven *cooling fan* and *thermal realy* for preventing over-loading. Besides these, they are fitted with *ignition* and *excitation anodes* and *grids*. These units if required can be operated in parallel to increase the ampere output.

For rectifiers of larger output, the glass bulb is replaced by a metal container. In this case insulators must be used for carrying the connecting wires into the chamber. But it is not possible to make these passages absolutely air-tight. Hence a *vacuum pump* is

1. Terminal
2. Cooler
3. Bushing
4. Cylinder shield
5. Anode
6. Grid
7. Tank
8. Auxiliary electrode
9. Cathode
10. Ignition device
11. Fan
12. Fan motor.

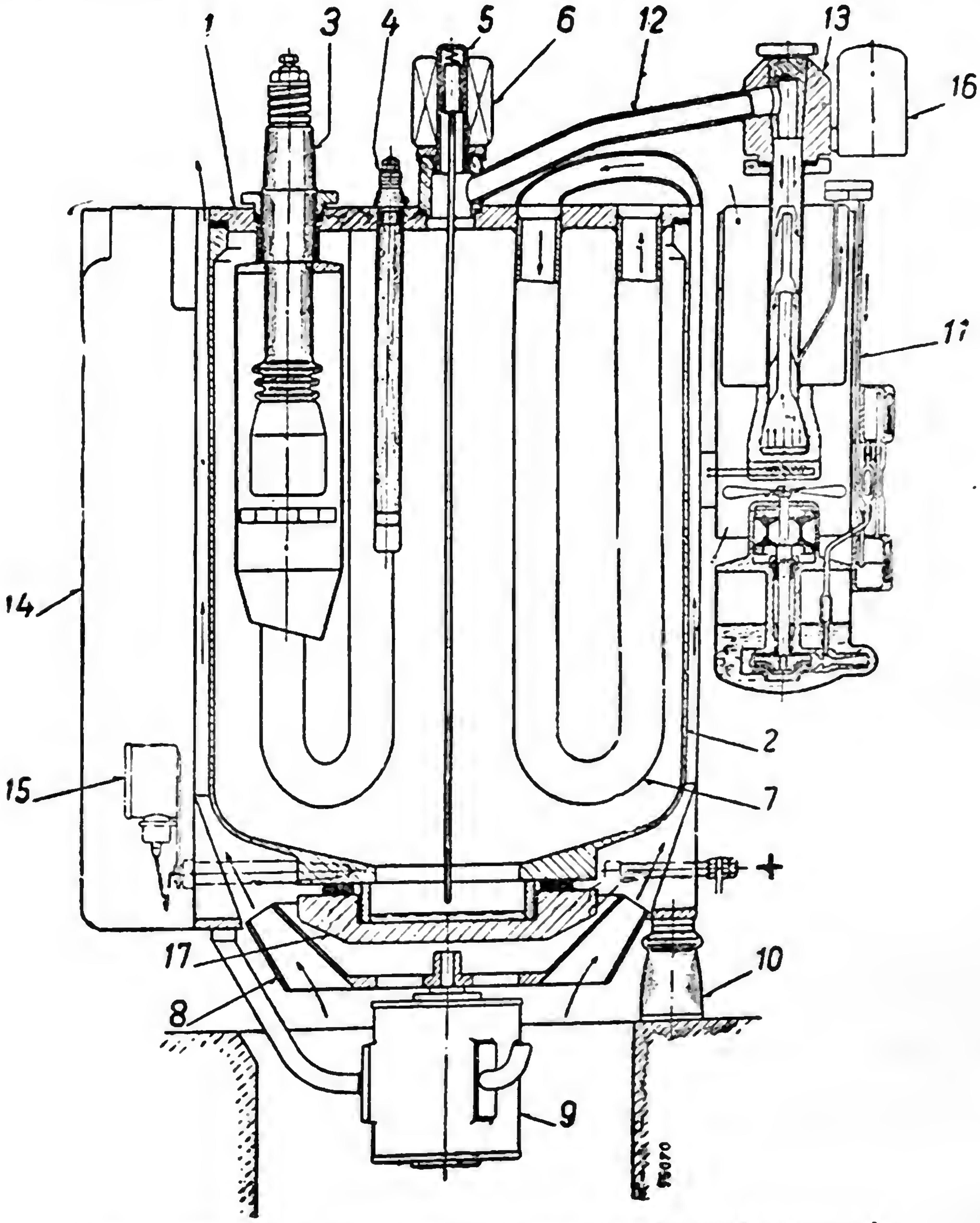


(*By courtesy Ateliers de Construction Oerlikon*)

Fig. 27. Air-cooled Single-anode Rectifier Tank.

necessary. The pump need not operate the whole time, but only for an hour or so during the day. Figs. 27 and 28 show, schematically represented, air-cooled rectifiers, the former is a single-anode type and the latter the multi-anode type.

To avoid increase in temperature of the glass bulb or the metal container, a cooling fan is employed. Where anode currents are heavy, cooling water through pipes is circulated. To avoid short-circuit between anodes they are surrounded by a *cylinder* in the case of metal containers. This cylinder also serves the purpose of avoiding



(By courtesy : Ateliers de Construction Oerlikon)

Fig. 28. Air-cooled Multi-anode Rectifier Tank.

1. Anode plate, 2. Tank, 3. Main anode, 4. Excitation anode, 5. Ignition anode, 6. Ignition coil, 7. Cooling tube. 8. Fan, 9. Fan motor, 10 Supporting insulator, 11. Jet pump, 12. Vacuum pipe, 13. Vacuum valve, 14. Apparatus pillar, 15. Fan thermostat, 16. Vacuum guage, 17. Cathode.

condensation of mercury on the anodes. In the case of multi-anode rectifiers, spring-suspended dipping (ignition) electrode is magnetically operated, which when retracted from the mercury pool starts the initial arc which is subsequently maintained by the excitation or auxiliary anodes.

17. Voltage and Current Ratios: Neglecting the voltage drop in the rectifier, the d. c. output voltage is a true reproduction of a part of a. c. voltage waves. Consequently the d. c. voltage is not steady as that from a battery of cells. It is pulsating in nature and has its maximum and minimum values. This is obvious from Figs. 23 and 24. The more the number of anodes the less is the percentage variation in d. c. voltage.

Fig. 29 shows the three voltage waves ***a***, ***b*** and ***c*** of a 3-phase system. The time reckoning starts from the vertical axis $y\ y'$. The d. c. output voltage is shown by the portion of a. c. waves by thick lines. The arc drop has been ignored for the present. P and Q are two consecutive points of intersection of the a. c. waves and the distance between them is $\frac{2\pi}{m}$ $\left(\frac{2\pi}{3} \text{ in this case}\right)$ radians, where m is the number of phases or anodes. If the equation of the wave ***b*** is

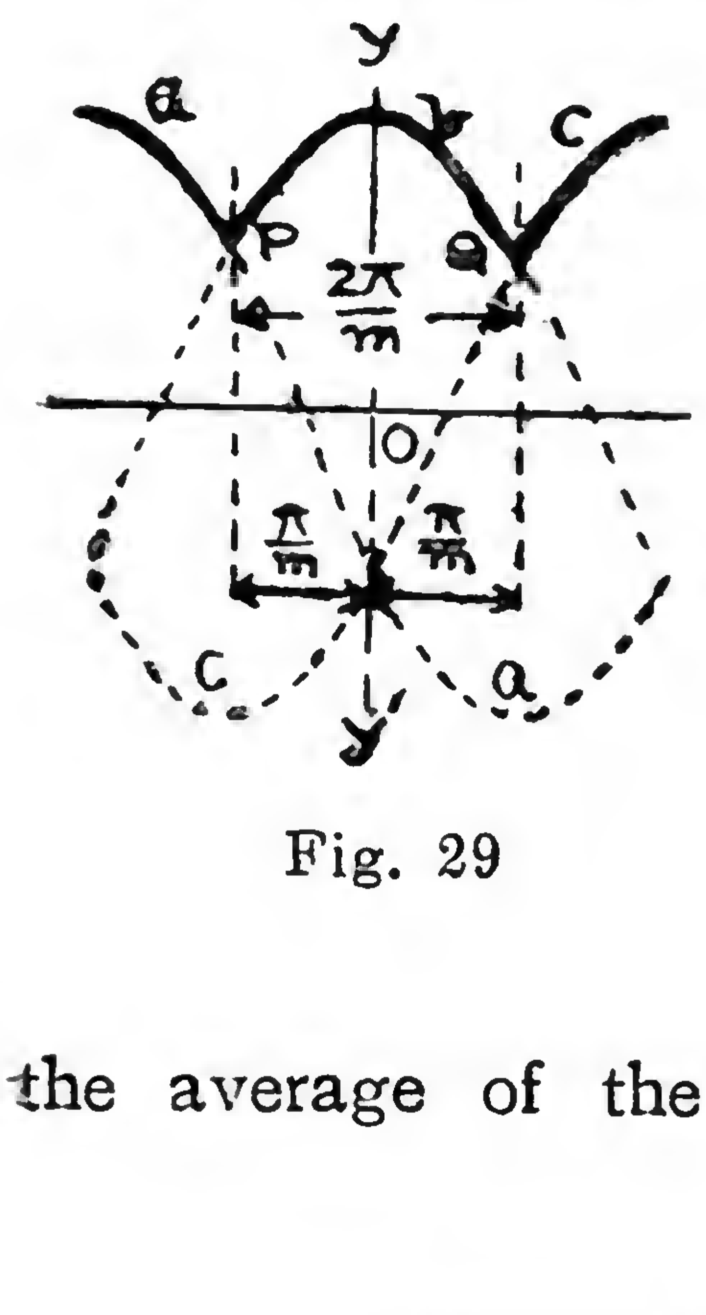


Fig. 29

$$e = \sqrt{2}\, E \cos\theta$$

the average of the portion of this wave between P and Q is

$$\text{average value} = \frac{\sqrt{2}\, E_{ph}}{\frac{2\pi}{m}} \int_{-\frac{\pi}{m}}^{\frac{\pi}{m}} \cos\theta\, d\theta$$

which gives the average value of the d. c. output voltage

$$E_{dc} = \sqrt{2}\, E_{ph} \frac{m}{2\pi} \times 2 \sin\frac{\pi}{m}$$

$$E_{dc} = \sqrt{2}\, E_{ph} \frac{m}{\pi} \sin\frac{\pi}{m} \qquad \ldots \qquad \ldots \qquad \ldots \quad (9)$$

where E_{ph} is the r. m. s. value of the secondary phase voltage.

The actual d. c. voltage is E_{dc} — *arc drop*.

In considering the current relationship, it must be remembered that each anode in turn contributes the d. c. current, so that the current per phase is a series of impules as shown in Fig. 30. Therefore the mean value of current per phase from the figure is

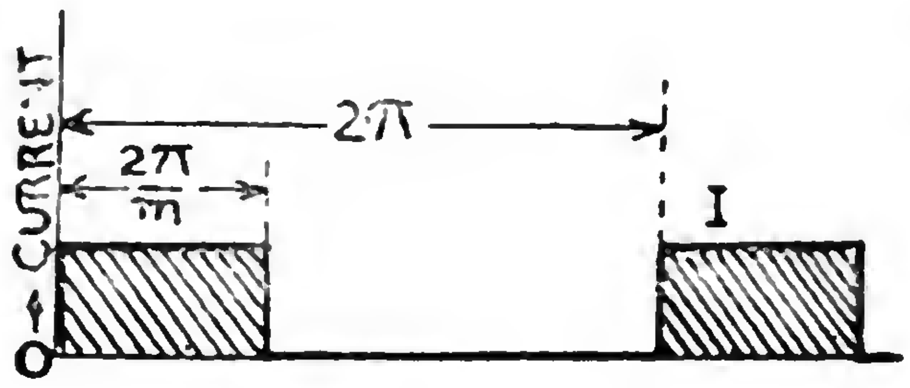


Fig. 30

$$average\ I_{ph} = \frac{I_{dc}}{m} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

and the effective or r. m. s. value of phase current is

$$(r.m.s.)\ I_{ph} = \frac{I_{dc}}{\sqrt{m}} \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

where m is the number of secondary phases and I_{dc} is the value of the rectified current on the d. c. side.

Example: A 3-anode rectifier supplies a d. c. load of 50 kW at 440 volts. The transformer h. v. side, which is delta connected, is supplied by a 3300-volt line. If the arc drop is 25 volts, calculate the bulb efficiency and the turns ratio of the transformer.

Solution: $E_{ph} = (E_{dc} + 25)\frac{\pi}{m} \times \frac{1}{\sqrt{2}\sin\frac{\pi}{m}}$

In this case $m = 3$; $\sin\frac{\pi}{m} = \sin 60° = 0{\cdot}866$; $E_{dc} = 440V$

$$\therefore \quad E_{ph} = (440+25) \times \frac{\pi}{3} \times \frac{1}{\sqrt{2} \times 0{\cdot}866} = 399V$$

The transformer secondary is in star.

Hence the turns ratio $= \left(\frac{\text{h. v. side}}{\text{l. v. side}}\right) = \frac{3300}{399} = 8{\cdot}27.$

The bulb efficiency $= \frac{440}{440+25} \times 100 = 94{\cdot}6\%$

Example: The transformer of the last problem has a no load loss of 1000 watts and copper loss of 1800 watts at 75 amperes. The reactance causes a loss of 500 watts in the rectifier circuit. Calculate the over-all efficiency.

Solution: The d. c. load current $= I_{dc} = \frac{50 \times 1000}{440} = 113{\cdot}6A$

$$\therefore \quad I_{ac} = \frac{113{\cdot}6}{\sqrt{3}} = 65{\cdot}6\ A$$

$$\text{Copper loss at } 65{\cdot}6A = 1800 \times \left(\frac{65{\cdot}6}{75}\right)^2 = 1380\ W$$

$$\text{Bulb loss} = 25 \times 113{\cdot}6 = 2840\ \text{W}$$

$$\text{Total losses} = 500+1000+1380+2840 = 5720\ \text{W}$$

$$\therefore \quad \text{overall efficiency} = \frac{50 \times 1000}{50 \times 1000 + 5720} \times 100 = \mathit{89{\cdot}7\%}$$

18. Methods of Voltage Control: Three different methods are used in mercury arc rectifier plants to vary the voltage. They are

(*a*) altering the anode voltage,

(*b*) shifting the instant of firing of the anode by controlling the grid voltage, and

(*c*) varying the duration of the total firing period of the anodes.

Method (*a*): The anode voltage can be varied by either a regulating transformer (with a tap-changer) or by an induction regulator.

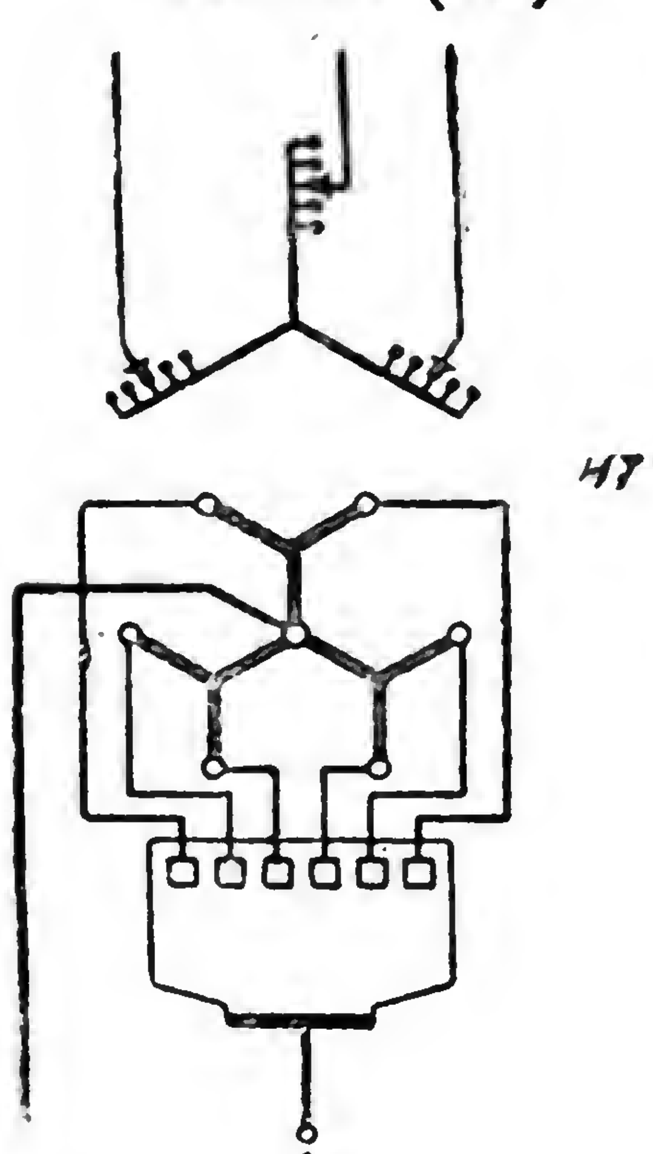
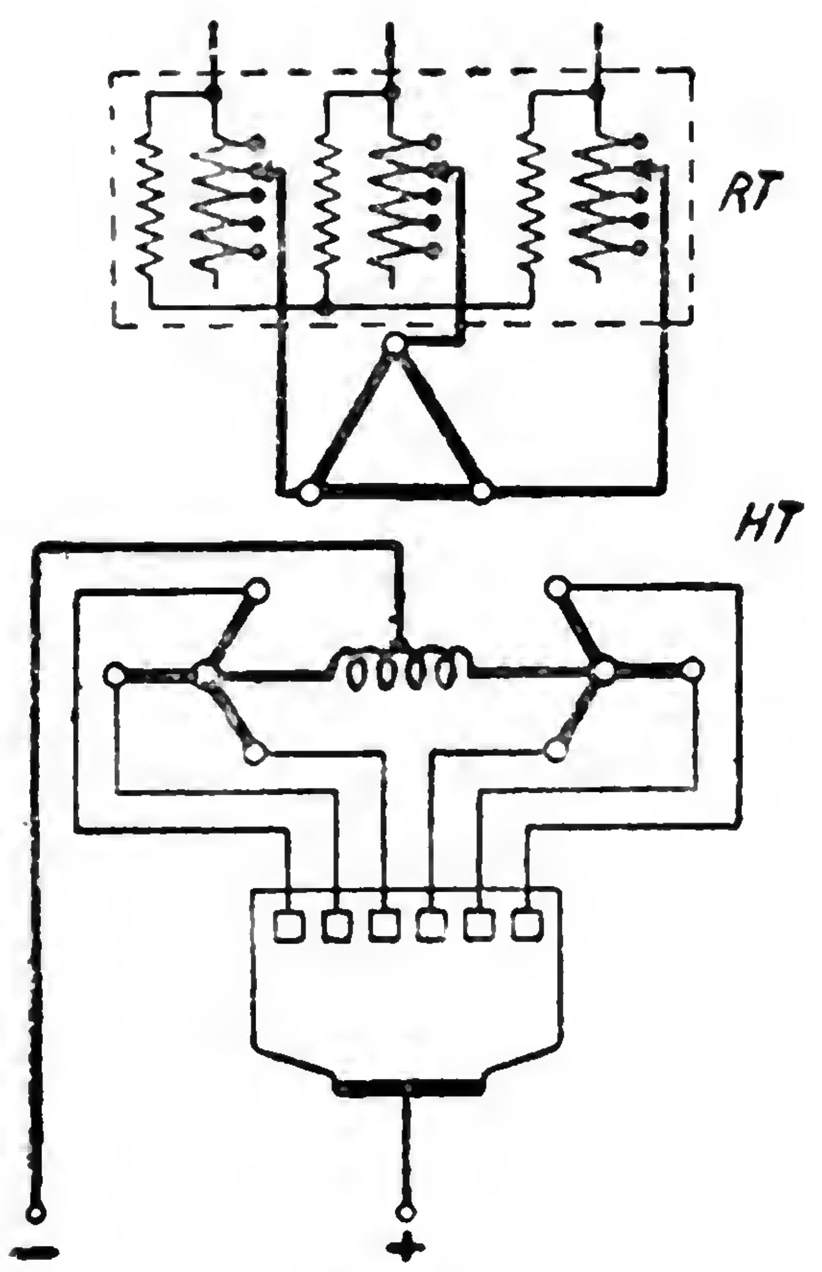
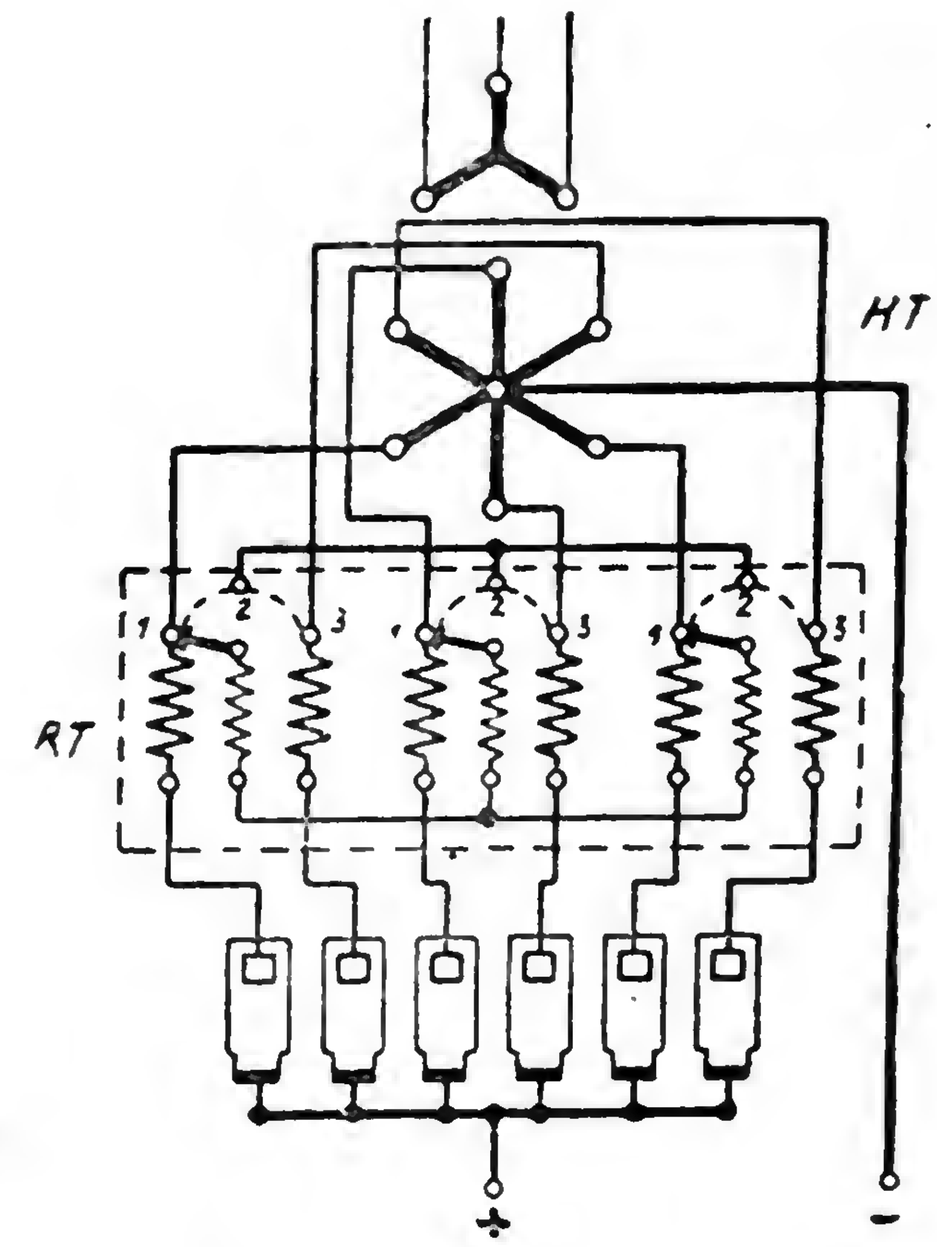


Fig. 31. Taps on Rectifier Transformer

Fig. 31 shows the usual tappings on the primary of the rectifier transformer as used in the case of small capacity plants and where there is no need to use a separate transformer for regulating the voltage. In the case of large capacity plants, the regulating transformer is connected on the primary side of the rectifier transformer as shown in Fig. 32, or if the supply voltage is too high it is connected on the l. v. side of the rectifier transformer as shown in Fig. 33. In the latter case it is operated by a low voltage air-break switch. In *position 1* of the switch the anode voltage is given a boost and in *position 3* the anode voltage is lowered. But in *position 2* there is neither boost not buck because the regulating transformer is inoperative, it being short-circuited. This arrangement is simpler and reliable.

Method (*b*) : Voltage regulation by delaying the firing point is carried out by *grid control.* Since the grid is in the path of electrons





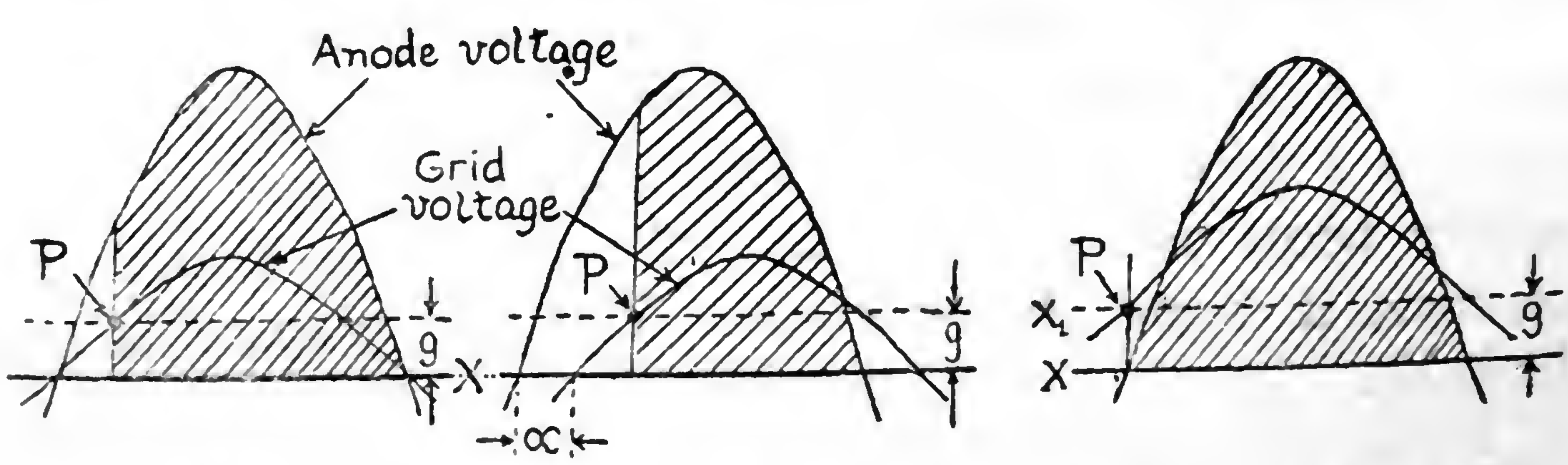
(*By Courtesy : Atelier de Construction Oerlikon*)

Fig. 32. Separate Regulating Transformer with Tap-changer.

Fig. 33. Regulating Transformer on the L. V. Side.

flowing from the mercury pool, the cathode, to anode, the potential of the grid with respect to the cathode plays an important part in the flow of electrons. The action is similar to an electronic triode valve. For instance, if the potential of the grid is positive to the cathode the electrons will be accelerated towards the anode, but if the potential of the grid is lowered, but still positive, the electrons will be slowed down. If the grid potential is made negative the electrons will not reach the anode but will be turned back even before reaching the grid. This is called *blocking*. The supply to the grid is given from a low-voltage transformer. Its frequency must be the same as that of the anode voltage.

In one type of grid control, a small induction regulator is used for giving a phase shift to the grid voltage. The arc is struck at the instant when the grid voltage is equal to the ignition voltage " *g* " of

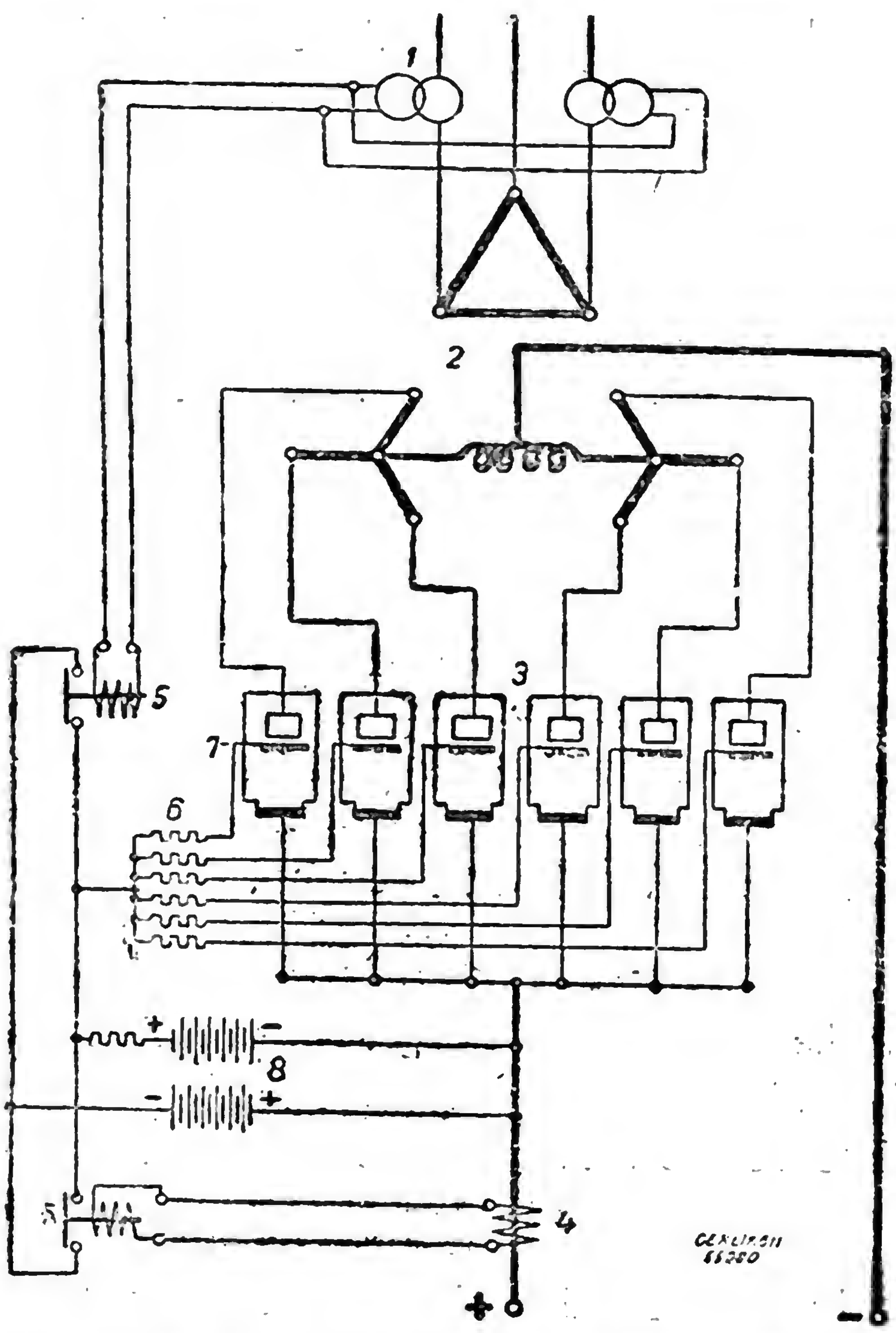


(a) (b) (c)

Fig. 34 Grid Control.

the anode, as shown in Fig. 34 (a), where the grid and anode voltages are in phase. The average d. c. voltage is proportional to the shaded area under the curve. In Fig. 34 (b) the voltage has been given a phase shift of α, the instant of firing is much later in the cycle and the period of firing of the anode in this case is shorter. Therefore the average d. c. voltage is lower than in the first case, as seen by the shaded area.

In another type of grid control, the grid voltage is given a d. c. bias, i. e. a d. c. voltage is superimposed upon the a. c. grid voltage. The firing point P is shifted according to the amount of d. c. bias, and blocking can be effected by giving a negative d. c. bias. In



(By courtesy: Ateliers de Construction Oerlikon.)

Fig. 35. Grid Blocking Device.

1. Current transformer. 2. Rectifier transformer. 3. Rectifier. 4. Impulse current transformer. 5. High speed relay. 6. Grid resistance. 7. Grids 8. Battery for grid control.

this case the axis of reference of the grid voltage can be changed at will but grid and anode voltages are always in phase. Fig. 34 (a) therefore may be considered as that wherein no d. c. bias is given to the grid while Fig. 34 (c) shows a positive d. c. bias given to the grid. The firing point P now occurs at an earlier stage than before, hence the d. c. voltage will be higher. The horizontal axis of reference of the grid voltage is now shifted from X to X_1.

It is not possible to increase the voltage higher than what is shown on Fig. 34 (c) by grid control. Moreover grid control is now used only over a range of 5 to 10 % reduction of voltage, preferably values of 0 to 5 % are provided, combined with a tap-changer.

Blocking of the anodes using grid control has become general as a protection against d. c. short-circuit. Using a high speed relay which operates on the occurrence of a short-circuit whithin 2–3 thousandths of a second, the control grids are connected to a negative voltage of a few hundred volts relative to the cathode whereby the anodes are prevented from firing once they become extinguished and the short-circuit is cleared in about $\frac{1}{50}$ second. The connection diagram is shown in Fig. 35.

Method (*c*): A further possibility of voltage regulation results from the fact that with increased duration of the firing period of the anodes, at constant anode voltage, the average d. c. voltage falls. The variation of the duration of the firing period can be effected by a suitable change-over from one type of transformer connection to another giving a longer or shorter firing period. These connections have already been shown in Figs. 25 and 26.

17. Hot Cathod Rectifiers: The thermionic valve, in its simplest form, the diode, is a hot cathod rectifier. The cathod is a filament wire heated by passing a current through it and the anode is a disc of graphite for low ampere work or a metal disc for large currents.

The glass bulb is filled with an inert gas, usually argon at low pressure for low output, and for larger output, the evacuated bulb contains a very small quantity of mercury. The action of emission of electrons is similar to that in a mercury arc rectifier. Full wave rectification is possible by using two units with a centre-tapped transformer. The voltage drop in the bulb is about 12 to 15 volts.

The hot cathod rectifiers have been developed for outputs upto 250 A and for 7000 volts or more. The future development of this type may be such as to render it suitable for reasonably large capacities at normal commercial voltages. The "Tungar" rectifier unit of this type is largly used for charging car batteries.

18. The Copper Oxide Rectifier: This is a dry contact type of rectifier. This rectifier is built up from a large number of relatively small capacity units. For high voltage and low working current the units are connected in series. For heavy current and low voltage work they are connected in parallel. Or these units can be connected in series-parallel to supply d. c. power at desired voltages and currents.

The valve action depends on the fact that a thin layer of cuprous oxide on a sheet of copper permits the electrons to pass only from copper to oxide. The action is atomic the reversed voltage required to break down the film of oxide is about 9 volts. One element consist of copper discs, lead washers and cooling fins, separated by steel spacers mounted on an insulated metal spindle.

Full wave rectification from a single-phase source is obtainable by connecting 4 rectifier units in the form of a bridge arrangement. The rectifier efficiency is about 75%. By suitably arranging these units there is no limit, so to say, to the voltage obtainable or the output currents available. For instance, for electroplating work these units will give several thousand amperes at 4 to 6 volts or for *X*-ray work a few milliamperes at several hundred thousand volts.

CHAPTER XIV

ILLUMINATION

1. Types of Electric Lamps : There are three ways of producing light by electricity for general illumination. They are

(i) By an arc between two electrodes.

(ii) By raising the temperature of a thin filament of wire placed in vacuum or in a gas.

(iii) By causing a discharge through a gas or a vapour between two electrodes, placed at the two ends of a tube.

(*a*) *The Carbon Arc Lamp* consists of two round rods of carbon placed across a d. c. supply as shown in Fig. 1. When the two rods are brought together and then separated an arc is established between the two rods. The positive rod *A*, as it is consumed, forms a crater of intense brightness at its tip, while the tip of *B* assumes a tapered shape and is not so bright as the other tip.

If the supply voltage is 220 volts a stabilising resistance *SR* is necessary, since the voltage across the arc is about 55 volts. When the supply voltage is alternating both tips assume a tapered shape, but the brightness is much less than that of the crater. The flame of the arc contributes only 5% of the total illumination. The flame can be made intensely luminous by *coring* the positive rod with a mixture of mineral salt and carbon. The lamp is then called a *flame arc lamp.*

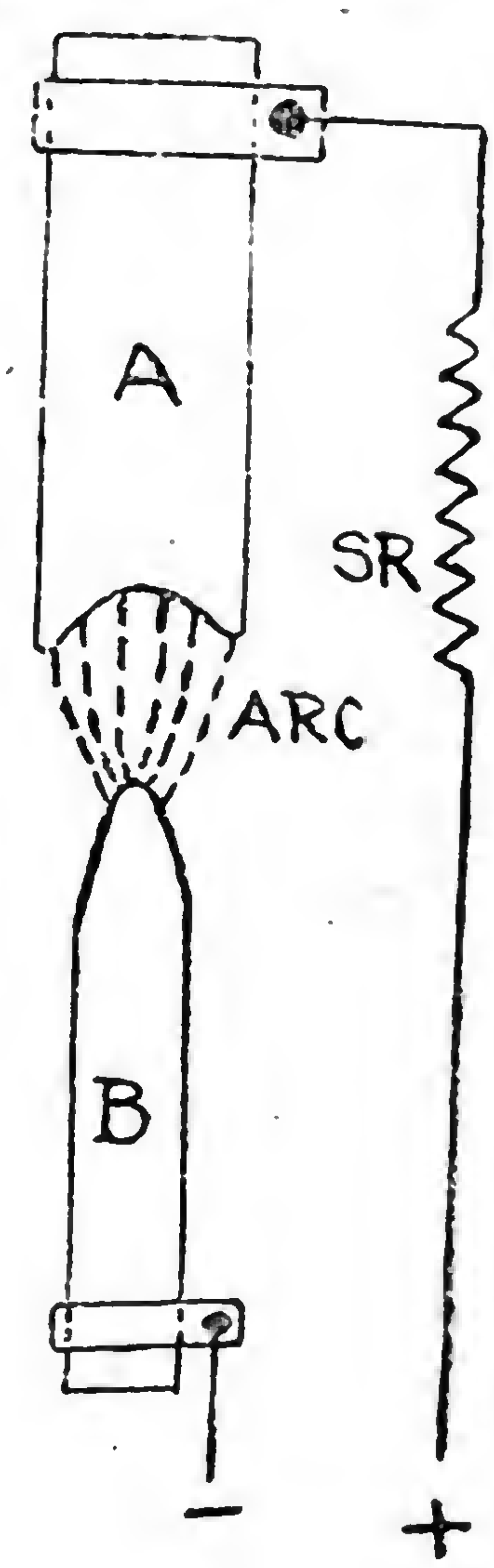


Fig. 1

These lamps were used formerly for street lighting. Their chief disadvantages are that they are bulky, their maintenance is costly and the renewals of rods are too frequent.

(*b*) *Carbon Filament Lamps* have a filament of carbon placed in an evacuated bulb. The first lamp to be produced and marketed

was of this type. Its working temperature must not exceed 1800° C, otherwise the carbon evaporates very rapidly and blackness the glass. Its resistance at working temperature is 2/5 of its resistance when cold. This is due to carbon having negative coefficient of temperature. This lamp is very inefficient and is no more used for general illumination. These lamps may be used as pure resistance load or in places where humidity is harmful, e. g. banks of relay instruments in a telephone exchange.

(*c*) *The Tungsten Filament Lamp* is more efficient, because tungsten is the most suitable metal for lamp filaments. The metal can be worked at 2000° C when in vacuum, but if an inert gas is introduced in the bulb and the filament is *coil-coiled* it can be worked at about 2500° C. This reduces loss by convection and greatly increases the lamp efficiency. The gas used is either argon or nitrogen. The life of the lamp is about 1000 hours. Coil-coiled filament is used in bulbs whose rating is more than 40 watts.

Where intense illumination is required, such as in photography and cinema projection work, the coil-coiled filament is worked at much higher temperatures, but the life span of the lamp is reduced to 100 hours.

All filament lamps are very susceptible to voltage variations, i. e.

$$\textit{candle power} \propto V^n$$

where V is the applied voltage and the value of n for tungsten lies between 4 and 5, while for carbon it is between 6 and 7. Thus if the applied voltage is more than the rated voltage of the lamp, the life of the lamp reduces considerably. On the other hand if the applied voltage is less the luminous power of the lamp is very greatly reduced and the percentage reduction in power consumption is very small. Hence to safeguard the public from financial loss, the Government compels all Power Supply Companies not to exceed ± 6% variation from the "declared" voltage.

(*d*) *The Neon Lamp* is a gas-discharge lamp. Its two electrodes are placed close together so that the lamp can operate on low voltages, i. e. 110 volts a. c. or 150 volts d. c. The bulb contains neon gas, and the pressure inside is reduced to 10 mm of mercury. For a. c. work the two electrodes are of equal size, but for d. c. operation, the negative electrode is made larger, since in d. c. only the negative electrode glows. The electrodes are usually flat plates. These lamps are used where only a very small amount of light is

required. The power consumption of these lamps is very low (about 5 watts) and some indicator lamps consume only ½ watt.

(e) *The Mercury Vapour Lamp* is also a gas-discharge lamp. Fig. 2 (a) shows the general arrangement in which the inner tube contains argon gas at low pressure and a small quantity of mercury. At each end of the tube is a main electrode and an auxiliary electrode near one of them. There is also a stabilizing resistance in series with the auxiliary electrode.

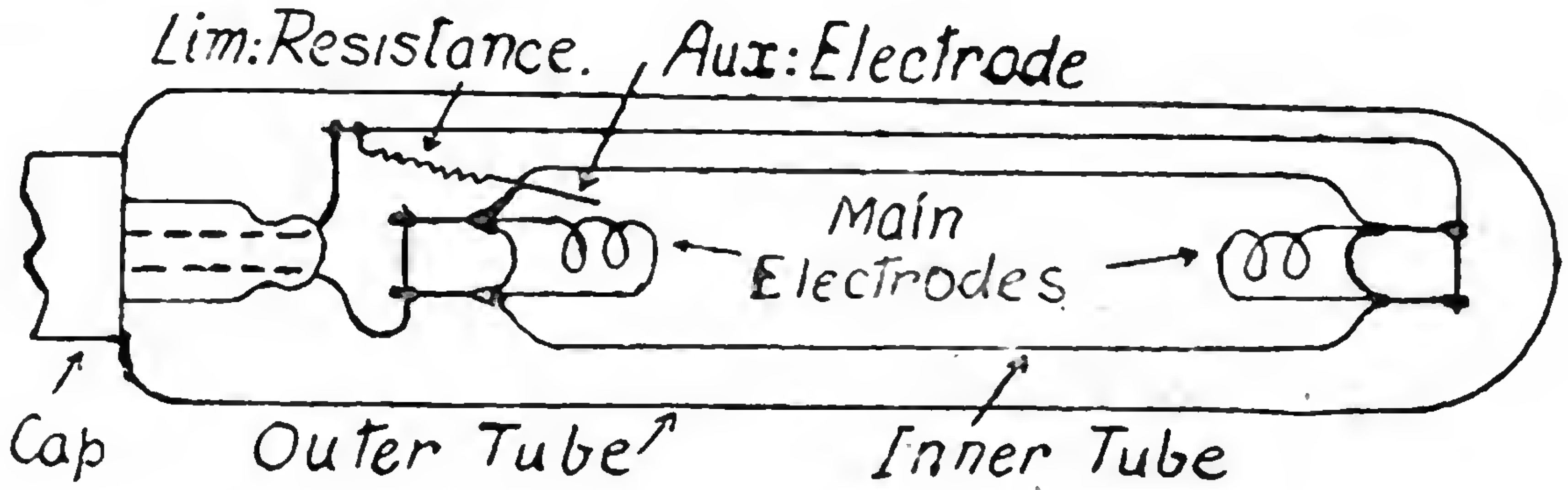


Fig. 2 (a). Mercury Vapour Lamp.
(By Courtesy of Siemens Brothers & Co. Ltd.)

On starting, say at 230 volts, an initial discharge takes place between the auxiliary electrode and the main electrode near it through the argon gas. This voltage of 230 volts is not sufficient to start any discharge between the two main electrodes. The heat from the initial discharge vapourises the mercury and this starts the flow of electrons between the two main electrodes. In the initial stages the illumination is poor, but when all the mercury has been vaporised and as the flow of electrons increases to full strength in about 7 or 8 minutes, the lamp begins to give its full luminous output. The average life of the lamp is 1000 hours. These lamps are used for street lighting and for other open air places. Fig. 2 (b) shows the connection diagram for the lamp. The choke is for stabilizing purpose and the condenser is for correcting the power factor.

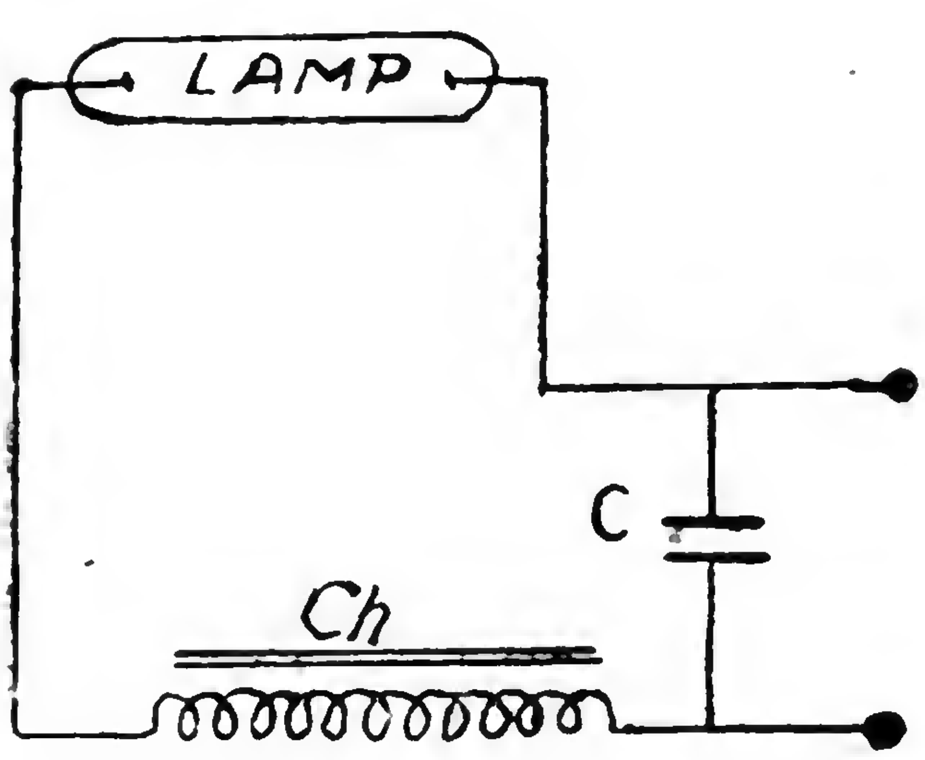
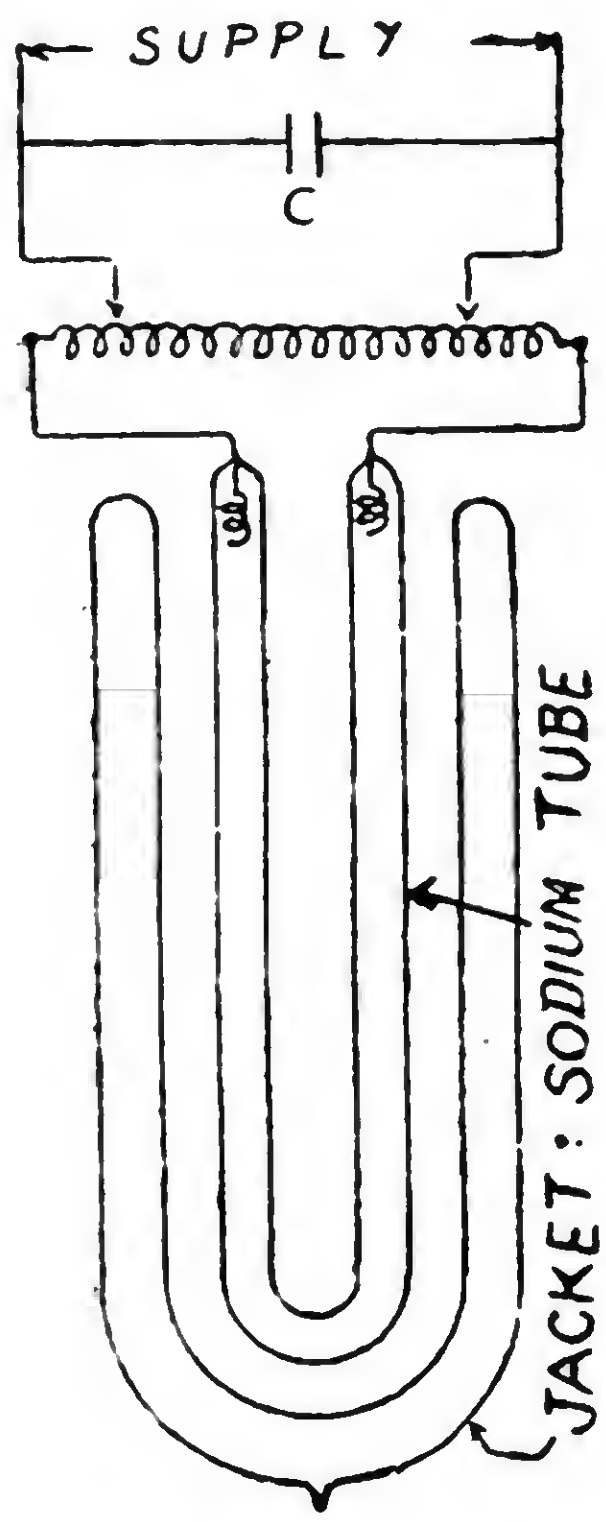


Fig. 2 (b)

(f) *The Sodium Vapour Lamp* is the third type of gas-discharge lamps. A special quality of glass is used for making the inner tube which is bent to an U-shape. The tube contains neon gas and a small quantity of sodium. This is a low vapour-pressure lamp

and therefore it has low intensity. Hence to get more illumination from the lamp the tube must be very long. But the lamp is very sensative to changes of temperature in the tube. Hence the inner tube must be enclosed in a double-walled jacket like a vacuum flask. But this jacket cannot be manufactured in such lengths, hence the inner tube is *U*-shaped. The neon gas is for the purpose of starting the initial discharge which gives sufficient heat to vapourise solid sodium, hence the initial colour is pink.

Voltage required for the initial discharge is between 410 and 480 volts, depending upon the design. This high voltage is obtained from an auto-transformer having high leakage reactance. After the initial discharge, the electrons start flowing between the two electrodes, but it takes nearly 15 minutes before the lamp gives its full luminous output when the colour of the light is yellow.



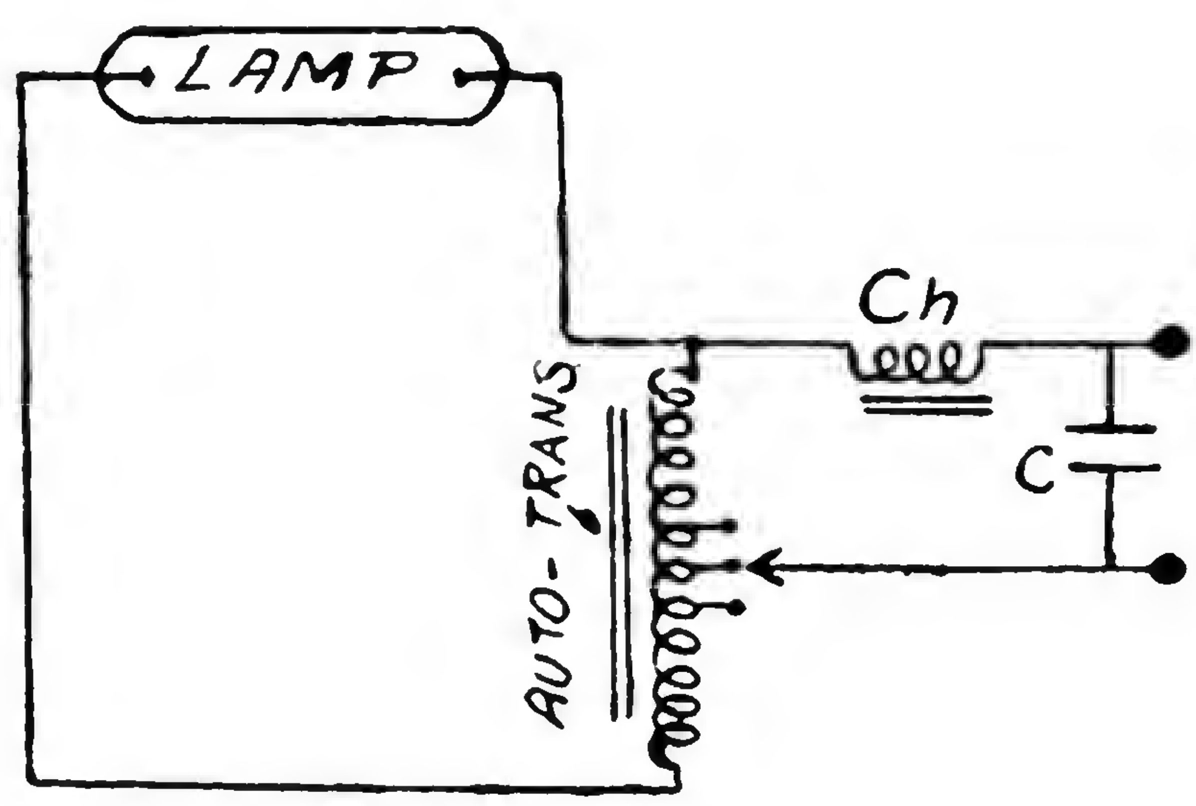


Fig. 3 (b)

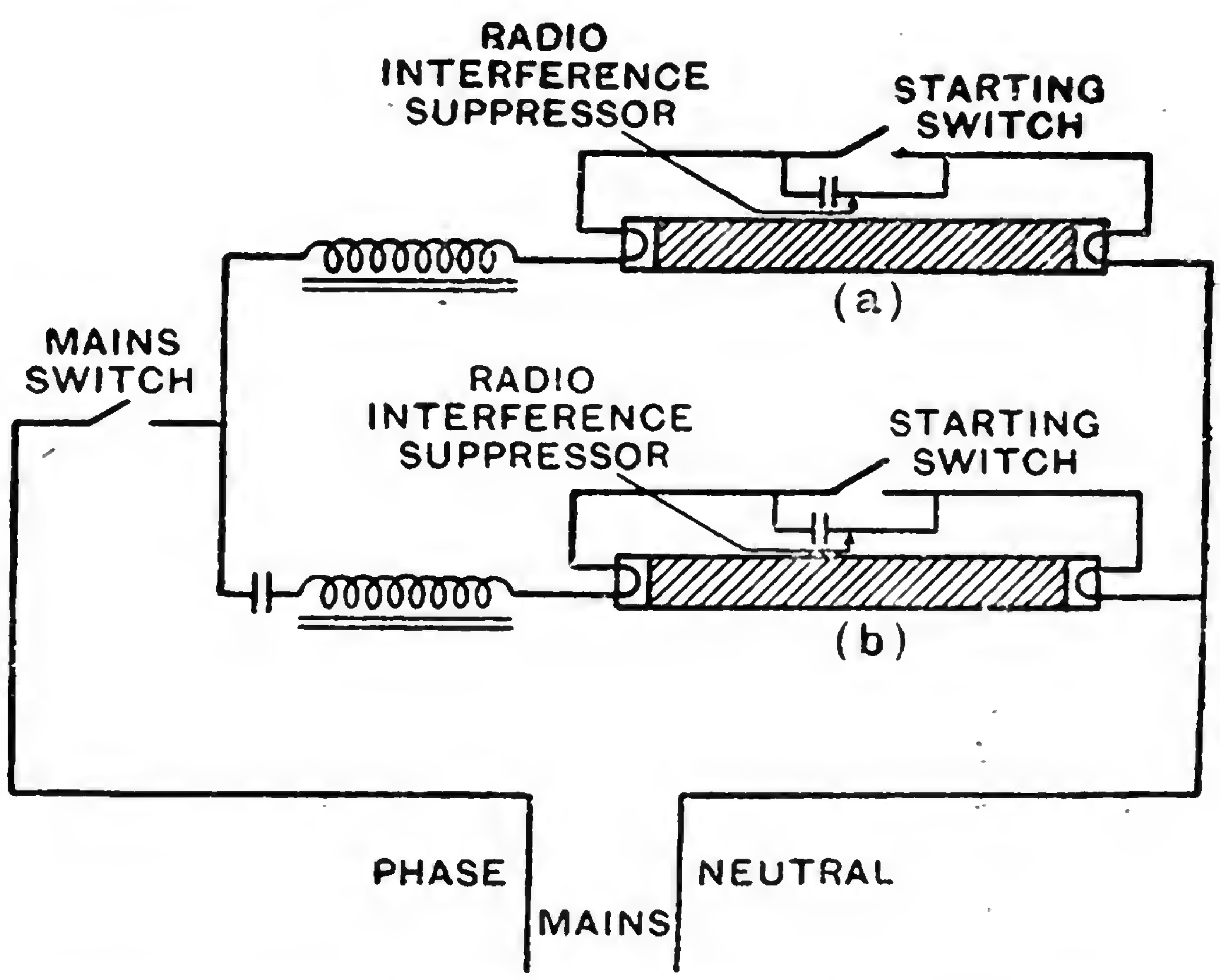
Fig. 3 (a) *(By Courtesy of Siemens Brothers & Co., Ltd.)*

The life of these lamps is about 3000 hours and they are mostly used for outdoor lighting. Fig. 3 (a) shows the lamp construction and Fig. 3 (b) the connection diagram. The choke is necessary as a stabilizer and the condenser for correcting the power factor.

(g) ***The Fluorescent Lamp*** has been developed from the mercury vapour lamp. This lamp is in the form of a long tube, 4 to 5 feet in length, contains a globule of mercury and argon gas at a

very low pressure. At each end of the tube there is a coil-coiled filmament of tungsten which is coated with oxides. On an a. c. supply each end acts alternately as anode and cathcde. There is also at each end a metal plate which acts as anode to facilitate the passage of electrons. This is necessary because the filaments, which are coated with an oxide, do not readily act as anodes.

The whole length of the glass tube is coated from inside with a fluorescent material, i. e. a *phosphor*, which has a property of converting radiant energy from one wave-length to another of lower wave-length. Each phosphor re-emits energy giving a particular colour of its own. Thus it is possible to get a wide range of colours by using a suitable phosphor or a mixture of phosphors.



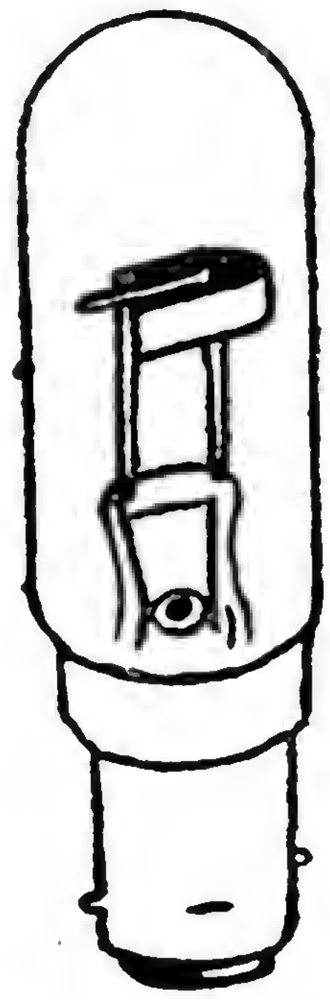
(*By Courtesy of The General Electric Co., Ltd.*)

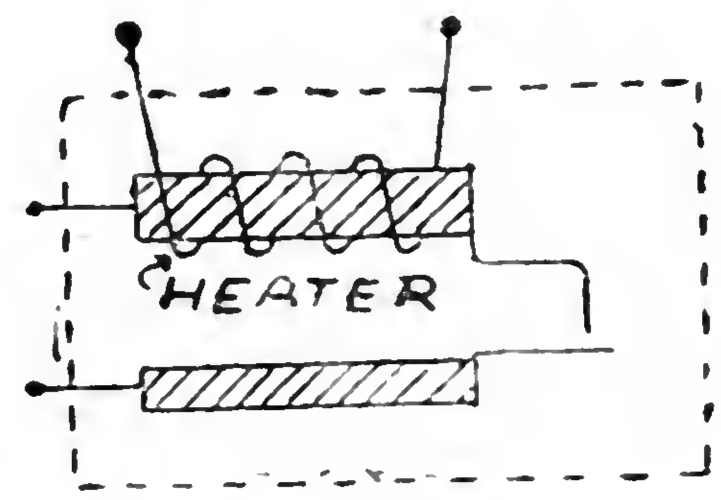
Fig. 4 Circuit Diagram for Twin Tubes

These lamps operate on 200 to 250 volts. Fig. 4 shows a diagram of twin lamp circuit. Lamp (a) is inductively or choke controlled, while lamp (b) is capacitively controlled. A choke is essential in series with each tube. A special *starter switch* is also necessary for each tube. There are two types of starter switches :— (i) the *glow discharge type*, Fig. 5 (a) and (ii) *thermal type*, Fig. 5 (b).

Owing to the very low pressure in the tube, the initial discharge cannot start by switching on the supply in the absence of a starter switch. A high voltage impulse is necessary to break down the long gap between the two lamp electrodes. This high voltage (about 1000

volts) is obtained by an interruption of current in the highly inductive circuit and is caused by the action of the starter switch.





(a) Fig. 5 (b)

These tubes are available in the following units and colours :—

	Colour		Wattage
I.	Daylight	...	80 W, 40 W.
II.	Warm White and Mellow		80; 40; 30; 20 and 15 W.
III.	Natural		125; 80; 40; 30; 20 and 15 W.
IV.	Red	...	80 W.
V.	Blue	...	80 W.
VI.	Green	...	80 W.
VII.	Yellow	...	80 W.

The 80 W tube is 5 ft. long and is the most popular unit in use, therefore, its performance figures are given below :—

Voltage	...200/250 V;	Life	... 2500 hrs;
Current	...0·8 A;	Power factor	... 0·45;
Brightness	...0·51 lm/cm²;	Output	... 35 lm/W.

2. Visual Range : There are various sources which give out radiant energy in wave-lengths ranging from less than 10^{-12} cm. to more than 10 kilometres. Radiant energy having wave-lengths between $3{\cdot}8 \times 10^{-5}$ cm. and $7{\cdot}6 \times 10^{-5}$ cm. produce visual sensation on the human eye. This is light and this range is called visual spectrum.

To express the wave-lengths in a convenient form, two units are usually used, either the *micron* or the *angstrom.*

$$1 \text{ micron} = 10^{-4} \text{ cm.}$$
$$1 \text{ angstrom} = 10^{-8} \text{ cm.}$$

The symbol used for micron is μ and for angstrom is Å. The human eye is able to detect the change in wave-length by noticing the change in colour. For instance, at a wave-length of 4000 Å the colour is violet, at 4700 Å it is blue and so on.

3. Standards of Luminous Intensity (I) : (i) A candle, made of spermaceti wax weighing 6 to the pound and burning at the rate of 120 grains per hour, has in a *horizontal direction* a luminous intensity of 1 candle power (c. p.).

(ii) The Harcourt Pentane lamp has a luminous intensity of 10 candle power.

(iii) The Heffner lamp has a luminous intensity of 1 candle power. This lamp burns amyl-acetate at a wick.

But none of these or others are convenient for use in an ordinary laboratory. For experimental work the best practice is to have a sub-standard filament lamp that has been certified by a Standards Laboratory.

4. Luminous Flux (F) : This is defined as the quantity of light emitted by a source and it is measured in *lumens.*

The Lumen is the luminous flux emitted by a source of 1 candle power in a unit solid angle. Symbol used is *lm.*

Hence if a point source of 1 candle power is placed in the centre of a hollow sphere of 1 foot radius, the total luminous flux emitted by this source is 4π lumens, since there are 4π solid angles emerging from the centre of a sphere. The area of the surface of this sphere intercepted by 1 solid angle is 1 sq. foot. *The illumination* on the inside surface of this sphere = 1 *foot-candle.* Therefore

1 foot-candle (ft-c.) = 1 lumen per sq. ft.

The foot-candle is a unit of illumination, and may be defined as *the illumination produced on the surface of a sphere of 1 ft. radius by a point source of 1 candle situated at the centre of the sphere.* Hence

$$1 \text{ candle power} = 4\pi \text{ lumens} \quad \dots \quad \dots \quad \dots \quad (1)$$

Lux or Metre-Candle is the illumination produced on the surface of a sphere of 1 metre radius by a point source of 1 candle at the centre of the sphere. Since 1 sq. metre 10·76 sq. ft. it is obvious that

$$1 \text{ foot-candle} = 10{\cdot}76 \text{ lux} \quad \dots \quad \dots \quad \dots \quad (2)$$

5. **Laws of Illumination :** The symbol used for illumination is *E*. The first law says that

"the illumination varies inversely as the square of the distance from the source."

This is known as the *Law of Inverse Square.*

The second law says that

"the illumination varies directly as the cosine of the angle between the normal to the surface and the direction of the incident ray."

This is known as ***Lambert's Cosine Law.***

Expressing the above two laws in the form of an equation

$$E = \text{c. p.} \times \frac{\cos\theta}{d^2} \quad \dots \quad \dots \quad \dots \quad (3)$$

In Fig. 6 the surface illuminated is *S* whose distance *d* from the source *L* is shown by a full line. The dotted line is the normal (i. e. at 90°) to the surface. Both these lines are in the same plane. The angle between these two lines is the angle θ.

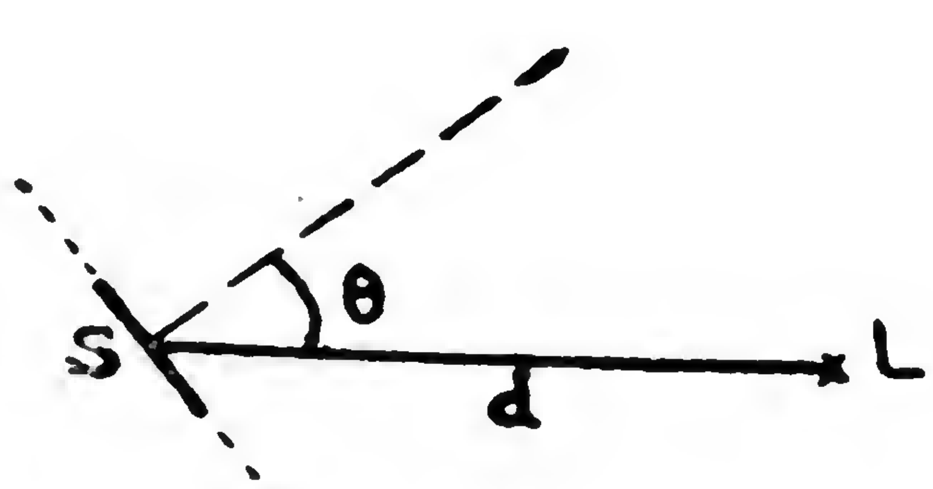


Fig. 6

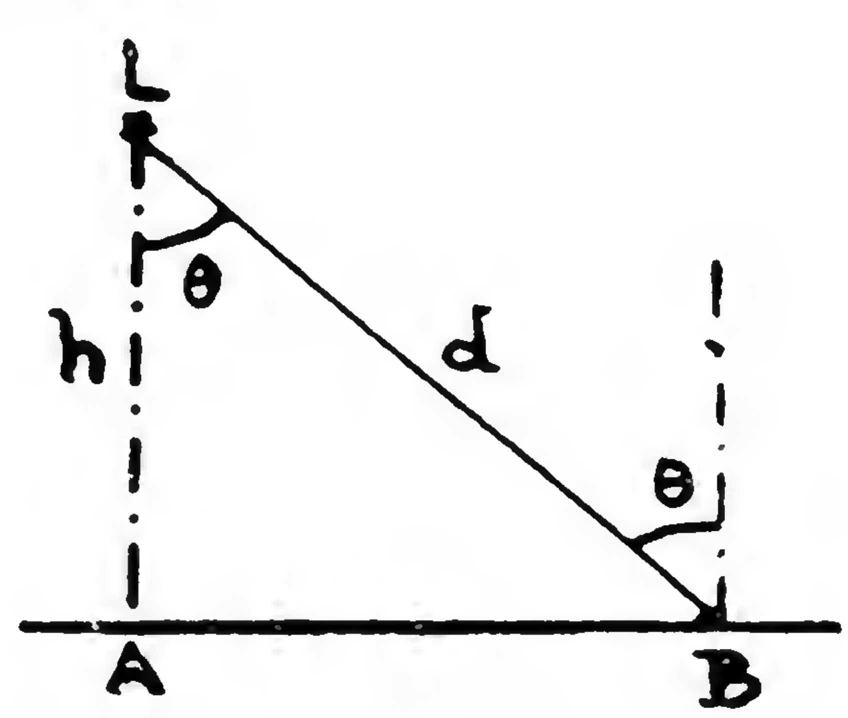


Fig. 7

Fig. 7 shows a source of light L suspended at a height of h ft. from the floor AB. Assuming the source to have a uniform luminous intensity of I candle power in all directions, the illumination at A will be

$$E_A = \frac{I}{h^2} \text{ ft.-}c \quad \ldots \quad \ldots \quad \ldots \quad (3a)$$

and the illumination at B will be

$$E_B = \frac{I}{d^2} \cos\theta \text{ ft.-}c.$$

where d is the distance in feet of B from L. But $d = \dfrac{h}{\cos\theta}$

$$\therefore \quad E_B = \frac{I}{h^2}\cos^3\theta \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

$$\text{and} \quad E_B = E_A \cos^3\theta \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

Example: A 500 c. p. lamp is suspended in the centre of a room at a height of 10 ft. from the floor level. The floor is 24 ft. long and 18 ft. wide. Calculate the illumination (i) at the centre of the floor and (ii) at the corners of the floor.

Solution: (i) At the centre of the floor, the distance of the centre of the floor from the lamp is 10 ft. and the angle θ of the incident ray is 90° therefore $\cos\theta = 1$,

$$\text{at the centre, } E = \frac{500}{10^2} = 5 \textit{ ft-candles.}$$

(ii) At any one corner, the distance between the centre of the floor and a corner $= \sqrt{12^2 + 9^2} = 15$ ft.

$$\tan\theta = \frac{15}{10} = 1{\cdot}5$$

From Tables, $\cos\theta = 0{\cdot}555$ and $\cos^3\theta = 0{\cdot}171$

$$\text{at one corner, } E = \frac{500}{10^2} \times 0{\cdot}171 = 0{\cdot}855 \textit{ ft-candles.}$$

Mean Values of Intensity (or Candle power): The intensity of illumination, or candle power, of a source of light is not the same in every direction. Therefore if photometric measurements are made of candle power of a source in all directions in the horizontal plane, the mean of all the values is called the *mean horizontal candle power* (*m. h. c. p.*). Similarly, if measurements of candle power of

a source made in all directions in all planes, their mean value is called the *mean spherical candle power* (*m. s. c. p.*).

$$m.\ s.\ c.\ p. = \frac{\textit{total flux in lumens}}{4\pi} \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

where 4π is the number of solid angles in steradians.

Another useful term is the *mean hemi-spherical candle power* (*m. h-s. c. p.*) and may be stated as

$$m.\ h\text{-}s.\ c.\ p. = \frac{\textit{flux in lumens in the hemisphere}}{2\pi} \quad \ldots \quad (7)$$

The hemisphere is usually the one below the horizontal plane and is commonly called the lower hemisphere.

The output of a lamp is stated in lumens per watt. This is sometimes called the efficiency of the lamp. The earlier method of expressing the output was in watts per candle power. The following are the output of some of the metal filament lamps. These figures are dependent upon the size of a lamp:—

	Lm/W	*W/c. p.*
Carbon	3·6	3·5
Tungsten	12—20	0·6—1·2

7. Measurement of Candle Power: For the measurement of candle power of a source a sub-standard lamp is required. The laboratory always uses a lamp whose intensity has been carefully calibrated with a standard of luminous intensity.

The measurement of candle power of a source is done by the use of one of the various types of photometers. The most popular and convenient type is the *Lummer-Brodhun Photometer* consisting of a screen of plaster of Paris and prisms. One side of the screen is illuminated by a sub-standard lamp and the other side by the lamp under test.

By the use of prisms or silvered glass mirrors M_1 and M_2 the light from the two sides of the plaster screen *PS* is brought to two *main prisms*, *X* and *Y*, of the photometer, see Fig. 8. An eye-piece

with a lens, as in a telescope, is provided for the purpose of observation. From the eye-piece the rays of one side of the screen appear in a circular area, while those from the other side appear round this

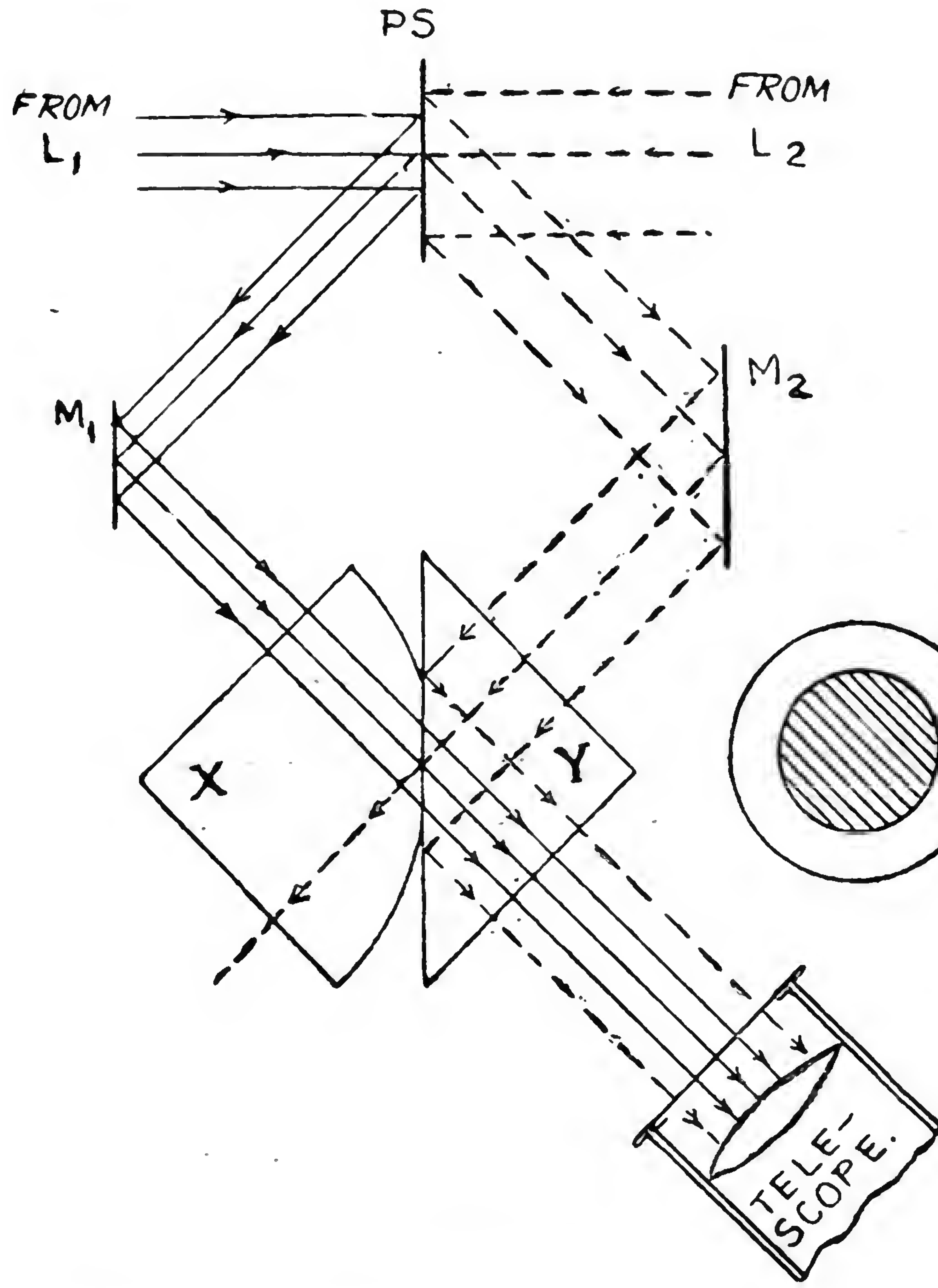


Fig. 8. Lummer-Brodhun Photometer.

area in the form of an annulus. When the two sides of the plaster screen are equally bright, the annulus and the inner circular area disappear, because both these areas are now equally bright and there is no demarcation visible any longer. This happens at the condition of balance.

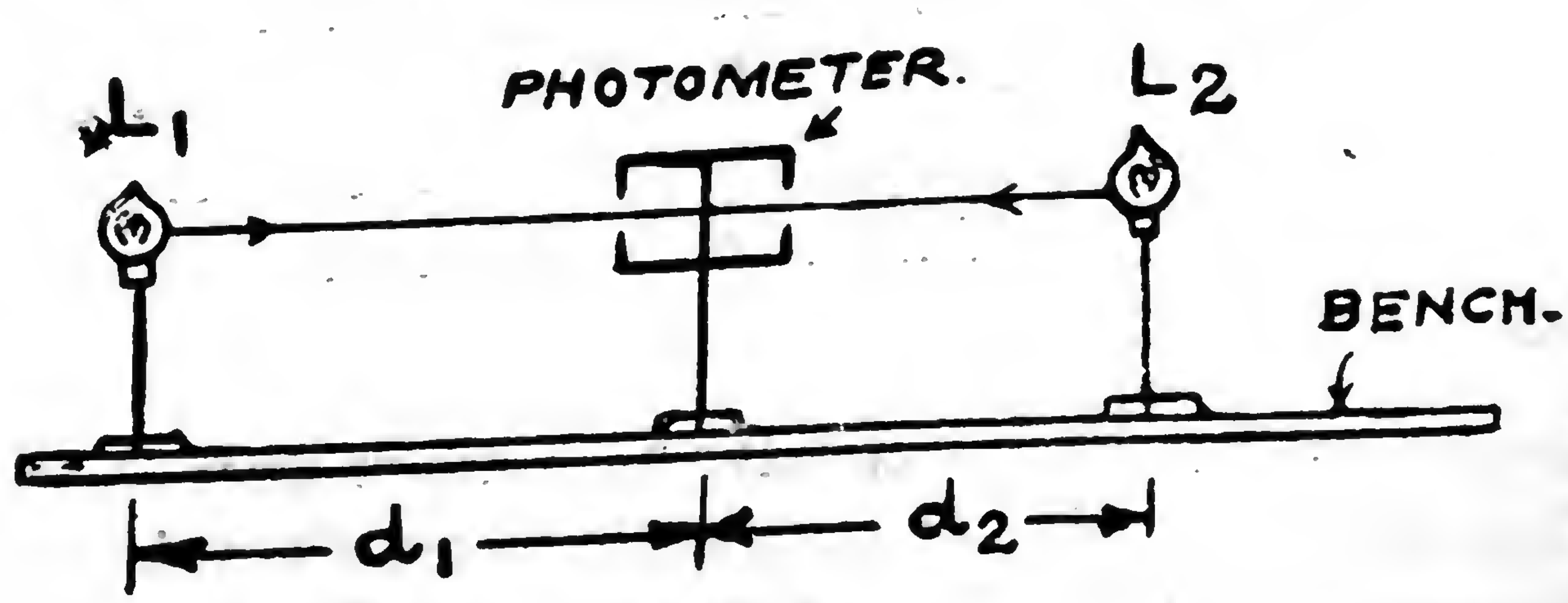


Fig. 9

Balance is obtained by altering the ratio of distances of the two lamps from the photometer. Fig. 9 shows an arrangement of the test bench. L_1 is the sub-standard lamp while L_2 is the lamp under test. The distances of L_1 and L_2 from the photometer head are d_1 and d_2 respectively. If the candle power of L_1 is I_s, the candle power I of the lamp L_2 can be easily calculated.

Under the condition of balance, the illumination E on both sides of the screen is the same, i. e.

$$E = \frac{I_s}{d_1^2} = \frac{I}{d_2^2}$$

$$\therefore \quad I = I_s \times \frac{d_2^2}{d_1^2}$$

where I is the candle power of the lamp under test.

8. Measurement of M. S. C. P. : If the m. s. c. p. of a source is known it is an easy matter to find its total luminous flux. All that one has to do is to use Eq. (6), i. e.

$$\textit{total flux in lumens} = 4\pi \times m.\,s.\,c.\,p.$$

Unfortunately no source has the same value of c. p. in every direction. Hence it is necessary to make actual measurements of c. p. in such a plane in which the source has some sort of symmetry as regards its polar curve.

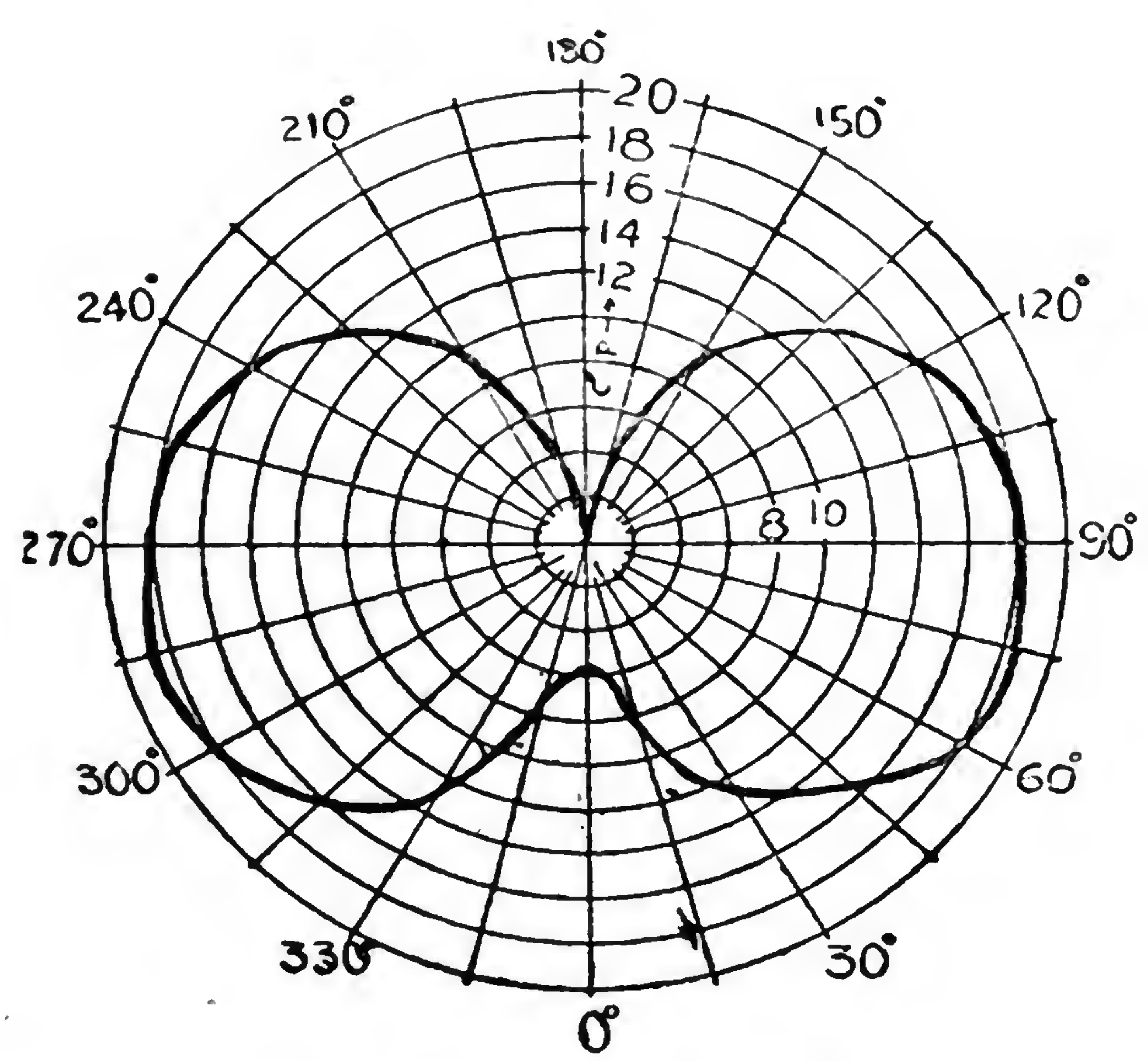


Fig. 10. Polar Curve

The lamp is turned through regular angular intervals and in each position its c. p. is measured, and the results are plotted as shown in Fig. 10. This graph is called the *polar curve* and is almost symmetrical about the line $180^\circ - 0^\circ$. The two portions of the curve are circular. The lamp is turned through 15° at a time. The length of the radius vector at any angle is proportional to the c. p. of the source at that angle.

From Fig. 10 another diagram, known as *Rousseau diagram*, is constructed as shown in Fig. 11, from which the m. s. c. p. of the source is calculated by determining the area under the curve. If the polar curve scale for c. p. is $1'' = 10$ c. p., then the numerical value of m. s. c. p. is $m.\ s.\ c.\ p. = \dfrac{area\ in\ sq.\ inches}{length\ of\ base\ in\ inches} \times 10$, the area mentioned in the equation being that of the Rousseau diagram and it can be calculated either by Simpson's Rule, or computed by using a planimeter.

The right hand curve of Fig. 12 is the same as the Rousseau diagram of Fig. 11. The difference between the two is that the base of Fig. 12 is divided into 10 equal parts. The projections from the mid-points of these parts are taken to the polar curve and they cut the outer semi-circle at definite points. If these points are joined to the centre point i. e. the origin as shown in the figure, a series of angles are obtained. These are called *Russell Angles.*

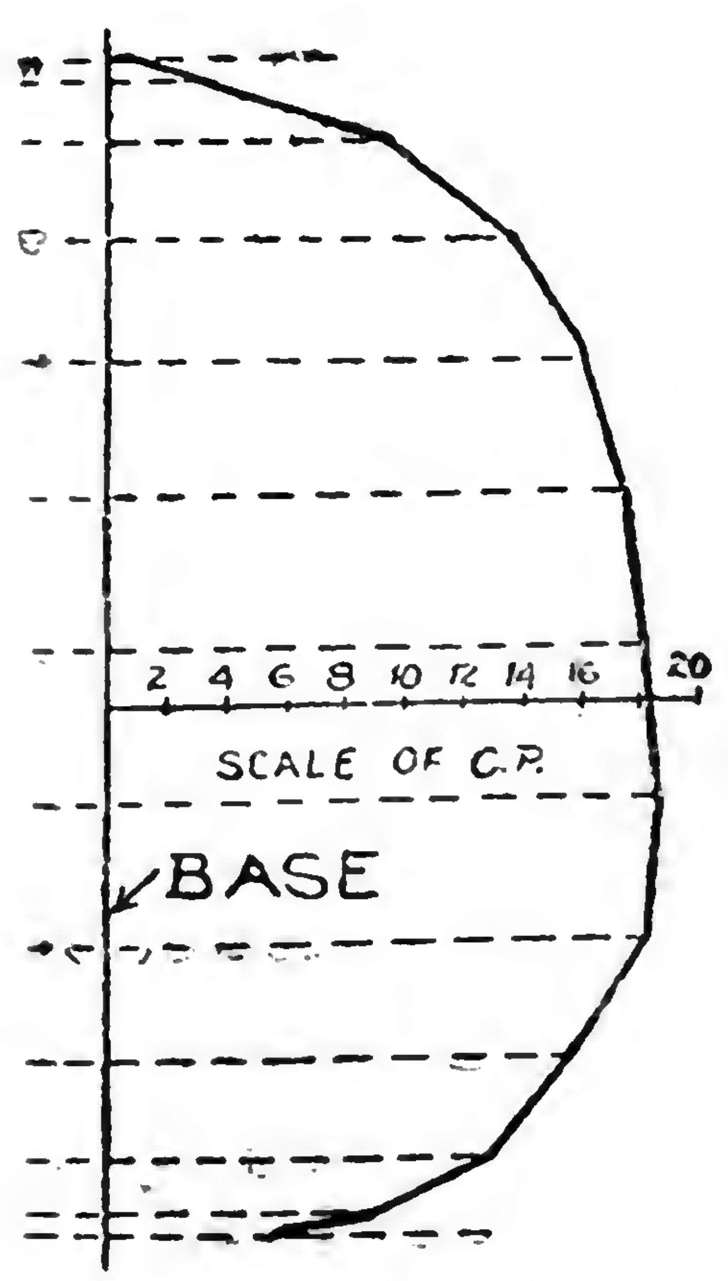


Fig. 11 Roussean Diagram

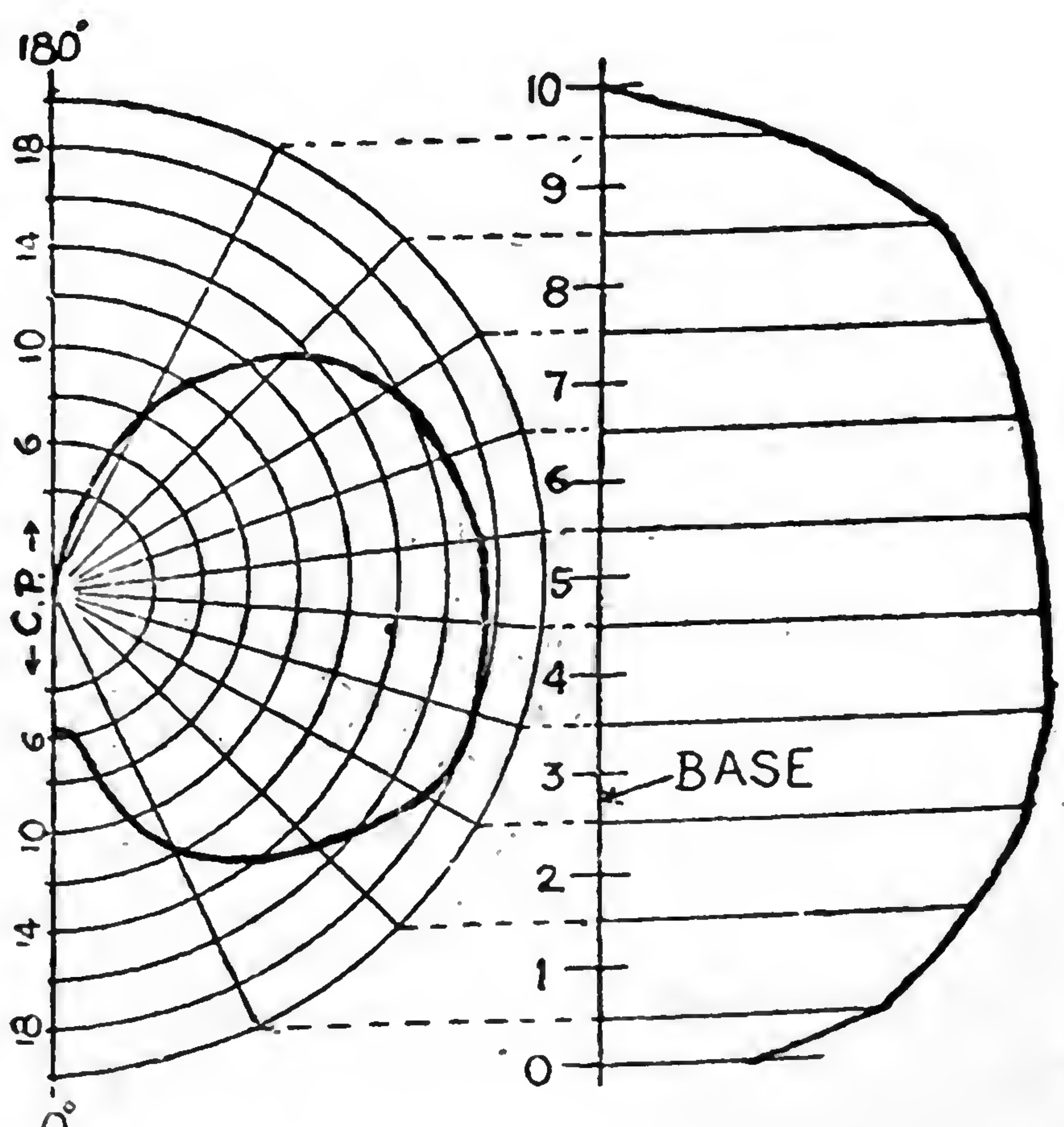


Fig. 12

If the intensity (i. e. candle power) of the source is measured at each of these angles, the arithmetic mean of these values gives the *m. s. c. p.* Thus there is no need to draw any polar curve at all.

9. **Brightness** is the intensity of illumination of a surface per unit *apparent area.* The apparent area is the projected area of the surface on a plane perpendicular to the direction in which it is viewed. The symbol used is B and the unit in which it is usually expressed is *candles per sq. cm* or *candles per sq. ft.*

When the object is very bright, such as a carbon arc, the brightness is stated in candles per sq. cm., but in the cases of walls and ceilings the brightness is stated in candles per sq. ft. because of the low value of brightness.

In the cases of mirrors and highly polished surfaces, the angle of reflection is equal to the angle of incidence, and there is always an image of the source. This sort of reflection is called *specular.* But distempered or painted surfaces, frosted glass, paper etc. scatter the light in all directions and there is no image of the source. This sort of reflection is called *diffuse,* and the surfaces are called *diffusing surfaces.* The ratio

$$\frac{\textit{reflected light}}{\textit{incident light}}$$

is called the *reflection ratio* (ρ).

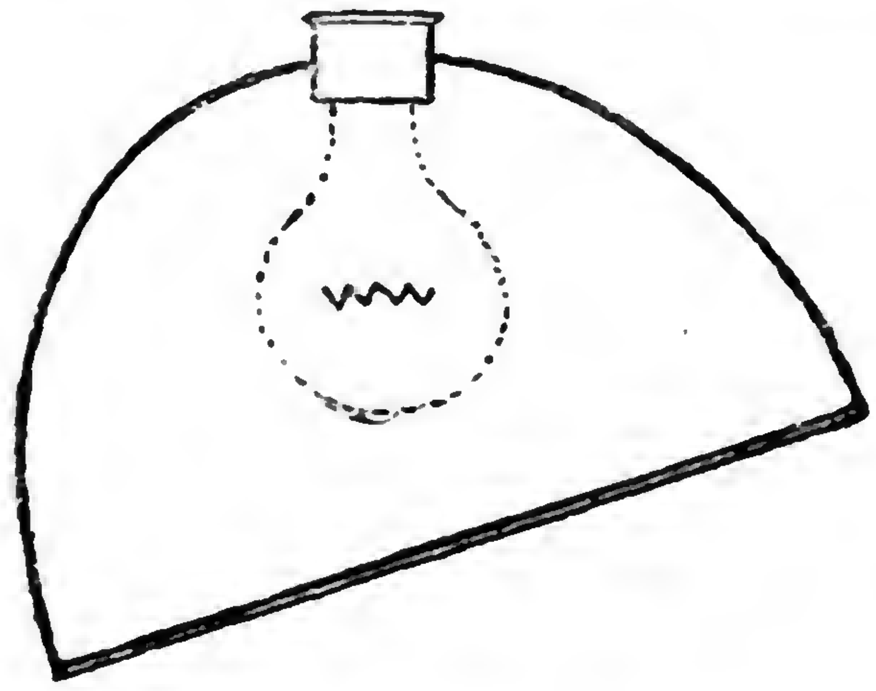
If E is the illumination of a diffusing surface and ρ is the reflection factor, then the surface reflects ρE lumens per unit area, i. e.

$$\rho E = \pi B$$

Hence the brightness of the surface is

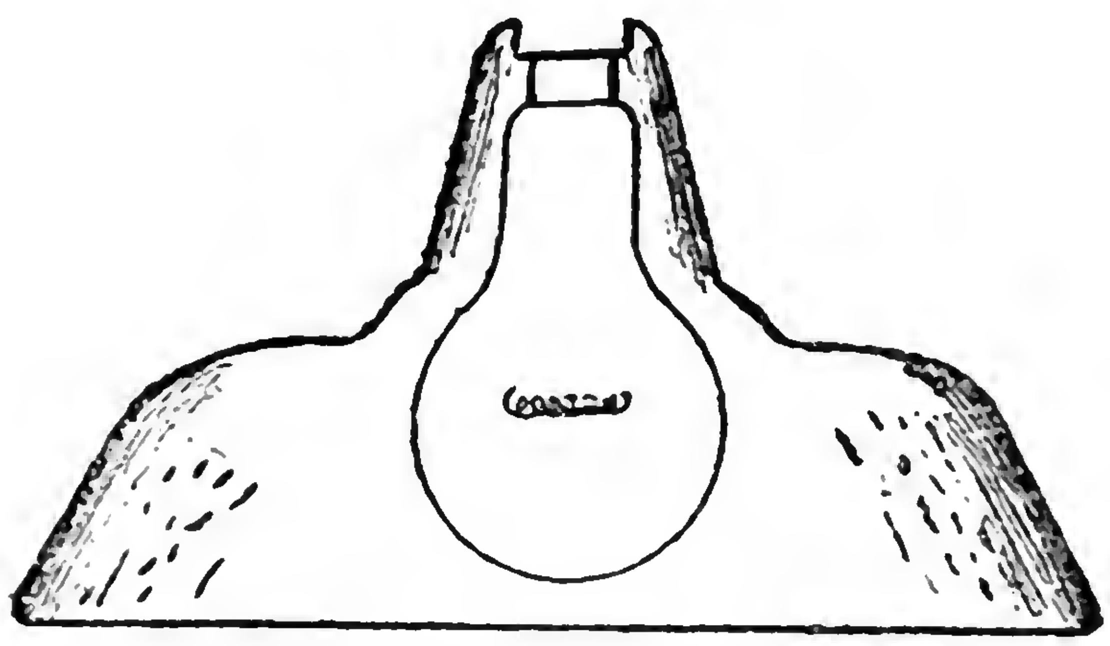
$$B = \rho \frac{E}{\pi} \text{ candles per unit area.}$$

10. **Reflectors** are used for distributing or directing the light of a source in a desired manner. In other words, the shape of the polar curve of the source is altered. The choice of a particular type of reflector depends upon the distribution of light required at a place, such as, a shop window; a corridor; a street; a reading room etc.

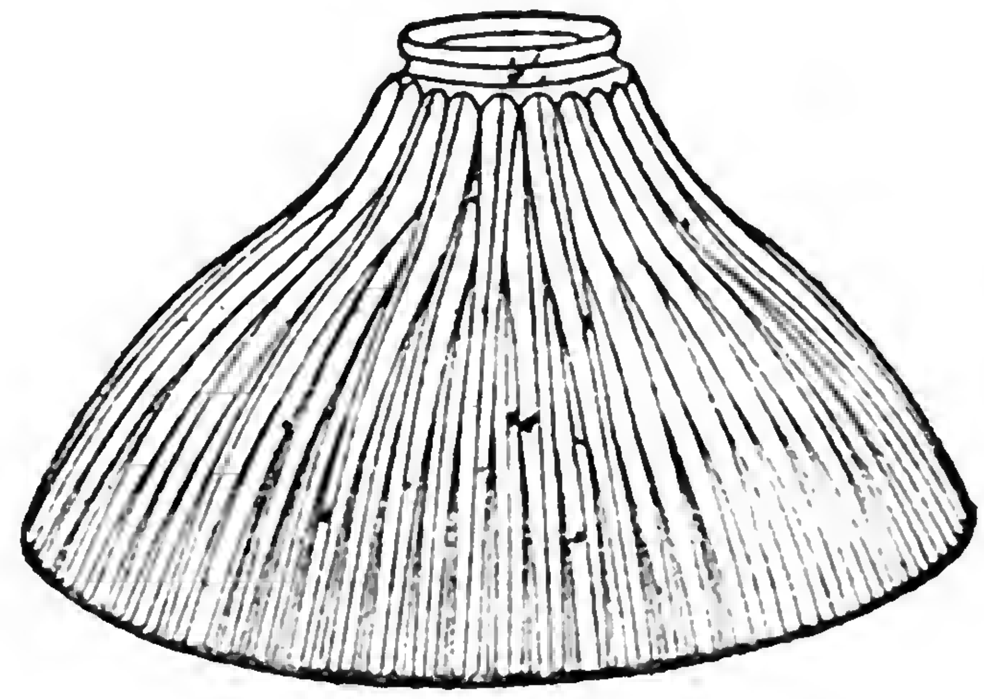


Shop-window Reflector
Fig. 13 (a)

The materials used for high-class reflectors are (a) frosted glass; (b) opal glass; (c) mirrored surfaces; (d) prismed surfaces (holophane type); (e) vitreous enamel coated surfaces etc. Fig. 13 shows three types of reflectors and Fig. 14 polar curves for three different types of service.



Vitreous Enamelled-Reflector.
Fig. 13 (b)



Holophane Reflector.
Fig. 13 (c)

11. Interior lighting schemes are classified as

(a) *direct lighting*—light reflected in the lower hemisphere:

(b) *indirect lighting*—light reflected in the upper hemisphere;

(c) *general lighting*—light reflected evenly;

(d) *semi-direct* } between 60 to 80 % light reflected in the direction concerned.

(e) *semi-indirect* }

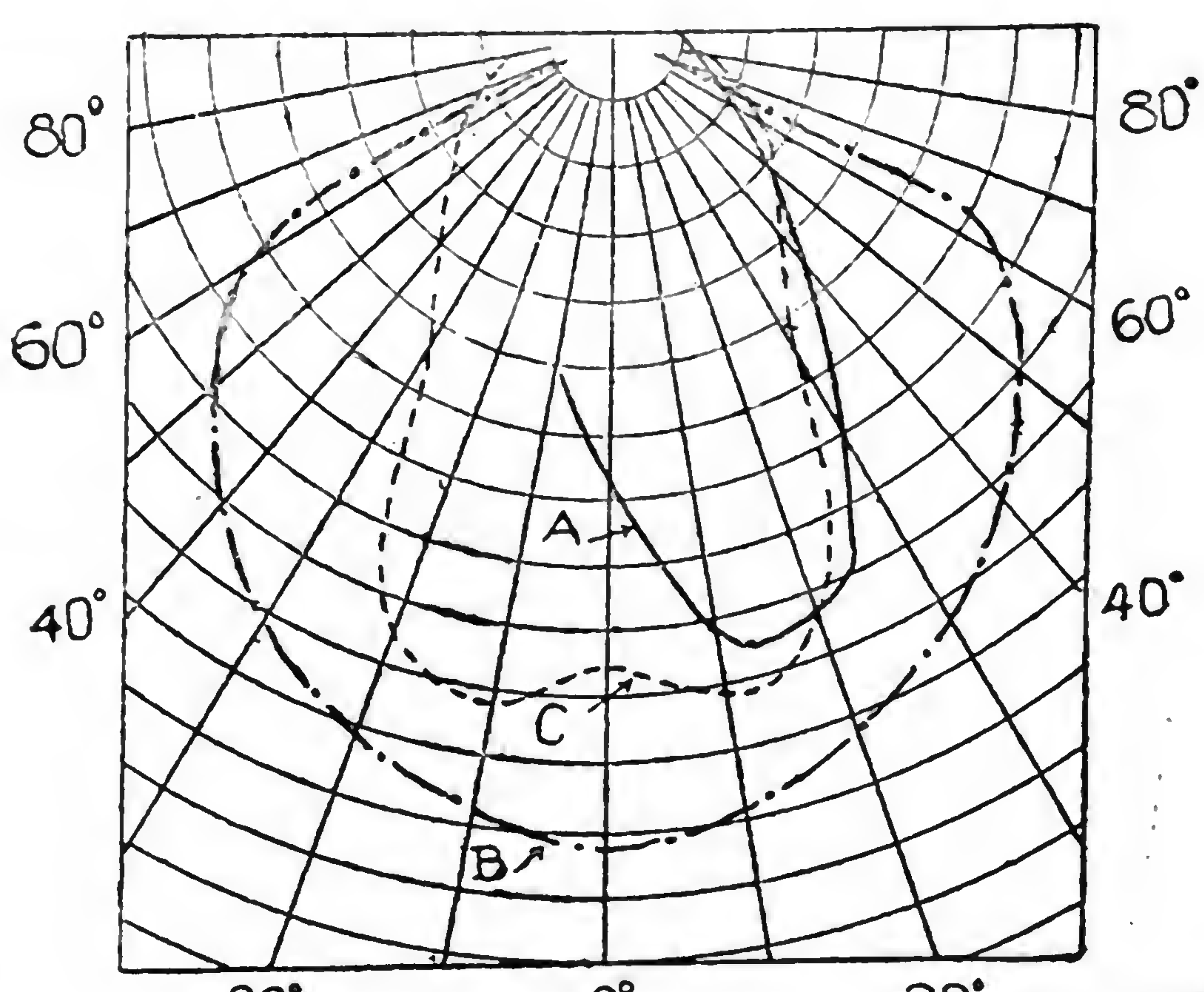


Fig. 14. Polar Curves.

In the design of any interior lighting scheme, the problem is to etermine (i) the number of lighting units; (ii) the wattage or rather the lumen output of each lighting unit and (iii) the disposition of lighting units. Further, the sort of illumination a particular place requires is governed by

(a) the actual size of the object and the distance from which it is viewed;

(b) the extent to which the details of the object should be clear, so that

(c) the degree of contrast is pronounced.

The *Coefficient of Utilisation* is a fraction of total light flux, or lumens, reaching the area to be illuminated. Its value depends upon (i) the total area to be illuminated; (ii) the mounting height of light units and (iii) the colour of walls, ceiling and room furniture and fittings. Its value, in the case of direct lighting schemes, is 0·3 to 0·6; for general lighting it is 0·25 to 0·5 and for indirect lighting the value lies between 0·1 to 0·25.

$$\textit{Depreciation Factor} = \frac{\textit{illumination under normal condition}}{\textit{illumination when everything is clean}}.$$

The value of this factor lies between 0·7 and 0·8.

Illumination is always specified in *lumes per sq. ft.*

A few recommended values for common situations are :—

Drawing Offices	30 lm/sq. ft.
Reading Rooms and Offices	20 ,, ,,
Work Benches	15 ,, ,,
Residential Halls	8 ,, ,,

The total area in sq. ft. *multiplied by* the required illumination gives the total lumens that must reach the area. Therefore the total gross lumens that must be provided is

$$\textit{gross lumens} = \frac{\textit{illumination required} \times \textit{area in sq. ft.}}{\textit{coefficient of utilisation} \times \textit{depreciation factor}}$$

$$\textit{Spacing-height Ratio} = \frac{\textit{distance between lamps}}{\textit{mounting height of lamps}}$$

and depends upon the nature of polar curve of lamps ***with their reflectors***. If uniform illumination is required the above ratio must

have a value between 1·25 and 1·8. These values are for reflectors used for interior lighting.

Example : The average illumination required for an Assembly Hall, 80 ft. long and 50 ft. wide, is 6 lumens per sq. ft. Assume the coefficient of utilisation equal to 0·4 and the depreciation factor equal to 0·75. Calculate the wattage of each lighting unit and the number of units, and state how these will be arranged. The Table below gives the lamp efficiencies :—

Wattage of lamps ...	200	300	500
Lumens per watt ...	13·6	14·2	15·9

Solution : Net lumens = area × 6 = $80 \times 50 \times 6 = 24000$ lm.

$$\textit{gross lumens} = \frac{24000}{0{\cdot}4 \times 0{\cdot}75} = 80000 \text{ lm.}$$

If 500-watt lamp is considered, one unit gives

$$15{\cdot}9 \times 500 = 7950 \text{ gross lumens.}$$

$$\text{total number of units} = 80000/7950 \doteq 10 \text{ units.}$$

If the area is divided into 10 rectangles, each 25 × 16 ft. one unit can be installed in the centre of each rectangle.

[The student should attempt the solution, taking 200 and 300 watt lamps.

As a general rule for interior lighting, it is better to have 50% diffuse and 50% direct light. But in places such as Drawing Offices, Machine Shops etc. more than 80% light should be diffuse. For places where delicate hand-work is done, such as Embroidery Works, Goldsmithy etc. direct light is essential to produce contrast and diffusion must be avoided.

Example : Street lights are suspended at a height of 20 ft. along the centre line of a road at regular intervals of 120 ft. Calculate the intensity of illumination along the centre line of the road surface between adjacent lamps and plot graph of ft. c. against distances in feet.

The polar curve of a lamp and its reflector is obtained by plotting the following figures:

Angle to the vertical	0°	15°	30°	45°	60°	75°	90°
Candle power	110	105	100	110	150	350	110

Solution: The polar curve is plotted in order to find the candle power of a lamp at any angle to the vertical.

Fig. 15 shows the method of calculating tan θ_1, tan θ_2 for a particular position. From this the values of $\cos^3\theta_1$ and $\cos^3\theta_2$ are determined. Similarly from the polar curve the values of c. p.$_1$ and c. p.$_2$ for various angles θ_1 and θ_2 are found. The results are tabulated as shown below.

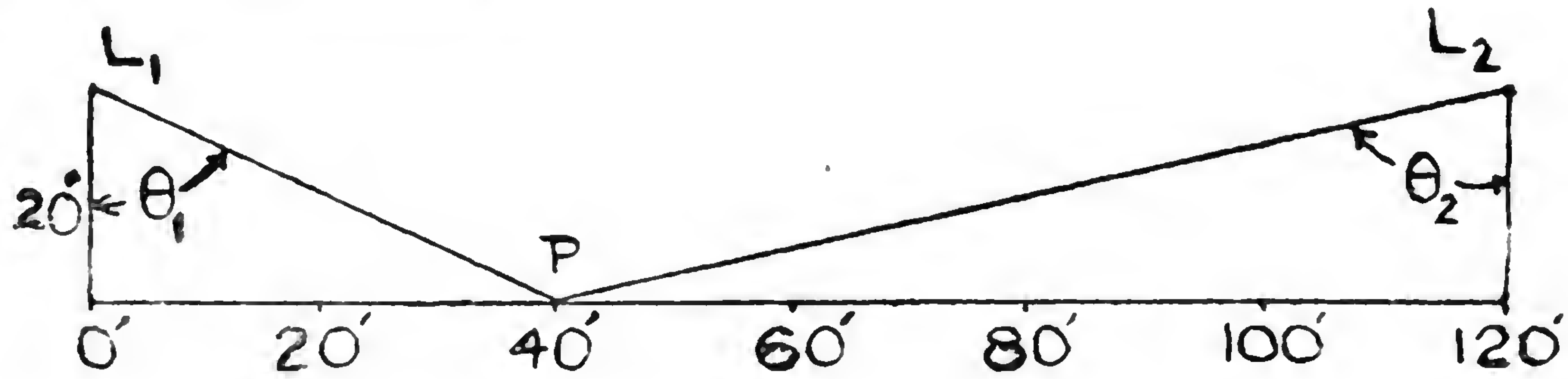


Fig. 15.

Distances	tan θ_1	θ_1	cos θ_1	$\cos^3\theta_1$	c. p$_1$	tan θ_2	θ_2	cosθ_2	$\cos^3\theta_2$	c. p$_2$
0'	0	0	1	1	110					
20'	1	45°	0·707	0·353	110	5	78°-42'	0·196	0·007	320
40'	2	63°-26'	0·447	0·089	165	4	76°-0'	0·242	0·014	345
60'	3	70°-36'	0·332	0·037	300	3	70°-36'	0·332	0·037	300

The next Table shows the results of calculations and Fig. 16 shows the graph of illumination.

Distance Ft.	c. p.$_1$ $\times \dfrac{\cos^3\theta_1}{h^2}$ + c. p.$_2$ $\times \dfrac{\cos^3\theta_2}{h^2}$	E in Ft.-C
0	0·275 + 0	0·275
20	0·097 + C·0056	0·103
40	0·038 + 0·012	0·050
60	0·0287 + 0·0287	0·057

The illumination at a distance of 120 ft. due to a lamp is not considered, since its value becomes very small.

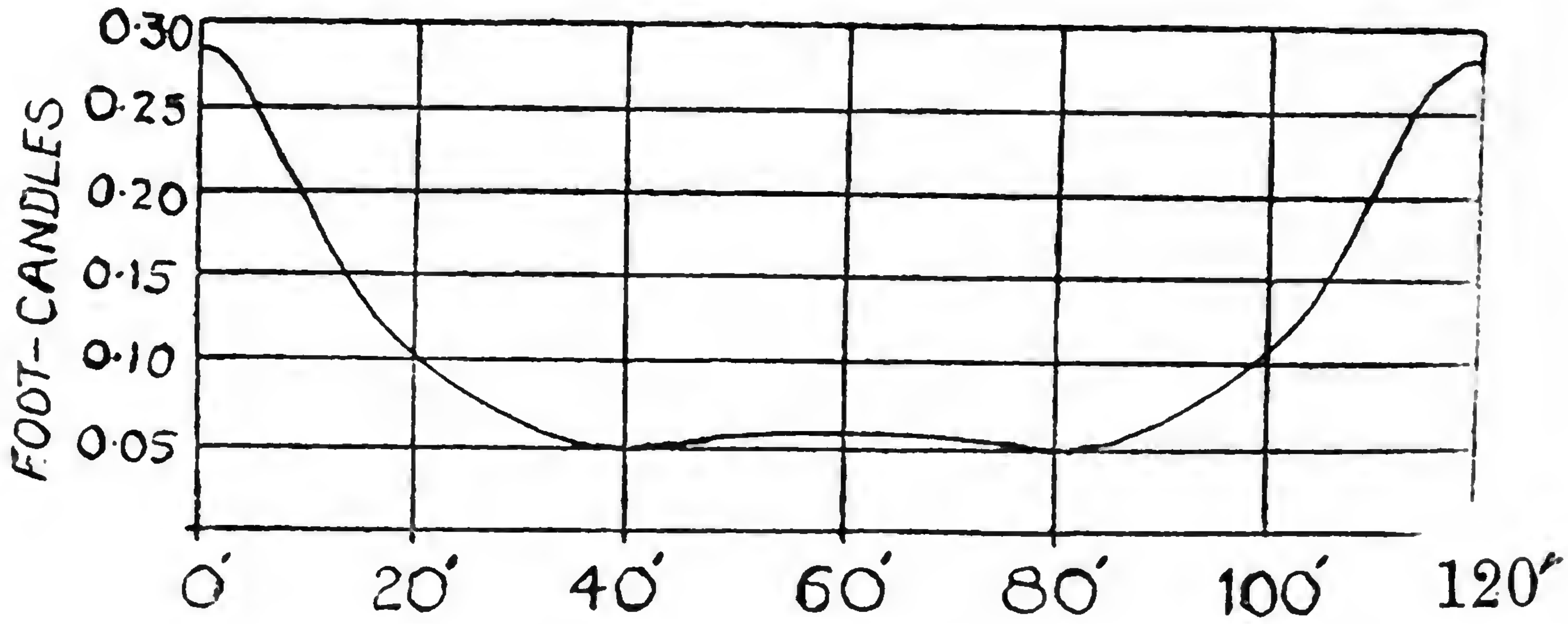


Fig. 16. Graph of Illumination.

CHAPTER XV

MEASURING INSTRUMENTS AND APPARATUS

1. General Features of Modern Instruments: All modern electrical measuring instruments have a fixed and a moving system, the latter is pivoted in jewelled bearings. A pointer attached to the spindle of the moving system sweeps over a graduated scale. In few cases the moving system is suspended by a fine wire of phosphor-bronz while the fixed system may compose of either a permanent magnet or an electro-magnet. The moving system consists of a coil through which a current is made to pass, or a plain metal disc as employed in an induction type instrument.

The moving system is subjected to

(a) a deflecting torque,

(b) a controlling (or restoring) torque, and

(c) a damping torque.

A. The Deflecting Torque is produced (i) magnetically by a force set up between two sets of fluxes, one due to the fixed system and the other due to the moving system. This is according to Maxwell's Law which states that

"every electro-magnetic system tends to change its configuration so that the existing system tends to embrace the largest number of lines of force".

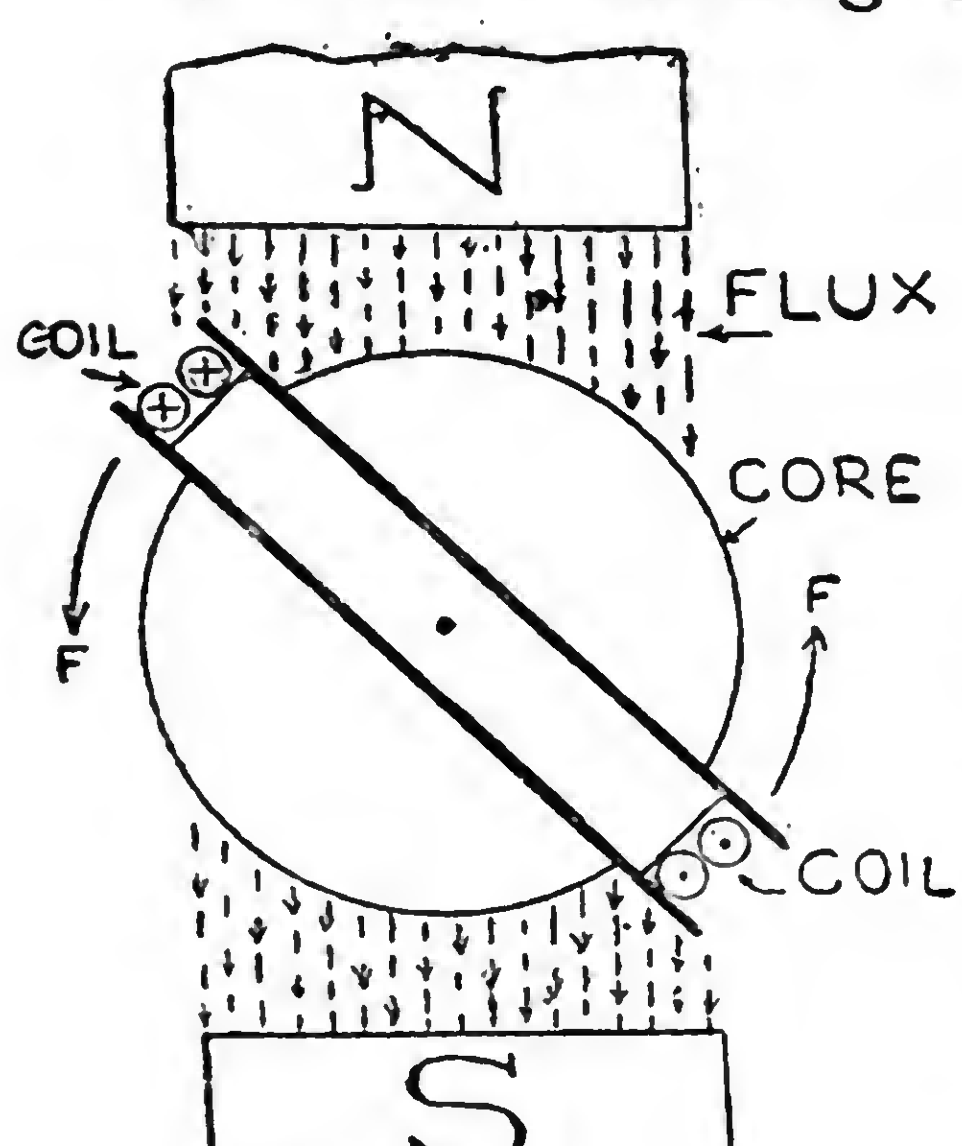


Fig. 1. Simple d. c. Moving Coil Instrument

Fig. 1 shows two poles of a permanent magnet. A coil, wound on a moving metal former, is free to move in the air-gap formed by the poles of the permanent magnet and a soft-iron cylindrical core. When a current passes through the coil a flux is produced having its own N- and S-poles. The coil moves in such a manner that its N-pole approaches the S-pole of the permanent magnet due to mutual attraction between the two. There

is a similar force between the other two poles. The result is that the coil tries to assume a position which will bring its plane at right angles to the direction of flux of the permanent magnet. In other words the system tends to embrace maximum lines of force.

The above phenomenon can be explained alternatively by considering the conductors of the coil carrying current in the direction shown in the figure. The force experienced by the conductors due to the field of the permanent magnet is indicated in the figure. This is really the motor action.

The deflecting torque is produced (i) electrostatically by a force between two electrically charged substances, one fixed and the other movable; or (ii) thermally by a mechanical force caused by the expansion of a wire in which a current is made to pass.

B. **The Controlling Torque** is absolutely necessary to ensure a definite position of the pointer corresponding with every value of the quantity being measured. Otherwise, under the influence of the deflecting torque alone, the deflection of the pointer would be indefinite. This torque is produced either (i) by gravity or (ii) by a spirally wound hair-spring. At the definite position of the pointer the deflecting and the controlling torques are equal and opposite.

C. **The Damping Torque** is necessary to help the pointer of the instrument to come to rest on the definite position in as short a time as possible. Because in the absence of the damping torque the moving system, due to the inertia of its mass, will oscillate under the influence of the deflecting and the restoring torques and will take a long time to come to rest.

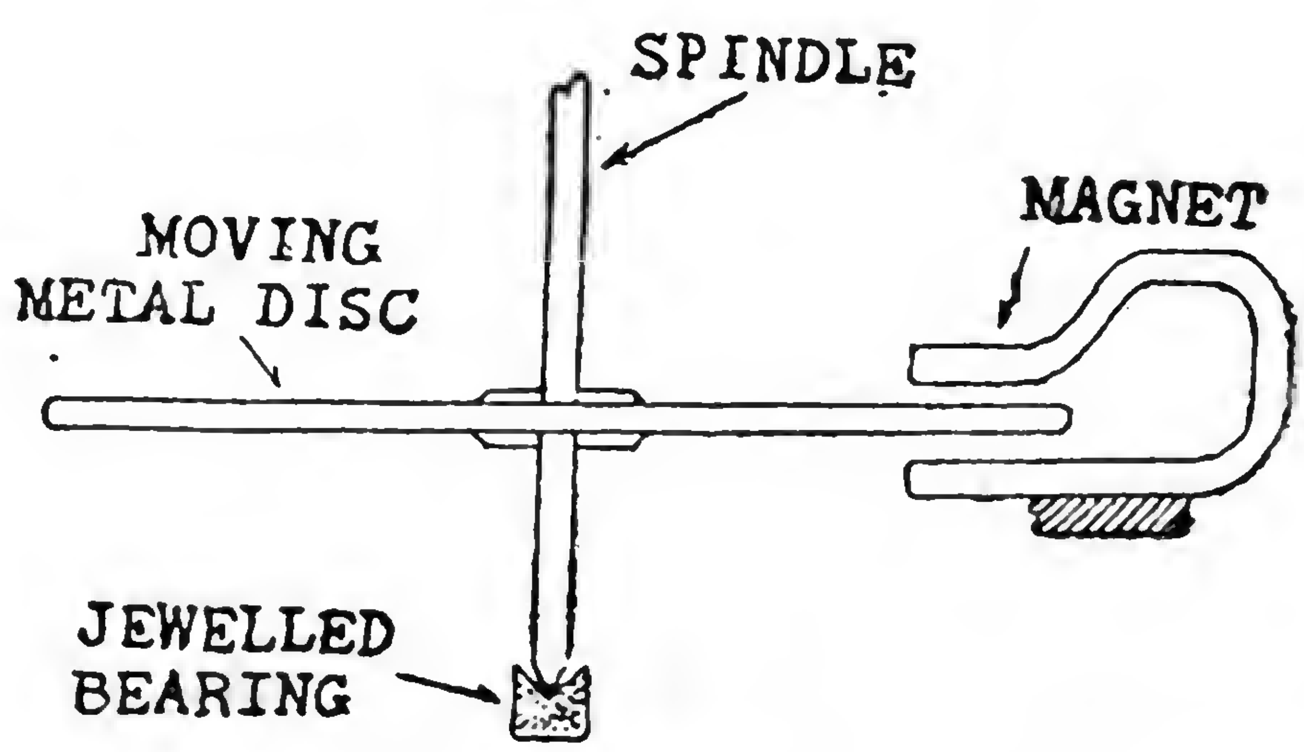


Fig. 2

The common methods employed in producing damping are

(*a*) electromagnetic,

(*b*) pneumatic and

(*c*) viscosity of liquids.

When a metal piece moves in a magnetic field an e. m. f. is induced in it. This e. m. f. circulates, within the piece, eddy currents which produce a damping torque, i. e. it opposes the motion of the moving system according to Lentz's Law. This is known as an *eddy-current brake.* See Fig. 2

In pneumatic damping a light weight piston or vane moves in a sector-shaped chamber with very little clearance. The chamber being closed on one side, the movement of the piston either increases or decreases the air pressure in the chamber depending upon the direction of motion. Thus the oscillations of the moving system are damped out in both directions of motion. This is called *air-friction brake.* See Fig. 3.

In liquid friction damping a vane, or a disc, attached to the spindle of the moving system, moves in a container filled with a viscous substance such as oil. The damping effect is similar in nature to the pneumatic damping. See Fig. 16.

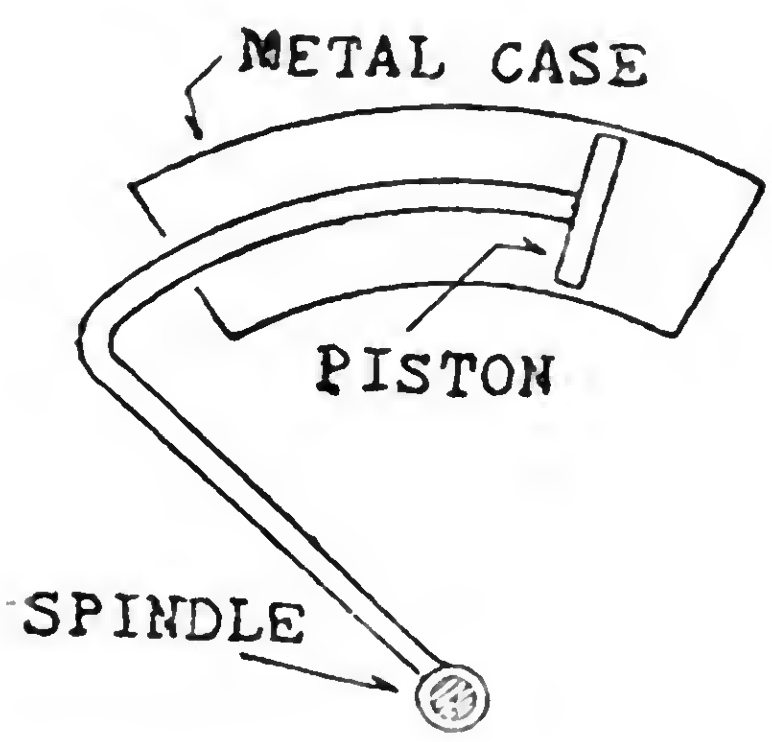


Fig. 3

Whatever method is employed it is essential that the damping torque exists only so long as the system is in motion. Hence the friction brake is inadmissible.

D. **Accuracy:** As regards the accuracy of their readings, instruments are classified under three groups, viz.:—

(1) 1st Grade; (2) 2nd Grade and (3) Substandard.

Substandard instruments are used where very high accuracy is required, such as in Testing Laboratories. The maximum permissible error of these is 0·5%. These instruments, being very costly and delicate, must be handled very carefully.

1st Grade instruments are used in Laboratories and other places, where fairly high accuracy is required. The maximum permissible error of these is 1%. 2nd Grade instruments are used where great accuracy is not needed such as in switch-board instruments. The permissible error is about 3%.

2. The Galvanometer: A galvanometer is very sensitive instrument for detecting very small currents in a circuit. With slight modification these can be used as ammeters or voltmeters.

The D'Arsonval galvanometer has superseded all the other types because of its simplicity and ruggedness. Moreover, it is not affected to any appreciable extent by stray magnetic fields. The description and action of this instrument is given below.

A coil of very thin wire, usually wound on a rectangular aluminium bobbin, is placed in the air-gap formed by the cylindrical pole-faces of a permanent magnet and a cylindrical soft-iron as shown in Fig. 4. The cylindrical form of the pole-shoes and the core makes the flux distribution in the air-gap uniform throughout, so that the deflection of the coil is proportional to the current in it.

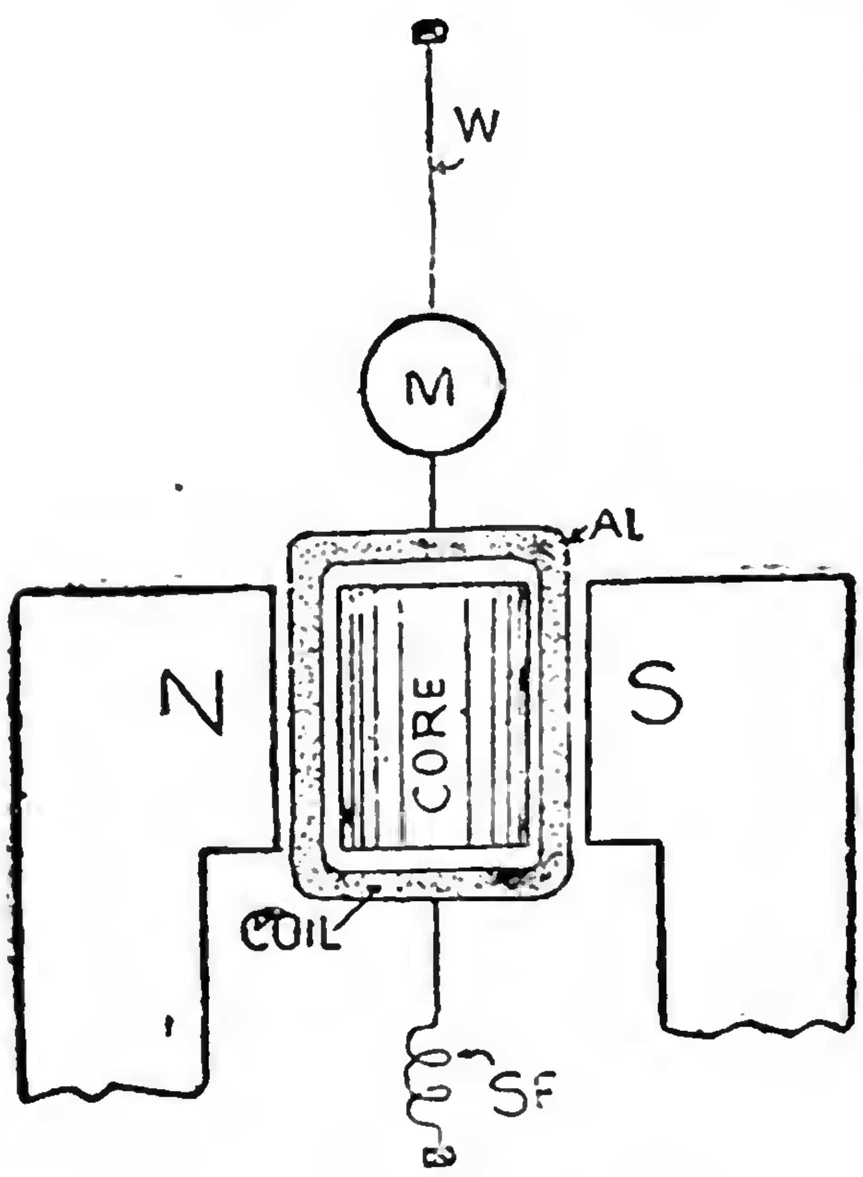


Fig. 4 (a)

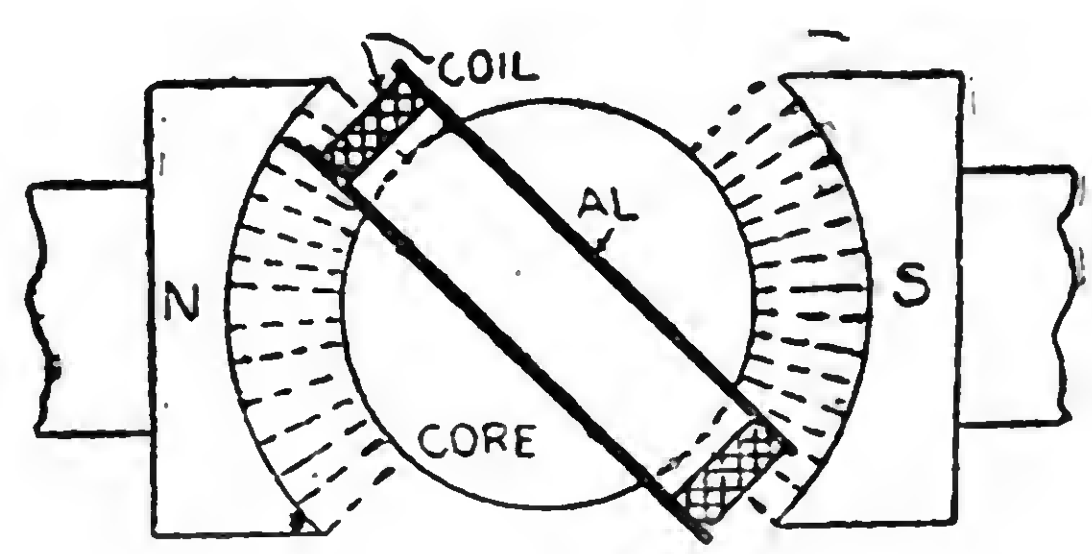


Fig. 4 (b)

The coil is suspended by a phosphor-bronze wire W which serves as one of the lead-in wires, and at the bottom a flexible spiral filament SF serves as the other lead-in wire. The action of the coil is explained in Section 1.

The method of reading the deflection is by a beam of light thrown on to a graduated scale. A small concave mirror M is fixed to W and when torsion in it takes place, the plane of the mirror changes with it. A light from a tiny filament lamp is thrown on M and the reflection from the mirror is thrown on the graduated scale. The aluminium bobbin AL acts as the damper.

The lamp and the graduated scales are placed at some distance from the instrument. The beam takes some time to come to rest on the graduated scale. Shunts are used in conjunction with galvano-

meters. These shunts are so designed that the galvanometer carries $\frac{1}{10}$th, $\frac{1}{100}$th or $\frac{1}{1000}$th part of the current in the external circuit.

Galvanometers therefore must have their own shunts which cannot be used with any other instrument. But the *Ayrton Shunt* is so designed that it can be used with any instrument. A schematic diagram of the Ayrton Shunt is shown in Fig. 5, where R_g is the galvanometer coil resistance, AB is the Ayrton Shunt having a total resistance of R_s. R is the resistance between A and a tapping point. The whole resistance AB is divided into $\frac{1}{1000}$th, $\frac{1}{100}$th, $\frac{1}{10}$th and 1 of AB. If the Slider S is at a tapping point which indicates $\frac{1}{100}$th of AB the deflection of the galvanometer is $\frac{1}{100}$th of its maximum deflection. In the figure where S is shown *the multiplying factor* is $\frac{R_s}{R}$.

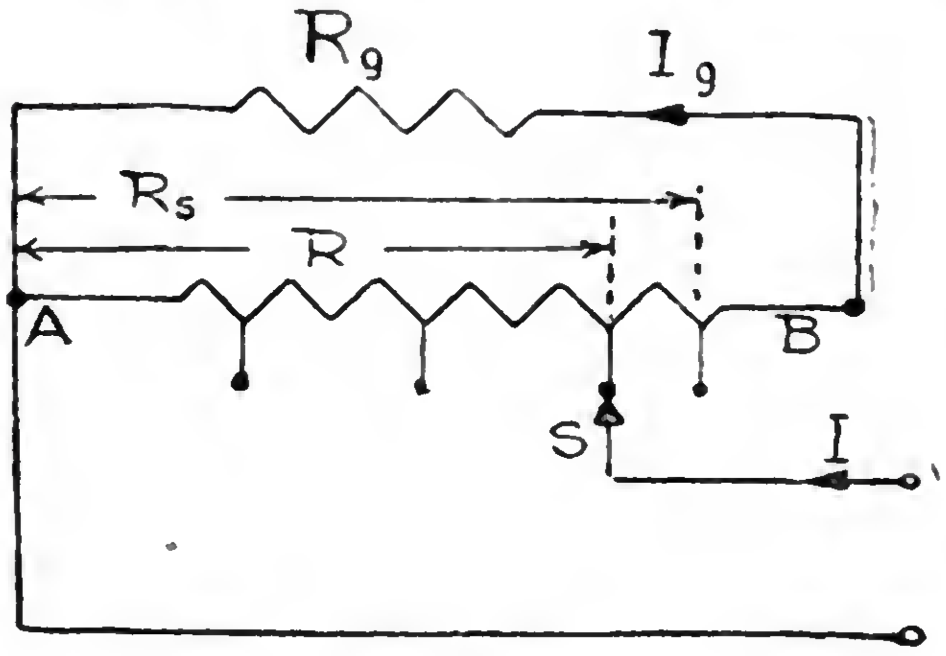


Fig. 5

3. **Moving Coil Type—Ammeters and Voltmeters:** The most common type of measuring instruments are the Weston type which work on the same principle as that of the D'Arsonval Galvanometer. The construction however is slightly different. Fig. 6 shows an instrument of this type as manufactured by Messrs. Evershed and Vignoles Ltd.

Instead of being suspended by a wire, the coil is supported at the top and bottom by sapphire jewel bearings. There are two flat hair-springs, one at the top and the other at the bottom. The function of these springs is two-fold—(1) they serve as lead-in wires for the coil, and (2) they act as a controlling force. The two springs are coiled in opposite directions. At the top there is a pointer which sweeps over a graduated scale. The pointer is perfectly balanced by a counterweight and is very light, being made of very thin sheet of aluminium.

The deflecting torque is produced electro-magnetically and the damping torque is provided by eddy currents induced in the metal former on which the coil is wound. The accuracy of these instruments is 1st grade. Since the coil moves in a uniform field the deflection is

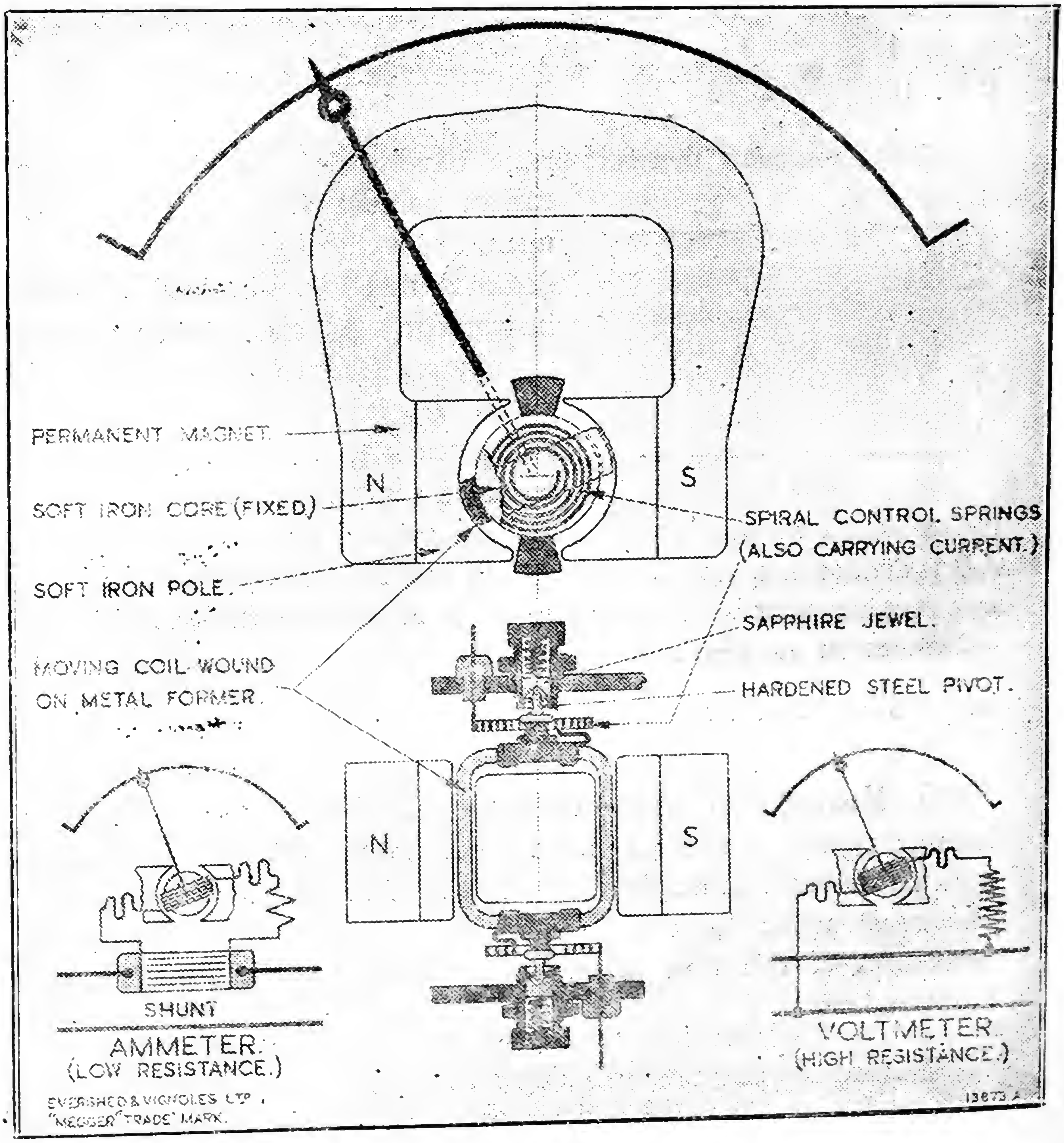


Fig. 6

(By courtesy : Evershed and Vignoles Ltd.)

proportional to the current in the coil. Hence the scale is uniform, but its angular extent is about 60° only.

Since the field of the permanent magnet is very strong, these instruments are not affected by stray fields and **they are suitable for D. C. circuits only.** Full scale deflection is obtained when few milli-amperes pass through the coil. Hence when used as ammeters suitable shunts must be provided.

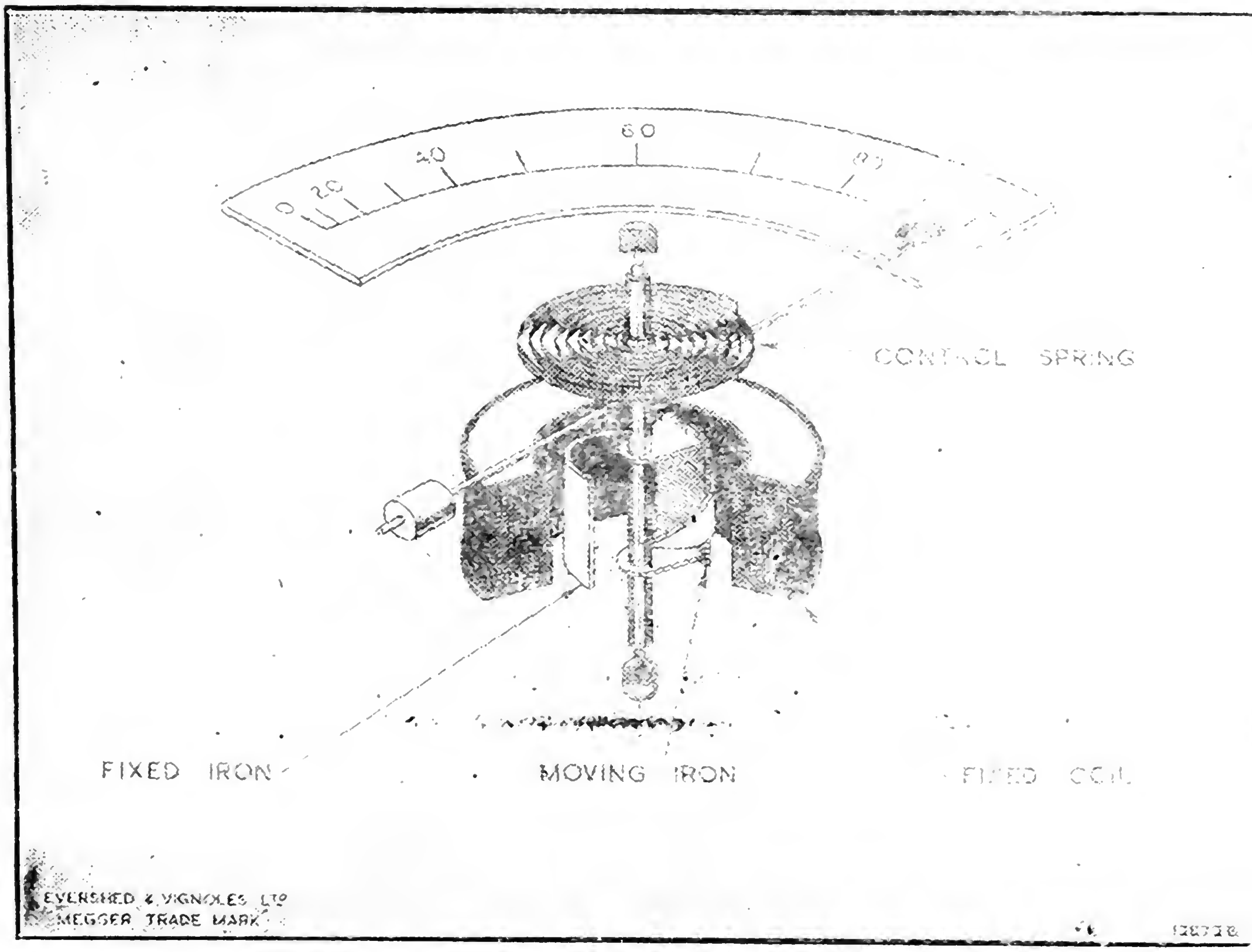


Fig. 7

(*By Courtesy : Evershed and Vignoles Ltd.*)

4. Moving Iron Type—Ammeters and Voltmeters : Fig. 7 shows an instrument of this type. It consists of a fixed coil wound on a brass bobbin inside which there are two bent strips of soft iron, one is fixed on the main frame and the other is attached to the spindle which is supported at the top and bottom in jewelled bearings.

When current passes in the coil the two iron strips are magnetised in the same direction. Hence they repel each other. This force of repulsion produces the deflecting torque which is proportional to the square of the current. Hence the scale is uneven.

These instruments can be used on A. C. as well as D. C. circuits. The restoring torque is either due to gravity or spring. Pneumatic damping must be used since the instrument field is weak and the presence of any other field would affect the deflection. And because of the weak field these instruments must be shielded from the effects of any stray field. The shielding is effected by using a cast iron case for the instrument. The accuracy is 1st grade on A. C. and 2nd grade on D. C. circuits.

5. Dynamometer Type—Ammeters, Voltmeters and Wattmeters: Fig. 8 shows the arrangement of this type of instrument. There is a fixed coil of fairly thick wire, and a movable coil of very thin wire, which moves in the magnetic field of the fixed coil. For this reason the fixed coils is usually split into two sections, as shown in the figure.

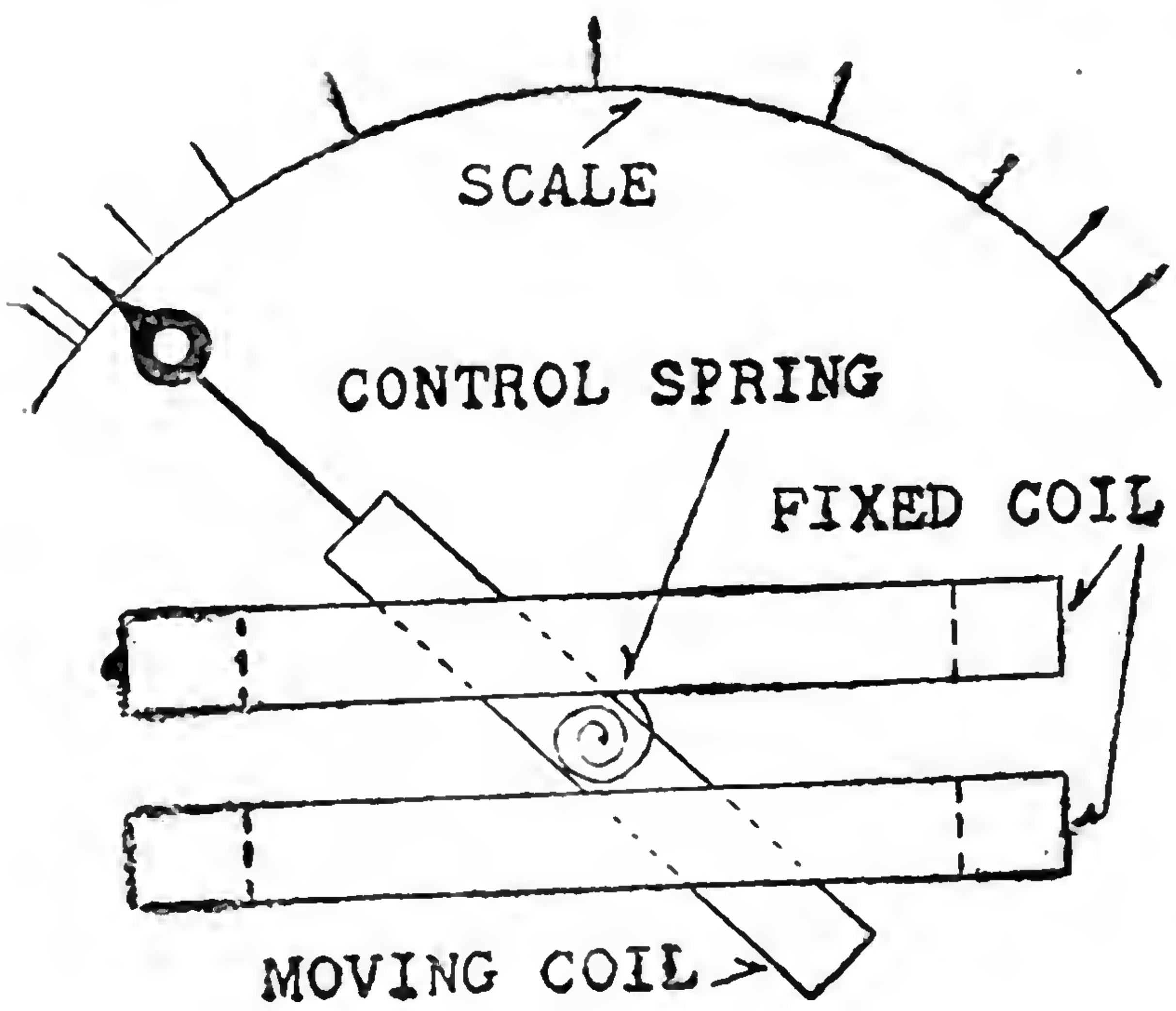


Fig. 8

The deflecting torque is produced by the force between the respective fields of the two coils, and the controlling torque by spiral springs. Damping must be pneumatic, since the field of the fixed coil is not very strong. Accuracy is 1st grade and the instrument can be used both on A. C. and D. C. circuits. In the case of ammeters and voltmeters of this type the scale is uneven, since

operating torque $\propto$ (current $)^2$ or (p. d. $)^2$.

In the case of wattmeters the scale is even. The reason is that the flux of the stationary coils is proportional to the line current and that of the moving coil is proportional to the p. d. Hence in the case of D. C. circuits

operating torque $\propto$ current $\times$ voltage $\propto$ power.

In the case of A. C. circuits the moving system takes up a position corresponding with the *average* value of the torque, since, due to the inertia of its mass, it is prevented from following the variations in the deflecting torque from instant to instant. Hence

operating torque $\propto$ average power.

6. Hot-Wire Type—Ammeters and Voltmeters: The general arrangement of this type of instrument is shown in Fig. 9. The "hot-wire" *HW*, made of platinum silver, is stretched between two points *A* and *B*. Its tension is adjusted at *A* by means of a screw. *W* is a phosphor-bronze wire soldered to *HW* at *C* and fixed at *D*.

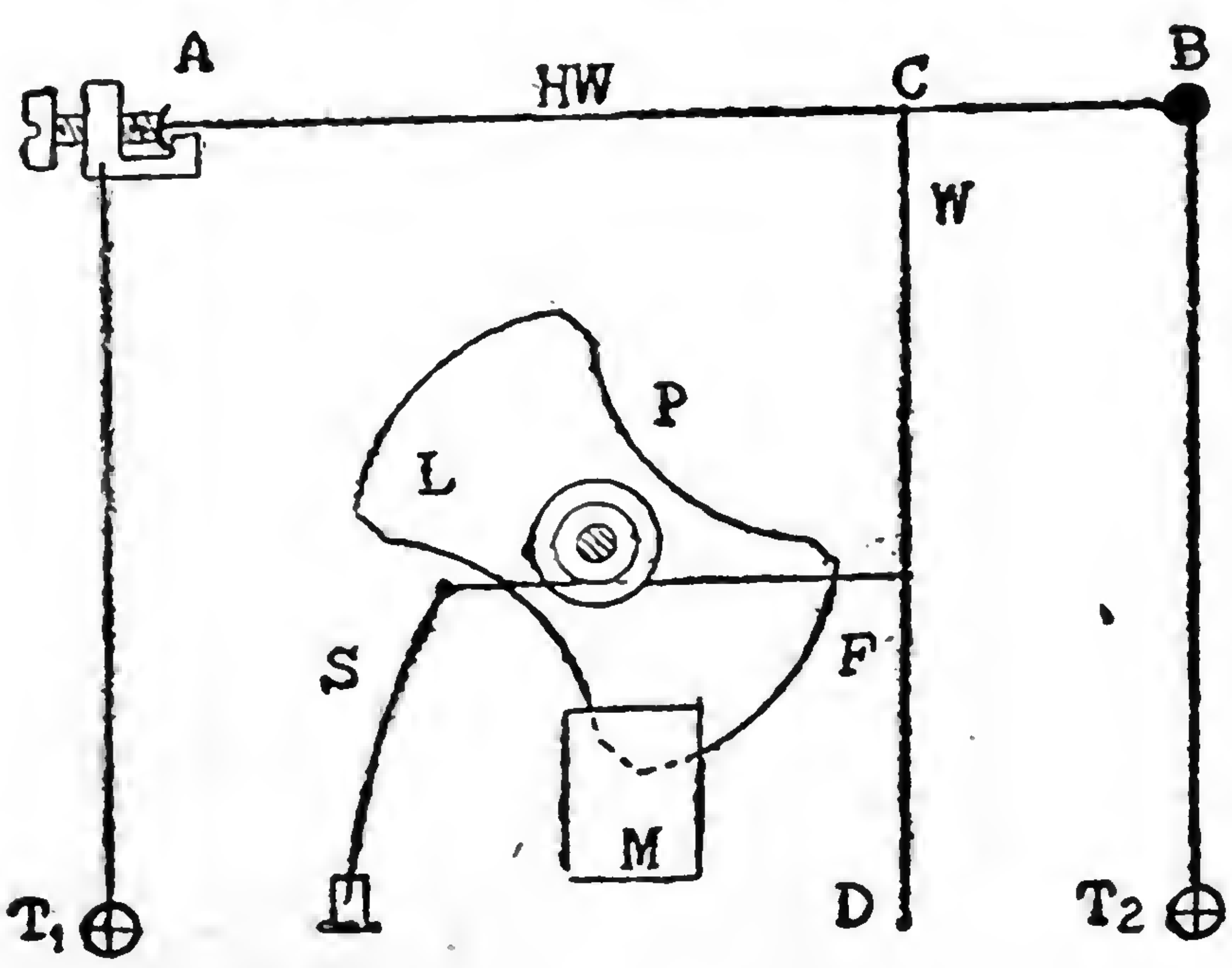


Fig. 9

When *HW* gets heated by a passage of current its sag

has the effect of making W slack. The slack in W is taken up by the fibre thread F one end of which is attached to the spring S. The fibre thread passes round a pulley P, to the spindle of which is attached a pointer. This pointer therefore greatly magnifies the small sag in HW. An aluminium piece L is fixed to the spindle of the pulley. The damping torque is provided by L as it passes in the field of the permanent magnet M.

Since the sag in HW is proportional to the square of the current the scale is uneven. Due to the following reasons these instruments have gone out of favour and should be considered as obsolete:

(a) they are sluggish in action ;

(b) they are very fragile ;

(c) their power consumption is higher than most of the other instruments ; and

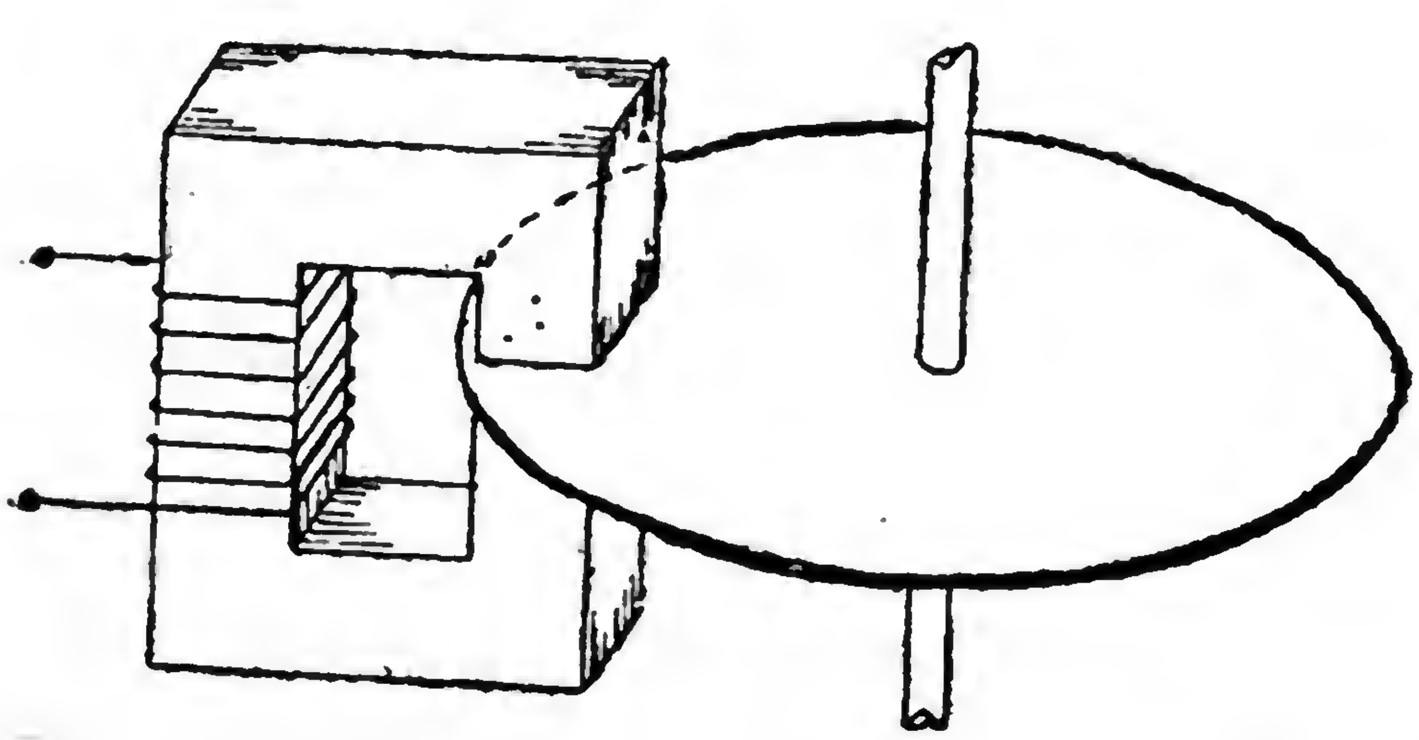
(d) the pointer of the instrument requires frequent adjustment to zero position.

However these instruments possess the following advantages :—

(i) they can be used on both A. C. and D. C. circuits ;

(ii) the deflection depends solely upon the r. m. s. value of the current and is independent of wave form or frequency.

7. Induction Type—Ammeters and Voltmeters : In order to understand the working principles of these instruments it is neces-



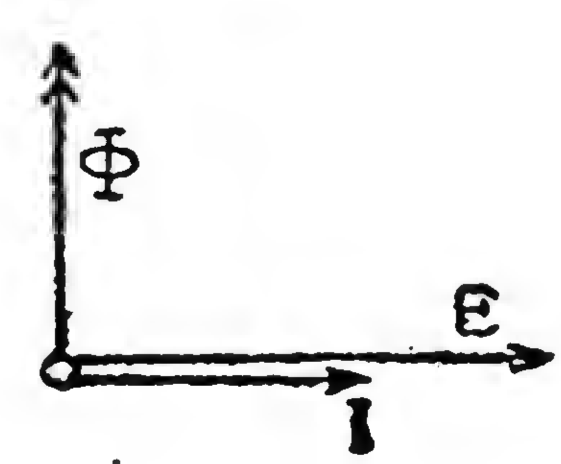


Fig. 10

sary to know how the deflecting torque is produced in the case of ammeters and voltmeters which are worked on one phase only.

Imagine a metal disc, which is free to rotate, placed between the poles of an A. C. electro-magnet, as shown in Fig. 10. The alternating flux sets up eddy currents in the disc and since the path through which these currents circulate is purely non-inductive these currents are in phase with the induced e. m. f. and therefore lag behind the alternating flux by 90°. The vector diagram is also shown in the figure. In order to produce torque it is necessary to have a component of these currents in phase with the A. C. flux.

Now suppose another A. C. electro-magnet is placed next to the first one and supplied from the same source but having a highly inductive circuit. This second magnet will also produce eddy currents in the disc, but the phase of the flux of the second magnet will lag behind the flux of the first magnet. The vector quantities of the two are shown in Fig. 11. The subscripts denote the quantities belonging to the two electro-magnets. With this arrangement it is possible to have a deflecting torque, since I_1 has a component I_1' in phase with Φ_2, and I_2' in phase with Φ_1. The disc can therefore rotate under the influence of the resultant torque. This is known as *splitting the phase.*

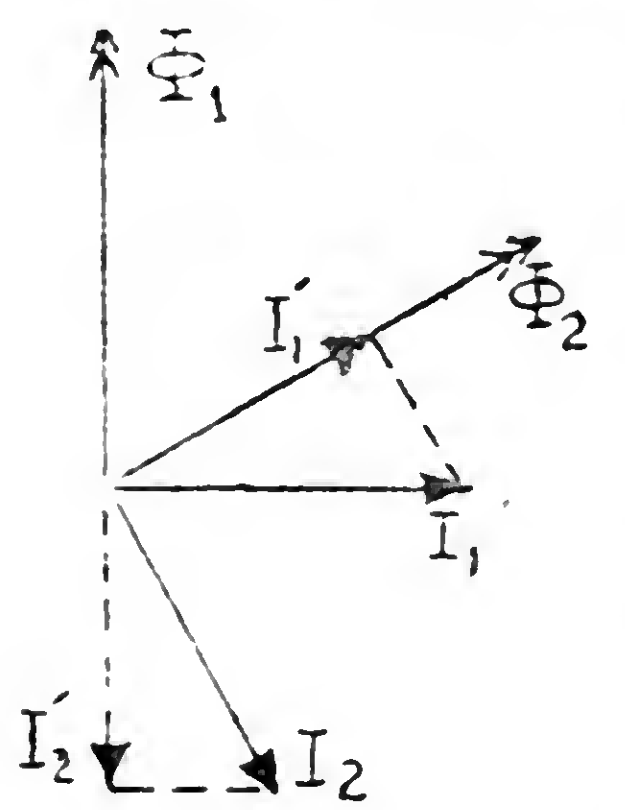


Fig. 11.

Note that the two electromagnets must not be placed diametrically opposite on the disc, otherwise the direction of torque will be through the centre of the disc and there will be no rotation of the disc.

Instead of using two electro-magnets to obtain deflecting torque on a single phase circuit, only one electro-magnet with a "shaded pole" is used. A shaded pole is shown in Fig. 12 where *M* is a portion of the laminated core of an A. C. electro-magnet. Its pole is split into two sections *O* and *P*. The *P* part is completely surrounded by a stout copper sleeve *S*. This sleeve or band acts as a short-circuited secondary and delays the decay of flux in the portion *P* according to Lentz's Law. In other words, the flux due to portion *P* lags behind that due to *O* and therefore the result is the same as that shown in Fig. 11.

The restoring torque is obtained by the use of spiral springs, and the damping torque by embracing another portion of the disc by a permanent magnet. The accuracy of these instruments is 2nd grade and since the deflecting torque is produced by eddy currents due to transformer action, any change in frequency will affect the readings. These instruments are used on A. C. circuits only and their chief advantage lies in the fact that the spread of the scale obtainable is almost 300°.

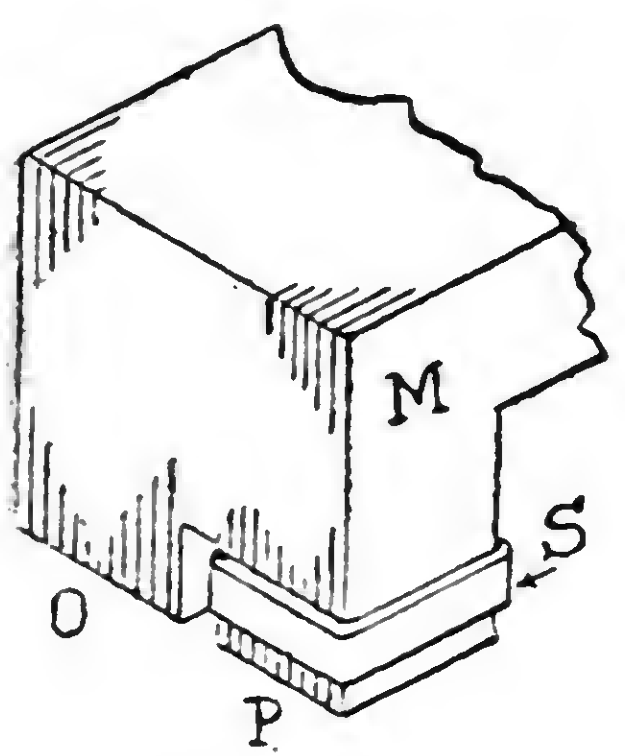


Fig. 12

8. **Induction Type—Wattmeters** : Fig. 13 shows the general arrangement of a single-phase wattmeter. Since there has to be two electro-magnets, one for the current element and the other for the voltage element, the device of the shaded pole is unnecessary. The core of one electro-magnet is similar to that of a shell type transformer and a coil is wound on the central limb. This is usually the voltage element. The core of the other electro-magnet is U-shaped and is placed on the other side of the rotating disc D. The current coil is placed on this core. The resulting vector quantities are shown in Fig. 14.

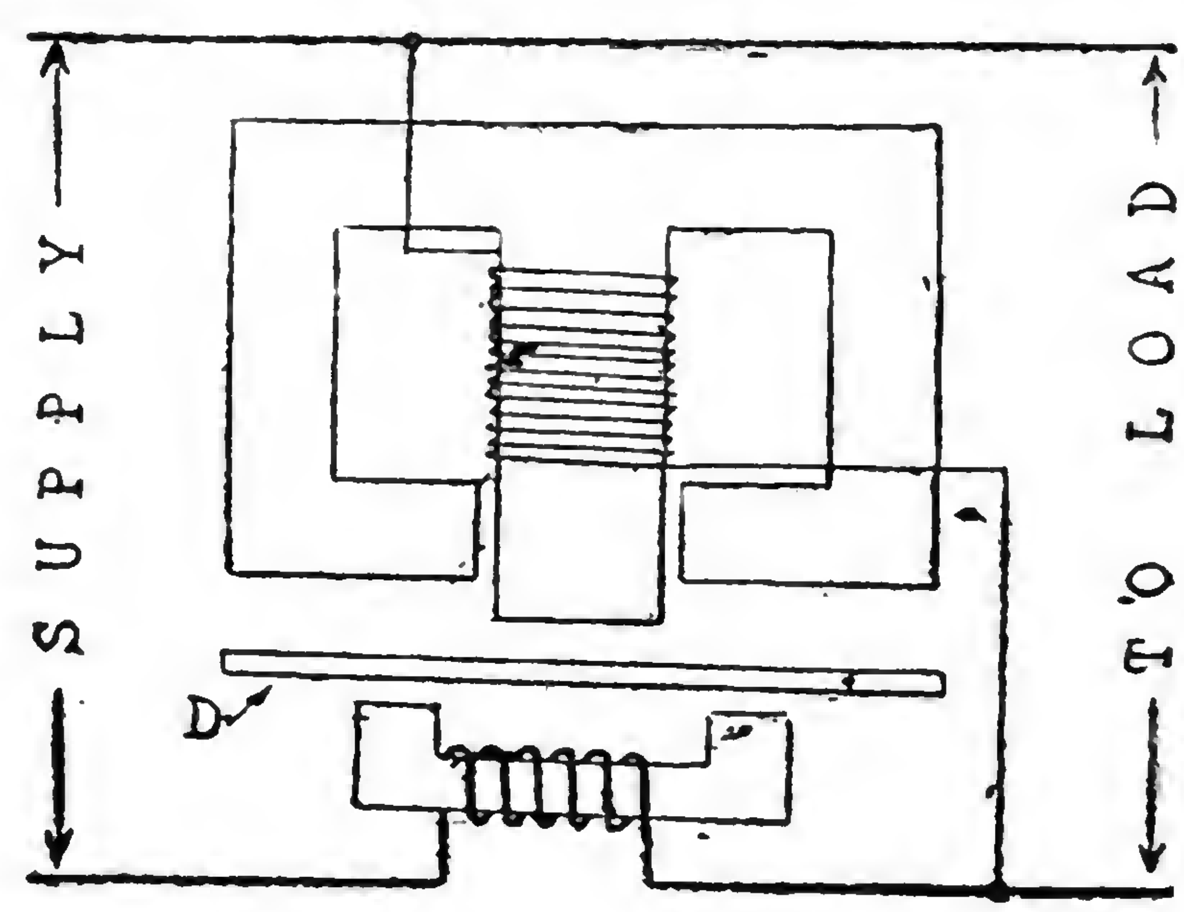


Fig. 13

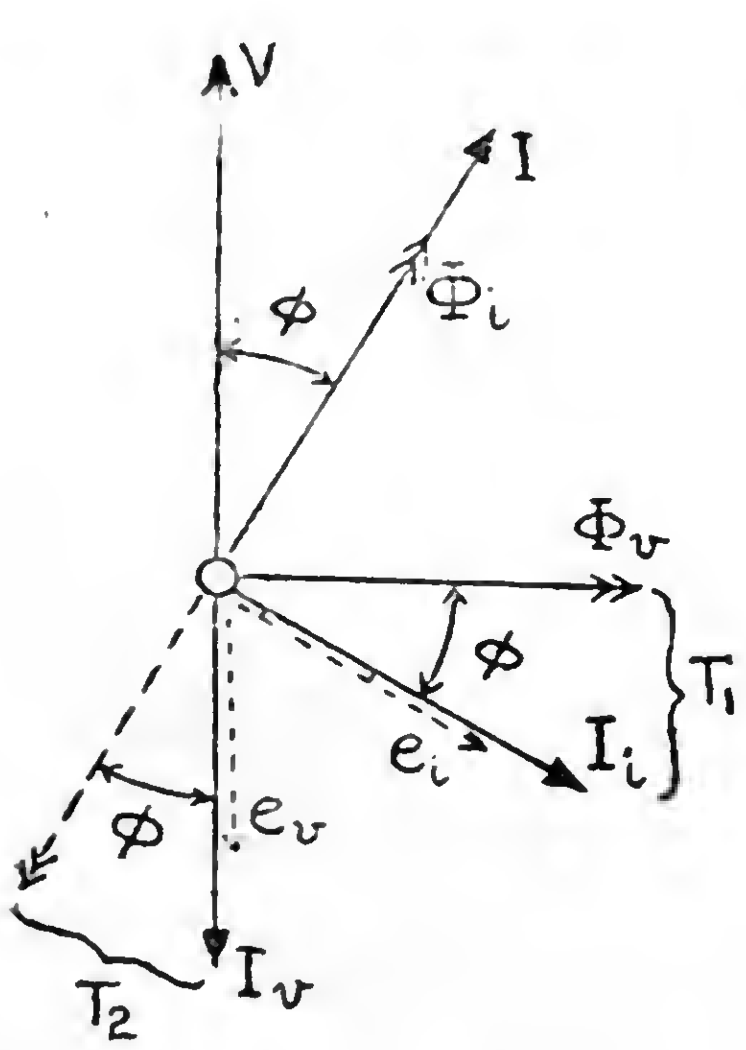


Fig. 14

It will be clear from the vector diagram that the disc experiences the torques T_1 and T_2. T_1 is due to Φ_v and I_i, i. e.

$$T_1 \propto \Phi_v I_i \cos \phi \quad \dots \quad (\text{i})$$

Similarly

$$T_2 \propto \Phi_i I_v \cos (180° - \phi)$$

$$\propto - \Phi_i I_v \cos \phi \quad \dots \quad (\text{ii})$$

Now Φ_v and I_v are proportional to V, and Φ_i and I_i are proportional to I. Both the expressions (i) and (ii) are therefore proportional to V. I. cos ϕ, where V and I are the circuit voltage and current and cos ϕ is the circuit power factor. Hence the net torque $\propto$ power.

These wattmeters are inferior in accuracy to the dynamo-meter-type of wattmeters, and they are chiefly used on switch boards on account of their scale having a spread of nearly 300°.

9. Electrostatic Voltmeters: The instrument consists of several sets of fixed metal sectors, such as *a*, *b*, *c* and *d* shown in Fig. 15. They are separated from each other by air-gaps. In between the sectors there are aluminium vanes *m* mounted on a spindle, which is either suspended by a phosphor-bronze wire or is mounted in jewelled bearings.

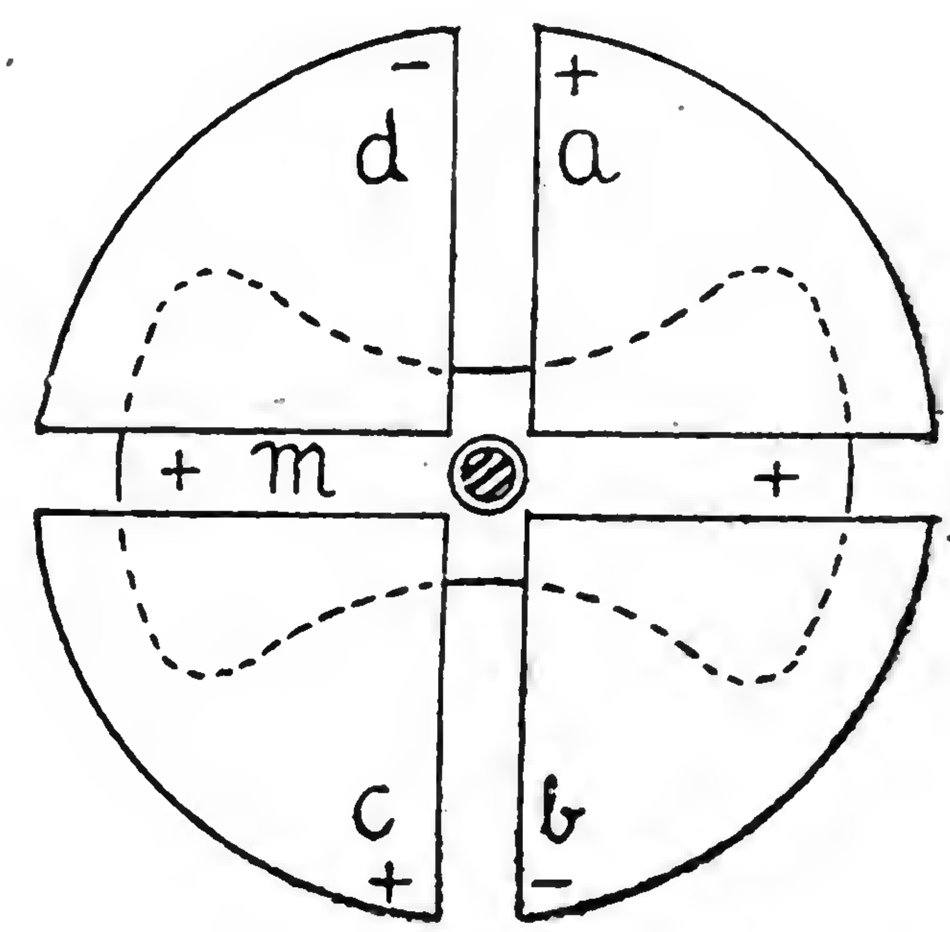


Fig. 15

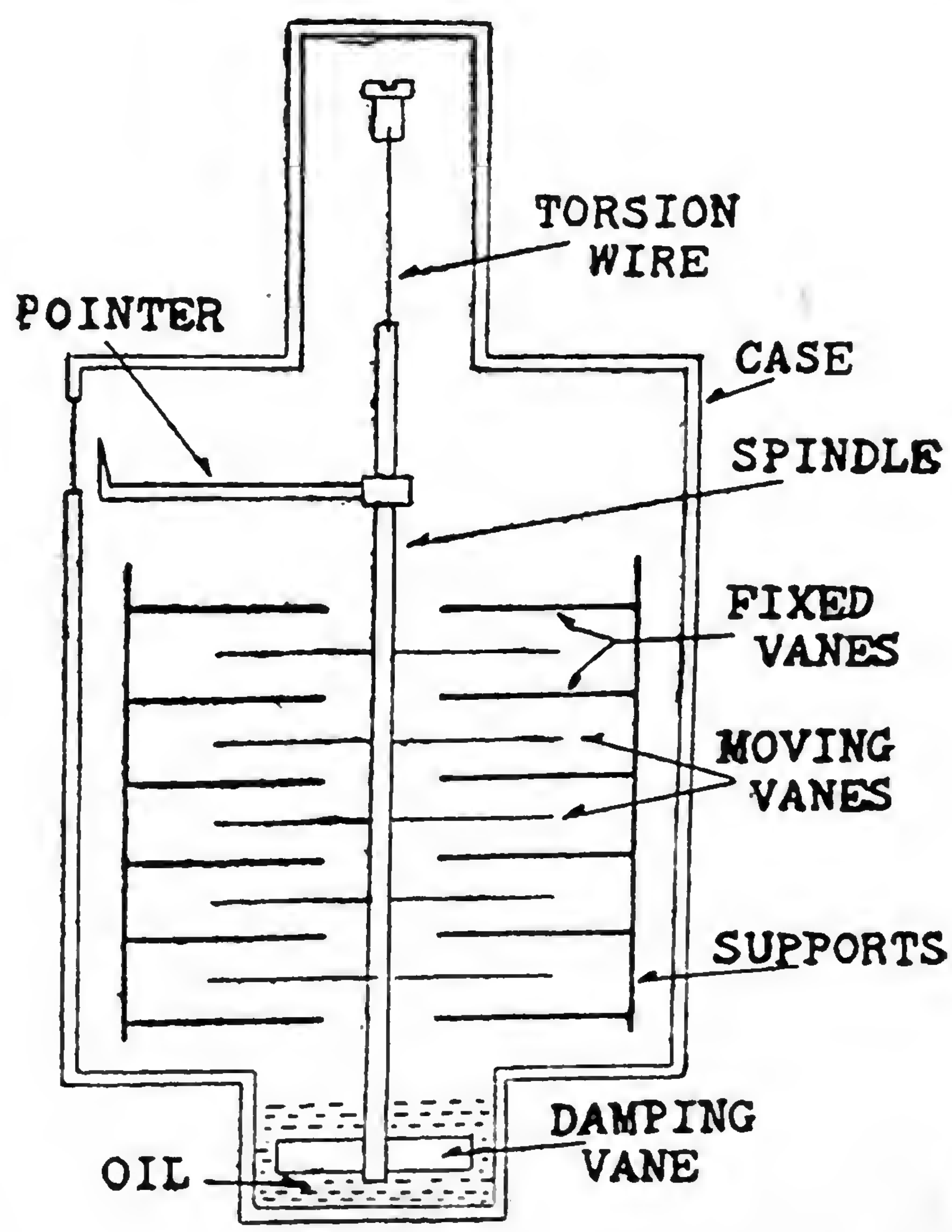


Fig. 16

a, *c* and *m* are in electrical connection with one of the supply lines, while *b* and *d* are connected to the other supply line. The deflecting torque is due to a force of attraction between those sectors and vanes between which a potential difference exists, i. e. between oppositely charged bodies.

In the suspended type the controlling torque is provided by torsion in the wire and the damping torque by a vane immersed in oil. See Fig. 16. In the pivoted type a spirally wound spring is the source of controlling torque and damping is pneumatic. The pointer of the instrument is attached to the spindle. The scale is uneven since the torque is proportional to the square of the voltage. The accuracy is 1st grade.

These are essentially laboratory instruments, and they are used on high voltage A. C. and D. C. circuits. However if the number of sets of sectors and vanes are increased the range of these instruments can be lowered to about 200 volts.

10. The "Megger": The "megger" is a ***true ohmmeter*** which is not a precision instrument, but most indispensible in determining whether a circuit is good or faulty, or whether an apparatus or a machine has sufficient insulation resistance. The following is a description of the instrument manufactured by Messrs. Evershed and Vignoles of London.

Bridge—Megger testing sets are self-contained testing instruments. They comprise the following components :—

1. ***A hand driven generator*** to produce the testing voltage. It is hand-driven through gearing and centrifugally controlled clutch, which slips when the speed of the generator exceeds a predetermined value. This ensures ***constant testing voltage*** and ***steady readings*** on the scale.

2. ***A change over switch*** which has normally two positions one of which, marked "Meg or Megger", is used when making insulation tests, and the other, which is marked "Bridge", for Wheatstone Bridge tests. Series 2 instruments, suitable for Varley Tests, have a third position marked "Varley".

3. *An ohmmeter* for measuring directly the value of the insulation resistance. It is of the true moving-coil type (D'Arsonval) without spring control, the accuracy of the indications being independent of voltage variation.

When the instrument is used as a Wheatstone Bridge, the ohmmeter movement acts as the galvanometer of the bridge.

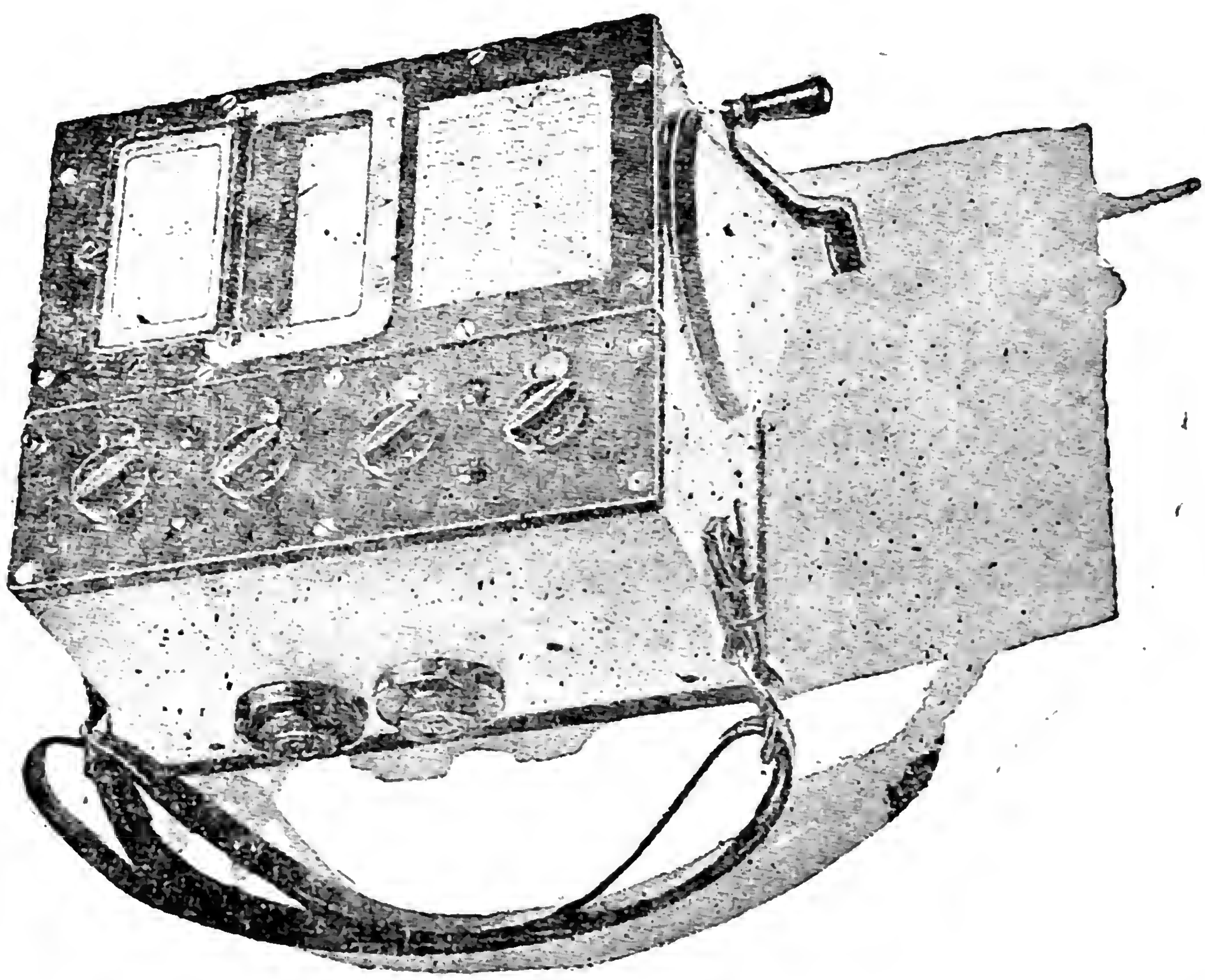


Fig. 17. Bridge–Megger Tester Series 2.
(By courtesy of Evershed & Vignoles Ltd.)

4. *A direct reading adjustable resistance* which is switch operated. The total value of resistance shown on four dials as a row of figures.

5. *A ratio switch* which adjusts the two remaining arms of the Wheatstone Bridge. When the bridge is in balance the value of resistance under test is equal to that shown on the four dials of the adjustable resistance multiplied or divided by the ratio in use.

Fig. 17 shows the *Bridge Megger Tester Series 2.* It has normally two terminals marked "Line" and "Earth" and these are used for both insulation and bridge tests. Instruments incorporating Varley Test facilities have a third terminal marked "Varley Earth".

Fig. 18 shows the *Bridge-Megger Testing Set, Series I*, which consists of two units, the adjustable resistance being contained in a separate case from the instrument to which it is connected by two short leads. There are two terminals, marked "Line" and "Earth", which are used for insulation tests, and at the end of the case are two pairs of terminals marked "R" and "X" for connection to the

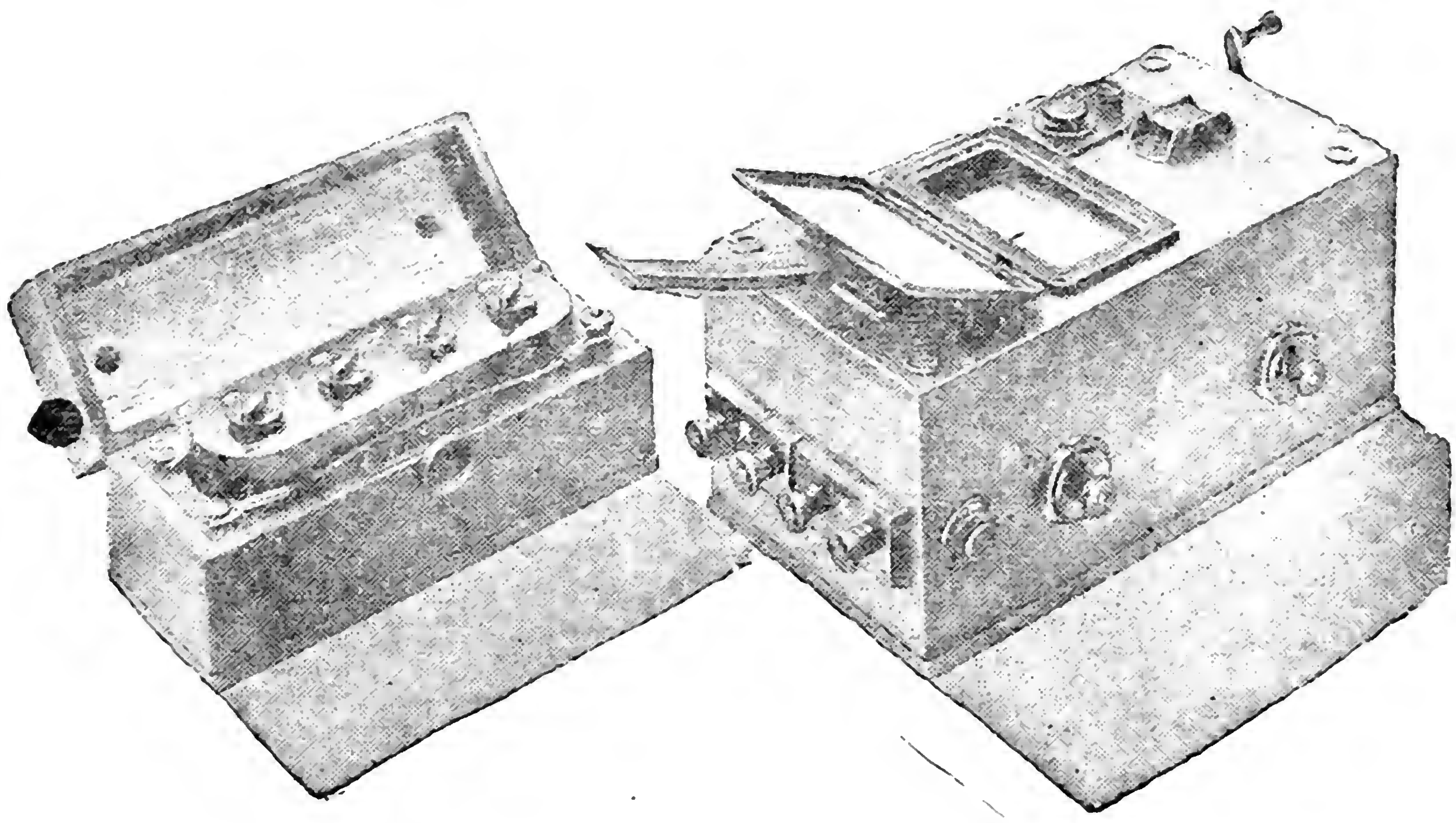


Fig. 18. Bridge-Megger Testing Set Series 1.

(*By courtesy of Evershed & Vignoles Ltd.*)

resistance box and the resistance under test when making bridge tests. Instruments scaled to 1000 megohms and over have a third terminal on the side of the case, marked "Guard", for eliminating the effects of surface leakage, and high sensitivity instruments are provided with levelling screws. All Bridge-Megger Tests, Series 1, are suitable for carrying out Varley Tests.

Insulation Tests

Principle of Operation: The moving element of the ohmmeter consists essentially of two coils, the *control* and the *deflecting coil*, See Figs. 19 and 20, both connected in parallel across the generator and arranged so as to oppose one another. The control coil is in series with a fixed resistance, whereas the deflecting coil is in series with the resistance under test.

There is no control spring, and the position of the movement is determined by the ratio of the currents in the two coils which varies solely with the value of the resistance under test, since changes in the applied voltage affect both coils in the same proportion. The instrument may therefore be calibrated directly in ohms and megohms and is essentially a *true ohmmeter.*

To guard against surface leakage between terminals, the "Line"

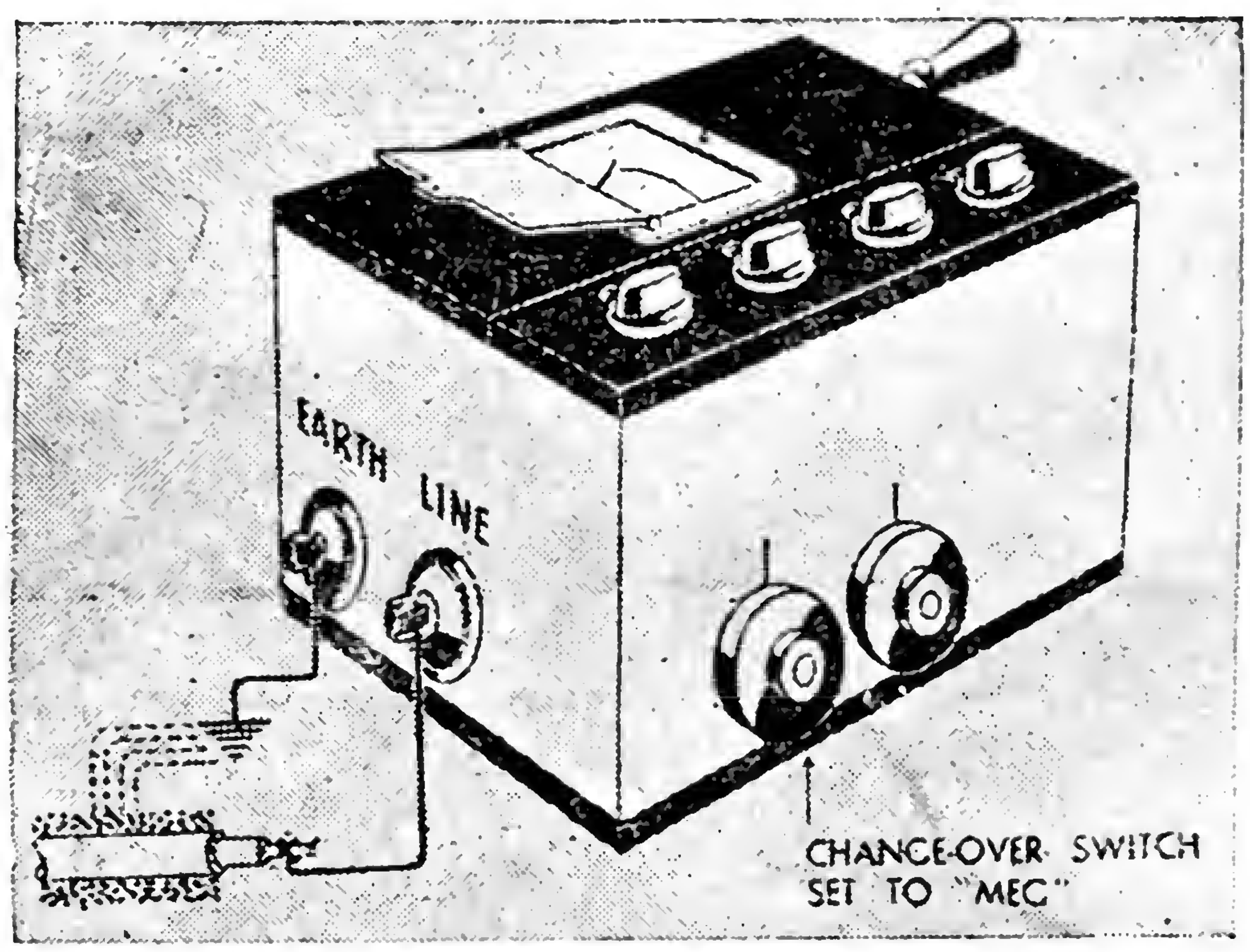


Fig. 19 (a)

terminal is surrounded with a metal *guard-ring*, so that any leakage current is returned directly to the generator without passing through the deflecting coil. In a similar way, the "Guard" terminal on a high sensitivity Series 1 Testing Set is used to guard against the effects of surface leakage on apparatus under test (Figs. 8 and 9).

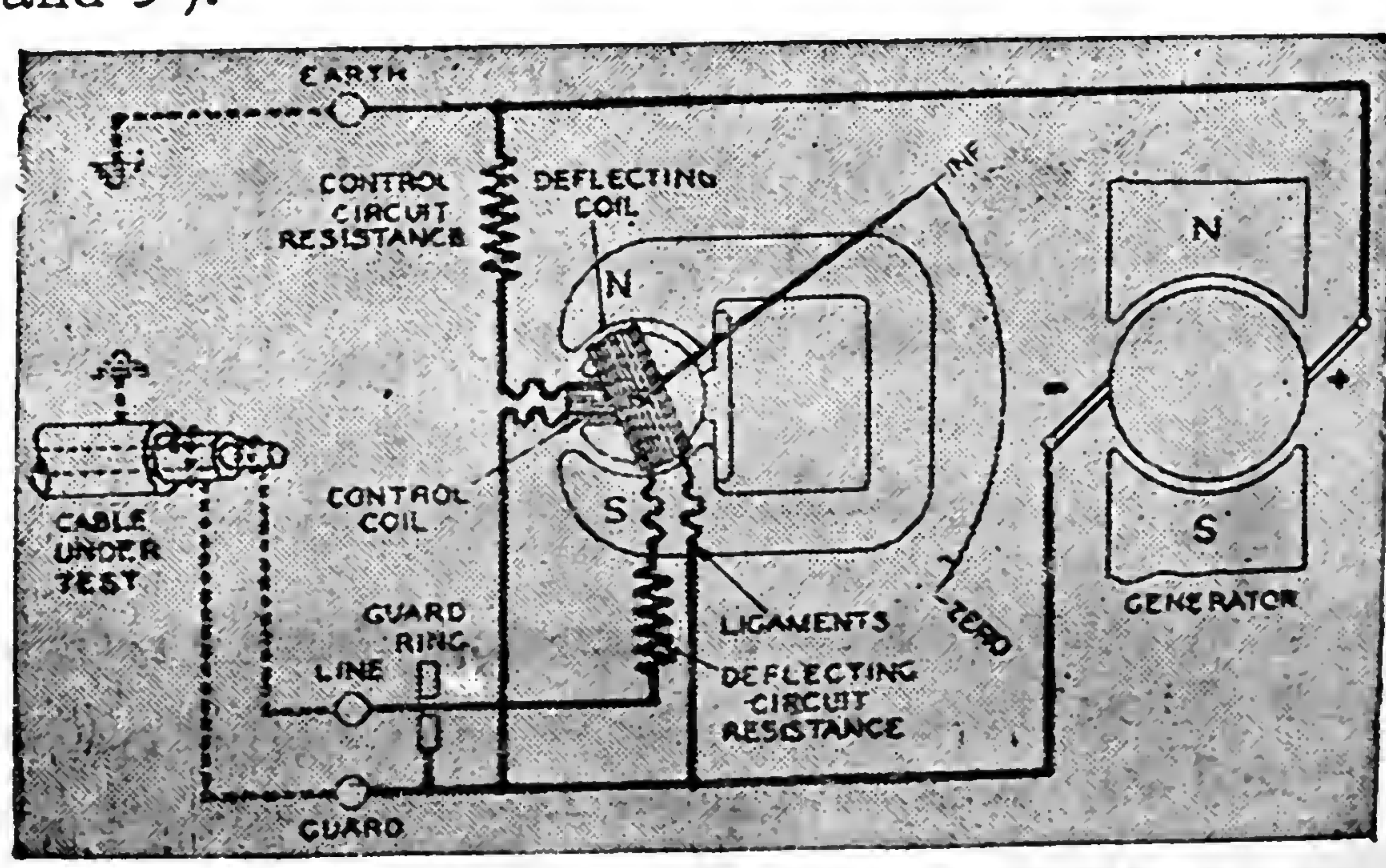
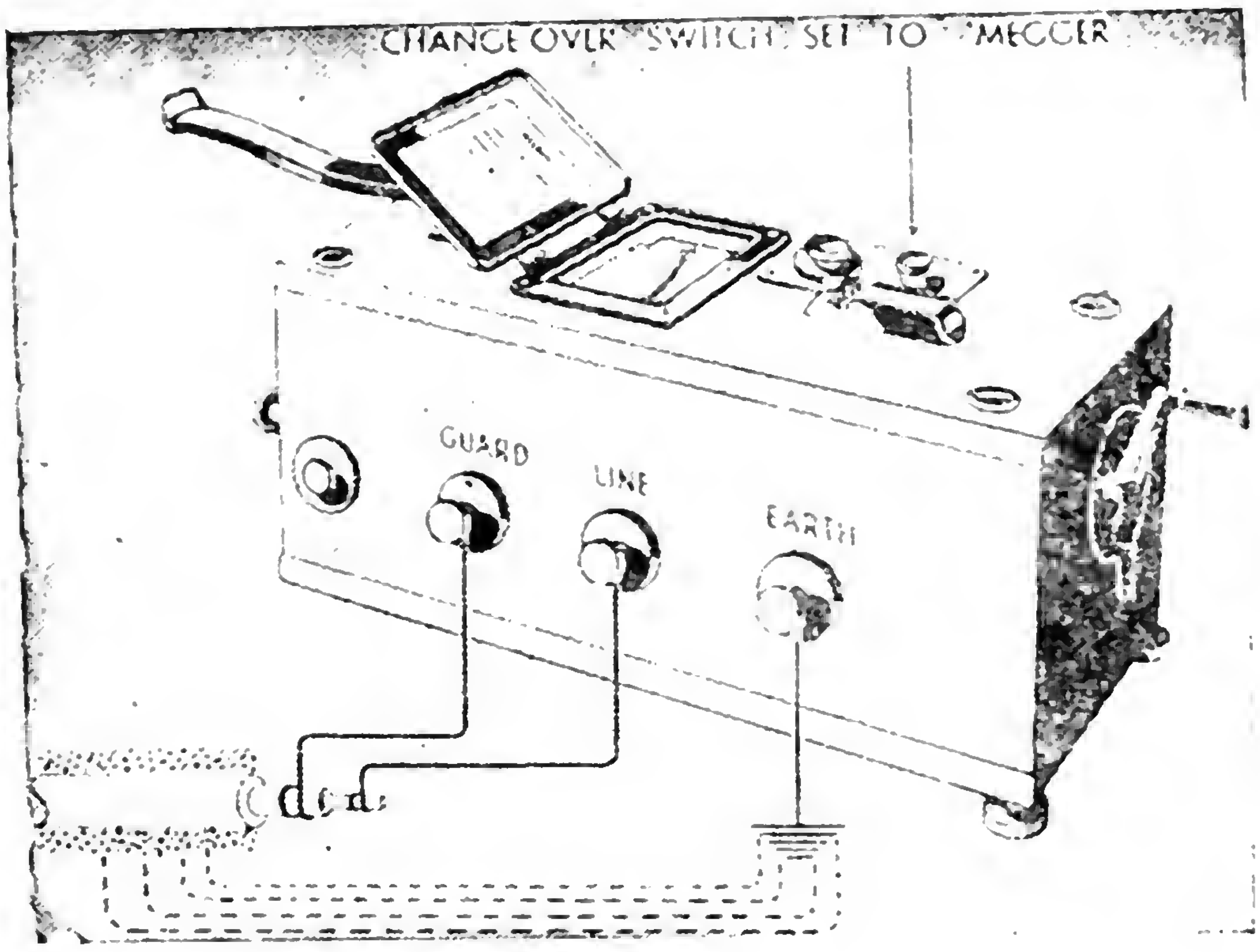


Fig. 19 (b) Insulation Tests–Series 2

(By courtesy of Evershed and Vignoles Ltd.)

Method of Use : Set the change-over switch to the position marked " Meg " or " Megger ". To test, for example, between circuit and earth connect the " Line " terminal to the circuit and the " Earth " terminal to a good earth. If the instrument is fitted with a "Guard" terminal connect as shown in Figs. 19 and 20.

Turn the handle at such a speed that the clutch is felt to slip and read the insulation value in megohms or ohms on the scale.



(a)

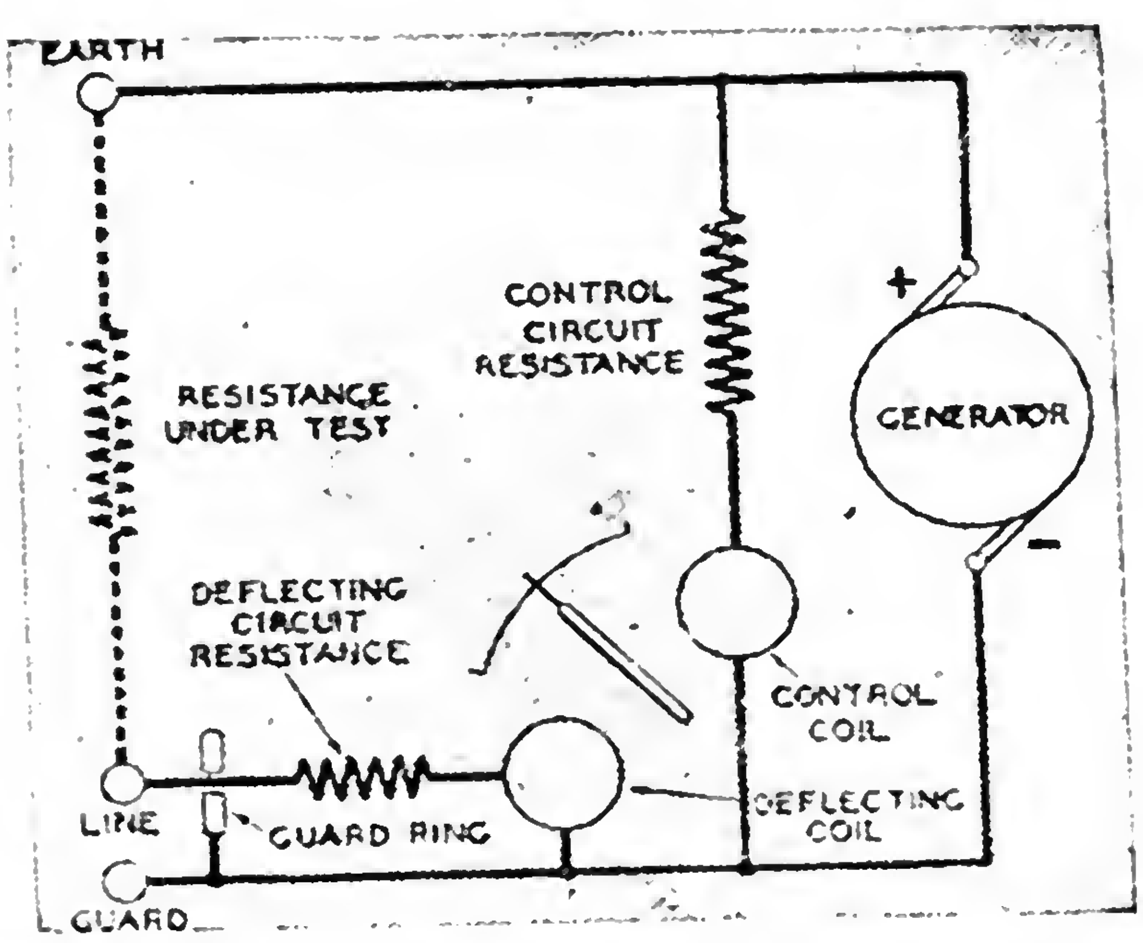


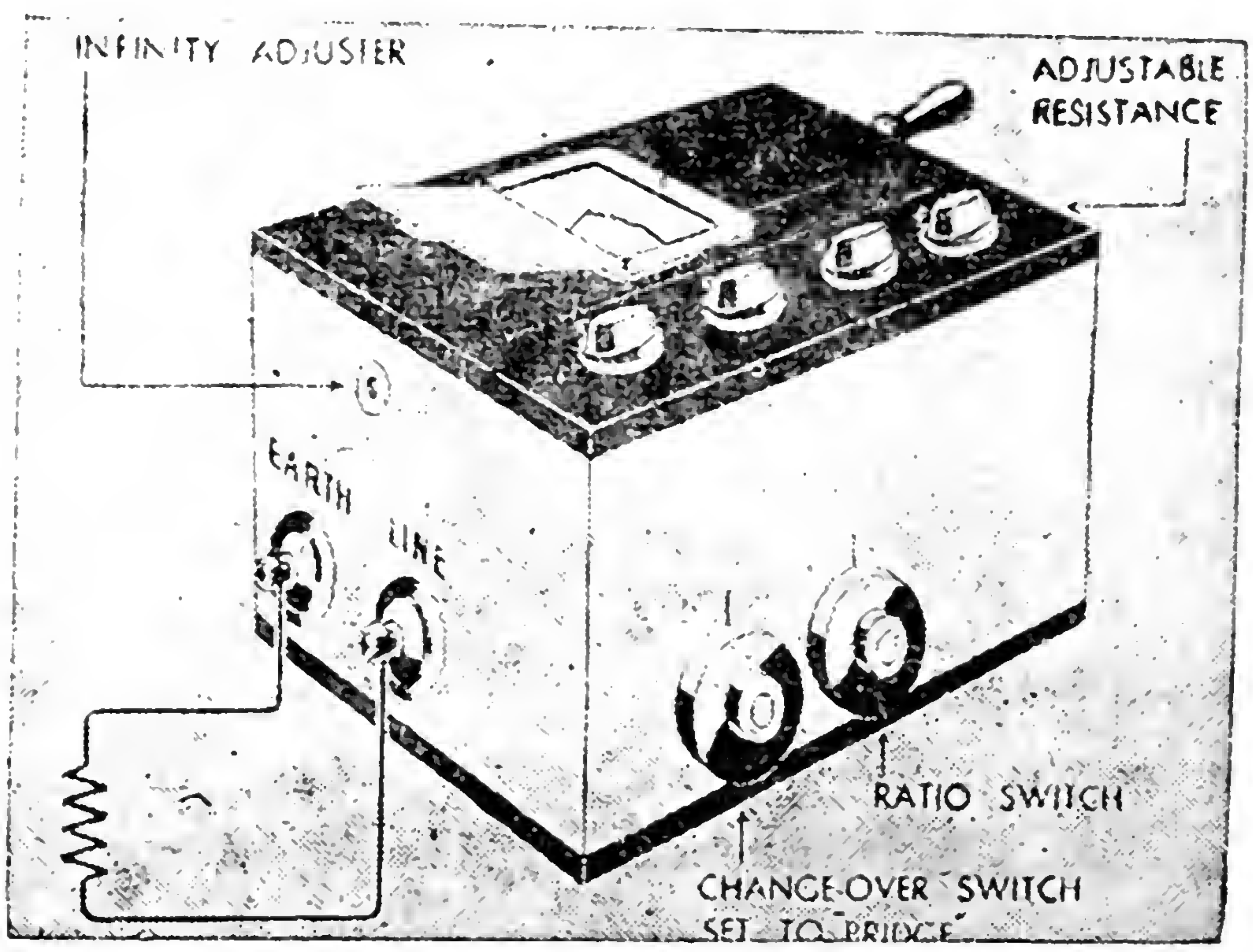
Fig. 20 (b) Insulation Tests–Series 1
(*By courtesy of Evershed & Vignoles Ltd.*)

Bridge Tests

Principle of operation : when the change-over switch is set to " Bridge ", the instrument is transformed into a Wheatstone Bridge. The deflecting coil serves as the galvanometer coil, and the control coil acts as a control spring. When no current passes in the deflecting coil, the force exerted by the control coil brings the moving system into the position in which the pointer reads " Infinity ", so that the " Infinity " reading corresponds to a balanced condition of the bridge.

Method of use : Before commencing tests see that the true point of balance is exactly on the " Infinity " mark on the scale. To

do this, set the change-over switch to "Meg" or "Megger", and with nothing across the terminals turn the generator at full speed. If the pointer is not exactly on the "Infinity" mark, bring it there by means of the "Infinity (or Index) Adjuster".



(a)

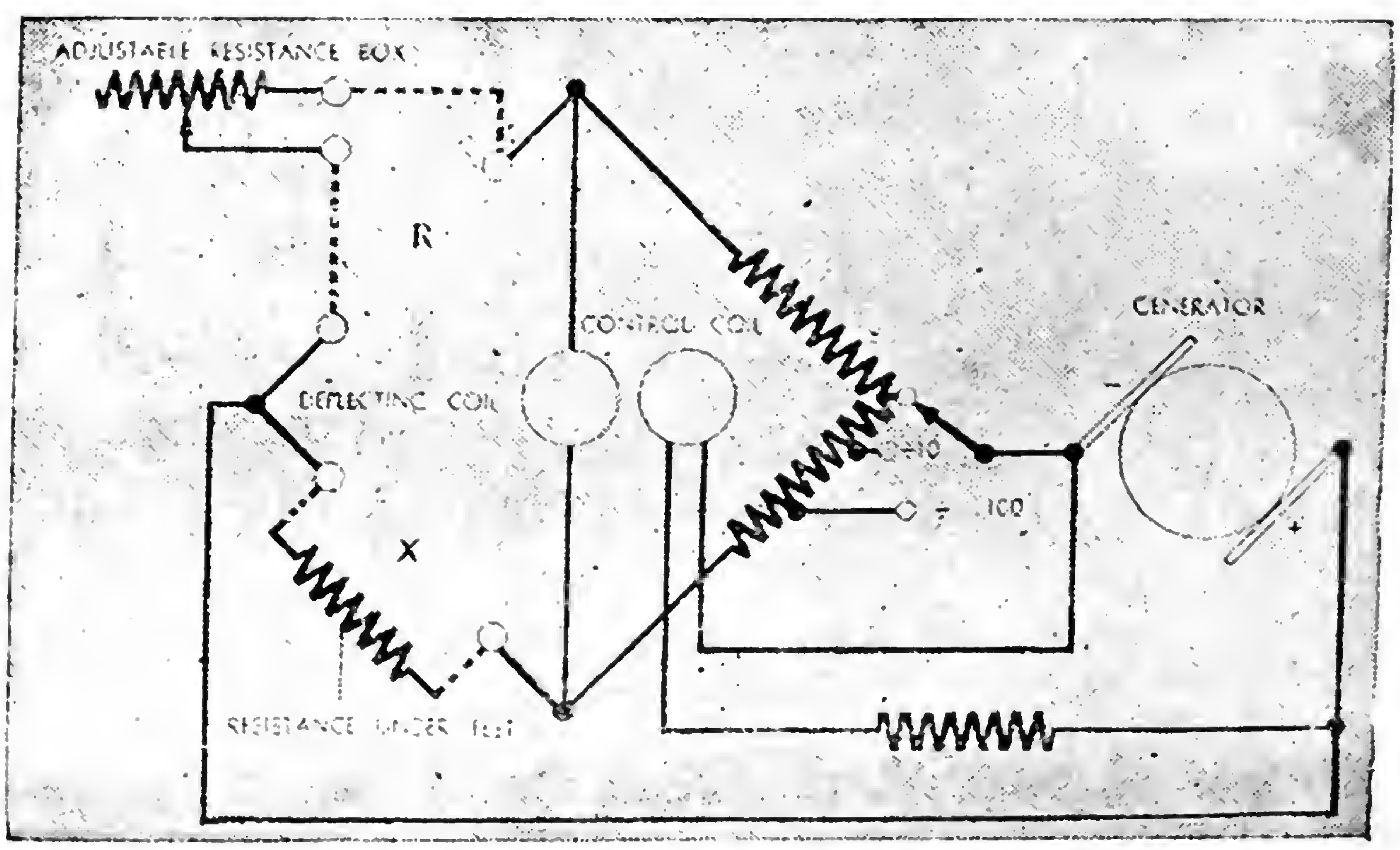


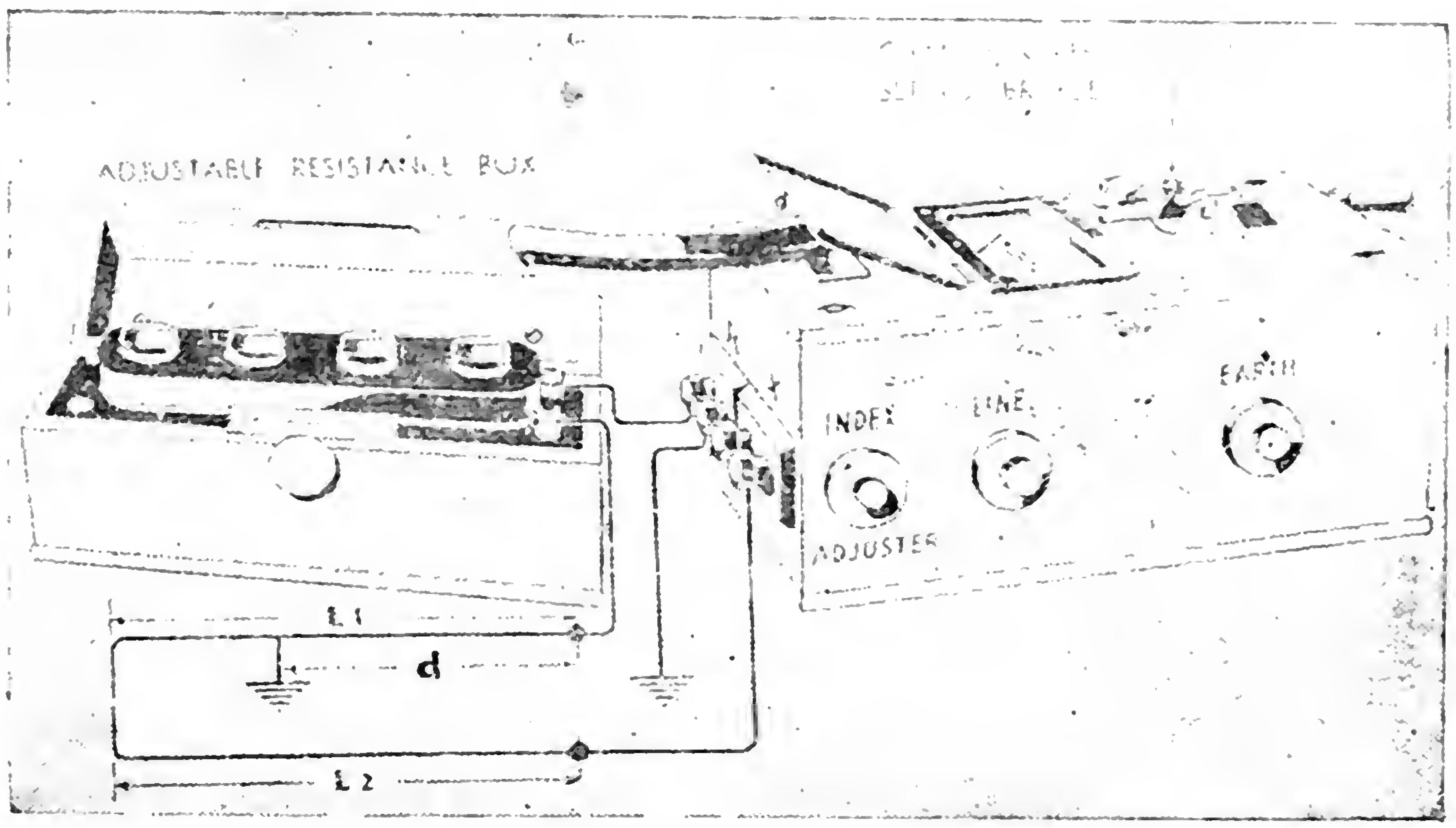
Fig. 21 (b) Bridge Test Series 2

(*By courtesy of Evershed & Vignoles Ltd.*)

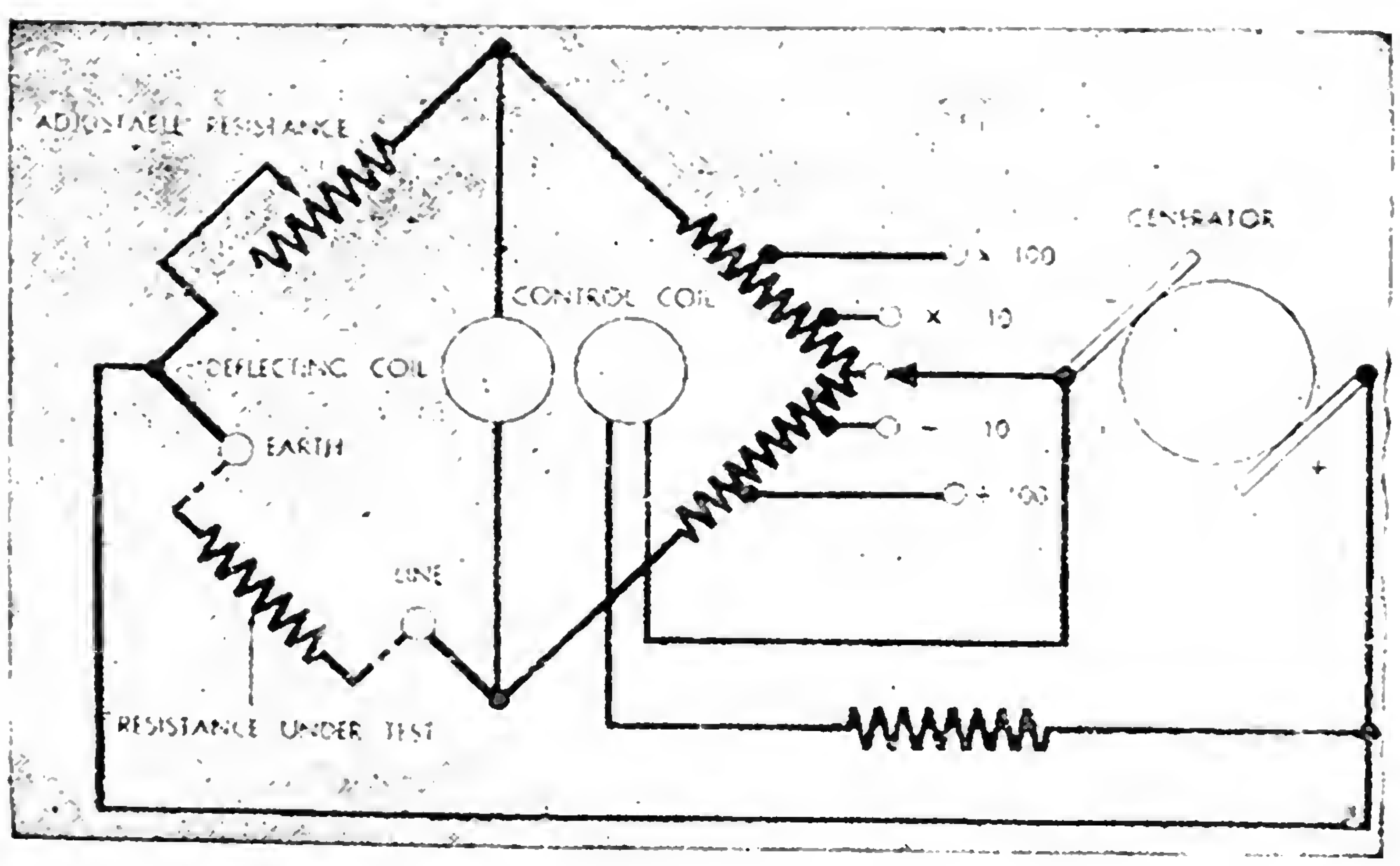
Then set the change-over switch to "Bridge", the ratio switch to "1", and the four resistance dials to "0". Connect up as shown in Figs. 21 and 22.

Turn the generator handle while raising the value of the adjustable resistance step by step until the pointer rests on the "Infinity" mark. Since the ratio switch is at "1", the value of the resistance

under test will then be equal to that shown on the four resistance dials.



(a)



(b)

Fig. 22. Bridge Tests Series 1
(Evershed and Vignoles Ltd).

If the resistance proves to be below 1000 ohms, greater accuracy is obtained by setting the ratio switch to " ÷ 10", and if below 100 ohmn to " ÷ 100". The value of the resistance under test is then equal to the reading of the dials divided by 10 or 100 respectively.

Similarly, when testing conductor resistances above 9999 chms, the ratio switch on the Series 2 instrument should be set to " × 10"

or "×100". With the series 1 Testing Set, the multiplying factor is obtained by interchanging the connections to the "R" and "X" terminals.

11. Wheatstone Bridge: This consists of an arrangement whereby resistances can be measured accurately. The unknown resistance X and three known resistances A, B and C are arranged to form a diamond as shown in Fig. 23. A galvanometer G is connected across two points of the diamond, while across the other two points a battery E is connected.

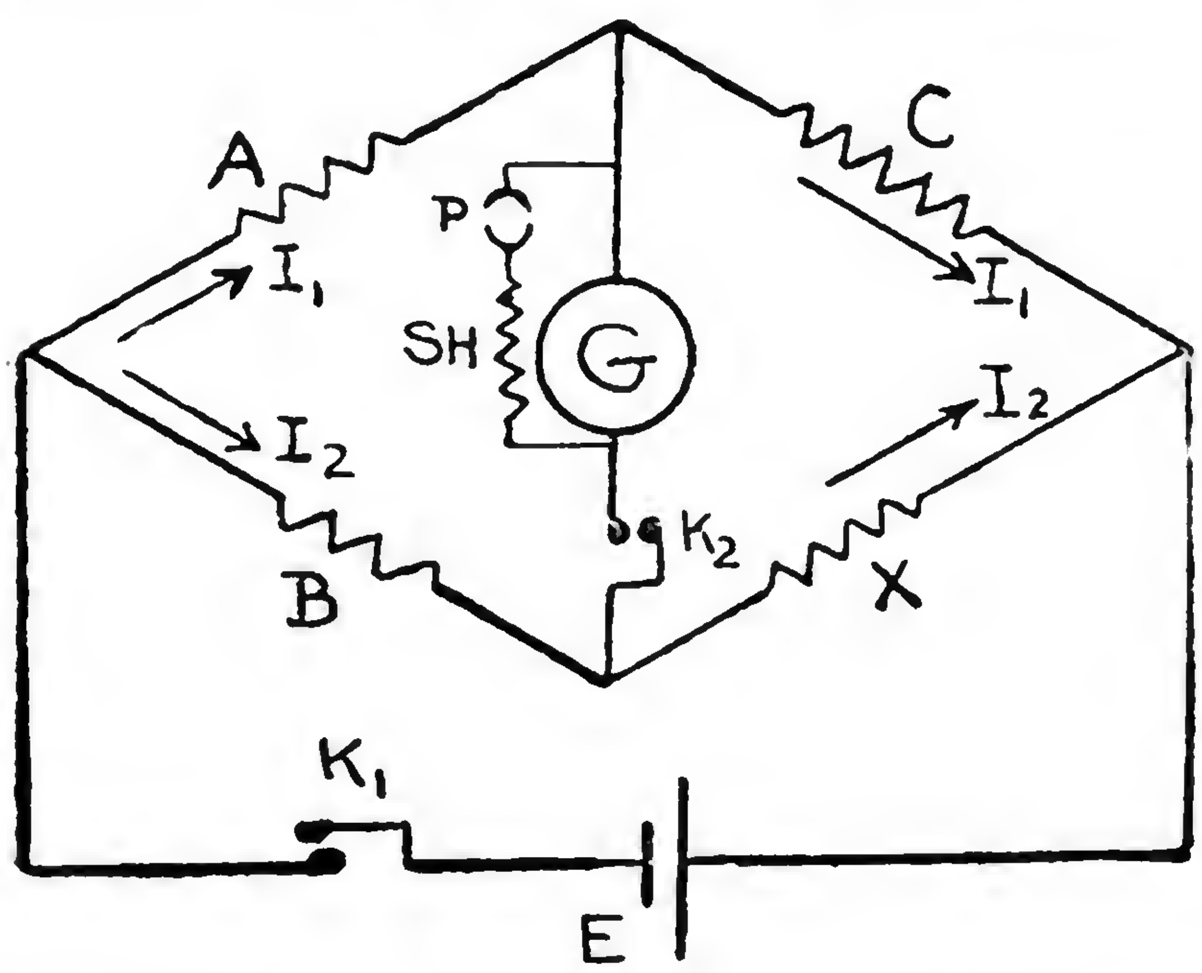


Fig. 23 Wheatstone Bridge.

The resistances A and B are of decimal values, such as 1, 10, 100 ohms. The resistance C is adjustable from 1 ohm to 1100 ohms. When the four resistances are balanced, no current flows through the galvanometer. In other words, the potential at both ends of the galvanometer is the same. Hence, the current I_1 flowing in A also flows in C, and I_2 flowing in B also flows in X. Hence

$$I_1 A = I_2 B \quad \dots \quad \dots \quad \dots \quad \text{(i)}$$

and

$$I_1 C = I_2 X \quad \dots \quad \dots \quad \dots \quad \text{(ii)}$$

Dividing (ii) by (i), $\dfrac{I_1 C}{I_1 A} = \dfrac{I_2 X}{I_2 B}$

cancelling and solving for X, $X = C\dfrac{B}{A}$,

which is the equation of the Wheatstone Bridge. A and B are the *ratio arms* and C is the *balance arm*. The position of the battery and the galvanometer are interchangeable. However the position of the galvanometer should be such as its sensitivity does not diminish by the interchange of position. The galvanometer has greater sensitivity if it is connected at those points, the equivalent resistance between which is nearer the galvanometer resistance under balanced condition.

Usually a tap-key is included in series with the battery and another in series with the galvanometer. For making observations

the battery key is pressed first then the galvanometer key. A shunt must be used with the galvanometer to protect it when the bridge is very much out of balance, or a high resistance in series with the galvanometer serves the same purpose. When balance is obtained the shunt or the series resistance should be out of circuit so as to get perfect balance.

The simplest form of Wheatstone Bridge is shown in Fig. 24. It is called the *Slide-wire Bridge.* It consists of a wire AB made of

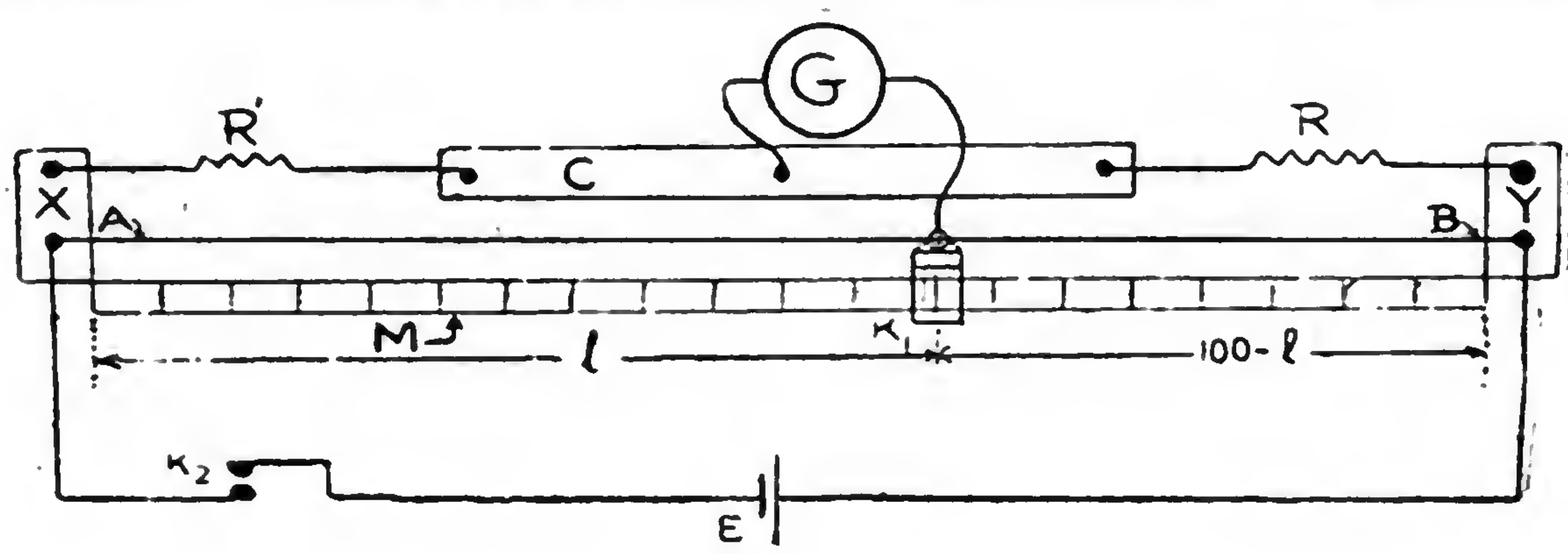


Fig. 24. Slide-Wire Bridge

some alloy having a high resistance and uniform cross-section. It is stretched between two copper blocks X and Y, and is 100 cm long. A metre scale M is either placed under the wire throughout its length or it is placed by its side, and a moveable contact key K_1 when pressed makes a knife edge contact with the wire. There is also a heavy copper bar C, to the mid-point of which one wire from the galvanometer G is connected. The other wire from the galvanometer is connected to K_1. The battery is connected between the blocks X and Y with a tap-key K_2 in series with it.

The unknown resistance R' is connected between X and C and a known resistance R is connected between C and Y. When balance is obtained, there is no current through the galvanometer, and the contact key K_1 divides the wire AB into two parts, l and $100-l$. If r is the resistance per cm length of AB, then from the figure

$$\frac{R'}{lr} = \frac{R}{(100-l)\,r}.$$

Cancelling r from both sides and solving for R',

$$R' = R\frac{l}{100-l}$$

12. The Potentiometer and its Uses: A potentiometer is an apparatus for making accurate measurements of voltages or e. m. fs.

The potentiometer makes the measurements with high precision because it takes practically no current and the results obtained with the potentiometer measurements are far superior to those obtained by standard voltmeters.

The chief principle of the potentiometer is to oppose a known standard e. m. f., such as that of a Weston Cadmium cell, by an unknown e. m. f. A galvanometer is included in the circuit. Fig. 25

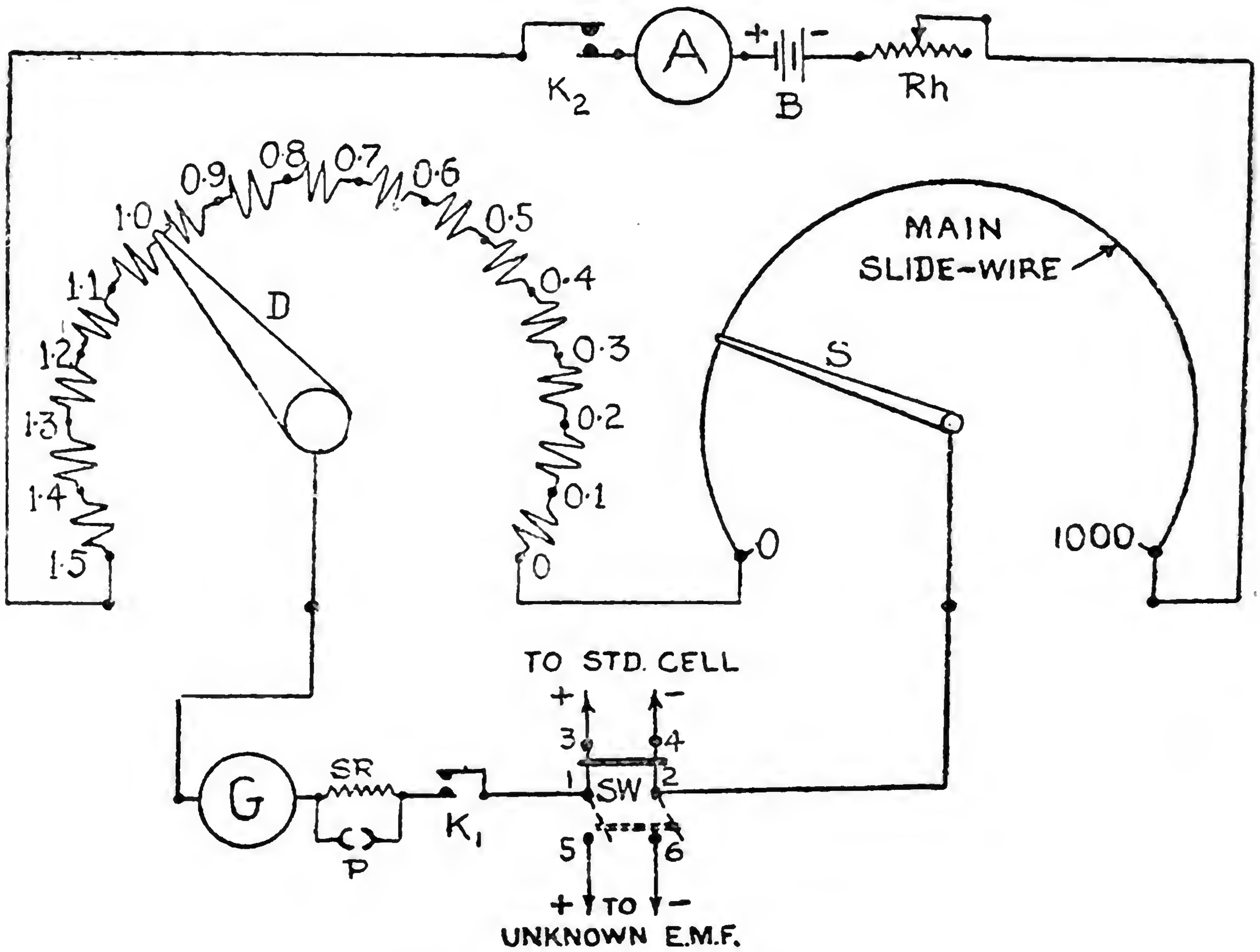


Fig. 25. Potentiometer.

shows a potentiometer in its simplest form. It consists of a *dial switch D* and a *main slide-wire* both connected in series. A low voltage battery sends any desired current through both of these. The dial switch is divided into 15 sections. The resistances of all the sections are the same. The resistance of the slide wire is equal to the resistance of one section on the dial switch, and it is divided into 1000 parts, which can be read off from a scale.

When the current flows through the resistors, the dial switch and the slide-wire, each resistor section will have a certain definite voltage drop across it. For instance, if the value of resistance of each section is 5 ohms and a current of 0·02 ampere passes through

the whole slide-wire and resistors, the voltage drop across each resistor will be $0{\cdot}02 \times 5 = 0{\cdot}1$ volt.

But whether this 0·1 volt is standard or not is as yet not known since neither the ammeter is standardised nor the resistance values of the resistors. Therefore to standardise the voltage drop across each resistor and the slide-wire the potentiometer must be *calibrated* with the help of a *standard cell.* To calibrate the potentiometer the procedure is as follows :—

The potentiometer is connected as shown in Fig. 25. Rh is a rheostat which varies the current of the battery B. The ammeter A is placed in the circuit merely as a guide to know the approximate value of current passing through the potentiometer. The resistance SR, in series with the galvanometer, is necessary to safe-guard the galvanometer. There is, however, a plug P provided so that finally SR is out of circuit or is shunted. If the voltage of the standard cell is, say, 1·018 volt, the dial switch arm D is placed on index 1·0, as shown in the figure, and the slide-wire is so set that it reads 0·018 or 18 divisions of the scale. Key K_2 is pressed and the rheostat is so adjusted that the ammeter reads 0·02 ampere. The standard cell is connected to terminals 3 and 4 of the double-pole double-throw switch SW in such a manner that it must oppose the voltage of the battery or that portion of the potentiometer against which the arms D and S are standing. In the figure, terminal 1 of the D-P D-T switch is positive, hence the positive terminal of the standard cell must be connected to terminal 3.

Close the D-P D-T switch on terminals 3 and 4, and with the plug out of P, press K_1 and K_2. The galvanometer will deflect from its centre zero position to one side. Adjust the rheostat Rh until the pointer of the galvanometer returns to zero. On inserting the plug, the galvanometer may deflect to any one side. Bring the pointer of the galvanometer back to zero by adjusting the resistance of Rh. When balance is obtained in this manner, the potentiometer is said to be calibrated.

Any e. m. f. whose value is less than 1·5 volt can now be measured if it is connected across terminals 5 and 6, 5 being the positive terminal. While performing the test, if it is found that the ammeter reading has slightly changed, the potentiometer must be

calibrated over again, by throwing the switch to terminals 3 and 4, and proceeding as before.

If the resistance of each section of the potentiometer is 10 ohms the battery current passing through it should be 0·01 ampere. The battery should not be in a discharged condition, nor must it be freshly charged, for under both conditions the battery voltage keeps on changing, and though the change in voltage may be very small, it is sufficient to give wrong results.

The potentiometer is used for calibrating d. c. voltmeters and ammeters. The methods of calibrating these instruments are described in what follows.

13. Calibration of a Voltmeter and an Ammeter: It is clear from the last Section and Fig. 25 that an e. m. f. higher than 1·5 volt cannot be opposed by the potentiometer, the limit being set by the standard cell and the construction of the potentiometer. However, with the help of a *volt-box*, which is really a voltage reducer, mesurements of voltages or e. m. fs. upto 150 or 300 volts can be made. This depends on the design of the volt-box.

Fig. 26 shows the diagram of connections for calibrating a voltmeter. Calibration means to determine the extent of error in the voltmeter reading throughout its range. A high resistance tubular rheostat *CD* is connected across the supply terminals T_1 T_2. This

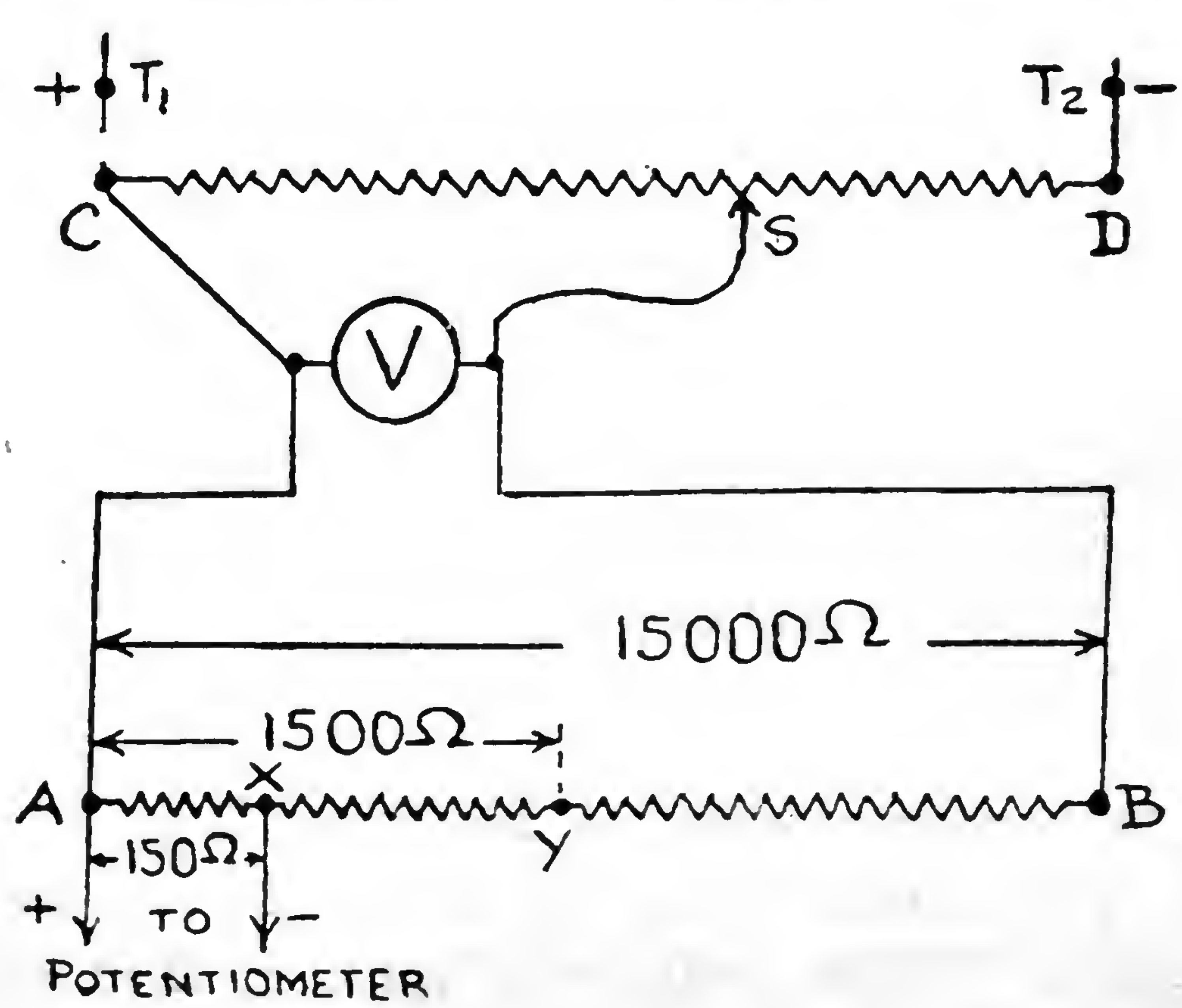


Fig. 26. Calibration of Voltmeter.

acts as a voltage divider. The volt-box consists of a high resistance resistor AB, usually 15,000 or 30,000 ohms. In the figure, two tapping points X and Y are such that the resistance of the portions AX and AY are $\frac{1}{100}$th and $\frac{1}{10}$th respectively of the resistance of AB. Clearly then, whatever the p. d. across AB, the corresponding p. d. across AX will be $\frac{1}{100}$th and across AY will be $\frac{1}{10}$th that across AB. When the volt-box resistance is 15,000 ohms and the p. d. across it is 150 volts, the p. d. across AX will be 1·5 volt. So that if AX is connected to terminals 5 and 6 of D–P D–T switch of the potentiometer of Fig. 25, this p. d. can be balanced on the potentiometer.

Suppose the voltmeter reads 110 volts in the position shown in the figure and that the potentiometer dial switch and the slide-wire reading together gives 1·098 volt, the true reading of the voltmeter is 109·8 volts. Hence there is an error of 0·2 volt in the reading of the voltmeter at 110 scale division.

Thus by sliding S and bringing the pointer of the voltmeter on to the principal divisions of its scale, the voltmeter can be calibrated throughout its range. The error should be plotted on a graph paper.

In calibrating an ammeter, the principle of Ohm's law is made use of, namely,

$$V = IR$$

If a current of 10 amperes passes through a standard resistance, say of 0·1 ohm, the voltage drop across the standard resistance will be 1 volt. If the terminals of the standard resistance are connected to terminals 5 and 6 of the D–P D–T switch of the potentiometer of Fig. 25, this p. d. can be balanced, and its true value can be determined. Suppose the potentiometer reading is 1·001 volt, the true current is 1·001 divided by the resistance,

$$I = \frac{1{\cdot}001}{0{\cdot}1} = 10{\cdot}01 \text{ amperes}$$

Hence there is an error of 0·01 ampere in the reading of the ammeter at the scale division marked 10. The instruments thus can be calibrated for its entire range of 0 to 10 amperes. If the ammeter happens to be of 0 to 25 ampere range, the value of the standard resistance should be 0·05 ohm. The maximum drop across the standard resistance in this case will be $0{\cdot}05 \times 25 = 1{\cdot}25$ volt which can be balanced on the potentiometer. Fig. 27 shows the ammeter

and the standard resistance connected to a circuit supplying a load.

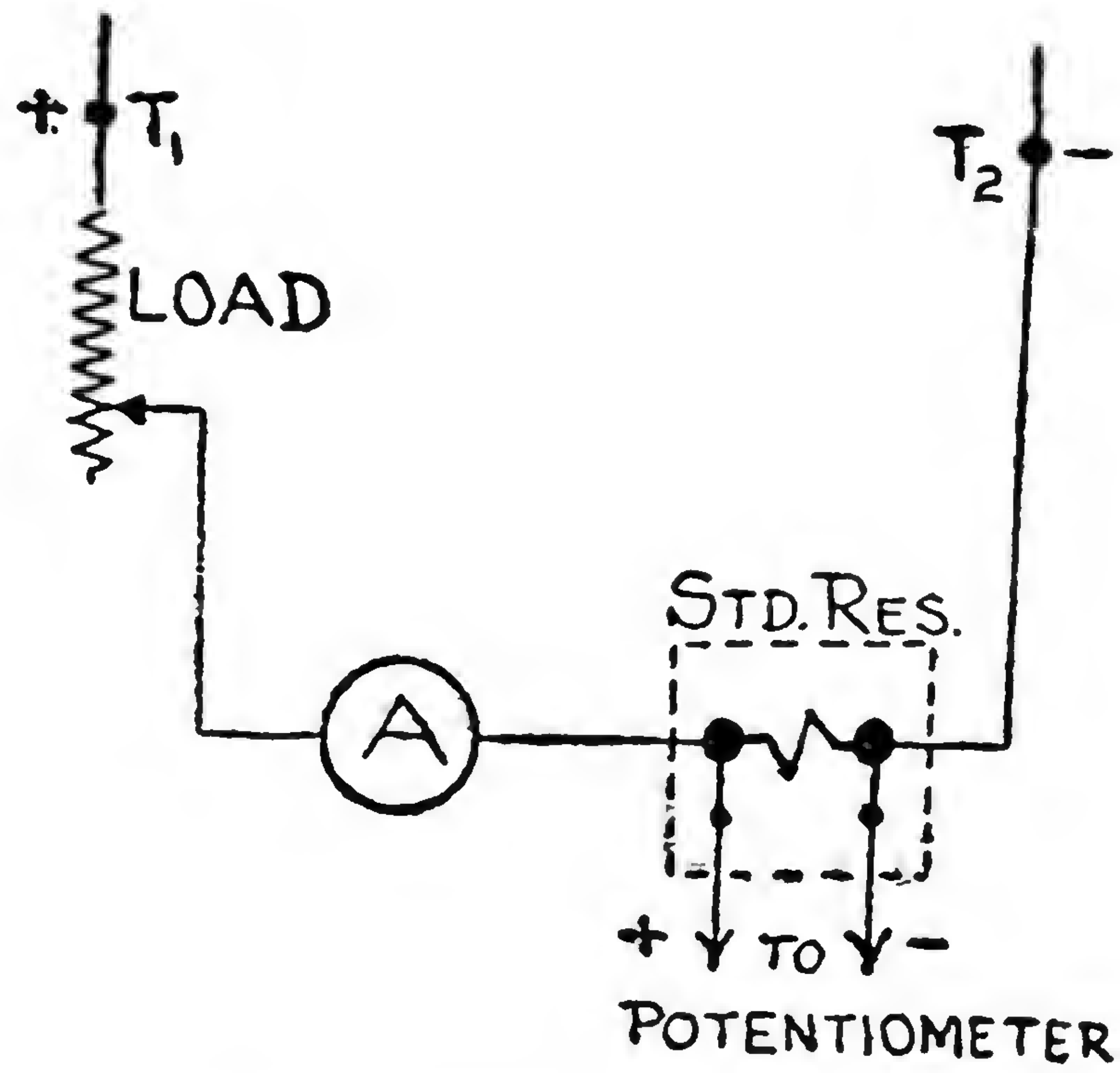


Fig. 27. Calibration of Ammeter.

Calibration or *Correction Curve,* is a graph which shows the departure of the instrument reading from the true reading. The curve may be on the positive or negative or on both sides of the y-axis. The scales on the x-axis represent the instrument readings and the departures from the true reading are marked on the y-axis. If the instrument reads at any scale point *more* than it should, the difference is marked on the *positive side.* Therefore, to get the true-reading the excess amount must be *subtracted* from the reading of the instrument.

14. **Murray's Loop Test:** This is a very useful test in locating a fault in a cable of low resistance and of short length. The most common fault is the *earth fault,* i. e. a leakage path occurs between the core and earth due to deterioration of cable insulation.

Fig. 28 shows the diagram of connections for the test. CD is a healthy cable running along the faulty cable EF. Ba is the battery supplying the current for the test. The diagram resembles the Wheatstone Bridge connection, in fact the connections are almost identical. The return path for the current is through the earth fault. Ends D and F are joined by a thick wire link.

Let P be the point where the fault has occurred in cable EF. If l yds. is the length of the cables and the distance of P from the test

end is X yds. and assuming that the cross-sectional area of each cable core is the same, then the resistance between C and P is $(2l - X)\,r$,

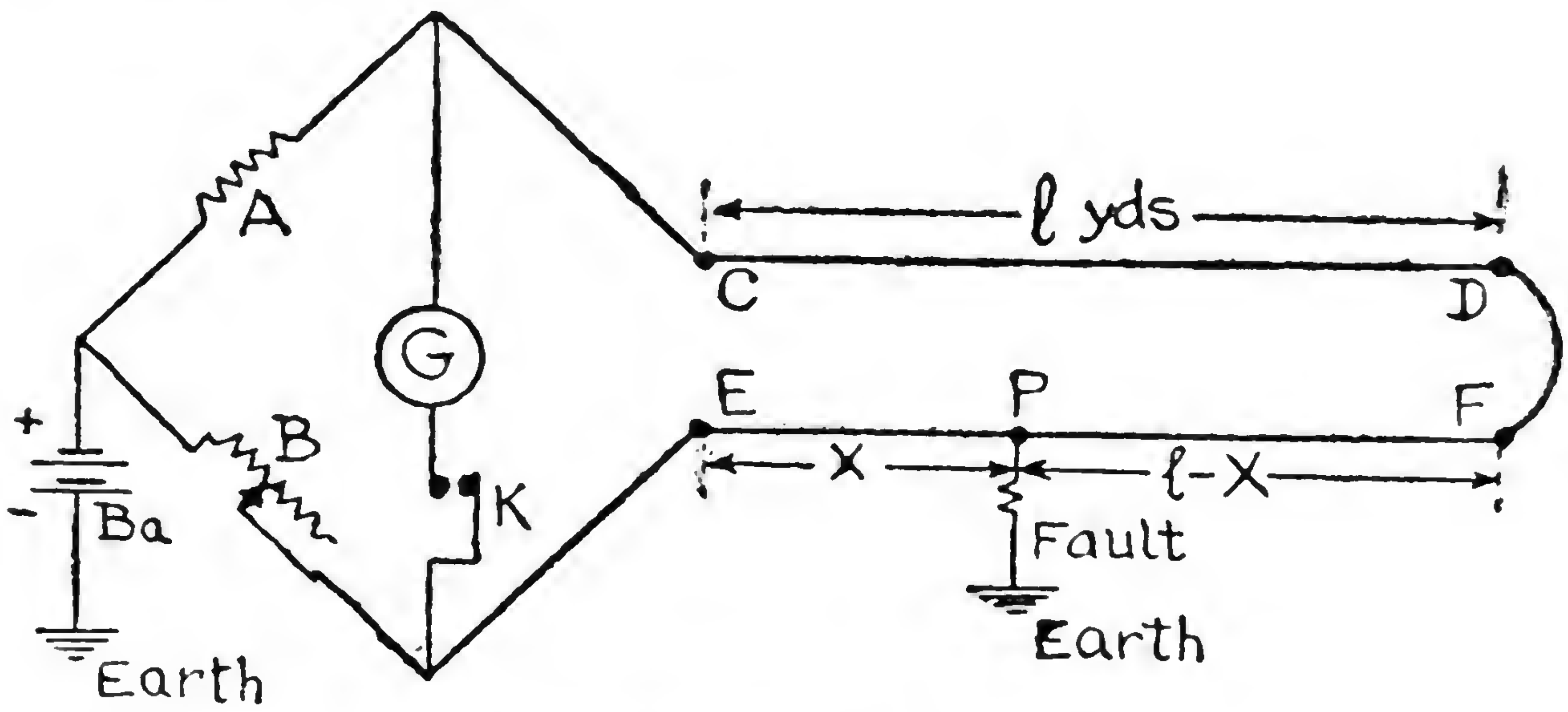


Fig. 28 Murray's Loop Test.

and between E and P the resistance is Xr. When balance is obtained by varying B, $\frac{A}{B} = \frac{(2l - X)\,r}{Xr}$. [r = resistance per yd.]

Cancelling r and solving for X, $X = 2l\,\frac{B}{A + B}$ yds.

This test can be performed with the help of a Slide Wire Bridge also.

If the healthy cable CD has a different cross-sectional area, then let r ohms be the resistance per yd. of the faulty cable and r_1 ohms the resistance per yd. of the healthy cable. Under the condition of balance
$$\frac{A}{B} = \frac{l r_1 + (l - X)\,r}{Xr}$$

Solving for X, $X = l\left(\frac{r_1 + r}{r}\right)\left(\frac{B}{A + B}\right)$

15. Varley's Loop Test : This is also a very useful test for cable lengths of high resistance, such as long telephone or telegraph lines.

Fig. 29 shows the diagram of connections for the test. As in Fig. 28, CD is a healthy cable and EF is the faulty one running along the same route. The ends D and E are joined by a stout wire of negligible resistance. The single-pole double-throw switch S is necessary to determine the resistance of the two cables in series, if their resistances are unknown.

Let the total resistance of both the cables be R ohms as found by the ordinary Wheatstone Bridge connection, when the switch S is connected to g. The resistance of the two cables is the unknown quantity. After the value of R is found, the switch S is thrown

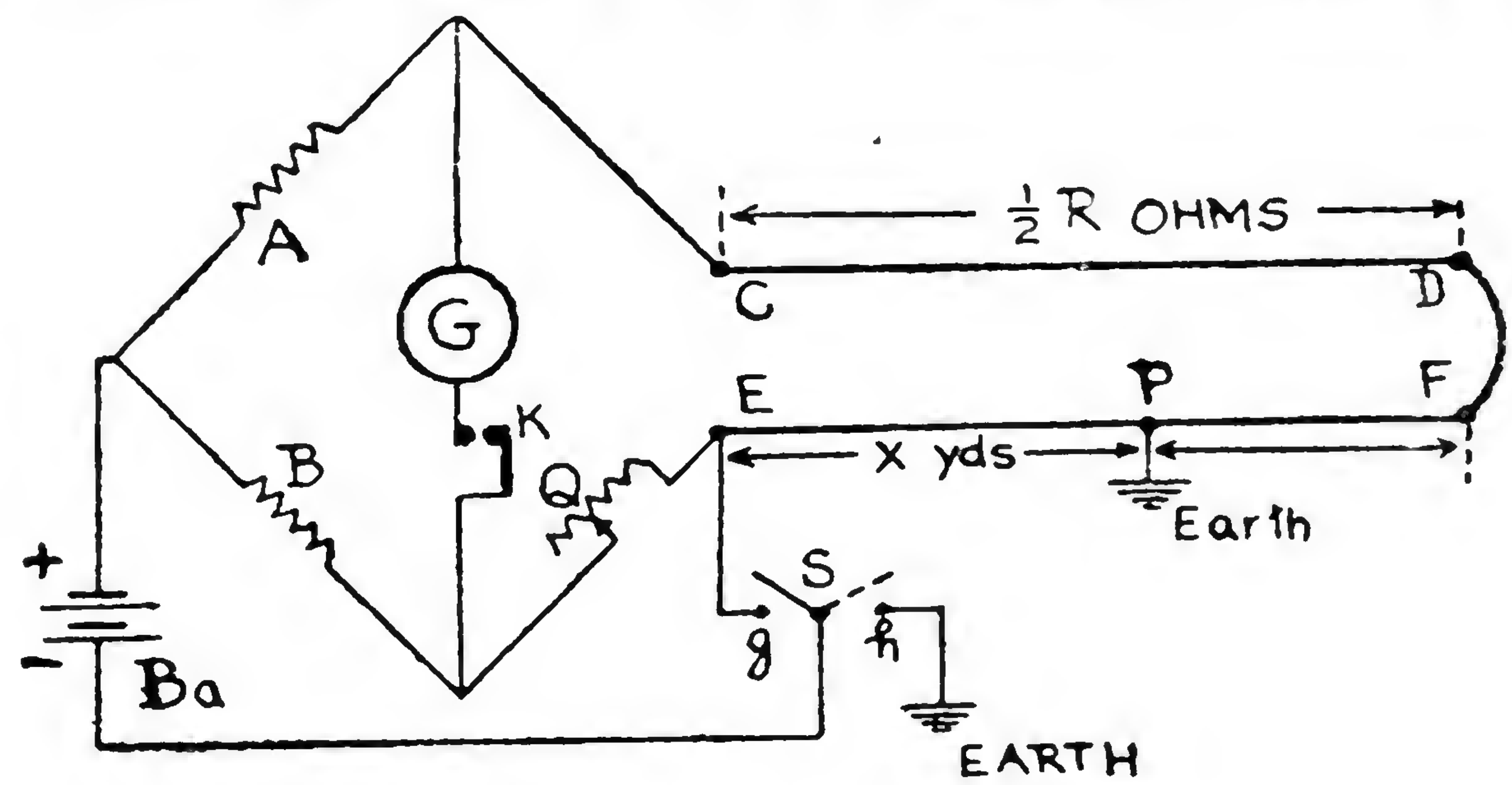


Fig. 29 Varley's Loop Test.

over to h and balance again is found by adjusting Q. Under this condition,

$$\frac{A}{B} = \frac{R - \dfrac{R}{2l}X}{Q + \dfrac{R}{2l}X},$$

where l is the distance in yds. of each cable length. Solving for X,

$$X = \frac{2l}{R}\left(\frac{B \times R - A \times Q}{A + B}\right) \text{ yds.}$$

16. Measurement of Capacitance: *Wheatstone Bridge Method.* Fig. (a) and (b) show the diagram of connections. In (a) the source of supply is alternating current and instead of the usual galvanometer a telephone receiver is used, When balance is obtained there is no buzzing sound in the receiver. A and B are the ratio arms as usual, C is a standard capacitor and C_x is the capacitor under test. The balance may be obtained by either adjusting the resistance of B, or by varying C. Since the current in a condenser is proportional to its capacitance, the equation under balance condition becomes,

$$\frac{A}{B} = \frac{C}{C_x}, \quad \text{or} \quad C_x = C\,\frac{B}{A}.$$

If alternating current is not available, a battery is used in conjunction with a double-contact key as shown in Fig. 30 (b). Under

balance, the galvanometer does not deflect if the key is operated in either direction several times. When the key is pressed down, the two capacitors get charged and when the key makes the two upper contacts the condensers are discharged. The equation for balanced condition for (*b*) is the same as that for (*a*).

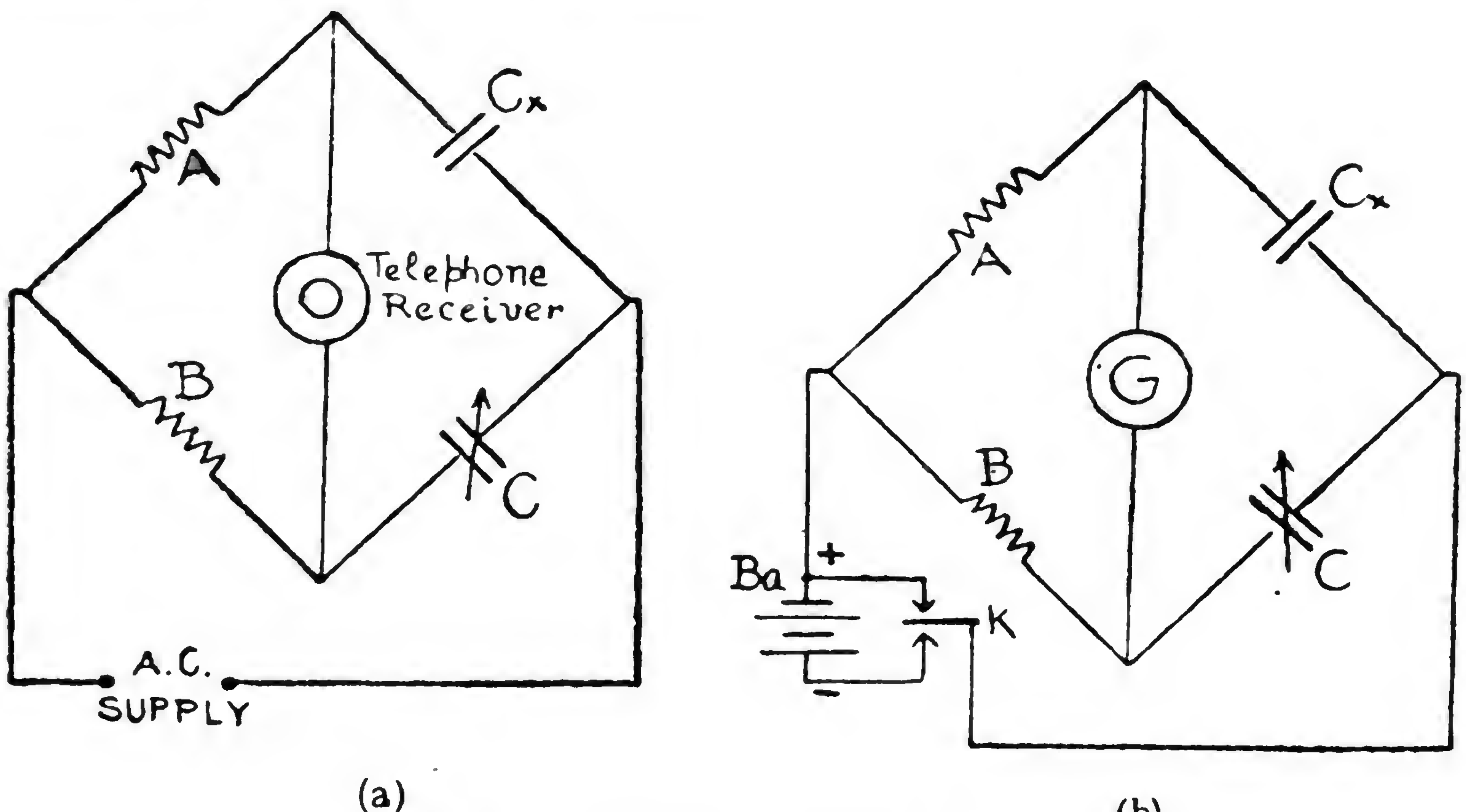
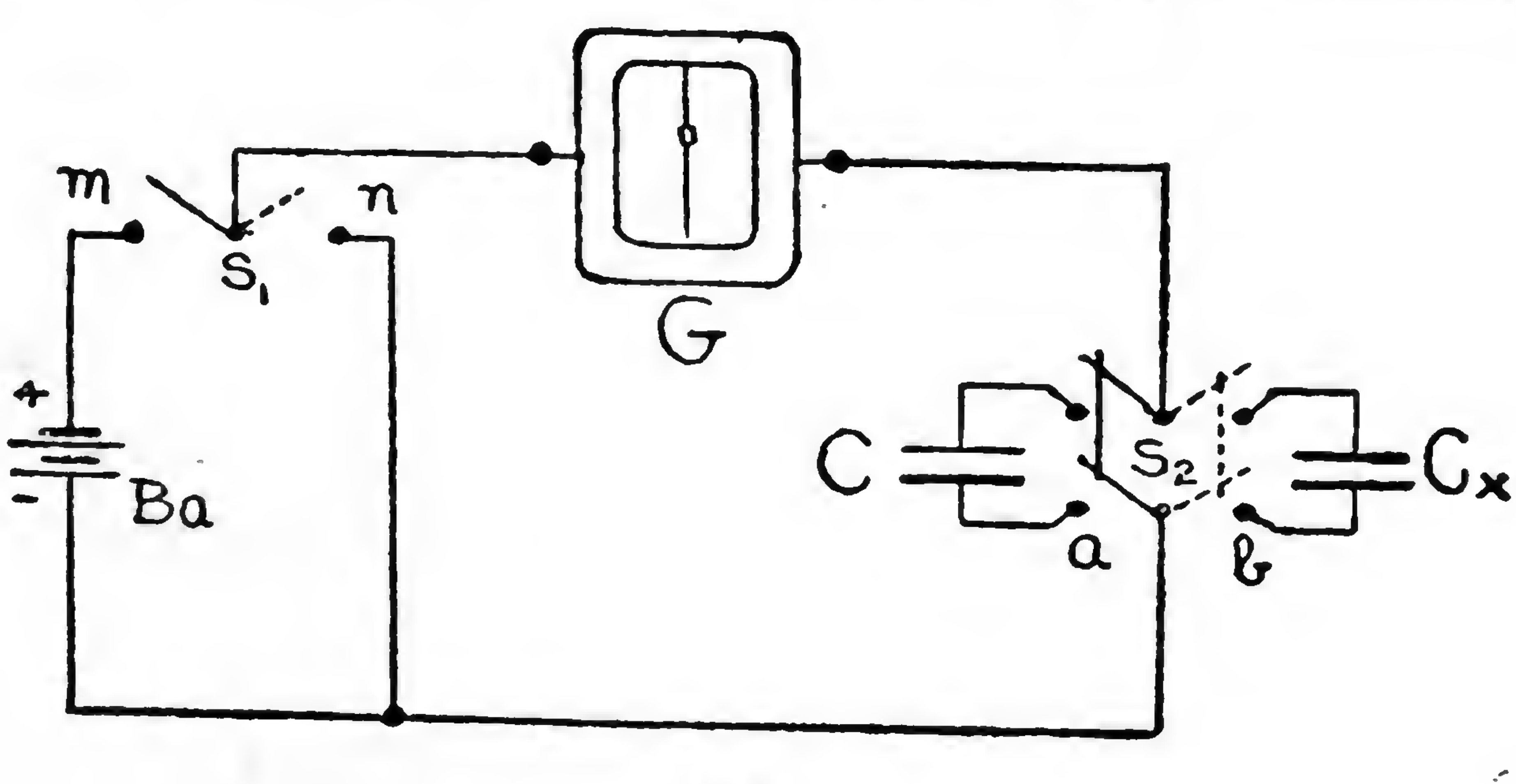


Fig. 30. Measurement of Capacitance

Method of Substitution: There is another method where the galvanometer is used ballistically, and where the deflection of the galvanometer, when C is in circuit, is compared with the deflection when C_x is in circuit. This method is known as the *method of substitution*, or *Ballistic method.*

Fig. (30) (*c*) shows the diagram of connections. S_2 connects either C or C_x in the circuit. When S_1 is on *m*, the capacitor in the



(c)

Fig. 30. Measurement of Capacitance.

circuit is charged and when S_1 is on n, the capacitor is discharged. The galvanometer deflects under both conditions. The sweep of the light beam is measured on a graduated scale. The maximum deflections for C and C_x are then compared, both on charge and on discharge. If for C or C_x the discharge deflection is less than the deflection on charge, a leaky capacitor is indicated. Under this condition it is better to compare deflections on discharge.

Let d_1 be the deflection when C_x is connected and Q_1 the quantity of charge on C_x. If the voltage across C is V, the battery voltage, then

$$Q_1 \propto d_1 \text{ and } Q_1 = C_x V$$

When the standard capacitor is substituted by throwing S_2 on the other side

$$Q_2 \propto d_2$$

$Q_2 = CV$, where d_2 is the deflection when C is in circuit. Combining these relations,

$$C_x V = Q_1 \propto d_1$$

$$CV = Q_2 \propto d_2$$

$$C_x = C\,\frac{d_1}{d_2}.$$

It is assumed that the inertia of the moving coil of the galvanometer is such that the entire charge passes through the coil before any deflection begins, and that the deflection of the sweep of the coil is proportional to the quantity of charge.

17. Measurement of Inductance, (*Bridge Method*): This is due to Maxwell. Fig. 31 (a) shows the necessary diagram of connections for the test. Alternating current must be used.

To obtain balance, not only must the p. ds. be equal in magnitude but they must also be in phase. Hence, when balance is obtained,

(i) the p. d. across circuit I = the p. d. across circuit II;

(ii) the p. d. across circuit III = the p. d. across circuit IV;

(iii) $i_1 = i_3$; and $i_2 = i_4$ (these are circuit currents).

Therefore,

(a) $i_1 (R_1 + j\,2\pi f\,L_1) = i_2 (R_2 + j\,2\pi f\,L_2)$

(b) $i_3 R_3 = i_4 R_4$; and

(c) $i_1(R_1 + j\,2\pi f L_1) \times i_4 R_4 = i_2(R_2 + 2\pi f L_2) \times i_3 R_3.$

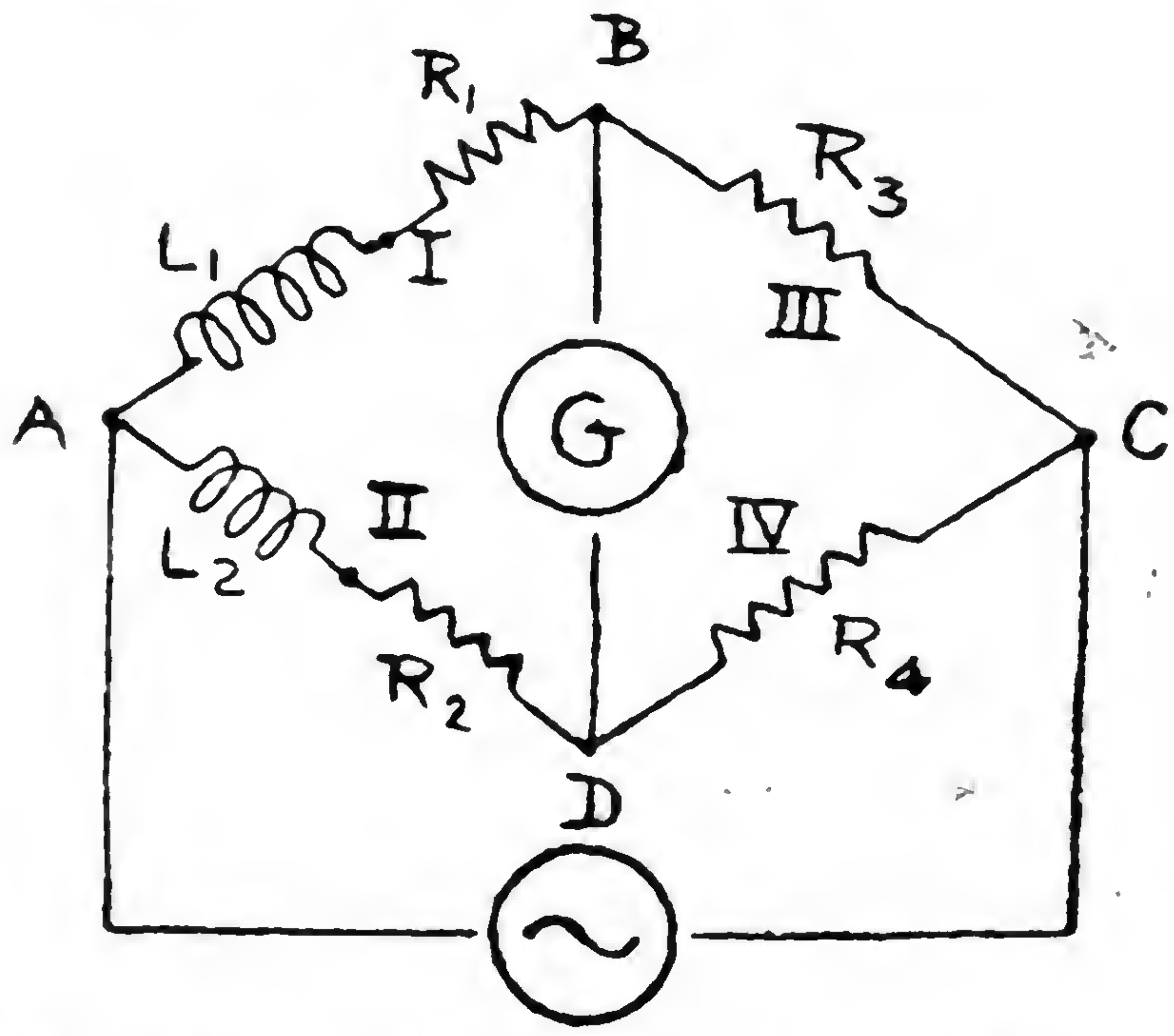


Fig. 31 (a) Measurement of Inductance

In order to fulfil the requirements of magnitude and phase, the following relations must hold :—

$$R_1 R_4 = R_2 R_3; \qquad L_1 R_4 = L_2 R_3, \text{ i. e.}$$

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} = \frac{L_1}{L_2}$$

(*Second Method*): This is a simpler method. The diagram of connections is shown in Fig. 31 (b). Alternating current must be used. R is the resistance of the coil and L is its inductance. R_1 is a non-inductive resistance whose value is known. V_1 and V_2 are measured by a voltmeter. Therefore

$$I = \frac{V_1}{R_1}; \qquad Z = \frac{V_2}{I}, \text{ substituting the value of } I$$

$$Z = \frac{V_2}{V_1} R_1; \quad \text{or} \quad R^2 + (2\pi f L)^2 = \left(\frac{V_2}{V_1}\right)^2 R_1^2$$

from which $$L = \frac{\sqrt{\left[\left(\frac{V_2}{V_1}\right)^2 R_1^2 - R^2\right]}}{2\pi f}$$

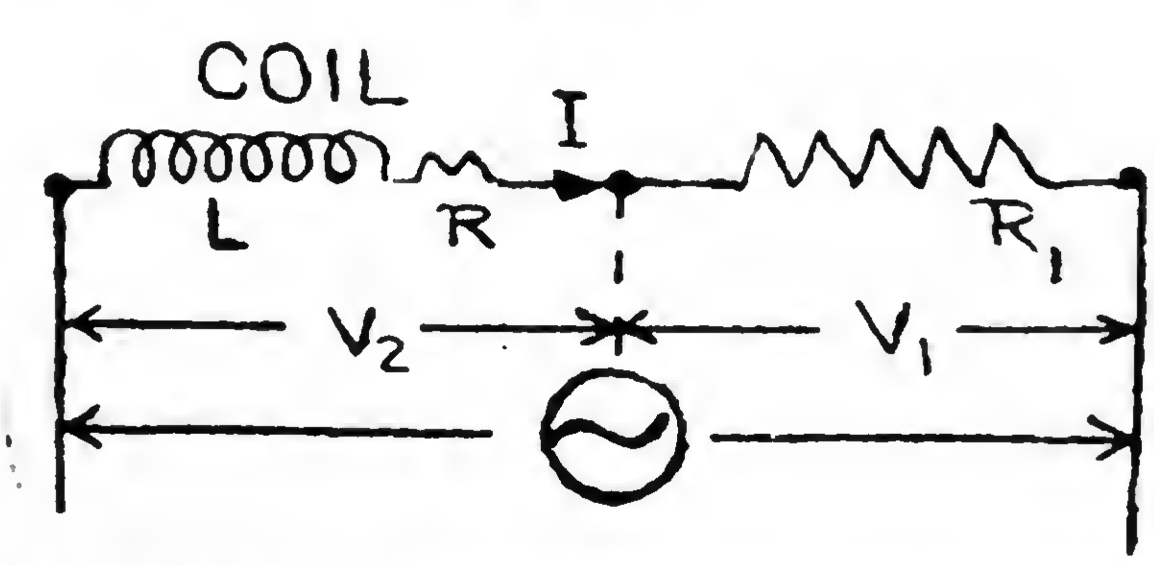


Fig. 31 (b) Measurement of Inductance

The resistance R of the coil, can be measured by the ordinary voltmeter and ammeter method by passing direct current through the coil.

The power factor of the coil, $\cos\phi$, can be found as follows:

$$\frac{R}{Z} = \cos\phi = \frac{R}{R_1} \times \frac{V_1}{V_2}$$

Power consumed by the coil,

$$P = V_2 I \cos\phi = \frac{V_2 V_1}{R_1} \cos\phi = V_1^2 \frac{R}{R_1^2}$$

NUMERICAL PROBLEMS

CHAPTERS I, II and III

1. A resistance wire has a specific resistance of 36 microhms per centimetre cube and a cross-sectional area of 0·1 sq. cm. What length of this wire will have 4 ohms resistance? **[111·1 metres.**

2. If the resistance of a conductor increases 20% while the temperature rises from 0°C to 50°C, what is the temperature coefficient for the material (a) at 0°C and (b) at 50°C?

[(a) 0·004 Ω per °C; (b) 0·00333 Ω per °C.

3. A field coil of a d. c. generator takes 5 A at 200 V when its temperature is 25°C. If the voltage remains constant, calculate the temperature of the coil when the current through it falls to 4·2 A.

[74·9°C.

4. Calculate the required resistance of a water heater which has to raise the temperature of 5 gallons of water by 40°C in 15 minutes. Assume the efficiency to be 90% and the supply voltage 220 V.

[10·36 ohms.

5. The resistances of four branches, **AB, BC, CD** and **DA** of a Wheatstone Bridge network **ABCD** are respectively 200, 100, 150 and 250 ohms. A detector is connected across **BD**, and **A** is maintained at a potential 2 volts above that of **C.**

Calculate the potential of **B** and **D** relative to that of **C,** if the detector has

(a) a very high resistance, and

(b) a very low resistance

in comparision with the resistances of the branches.

[(a) B $\frac{2}{3}$ V and D $\frac{3}{4}$ V above C; (b) B and D $\frac{7}{10}$ V above C.

6. An iron ring has a mean circumference of 100 cm and cross-sectional area of 12 sq. cm. It is wound with 600 turns of

wire and when 1·2 A pass through the coil the flux produced is 10^5 maxwells. Calculate the permeability of the iron at this density.

[921.

7. A steel ring, having a cross-sectional area of 4 sq. cm, has a mean diameter of 100 cm. Calculate the ampere-turns required to produce a flux of 40,000 lines in the ring. Take μ of ring as 1900.

If an air-gap 2 mm. wide is made in the ring, determine the ampere-turns required to produce the same flux of 40,000 lines in the air-gap. Neglect leakage. **[1316; 1590 Amp-turn in air-gap.**

8. Calculate the force in lb. needed to separate two iron surfaces each of 1 ft. in diameter when the total flux normal to the surface is 10 megalines. **[12,240 lb. wt.**

9. The hysteresis loop of a transformer core material, plotted with a scale of 1 cm = 10 ampere-turns per cm against 1 cm = 1000 gauss, has an area of 6·5 sq. cm. Find the loss due to hysteresis at 50 cycles per second for a core weighing 25·5 kg and subjected to a density of 12,800 gauss (max.). Take specific gravity of core material as 7 6. **[109 W.**

10. An air-cored solenoid 1 cm in diameter and 1 metre long is wound with 2000 turns. Find the inductance of the solenoid.

[0·395 mH.

11. A coil having a resistance of 10 ohms and an inductance of 25 henrys is connected across a 100 volt d. c. supply mains. Calculate

(a) the initial rate of change of current;

(b) the time constant of the circuit;

(c) the value of current at (i) $t=2$ seconds and (ii) $t=5$ seconds.

[(a) 4 A per second; (b) 2·5 seconds;
(c) (i) 5·5 A, (ii) 8·65 A.

12. The reluctance of a transformer core at a particular flux density is 0·0008. Find the mutual inductance between the two coils wound on the core. Their respective number of turns are 1120 and 50. Neglect magnetic leakage. **[880 mH.**

13. Two coils, having 100 and 150 turns respectively, are wound side by side on a closed iron circuit. The cross-sectional area

of the core is 125 cm^2 and its length 200 cm. Calculate the mutual inductance between the coils and the self-inductance of each coil. Assume permeability of iron as 2000.

If the current changes from 0 to 5 A in 0·02 seconds in the 100 turn coil calculate the induced e. m. f. in the other coil.

[M = 2·36 H ; e. m. f. = 59 V.

14. Determine the capacity of a condenser with 15 plates each 10 cm square. The clearance between the plates is 0·5 mm and the permittivity of the dielectric (between the plates) is 5.

[0·0123 μ F.

15. Two condensers give a joint capacitance of 2 μ F when in series and 9 μ F when in parallel. Calculate the individual capacitance.

[6 μ F and 3 μ F.

16. A condenser of 50 μ F is in series with a resistance of 20,000 ohms. If the above combination is switched suddenly on to a 400 V d. c. supply, determine (a) the initial current, (b) the time constant of the circuit, (c) the equation of current as a function of time and (d) the equation of electric charge as a function of time.

[(a) 0·02 A; (b) 1 second; (c) $i = 0{\cdot}02\ (e^{-t})$;
(d) $q = 0{\cdot}02\ (1 - e^{-t})$.

17. Plot the graphs of i and q from the equations obtained in the last problem, and determine the energy stored in the capacitor when it is charged fully. **[4 joules.**

18. A 2–plate condenser is made up of (metal) sheets, each having an area of 120 cm^2 and separated by a dielectric of 1·5 mm thickness. Its capacitance is $2{\cdot}83 \times 10^{-4}$ micro-farads.

If a p. d. of 20 kV is applied across the plates, find (a) the potential gradient in kV/cm; (b) the permittivity of the dielectric; (c) the intensity of electric field and (d) the total dielectrix flux.

[(a) 133·33 kV ; (b) 4; (c) 445 e. s. u.; (d) 213×10^3 lines.

19. A 1 mile long, 1–core metal sheathed cable has a copper conductor of 1·5 cm diameter and the internal diameter of the sheath is 3 cm. The specific resistance of copper is $1{\cdot}7 \times 10^{-6}$ ohm per cm^3

and that of the insulating material 6×10^9 ohms per cm^3. Calculate (a) the ohmic resistance and (b) the insulation resistance of the cable.

(c) If the above cable is cut into two pieces *A* and *B*, what will be the ohmic and insulation resistances of the pieces when *A* is twice as long as *B* ?

[(a) 0·155 Ω; (b) 420 M Ω; (c) *A*: 0·1033 Ω, 630 M Ω; *B*: 0·0516 Ω, 1260 M Ω.

CHAPTERS IV and V

1. Draw developed winding diagrams, showing the location of brushes for a drum armature of a machine. The particulars are :—

Number of poles = 4; number of slots = 21; coil-sides per slot = 2. Type of winding—(a) simple lap and (b) simple wave.

2. Determine a suitable lap winding for an armature drum with 18 slots. Following is the data of the machine :—

no. of poles = 4; flux per pole = $1{\cdot}2 \times 10^6$ lines

induced e. m. f. = 260 V; speed = 1000 r. p. m.

3. In a uniform magnetic field of 8000 gauss a straight conductor, 50 cm. long and carrying a current of 40 A, is made to move against the mechanical force at a uniform speed of 5 metres per second. Calculate (a) the e. m. f. generated in the conductor, (b) the mechanical force acting on the conductor and (c) assuming no friction, the power required in watts to move the conductor.

[(a) 2 volts; (b) $1{\cdot}6 \times 10^6$ dynes; (c) 80 watts.

4. The armature of a 500 V, 4-pole, lap-wound generator rotates at 600 r. p. m. and has 400 conductors. Each field winding has 1000 turns.

Calculate (a) the flux per pole, (b) the induced e. m. f. in each field coil on breaking the field circuit if the flux dies away in 0·1 second. **[(a) 12·5 megalines; (b) 1250 volts.**

5. The armature of a 220 V d. c. generator is 38 cm. long and its external and internal diameters are 76 and 44 cm. respectively. The machine has 6 poles, runs at 1000 r. p. m. and the winding is lap. Calculate the number of conductors on the armature if the flux density in the armature core is 5000 gauss. **[216.**

6. A d. c. generator has an armature resistance of 0·1 ohm and gave the following open circuit characteristic at 800 r. p. m. when excited separately :—

Field current (*amps*) :	0·5	1	1·5	2	2·5	3
Induced e. m. f. (*volts*) :	70	118	155	186	211	232

The generator is run as a shunt machine at 1000 r. p. m. Determine (a) the total resistance of the field circuit so that the no load voltage of the generator is 250 volts at 1000 r. p. m. (b) The speed at which the generator just fails to build up (*critical speed*). (c) The terminal voltage and field current when the armature current is 80 A, and (d) the *critical resistance* at 1000 r. p. m.

[(a) 111 ohms; (b) 630 r. p. m.; (c) 233 V : 2·1 A ; (d) 175 ohms.

7· A series generator is used as a booster between station bus-bars and an outgoing feeder whose total resistance is 0·5 ohm. Calculate the difference of voltages between the bus-bars and the far end of the feeder when the station supplies (i) 25 A ; (ii) 150 A through the feeder.

The external characteristic of the booster is a straight line from 0 to 60 volts at 150 A. **[(i) 2·5 V ; (ii) 15 V.**

8. A 400 kW compound generator gives 525 volts on full load and 480 volts on no load, the respective ampere-turns being 12000 and 7000 per pole. The shunt field winding is designed to give 480 volts at no load and at a room temperature of 25°C, and it reaches a steady temperature of 60°C on full load. Calculate the series turns per pole. **[7.**

9. Two shunt generators, *A* and *B*, operate in parallel on a bus-bar and supply 1000 A to consumers. Each machine has an armature resistance of 0·02 ohm and a field resistance of 30 ohms. If the induced e. m. f. of *A* is 465 volts and that of *B* 460 volts calculate (a) the bus-bar voltage and (b) the output current of each machine.

[(a) 452·2 V ; (b) *A* : 625 A : *B* : 375 A.

10. A belt-driven 50 kW shunt generator, running at 500 r. p. m., delivers full load to a 240-volt bus-bars. Its armature and

field resistances are 0·04 and 80 ohms respectively, and the brush contact drop is 0·9 V per brush. When the belt breaks the machine continues to run taking 5 kW from the bus-bars. Calculate its speed. **[475 r. p. m.**

11. Calculate the percentage speed regulation of the shunt motor having the following data:

H. P. ... 30; *rated voltage* ... 240 V; *number of poles* ... 4; *armature resistance* ... 0·04 ohm; *field resistance* ... 40 ohms; *flux per pole* ... 4·5 megalines; *wave-wound armature conductors* ... 190; *no load current* ... 10 A; *full load current* ... 115 A.

Neglect armature reaction. **[1·78 %.**

12. Calculate (a) the overall efficiency, (b) the lost torque in lb.-ft., (c) the total iron and friction losses in watts, for the motor of the last problem when fully loaded.

[81 %; 28 lb.-ft.; 3282 W.

13. The magnetisation curve of a 250-volt, 4-pole series motor is obtained by the following figures:—

Excitation amps.:	7·5	15	30	45	60	75
Flux per pole: (*in megalines*)	0·48	0·88	1·36	1·58	1·69	1·75

The armature has 782 wave-wound conductors and the total armature and field resistance is 0·14 ohm. Estimate the speed when the motor takes a current of (a) 40 A and (b) 70 A.

[(a) 627 (b) 528 r. p. m.

14. Plot graphs of (i) h. p. (ii) torque in lb.-ft. and (iii) speed in r. p. m. against current for the motor of the last problem.

15. A d. c. shunt motor drives a centrifugal pump whose torque varies as the square of the speed. The motor takes 50 A at 200 V and runs at 1000 r. p. m. The armature resistance is 0·1 ohm and that of the shunt field 100 ohms. It is required to reduce the speed to 800 r. p. m. Calculate the necessary resistance required in series with the armature. **[1·32 ohm.**

16. A 10 h. p., 250 volt d. c. shunt motor has an armature resistance of 0·4 ohm and field resistance 125 ohms. Estimate

approximately the current taken by the motor on no load if its full load efficiency is 85%. **[3·5 A**

17. In a brake test on a shunt motor the following reading are taken :—

Supply volts	...	*200 V*	*Load on one band*	...	*88 lb.*
Line current	...	*12·5 A*	*Load on other band*	...	*28 lb.*
R. p. m.	...	*1000*	*Diameter of pulley*	...	*6 inches.*

Calculate (*a*) the torque; (*b*) b. h. p. and (*c*) the efficiency.

[(a) 15 lb. ft. ; (b) 2·86 ; (c) 77·5%

18. The Hopkinson's test on two shunt machines gave the following results on full load :—

Line voltage ... 200 V

Line current (excluding field currents) ... 10 A

Motor armature current ... 70 A

Field currents ... 1 A and 1·2 A.

The armature resistance of each machine is 0·2 ohm. Tabulate motor losses and generator losses and hence calculate the efficiency of each machine.

	Motor	*Generator*
[Armature loss	**980 W**	**720 W**
[Field loss	**200 W**	**240 W**
[Stray losses	**150 W**	**150 W**
[Efficiency	**90·63%**	**91·53%**

CHAPTER VI

1. Three cells, connected in series, supply a lamp having a resistance of 6 ohms. The e. m. f. and the internal resistance of each cell are 1·4 V and 0·3 ohm respectively. Find the power consumed by the lamp. **[2·05 W.**

2. Calculate the resistance of a load to consume 32 W. The source of supply is a 12-volt battery having an internal resistance of 1 ohm.

[2 or $\frac{1}{2}$ ohms.

3. A battery of e. m. f. of 12 V and internal resistance of 0·8 ohm is connected in parallel with another battery of e. m. f. 6 V and internal resistance of 0·5 ohm. This combination is used for sending a current through a load of 2 ohm resistance. Find the current through each battery and the load. Draw a neat sketch of the connection and put the direction of current in each path.

[6 A discharge through 12 V;
2·4 A charge through 6 V;
3·6 A through the load.

4. A battery of 20 cells is arranged to pass maximum current through a resistance of 2·4 ohms. Each cell has an e. m. f. of 2 volts and an internal resistance of 1·8 ohms. Determine the current in the resistance and state the arrangement of cells.

[I = 2·15 A; 4 rows of 5 cells in series.

CHAPTER VII

1. A d. c. feeder supplies 100 kW to a feeding point from a Station bus-bar of 500 V. Calculate the total resistance of the 2-wire feeder so that the feeding point voltage on full load shall be 460 volts.

[R = 0·184 ohm.

2. A 2-wire distributor *AB* is 1 mile long and is fed at *A*. Loads are connected as follows :—

30 A at *C*; 50 A at *D*; 100 A at *E* and 60 A at *F*. The distances of *C*, *D*, *E* and *F* from *A* are 100, 300, 1000 and 1500 yards respectively.

If the voltage at *A* is 250 V and at *F* 210 V, calculate (a) the cross-section of the conductor, (b) the voltages at *C*, *D* and *E*, and (c) the efficiency of transmission. Take $\rho = 0{\cdot}67 \times 10^{-6}$ ohm per inch cube.

[(a) 0·25 sq. inch; (b) V_C = 245·39 V;
V_D = 237·33 V; V_E = 215·83 V; (c) 89%.

3. If the distributor of the last problem is supplied at both ends with 250 V, calculate (a) The point of minimum voltage and (b) the efficiency of transmission.

[(a) E; (b) 96·8%

4. A 2-wire d. c. distributor AB, 600 yd. long, is loaded as under :—

50 A at C 130 yd. from A : 80 A at D 280 yd. from A ; 60 A at E 410 yd. from A and 40 A at F 500 yd. from A.

If it is fed at 230 V at A and 226 V at B, calculate the currents fed at A and B and the voltages at each load point. The total resistance of the distributor is 0·09 ohm.

[Current at A = 151·9 A; at B = 78·1 A; V_C = 227·04 V; V_D = 224·75 V ; V_E = 224·31 V; V_F = 224·83 V.

5. A 400 yd. 2-wire distributor AB, fed at A, is loaded at the rate of 0·75 ampere per yard. The resistance of each conductor is 0·2 ohm per 1000 yd. Calculate the voltage at A if the voltage at B is 210 volts. **[234 V.**

6. A 2-wire d. c. distributor AB, 1600 yd. long and having a total resistance of 1·2 ohm, is loaded uniformly at the rate of 0·1 A per yard. Feeding voltages at A and B are 250 and 240 V respectively. Find (a) the current at A and (b) the value of minimum voltage and (c) the distance from A of the point of minimum voltage.

[(a) 85 A; (b) 222·96 V; (c) 850 yd.

7. A 2-wire feeder, 400 yd. long, delivers 50 kW at 250 V at the load end, and the efficiency of transmission is 90 %. A third wire, having a cross-sectional area of existing conductors is added to convert the feeder into a 3-wire one. How much power the new feeder can deliver (a) at the original efficiency or (b) at the original current density ? Assume consumers' load voltage constant and balanced load. **[(a) 200 kW; (b) 100 kW.**

8. A 3-wire distributor PQ, 250 yd. long, is supplied at one end P with 500/250 V and is loaded as under—

Positive Side — 20 A 150 yd. from P; 30 A 250 yd. from P.

Negative Side — 24 A 100 yd. from P; 36 A 220 yd. from P.

The resistance of each "outer" is 0·02 ohm/100 yd. and the "neutral" is of half the cross-sectional area of the "outers". Find the voltage at each load point.

[Positive Side : 248·62; 247·83 V;
Negative Side : 248·4 ; 247·65 V.

9. In a 460/230 V, 3-wire d. c. distributor the load on the positive side is 600 kW and on the negative side 400 kW. The balancer machines at this loading have a loss of 8 kW each. Calculate the current in each armature of the balancer set.

[400 A; 469·5 A.

CHAPTER VIII

1. Draw the following waves extending ever one periodic time T :—(*a*) 10 sin ωt ; (*b*) 10 cos ωt ; (*c*) 10 sin $2\omega t$.

Take the value of $\omega = 100\pi$ (periodic time $T = \frac{2\pi}{\omega}$ in seconds.)

1 radian = 57·3 degrees or π radians = 180 degrees.

2. Find the values at instants when $\theta = 30°$; $45°$; $60°$; $120°$ and $240°$ for the following two waves, their expressions are

(*a*) 100 sin θ and (b) 100 cos θ.

3. A 50 cycle synchronous clock showed correct time on Monday at 8 a. m. but on Tuesday noon it was slow by two minutes. What was the average value of supply frequency ?

[49·94 c. p. s.

4. A single-phase alternator supplies 100 A to two loads *X* and *Y*. *X* takes 50 A which leads the alternator current by 16°. Calculate the load current of *Y*. **[53·7 A**

5. An iron-cored choke coil take 5 A when connected across 20 V d. c. supply and takes 5 A when connected across 100 V a. c. supply consuming 250 W. Determine (a) the coil impedance and (b) the core loss. **[20 ohms ; 150 W**

6. A coil of 0·1 H inductance and 20 ohm resistance is connected in series with a 250 μF condenser. This combination is put across 200 V, 50 cycle supply mains.

Calculate (*a*) the current, (*b*) the power, (*c*) the power factor, (*d*) the voltage across the coil and (*e*) the voltage across the condenser. **[(a) 7·32 A ; (b) 1070 W ; (c) 0·73 lagging ; (d) 270·8 V : (e) 93·2 V.**

7. Calculate the frequency at which the current will be maximum for the coil of the last problem, the voltage being 200 as before.

[$f = \dfrac{100}{\pi}$.

8. Plot the locus of extremities of currents in the following cases :—

(a) R + j 20 }
(b) R − j 20 } ; R to vary from zero to infinity.

(c) 20 + jX }
(d) 20 − jX } ; X to vary from zero to infinity.

9. An inductive circuit takes

(i) 10 A at 100 volts and 50 cycles.

(ii) 5 A at 100 volts and 200 cycles.

Calculate R and L of the circuit.

[R = 8·94 ohms; L = 14·22 mH.

10. A resistor is connected in series with a choke coil. The total voltage across the combination is 100 volts and the current is 5 A. If the voltage across the resistor is 60 volts and that across the coil is 63·24 volts, calculate (a) the power dissipated in the choke coil and (b) the power factor of the whole circuit.

[(a) 100 W; 0·8 lag.

11. (6 + j8) ohms and (8 − j6) ohms are in parallel across a 50 cycle supply of 200 volts. Calculate (a) the admittance, the conductance and the susceptance of the total circuit and (b) the total current taken from the supply mains.

[(a) Y = 0·1414; G = 0·14 and B = −0·02 mho;
(b) 28·28 A.

12. A small factory has a total load of 150 kW at 0·9 lagging power factor. If 100 kW of the load has a power factor of 0·8 lagging, calculate the power factor of the remaining load.

[0·9987 lag.

13. (80 − j60) volts produce a current of (10 − j5) amperes in a circuit. Calculate the impedance of the circuit and state it in

vector notation. Calculate also the power consumed and the power factor of the circuit. Draw the vector diagram.

[$(8{\cdot}8 - j1{\cdot}6)$; 1100 watts; $\cos\phi = 0{\cdot}984$ leading.

14. Two impedances Z_1 and Z_2 are connected in parallel. Another impedance Z_3 is connected in series with this parallel combination. The whole circuit is connected across 200 V, 50 cycle supply mains. If $Z_1 = (8+j10)$, $Z_2 = (20-j100)$ and $Z_3 = (10-j7)$ ohms, calculate (a) the total power consumed and (b) the power factor of the whole circuit. **[(a) 1972 W; (b) 0·991 lag.**

15. Three similar impedances are connected (a) in delta, (b) in star across 400-volt, 3-phase mains. Each impedance is $(20 + j15)$ ohms. Calculate for each case the phase current, line current and the total power consumed. Draw the vector diagrams for the two cases.

[*Delta* : I per phase = 16 A; I (line) = 27·7A; power = 15,360 W.

Star : I per phase = $\frac{16}{\sqrt{3}}$ A; I (line) = $\frac{16}{\sqrt{3}}$ A; power = 5,120 W.

16. If one of the impedances is removed from the circuit of the last problem, calculate the current in each phase, the line current, and the total power consumed in each case. (i. e. when in delta and when in star formation).

[*Delta* : 16 A, 16 A; 16 A, 16, 27·7 A; 10,240 W.
Star : 8 A, 8A; 8 A; 8 A, zero; 2560 W.

17. Two wattmeters, when connected to measure power in a 3-phase, 3-wire system read 1095 and 365 watts. Find the power factor of the circuit.

If the power consumed remained the same and the power factor changed to 0·9 leading, what will be the readings of the two wattmeters?

[$\cos\phi = 0{\cdot}756$ (lag.); $W_1 = 934$; $W_2 = 526$ watts.

18. Two wattmeters are used for measuring the power input and the power factor of an over-excited synchronous motor. If the readings are 7 kW and -2 kW, what is the input and the power factor of the motor? **[5 kW; 0·305 leading.**

19. Two wattmeters measure power input to a balanced 3-phase load. W_1 reads 10 kW and W_2 reads 5 kW. Calculate the total power input, the power factor of the circuit and the reactive power in the circuit.

[15 kW; 0·866 lag.; 8·66 kVAR.

CHAPTER IX

1. In a 50-cycle transformer the *induced volts per turn* = 5 and the flux density in the core is 10,000 gauss. Find the cross-sectional area of the core.

[225 sq. cm.

2. A sinusoidal flux of 2 megalines (maximum) links with 55 turns of a transformer secondary coil. Calculate the r. m. s. value of the induced e. m. f. in the secondary coil. The supply frequency is 50 c. p. s.

[244 V.

3. A 10 kVA, 1-phase, 50 cycle transformer has the following figures referred to the primary side of 400 V:—

R_0 = 750 ohms; X_0 = 320 ohms; total equivalent impedance of windings $\overline{Z} = (0{\cdot}32 + j\,0{\cdot}7)$ ohms, and the load impedance $\overline{Z}_L = (13 + j\,8)$ ohms.

The primary turns are 52 and the secondary 14.

Construct the equivalent circuit diagram of the transformer and calculate (a) the line current, (b) the power factor (primary) and (c) the secondary terminal voltage.

[(a) 26·2 A; (b) 0·82; (c) 98·8 V.

(4) Calculate the iron and copper losses of the transformer of the last problem at the given loading.

[Iron loss = 213 W; copper loss = 202 W.

(5) The open circuit and short circuit test readings on a 400/200 V single-phase transformer are :—

O. C. Test (*l. v. side*): 200 V; 0·7 A; 95 W

S. C. Test (*h. v. side*): 15 V: 20 A; 130 W.

Calculate (a) $\overline{R}$ and $\overline{X}$ and (b) R_0 and X_0 all referred to the l. v. side.

[(a) $\overline{R}$ = 0·081 ohm; $\overline{X}$ = 0·169 ohm;
(b) R_0 = 421; X_0 = 389 ohms.

6. A 5 kVA, 400/200 V, 50 cycle, 1–phase transformer gave the following results :—

No load : 400 V, 1 A, 50 W (*h. v. side*).

Short-circuit : 12 V, 10 A, 40 W (*h. v. side*).

Calculate on full load, the efficiency and regulation when the power factor is 0·8 lagging.

[Efficiency = 97·7 % ; regulation = 3·125 %.

7. A 10 kVA, 1-phase transformer has an efficiency of 90 % at $\frac{1}{2}$ load and full load, the power factor being unity. Calculate

(a) the iron and copper losses at full load

(b) efficiency at full load, 0·8 p. f. lagging

(c) efficiency at $\frac{3}{4}$ full load, unity p. f.

[(a) Iron loss = 370 W ; copper loss = 740 W ; (b) 87·81 % ; (c) 90·5 %

8. Find the all-day efficiency of a 500 kVA distributing transformer, whose copper and iron losses at full load are 4·7 kW and 3·3 kW respectively. During a day of 24 hours it is loaded as under :—

No. of hours	*Load in kW*	*Power factor*
6	450	0·9
6	250	0·75
4	100	0·9
8	0	...

[97·4 %

9. Calculate the weight of copper in an auto-transformer, as a percentage of weight of copper in a 2-winding transformer, the ratio of transformation being (i) 1·2, (ii) 1·5, (iii) 2 and (iv) 3.

[(i) 16·67%, (ii) 33·33%, (iii) 50%, (iv) 66·67%

10. A 1-phase load takes 16 A from 220 volts, 50 cycle mains. An auto-transformer is interposed between the 220 V mains and the load to reduce the current in the load to 10 A. Calculate (a) the ratio of transformation, (b) the current drawn from the supply mains, and (c) the current in the common portion of the transformer.

Neglect losses and magnetising current of the transformer.

[(a) 220 : 137·5 V, (b) 6·25 A, (c) 3·75 A

11. A 3300/400 V, 3-phase, 50 cycle transformer is star-connected on the primary (h. v.) side and delta connected on the secondary side. The core has a gross cross-section of 600 cm^2 and the flux density in the core is 12,000 gauss. Calculate the turns per phase on the h. v. and l. v. side.

[133 h. v. turns per phase; 28 l. v. turns per phase.

12. A 100 kVA, 3-phase, 50 cycle transformer, with a voltage ratio of 3300/400 V, is mesh-connected on the h. v. side and star connected on the l. v. side. The resistance of the h. v. winding per phase is 3·5 ohms and that of the l. v. winding per phase is 0·02 ohm. The iron losses at normal voltage and frequency total 1200 watts. Calculate the efficiency when the power factor is 0·8 lagging at (i) 3/4 full load and (ii) full load. **[(i) 96%; (ii) 95·8%.**

13. A 3-phase transformer is connected in star on the primary side and is supplied from 3300 V mains. The primary turns per phase = 159, while there are two equal sections per phase on the secondary side, each section has 17 turns.

By using all the secondary turns, determine the various supply voltages obtainable from the secondary side for (i) 3-phase working and (ii) 6-phase working. Draw for each case the connection and vector diagrams.

(i) Plain delta = 407·4 V; plain star = 705·6 V; Zig-Zag = 611·1V.

(ii) Ring mesh = double star = diametral = 203·7 V; double delta = 117·6 V.

14. A 1-phase transformer designed for 25 cycle supply has, on full load and unity power factor, the following losses :—

(i) Copper loss = 2%; (ii) hysteresis loss = 0·8% and (iii) eddy current loss = 0·6%.

If this transformer is used on 50 cycle supply (of the same voltage) calculate the above losses at the same loading.

[(i) 2%; (ii) 0·528%; (iii) 0·6%.

15. Two 1-phase transformers A and B, having following particulars, operate in parallel and supply a load of 400 kVA at 0·8 p. f. (lag.) :—

	kVA	%R	%X
A :	300	1	6
B :	200	1·2	4·8.

Calculate the kVA loading and the p. f. of each transformer.

[A : 218·4 kVA at 0·775 p. f. (lag.) ;
B : 181·6 kVA at 0·825 p. f. (lag.).

CHAPTER X & XI

1. The armature of a 1-phase alternator is completely wound with 100 single-turn coils distributed uniformly and the induced e. m. f. per turn is 3 volts (r. m. s.). Calculate the total e. m. f. generated with all the coils in series. **[191V**

2. The speed of a 3-phase, 8-pole, star-connected alternator is 750 r. p. m. It has 72 slots and 10 conductors per slot. If the flux per pole is $5·5 \times 10^6$ lines or maxwells, calculate the line voltage at no load. **[2434 V**

3. Find the no-load terminal voltage of an alternater whose particulars are :

4 poles ; 1500 r. p. m. ; 48 slots ; 10 conductors per slot ; coil span 150° (electrical) : 9×10^6 maxwells per pole and star-connected 3-phase winding. **[2553V**

4. A 4-pole, 50 cycle, 50 kVA, 550-volt, 1-phase alternator has an armature resistance of 0·48 ohm. A field current of 8 A gives 160 A on short circuit and an e. m. f. of 500 V on open circuit. Calculate (a) the synchronous reactance and impedance and (b) the regulation for full load at (i) unity p. f. and (ii) 0·8 p. f. lag.

[(a) Z = 3·125 ohms ; X = 3·08 ohms ;
(b) (i) 18·6% ; (ii) 41·5%.

5. Two 500 kVA alternators operate in parallel to supply ;

(a) 250 kW at 0·9 p. f. lag. ;

(b) 150 kW at 0·8 p. f. lag. ;

(c) 300 kW at 0·75 p. f. lag. and

(d) 100 kW at 0·9 p. f. leading

One machine power factor is 0·9 lag. and supplies 400 kW, calculate the power factor of the other machine. **[0·844 lag.**

6. Two 500 kVA alternators run in parallel. The generators on the prime movers are so set that the frequency drops from 51 to 49 c. p. s. from no load to full load in one while in the other from 51 to 49·5 c. p. s. Calculate (a) the load on each machine when the total load is 750 kW, (b) the frequency at this load and (c) the maximum load the two can supply.

[(a) 428·6 and 321·4 kW ; (b) 49·7 c. p. s. ; (c) 875 kW.

7. A 2200 V, 500 h. p., 3-phase star-connected synchronous motor has a resistance of 0·3 ohm per phase and synchronous reactance of 3 ohms per phase. Determine the induced e. m. f. per phase if the motor works on full load with an efficiency of 94% and a power factor of 0·8 leading. **[1510 V.**

8. A 400 V, 3-phase, star-connected syn. motor takes 3·73 kW at normal voltage, and has an impedance of ($1 + j8$) ohms per phase. Calculate the current and the p. f. if the induced e. m. f. is 460 V.

[7·03 A; 0·763 (lead.).

9. A 400-volt, 3-phase, star-connected synchronous motor takes a constant power of 5 kW and has a reactance of 8 ohms per phase. Draw for this load (i) the armature current and (ii) power factor against field excitation. The magnetisation curve of the motor is obtained by the readings given below :—

Volts per phase	125	150	175	200	225	250	275	300	325
Excitation amperes	1·3	1·5	1·75	2·0	2·2	2·5	2·9	3·4	4·6

10. A load in a factory absorbs 1000 kVA at 0·8 p. f. lagging, and another load absorbing 170 h. p. is driven by a synchronous motor having an efficiency of 90·6%. The combined power factor is 0·94 lagging. Determine the total power factor if the load of 1000 kVA is suddenly reduced to 500 kVA. **[0·999 (lag).**

11. A synchronous motor absorbing 41·5 kW is in parallel with a load absorbing 250 kW at 0·8 p. f. lagging. Calculate the power factor at which the synchronous motor should work so that the combined power factor improves to 0·95 lagging. **[0·449 (lead.).**

12. The magnetisation curve of a 400 V, 50 cycle alternator is obtained by the following readings :—

Open circuit volts :	266	334	377	422	450	484	508
Field amperes :	2·0	2·5	3·0	3·5	4·0	4·5	5·0

Excitation current of 2 A gave full load current on short circuit. Calculate the percentage regulation at 0·8 p. f. lagging (i) by synchronous impedance method and (ii) by ampere-turn method.

[(i) 44·5%; (ii) 25%

13. Calculate from the following observations, taken on a 125 kVA, 400 V, 3-phase alternator, the percentage regulation for half load at 0·8 p. f. (lead.) by (a) ampere-turn method and (b) synchronous impedance method. State the assumptions made in the two methods :—

Field amperes :	0	5	10	15	20	25	30	35	40
O. C. volts :	0	80	140	200	250	300	340	370	400.

The short circuit characteristic is a straight line through the origin and the full load current is obtained for a current of 20 A in the field. Neglect resistance of armature.

[(a) −8·23%; (b) −16·45%

CHAPTERS XII and XIII

1. Below is a name-plate of an induction motor. Calculate its (i) number of poles, (ii) percentage slip on full load and (iii) full load current.

3 Ph.	400 V	50 ~
1440 R. P. M.		7·5 H. P.
cos ϕ 0·8		η 85 %

[(i) 4; (ii) 4 %; (iii) 11·88 A.

2. The rotor of a 3-phase induction motor is star-connected and has an induced e. m. f. of 50 volts between slip-rings at stand-still on open circuit, when connected to its normal supply voltage. The rotor impedance at stand-still is $(0\cdot5 + j\,3\cdot5)$ ohms per phase.

Calculate the current in each phase and the power factor *at the instant* of starting when (a) the rotor is connected to a star-connected external resistance of 4 ohms per phase, and (b) the slip-rings are short-circuited. **[(a) 5 A; 0·79; (b) 8·17 A; 0·14.**

3. A 20 h. p., 400 V, 3-phase induction motor runs at 970 r. p. m. at full load. The mechanical losses are 600 W. Calculate the full load copper loss and efficiency. The supply frequency is 50 c. p. s. and the stator losses are 750 W. **[795 W; 86·6%**

4. The power input to a 400 V, 3-phase, 50 cycle, 4-pole induction motor running at 1445 r. p. m. is 22·5 kW. The stator losses are 1·2 kW and mechanical losses due to friction, windage etc. equal to 900 W. Calculate (a) the b. h. p., (b) the rotor copper loss and (c) the efficiency.
[(a) 26·3 b. h. p.; (b) 781 W; (c) 87·2 %.

5. A delta connected 400 V, 40 h. p. squirrel cage induction motor takes 48 A on full load and runs at 955 r. p. m. The stator impedance per phase is 3 ohms. Estimate (a) the starting torque and (b) the starting current taken from the supply mains if (i) star-delta starter is used and (ii) an auto-transformer starter with 70 % tapping is used for starting the motor.
[(i) (a) 76·43 lb. ft.; (b) 77 A; (ii) (a) 111·5 lb. ft. (b) 113 A.

6. A 3-phase rotary converter supplies a d. c. load of 200 A at 600 V. Determine the line voltage and slip-ring current on the a. c. side. The a. c. power factor is 0·9. Neglect losses.
[367 V; 209 A

7. Find the voltage and current of one secondary phase of a 3-phase transformer feeding a 6-ring rotary converter and delivering 150 A at 500 V d. c. The efficiency of conversion is 90 % and the power factor is 0·85. Diametral tappings are used.
[354 V; 92·5 A

8. A 3-ring converter has 0·4 ohm reactance between each slip-ring and its transformer. Calculate the d. c. voltage when the d. c.

load current is 100 A and the p. f. measured at the slip-ring terminals is (a) unity, (b) 0·9 leading. The secondary voltage of the transformer is constant at 153 volts. **[(a) 226 V: (b) 278 V**

9. A 6-ring rotary converter supplies power to a d. c. network. A reactance of 0·15 ohm is inserted between each slip-ring lead and the transformer.

Calculate the d. c. voltages when (a) d. c. load currents are (i) 1C0 A and (ii) 500 A, the a. c. power factor being 0·9 lag.; (b) the d. c. load is 500 A and the power factor is (i) unity and (ii) 0·9 leading. **[(a)(i) 490 V, (ii) 462: (b)(i) 509, (ii) 555**

10. Determine the average d. c. voltages for mercury arc rectifiers with (i) 3, (ii) 6 and (iii) 12 anodes. The arc drop is 15 V. The secondary supplying the anodes has a maximum voltage per phase = $\sqrt{2} \times 240$ V.

[(i) 224 V, (ii) 309 V, (iii) 321 V

11. The turns ratio of a 3-phase delta-star transformer operating from 6600 V mains is 15:1 per phase. Find the mean value of the d c. output voltage of a 3-anode mercury arc rectifier, allowing 25 V for arc drop. **[490 V**

12. A 3-anode mercury arc rectifier supplies a d. c. load of 40 A at 500 V. The transformer no load loss is 560 W and its short-circuit loss at 25 A is 900 W and losses due to reactances etc. are 120 W. Calculate the bulb and overall efficiencies of the rectifier. Take arc drop equal to 15 V.

[Bulb eff. = 97·09 %; overall eff. = 90·7 %

CHAPTER XIV

1. Calculate the illumination from a 100 c. p. lamp at a distance of (a) 15 ft.; (b) 30 ft. and express ths results in foot-candles and (ii) in lux.

[(a) (i) 0·444 ft.-c.: (ii) 4·78 lux.

[(b) (i) 0·111 ft.-c.: (ii) 1·195 lux.

2. A 150-watt lamp is suspended from the centre of the ceiling of a room 25 ft. long and 18 ft. wide. The height of the lamp is 12 ft. above the floor. Calculate the intensity of illumination at

(a) the centre of the floor and (b) one corner of the room on the floor. This lamp gives 15 lumens per watt. **[(a) 1·24; (b) 0·31 ft.-c.**

3. Two Street lights are mounted 25 ft. above the road level and they are placed 100 ft. apart along the centre line of the road. Calculate the intensity of illumination at points (a) under a lamp and (b) midway between two lamps. The c. p. is 100 in the direction for (a) and 400 in the specified direction for (b).

[(a) 0·16 ft-c; (b) 0·114 ft-c.

4. An illumination of 4 ft-candles is required on the working plane of a factory bay of 240 ft. by 75 ft. floor space. The lamps should be 15 ft. above the working plane and the luminous output per lamp is 14 lumens per watt. Assuming a suitable spacing height ratio, a utilization factor of 0·4 and depreciation factor of 0·8, calculate the number and wattage of lamps to be installed. Show by a sketch the arrangement of fixtures.

[80 lamps, each of 200 W.

5. An illumination of 6 ft.-candles is required on the working plane in a Drawing Hall 80 ft. long and 40 ft. wide. The height of fixtures should be about 12 ft. above the working plane. Taking the depreciation factor of 90% and utilization factor of 50%, calculate the number of twin tube units of fluorescent fixtures necessary. Wattage per tube = 40 and the output per tube is 33·5 lumens per watt. **[16 units.**

CHAPTER XV

1. An ammeter has a resistance of 0·2 ohm and reads 0 to 5 amperes. Calculate the value of shunt to make this instrument read from 0 to 25 amperes. **[0·05 ohm.**

2. A voltmeter reads from 0 to 0·75 volt, the full scale deflection takes place when 50 milli-amperes pass through its coil. Calculate the resistance to be put in series with this instrument so that it will read upto 150 volts. **[2985 ohms.**

3. A moving coil voltmeter has 100 turns and a resistance of 9·6 ohms. The coil has an active length of 3 cm and a width of 2 cm. A p. d. of 50 millivolts produces full scale deflection and the flux density in the gap is 1500 gauss. Calculate the torque exerted by the control spring at full scale deflection. **[0·478 g-cm.**

4. A moving coil ammeter has a coil of resistance of 0·2 ohm at 20°C. Its shunt, made of platinoid, has a constant resistance of 0·004 ohm. The instrument reads correctly at 20°C. Calculate the percentage error in the reading at 40°C. Take α for the coil material $= \frac{1}{234 \cdot 5}$ per °C at 0°C.

[7·2% low·

5. A d. c. ampere-hour meter has its dial marked in **kwh.** Its disc makes 1 revolution per second when 10 amperes pass through it, and the recording is correct on 240 volt supply.

(*a*) Determine the number of revolutions of the disc to indicate a consumption of 1 kilo-watt hour on 240 volt supply.

(b) Calculate the percentage error in the recordings if the supply voltage is (i) 230 volts, (ii) 250 volts.

[(a) 1500 ; (b) (i) +4·348% ; (ii) −4%.

6. A 10-ampere watthour meter records 0·25 watthour per revolution of its disc. When tested the meter disc made 75 revolutions in 30·5 seconds. The average voltage being 230 volts and average current 9·8 amperes during the interval. Determine whether the instrument runs fast or slow and the percentage error.

[Slow ; 1·85%.

CORRECTIONS

Page	*Line*	
8	7	$15i_2 + 16(i_1 + i_2 - i_3) - 5 = 0$
	8	$20i_1 + 100(i_1 - i_2) - 15i_2 = 0$
21	18	$s = r \sec \theta$
27	5	if a unit pole is taken
31	2	"trial and error"
47	1	cobalt-chromium
56	13	and on the -ve plate
59	14	the capacitor, $V = 0$
67	Fig. 9	4-Pole LAP Winding
78	13	left
95	6	$E_b I_a = V I_a - I_a^2 R_a$
96	13	7·04
140	Fig. 8	$75+i$; $40+i$; i; $50-i$; $75-i$
147	30	feeder length, increasing the
169	19	I is in phase with V_R
194	2	10^{-8}
223	18	wattmeters
229	20	The breadth factor $= \dfrac{ag}{Nab}$
248	6	using Fig. 12(b)
256	13	shown in Fig. 3(a)
	20	shown in Fig. 3(b)
258	last	205·2
274	21	Eq. (21) or (22)
288	9	as per Eq. (24)
309	26	valve action

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